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OPTIMAL SIMPLE MONETARY POLICY RULES AND EQUILIBRIUM DETERMINACY UNDER INFORMALITY

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Abstract

This thesis depicts a two-sector closed-economy New Keynesian model with formal labor market frictions in order to study the effects of informality for the design of optimal monetary policy. We find that the informal sector helps the formal sector to adjust to shocks and this can result in welfare gains provided the interest-rate rule reacts to formal-only inflation and output. In this case, the informal sector appears to buffer the formal economy with respect to the shocks. This result suggests that simple, formal-only Taylor rules can do a good job in terms of welfare under informality and such rules are easily implementable without the need for the central bank to accurately measure the informal sector. However, in general terms, we find the following policy trade-off. While informality can improve the performance of optimal simple rules in terms of welfare, depending on the calibration it could lead problems of indeterminacy. With high inflation values, the model shows a bigger welfare loss but with lower inflation values the analysis cannot even be conducted because the multiple equilibria issues. Thus, economies with large informal sectors must target inflation more aggressive, because otherwise the probability of getting welfare-reducing multiple equilibrium is higher. Then, central banks must take into account this potential policy trade-off when designing monetary policy.

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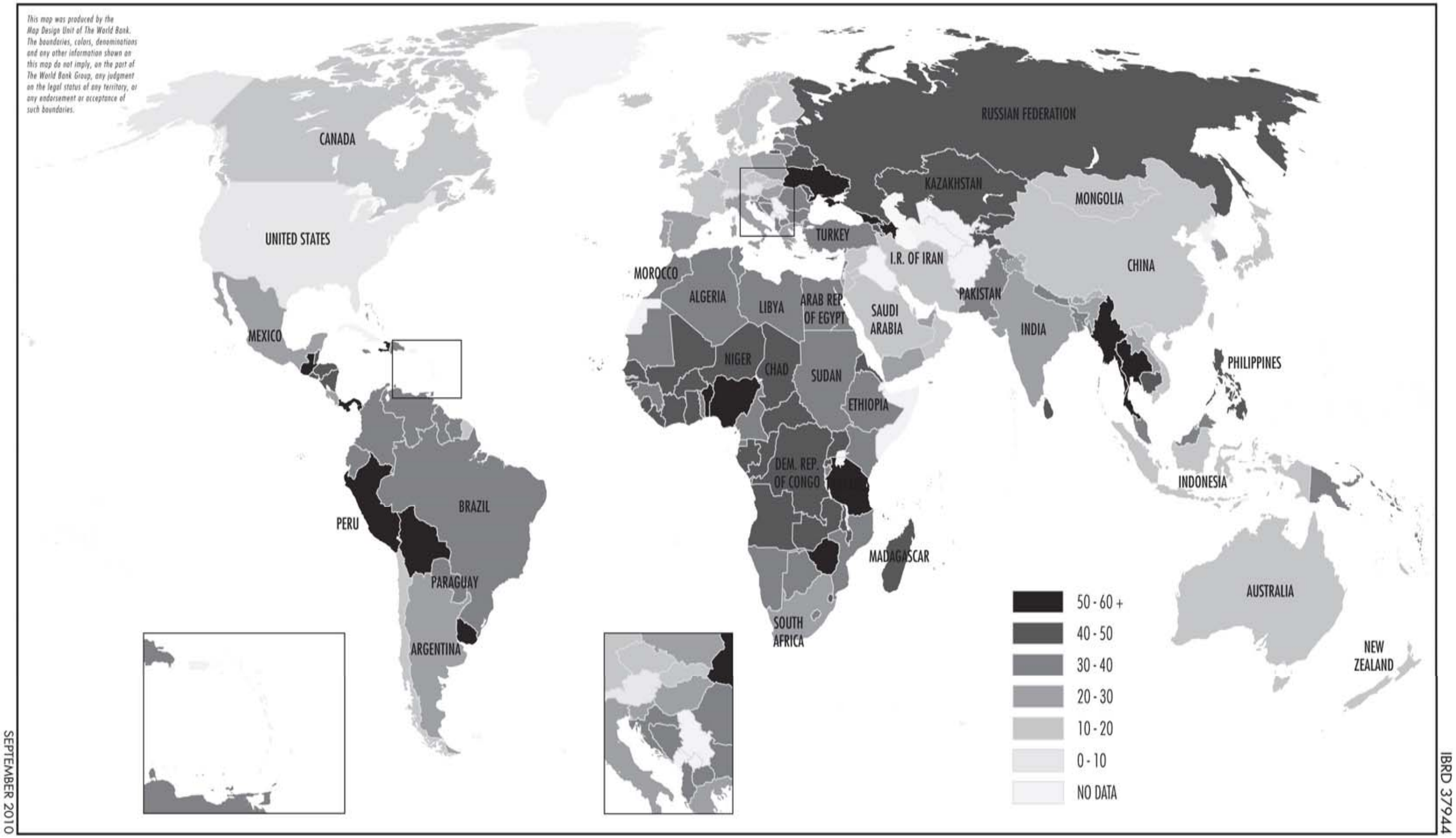
Introduction

Informality is a worldwide phenomenon which is present to a greater extent in developing countries. According to Schneider (2012), informality or the shadow economy can be understood as the production of goods and services, whether legal or illegal, that escapes detection in the official estimates of GDP. Then, the shadow economy can be defined as "those economic activities and the income derived from them that circumvent or otherwise avoid government regulation, taxation or observation". Then, informality usually has been associated to unregistered, hidden, shadow, unofficial or underground activities and consequently measuring the informal economy is inherently difficult (La Porta and Shleifer (2014)).

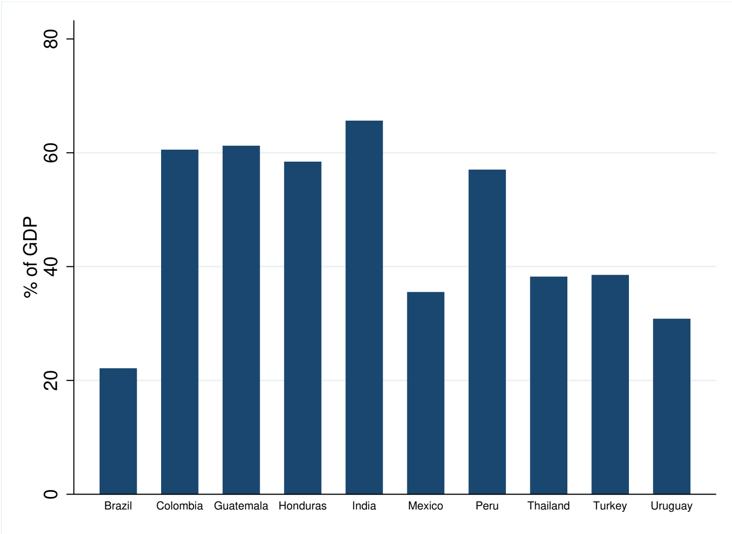
Figure 1 shows the estimated worldwide size of the shadow economy. We can see that Latin America and the Caribbean and Sub-Saharan Africa have the highest estimates for the shadow economy followed by Europe and Central Asia. As La Porta and Shleifer (2014) shows, especially in the poor countries, the informal sector is huge, accounting for a giant share of output and employment, whereas as countries develop, informality becomes less important. Figure 2 shows estimates for the GDP share of informality and its share of employment for selected emerging economies. In most of the cases, the share of the informal sector to GDP is more than 30 percent and the share of informal employment to aggregate employment is more than 20 percent, reflecting that for these economies the shadow economy is relevant in real terms.

Figure 1: Average Size of the Shadow Economy of 192 Countries over 1999–2007

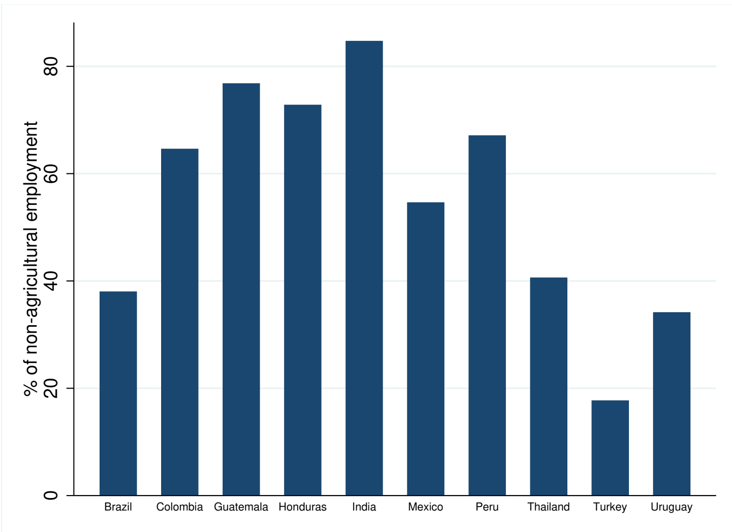


Source: taken from Schneider (2012).

Figure 2: Shadow Economy relevance in Selected Countries (2012)



(a) Share of the informal sector to GDP



(b) Share of informal employment to aggregate employment

Source: International Labour Organization (2016)

Therefore, as Batini et al. (2011) mention, it could be expected that transmission mechanism of monetary policy would be different in countries with large informal sectors. Therefore, this thesis aims to investigate how informality affects the design of monetary policy in terms of welfare and equilibrium determinacy. To date, the monetary policy literature has generally ignored this subject. As far as we know, Batini et al. (2011) and Bandaogo (2015) are the few studies that investigate the interaction between informality and optimal monetary policy but they do not consider the implications of informality for determinacy and informality. McKnight and De la O (2016) investigate the determinacy implications of informality in the presence of capital and investment spending but do not consider optimal monetary policy. This thesis tries to contribute to this literature proposing a two-sector model with labor market frictions and analyzing the design of monetary in this context.

A series of papers have modeled informality using the DSGE approach. As discussed by Leal-Ordoñez (2014), one advantage of this approach is that the informal sector is endogenized, enabling the investigation of how the informal sector affects the performance of aggregate variables.

Castillo and Montoro (2012) explore informality in a New Keynesian model. They extend the search and matching model presented in Blanchard and Galí (2010) allowing labor market frictions in a dual New Keynesian model with formal and informal contracts. In this model, informality is the result of hiring costs which depends on the degree of labor market tightness. They show that "a large pool of informal workers is a buffer stock of labor that allow firms to expand output without putting pressure on wages. In particular, firms at the margin can substitute formal jobs with informal ones and expand output without raising their marginal costs" (pp.18). They also show an employment transition between sectors: given a productivity shock, formal employment is lower but informal employment rises.

Restrepo-Echavarria (2014) looks at the effects of poor-quality measures of informality on the dynamics of relevant macroeconomic variables. Building a two-sector small open economy, she shows a greater volatility consumption than output volatility. This result contrast to the standard one-sector RBC theory which features consumption smoothing. She finds that when the informal sector is poorly measured agents can substitute formal for informal consumption over the business cycle. Since only formal consumption is seen, then we mostly observe movement in and out of the formal sector consumption.

Fernandez and Meza (2015) find using a small open economy RBC model calibration for Mexico that the presence of informal employment is countercyclical. This helps to explain why aggregate employment in Mexico displays a low variability over the business cycle. They also emphasize that a proper measurement of informal sector matters when quantifying this macro volatility.

This model presented in this thesis is also related with different literatures on monetary policy. First, with the literature relating optimal monetary policy and informality. Batini et al. (2011) study how the design of monetary policy is affected by informality in emerging economies. They built a New Keynesian model with two sectors an informal one and an formal one. The informal sector is more labor intensive, is untaxed, is perfectly competitive, faces high credit constrains in financing investment and is less visible in terms of output. They find that labor and financial market frictions the time inconsistency problem is worse-off. Therefore, the importance of a strong credible commitment is necessary in economies with large informal sector. Given the distortions associated to their model, they argue that the steady state output is lower that the social optimum. Considering this problem, they use the ”small distortions” quadratic approximation to the houselholds period utility described by Woodford (2003). The suggested quadratic approximation of the intertemporal expected welfare loss depends on aggregate consumption and formal and informal inflation and formal and informal labor. Bandaogo (2015) examines how does informality affect the conduct of fiscal and monetary policy. Using a small open economy DSGE model, he shows that informality significantly decreases the optimal tax rated levied on the formal sector. Thus, policymakers in countries with significant informality should try to keep low taxes in the formal sector in order to avoid formal goods becoming relatively more expensive than the informal good. Also find that in countries with neither the technology nor the credibility to commit to the optimal policy, it is desirable to peg the nominal exchange rate and adopt a flax tax rate. McKnight and De la O (2016) investigates the role of labor informality in the propagation of transitory shocks and its implications for interest rate policy in preventing self-fulfilling inflation expectations. They embedded search-matching frictions into a New Keynesian model with segmented labor markets and they show that while informality amplifies the propagation of demand shocks on inflation, it dampens the response of output, weakling the transmission mechanism of monetary policy to output.

Second, the literature on optimal monetary policy emphasizes the effects of multiple sectors in the design of monetary policy. Woodford (2003) studies the consequences of real disturbances that would affect equilibrium relative prices even in the case when all prices are fully flexible. This case has welfare implications because sectoral asymmetries imply that the different sectors of the economy will respond differently to shocks. In this case, he shows that the appropriate stabilization goals can be expressed by a welfare loss function depending on the inflation rate in each sector individually, the aggregate output gap and misalignments of the relative prices between the two sectors. Rychalovská (2007) analyses the stabilization objectives of optimal monetary policy and the trade-offs facing the central bank in a two-sector small open economy. She introduces multiple domestic sectors combined with a variety of sector-specific and foreign shocks. Her contribution is methodological since she derives a utility-based welfare measure and the optimal reaction function of the central bank and shows that the optimal targeting rule is represented by an expression that prescribes the response to appropriate measures of domestic inflation, sectoral output gaps, as well as to the relevant relative prices. Mattesini and Rossi (2009) analyze the design of optimal monetary policy in a New Keynesian model with indivisible labor and a dual labor market: a Walrasian market where wages are fully flexible and a unionized market where wages are the result of bargaining between firms and monopoly unions. They find a significant trade-off between stabilizing inflation and stabilizing unemployment, in response to technology and exogenous wage shocks. In the presence of real wage rigidities, an optimizing central bank must react to positive technological and wage shocks by increasing the interest rate, where the magnitude of the interest rate movement depends on the size of the Walrasian sector relative to the unionized sector.

Third, the thesis is also related to the optimal monetary policy and labor market frictions literature, which emphasizes the role of search and matching frictions in designing monetary policy. Blanchard and Galí (2010) extend the standard New Keynesian model by introducing labor market frictions similar to those found in the search and matching model of unemployment proposed by Pissarides (2000). Then, their model has labor market frictions, real wage rigidity, and staggered price setting which are common properties of sclerotic labor markets. They argue that the aforementioned distortions are needed in order to explain the unemployment movements caused by exogenous shocks. One of their principal findings is that labor market frictions are relevant for the design of optimal monetary policy, since social welfare loss is now determined by the degree of labor market tightness. The importance of labor market tightness on unemployment and it relation with social welfare is confirmed, among others, by Thomas (2008), Faia (2009), Ravenna and Walsh (2011) and Sunakawa (2015).

This thesis is related to the determinacy literature investigating the local stability of monetary policy rules. Schmitt-Grohé and Uribe (2007) find that the size of the inflation response coefficient in the interest-rate rule plays a minor role for welfare but has an important role in the equilibrium determinacy. Thus, they recommend larger values for the inflation response coefficient, mainly to ensure uniqueness of the rational expectations equilibrium. However, they do not consider informality in their analysis. As far as we know, McKnight and De la O (2016) is the only paper that considers the relationship between informality and determinacy. However, they consider a New Keynesian model with capital and investment spending. Since Carlstrom and Fuerst (2005), it is well known that the stability properties of models that include endogenous capital significantly differ from labor-only economies.

Therefore, this work depicts a two-sector closed-economy New Keynesian model with formal labor market frictions in order to study the effects of informality for the design of optimal monetary policy. As previously mentioned, we ignore investment and capital accumulation from the analysis. The economy consists of two sectors. The formal sector is characterized by monopolistic competition with staggered price-setting and the formal wage is determined via Nash bargaining. The informal sector behaves perfectly competitively and we assume that the informal productivity is lower than the formal productivity. The central bank conducts monetary policy according to a nominal interest-rate feedback rule. Finally, we assume that the business cycle of the economy is driven by formal technology shocks or by demand shocks.

The model is log-linearized around a unique steady state to obtain an equilibrium system of linear difference equations and we use the Blanchard-Kahn algorithm in `Dynare`[1] to solve the model. The parameters of the model are broadly calibrated to match the Mexican economy. The standard procedure for analyzing this model would be one that considers both sectors when constructing the social welfare loss function of the central bank, as Woodford (2003) and Rychalovská (2007) have shown. It would also be necessary to include the role of formal labor market frictions on welfare loss, as Blanchard and Galí (2010) have shown.

As a result, social welfare loss functions depend on sectoral output gaps, relative prices between sectors and the degree of formal labor market tightness. However, the aforementioned models assume that the multiple economic sectors and the formal labor market tightness could be easily observed by the central bank. We argue that with informality this is not realistic. Indeed, the informal sector and formal labor market frictions are very difficult to accurately measure in developing countries. Consequently, the actual output gap is not known and therefore the standard procedure of setting the nominal interest rate and the social welfare loss as functions of the output gap is not realistic for many policymakers in this case. As shown by Ehrmann and Smets (2003), the performance of Taylor rules in stabilizing the economy will deteriorate significantly in the face of high uncertainty about the true output gap.[2] The approach proposed here to deal with those problems is an empirically appealing one in which the central bank is only interested in stabilizing inflation and output (and not the output gap). In particular, we assume that

[1] `Dynare` is a set of `Matlab` libraries commonly used to study DSGE models.

[2] Ehrmann and Smets (2003) build a DSGE model for the euro–area economy to investigate the implications of incomplete information about the potential output for the conduct and the design of monetary policy. They show that simple Taylor rules are robust in the face of considerable uncertainty about the output gap as long as the central bank uses it best estimate of the output gap. This robustness breaks down when the estimate output gap is mis-specified.

the central bank has two possible choices when conducting monetary policy: reacting to aggregate inflation and aggregate output or reacting to formal-only inflation and formal-only output. Thus, in the first case, the central bank considers aggregate measures, which contains both formal and informal variables and, in the second case, the central bank just focuses on formal-only measures.

The key results of the thesis can be summarized as follows. First, the quantitative analysis finds a sectoral substitution effect, the sign of which crucially depends on the interest-rate rule adopted by the central bank. For productivity shocks, if the central bank reacts to formal-only inflation and output there is a transition from the formal sector to the informal sector, whereas when the central bank reacts to aggregate inflation and output the opposite transition occurs. With productivity shocks, we find that the key difference between the two policy rules is the dynamic behavior of the nominal interest rate. However, in aggregate terms, a positive formal productivity shock implies a larger response of aggregate employment and thus the negative relationship found in standard New Keynesian models between productivity shocks and employment disappears. When we consider the effects of aggregate demand shocks we find a similar labor market substitution effect and the pro-cyclicality of aggregate output on aggregate employment.

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This thesis proceeds as follows. Chapter 1 describes in detail the theoretical model used, Chapter 2 shows the derivation of the log-linearized version of the model and discusses its calibration. Chapter 3 analyzes the relationship between informality and welfare using optimal simple rules and Chapter 4 studies the implications of informality for equilibrium determinacy. Finally, some conclusions are presented.

Chapter 1

A New Keynesian Model with Informal Labor

The model is based on a version of the economy described in McKnight and De la O (2016). In their model endogenous investment and capital accumulation decisions are made by formal firms, while informal firms use only labor in the production process. Here, we assume a labor-only economy and we ignore investment and capital accumulation in order to simplify the analysis for the study of optimal monetary policy. The economy consists of a large number of identical infinitely-lived households, two kind of sectors (formal and informal) and a central bank responsible for monetary policy. It is assumed that the formal labor market has frictions because it takes time for a vacancy to be filled by a worker and this represents an economic cost.

There is no unemployment in the model because it is assumed that there are no barriers to entry in the informal sector. Thus, the informal sector is able to absorb any surplus of labor in the formal market. Informal firms operate in a perfectly competitive environment but formal firms operate in an imperfectly competitive environment. Formal firms set staggered prices according to Calvo (1983). The formal wage is determined by a Nash bargaining problem between formal firms and matched workers. The central bank conducts monetary policy by setting the nominal interest rate which responds to variations in inflation and output. The analysis considers two alternatives measures for these variables: aggregate measures which includes informality and formal-only measures which omits informal variables. Finally, the economy faces two types of shocks: a formal-sector productivity shock that affects formal firms and an aggregate demand shock that affects household consumption.

1.1 Labor Market Dynamics

The model used to describe the dynamics of labor market is a version of the basic infinite-horizon search and matching model based on Pissarides (2000). The informal sector is perfectly competitive and adjusts immediately in order to maintain the wage equal to the marginal product of labor. However, the formal market has search frictions, in the sense that opening a vacancy by a formal firm is not filled immediately; once the firm posts a vacancy some time is needed until a worker finds a match with that firm. Therefore, as in McKnight and De la O (2016), an informal worker is always queuing for a formal position but vacant jobs need not match instantaneously with the searching worker.

The quantity of total success matches in period t is given by a matching function $\mathscr{M}(V_t)$, where V_t is the aggregate measure of vacancies.[3] The job-filling rate is given by $q_t = \mathscr{M}_t/V_t$ and market tightness is defined as:

$$\theta_t = \frac{V_t}{L_t^i}. \tag{1.1}$$

So, by the linear homogeneity property we have:

$$\mathscr{M}(V_t, L_t^i) = V_t^{\vartheta} {L_t^i}^{1-\vartheta} \tag{1.2}$$

$$\frac{\mathscr{M}(V_t, L_t^i)}{V_t} \equiv m\left(\frac{1}{\theta_t}, 1\right) \equiv q(\theta_t) = M\theta_t^{\vartheta-1} \tag{1.3}$$

where $\vartheta \in (0,1)$ is the matching elasticity and $M > 0$ is the matching coefficient. With informality, the aggregate employment N_t is given by:

$$N_t = L_t^f + L_t^i \tag{1.4}$$

while the law of motion for formal labor is:

$$L_{t+1}^f = (1-\delta)L_t^f + \theta_t q(\theta_t) L_t^i \tag{1.5}$$

[3]It is supposed that the matching function is increasing, concave, and linearly homogeneous.

where L_t^i and L_t^f are measures of informal and formal employment in the economy and $\delta \in (0,1)$ is the job destruction rate in the formal sector.

1.2 Households

There is a continuum of identical households that derive utility from consumption C of final goods and disutility from working N. The representative household seeks to maximize the objective function:

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t u\left(C_t, N_t; Z_t\right),$$

where $\beta \in (0,1)$ is the subjective discount factor and $u(\cdot)$ satisfies standard assumptions. Workers are allowed to supply labor to both formal and informal sectors and have no financial market restrictions. Following Galí (2015), a separable specification for the period utility function is assumed:

$$u(C_t, N_t; Z_t) = \begin{cases} \left(\frac{{C_t}^{1-\gamma}}{1-\gamma} - \frac{{N_t}^{1+\nu}}{1+\nu}\right) Z_t & \text{for } \gamma \neq 1 \\ \left(lnC_t - \frac{{N_t}^{1+\nu}}{1+\nu}\right) Z_t & \text{for } \gamma = 1 \end{cases}$$

where $\gamma \geq 0$, $\nu \geq 0$. $\left(\frac{1}{\nu}\right)$ is the Frisch elasticity of labor supply and $\left(\frac{1}{\gamma}\right)$ is the intertemporal elasticity of substitution. Z_t can be interpreted as a discount factor shock and it is assumed that $z_t \equiv ln(Z_t)$ follows an exogenous autoregressive first order process:

$$z_t = \rho_z z_{t-1} + \epsilon_t^z$$

with $\rho_z \in (0,1)$ and ϵ_t^z is and *i.i.d.* shock such that $\epsilon_t^z \sim \mathcal{N}(0, (\sigma_z)^2)$ and $(\sigma_z)^2 > 0$. A representative household solves the following problem:

$$\begin{aligned} \underset{\{C_t, B_t, L_{t+1}^f\}}{\text{Max}} \quad & \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\frac{{C_t}^{1-\gamma}}{1-\gamma} - \frac{{N_t}^{1+\nu}}{1+\nu}\right) Z_t \\ \text{subject to} \quad & P_t C_t + \mathbb{E}_t Q_{t,t+1} B_t \leq B_{t-1} + W_t^f L_t^f + W_t^i L_t^i + \iota_t, \\ & N_t = L_t^f + L_t^i, \\ & L_{t+1}^f = (1-\delta) L_t^f + \theta_t q(\theta_t) L_t^i \end{aligned}$$

where W^f and W^i are the wages paid in the formal and informal sectors, B_t represents the pay-off of the portfolio held at the end of period t and ι_t are the profits from ownership of firms. The stochastic discount rate is defined as:

$$Q_{t,t+1} \equiv \frac{1}{1+i_t},$$

where i_t is the nominal interest rate received form holding portfolio B_t at the end of period t. Then, the household's optimality conditions are:

$$Q_{t,t+1} = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left(\frac{Z_{t+1}}{Z_t} \right) \left(\frac{P_t}{P_{t+1}} \right) \right], \tag{1.6}$$

$$N_t^{\nu} = \frac{W_t^i}{P_t} C_t^{-\gamma} + \theta_t q(\theta_t) \Psi_t, \tag{1.7}$$

$$\Psi_t = \beta \mathbb{E}_t \left(\frac{W_{t+1}^f}{P_{t+1}} C_{t+1}^{-\gamma} - N_{t+1}^{\nu} + (1-\delta) \Psi_{t+1} \right), \tag{1.8}$$

where equation (1.6) is the standard consumption Euler equation, equation (1.7) is the labor supply condition and equation (1.8) is the shadow price for a worker in the informal sector. Equation (1.7) implies that in equilibrium, the marginal utility of being part of the labor force must be equal to the informal payment plus the expected benefit of a formal match with probability $\theta_t q(\theta_t)$. The intuition of equation (1.8) is simple: if a formal match is successful, in the next period the worker will receive the formal payment minus the loss in utility of being part of the labor force plus the benefits of formal employment given that the job was not destroyed with probability $(1-\delta)$. Is important to note that workers decide their optimal labor supply but do not control its final destination, so the dynamics of the formal sector is determined by (1.5) subject to the labor frictions previously mentioned. Therefore, similar to McKnight and De la O (2016) in this set-up there is perfect risk sharing by households, not only in terms of consumption, but also in terms of leisure.

Aggregate consumption C_t is a CES aggregator defined as

$$C_t = \left(\chi^{\frac{1}{\tau}} [C_t^f]^{\frac{\tau-1}{\tau}} + (1-\chi)^{\frac{1}{\tau}} [C_t^i]^{\frac{\tau-1}{\tau}} \right)^{\frac{\tau}{\tau-1}}, \tag{1.9}$$

with associate aggregate price index P_t

$$P_t = \left(\chi {P_t^f}^{1-\tau} + (1-\chi){P_t^i}^{1-\tau}\right)^{\frac{1}{1-\tau}}, \tag{1.10}$$

where $\tau > 0$ is the intersectoral elasticity of substitution, $\chi \in (0,1)$ is the relative proportion of formal and informal goods and P_t^f and P_t^i are the price indices of formal and informal goods, respectively. Hence, the consumption demand conditions are:

$$C_t^i = (1-\chi)\left(\frac{P_t^i}{P_t}\right)^{-\tau} C_t, \tag{1.11}$$

$$C_t^f = \chi\left(\frac{P_t^f}{P_t}\right)^{-\tau} C_t, \tag{1.12}$$

and the demand for individuals goods are:

$$C_t^i(k) = C_t^i, \tag{1.13}$$

$$C_t^f(j) = \left(\frac{P_t^f(j)}{P_t^f}\right)^{-\varepsilon_f} C_t^f, \tag{1.14}$$

where $\varepsilon_f > 1$ is the elasticity of substitution within formal goods.

1.3 Informal Firms

Informal firms behave perfectly competitive with no impediments to price setting and their production only depends on labor:

$$Y_t^i(k) = A^i L_t^i(k) \tag{1.15}$$

where $A^i > 0$ is (constant) informal labor productivity identical for all informal firms. From the cost minimization problem, the wage paid in the informal sector is given by:

$$W_t^i = P_t^i A^i. \tag{1.16}$$

1.4 Formal Firms

Production in the formal sector only depends on formal labor:

$$Y_t^f(j) = A_t^f L_t^f(j), \tag{1.17}$$

where A_t^f is the level of technology common to all formal firms. It s assumed that formal firms are more productive that informal firms so $A_t^f > A^i$. It is also supposed that $ln(A_t^f) \equiv a_t$ follows an AR(1) process:

$$a_t = \rho_a a_{t-1} + \epsilon_t^a$$

with $\rho_a \in (0,1)$ and ϵ_t^a is and *i.i.d.* shock such that $\epsilon_t^a \sim \mathcal{N}(0, (\sigma_a)^2)$ and $(\sigma_a)^2 > 0$. Following traditional New Keynesian models, formal firms are assumed to be monopolistically competitive and set prices according to Calvo (1983). The aggregate demand faced by a formal firm j is:

$$C_t^f(j) = \left(\frac{P_t^f(j)}{P_t^f}\right)^{-\varepsilon_f} C_t^f.$$

Real marginal cost derived from cost minimization is:

$$mc_t = \left(\frac{W_t^f}{A_t^f P_t^f}\right). \tag{1.18}$$

Now, consider the price setting problem of a formal firm j. In Calvo's (1983) framework, in each period firms that adjust their prices are randomly selected, so a fraction $(1-\omega)$ adjust their prices while the remaining fraction ω do not adjust. The parameter ω captures the amount of nominal rigidity. If a firm is randomly selected to adjust its price at time t, the firm choose $P_t^f(j)$ in order to solve the following problem:

$$\underset{\{P_t^f(j)\}}{\text{Max}} \quad \mathbb{E}_t \sum_{s=0}^{\infty} (\omega\beta)^s \Delta_{s,t+s} \left[\left(\frac{P_t^f(j)}{P_{t+s}^f} \right) C_{t+s}^f(j) - mc_{t+s} C_{t+s}^f(j) \right]$$

$$\text{subject to} \quad C_t^f(j) = \left(\frac{P_t^f(j)}{P_t^f} \right)^{-\varepsilon_f} C_t^f$$

where $\beta^s \Delta_{s,s+t} = \beta^s \left(\frac{C_{t+s}}{C_t} \right)^{-\gamma}$.

Denoting $P_t^{f*}(j)$ as the optimal price chosen by firm j, the first order condition is:

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\omega\beta)^s \Delta_{s,t+s} \left[\left(\frac{1-\varepsilon_f}{p_{t+s}^f} \right) \left(\frac{P_t^{f*}(j)}{P_{t+s}^f} \right)^{-\varepsilon_f} + mc_{t+s} \left(\frac{\varepsilon_f}{p_{t+s}^f} \right) \left(\frac{P_t^{f*}(j)}{P_{t+s}^f} \right)^{-(1+\varepsilon_f)} \right] C_{t+s}^f = 0,$$

and it follows that

$$\mathbb{E}_t \left[\sum_{s=0}^{\infty} (\omega\beta)^s \Delta_{s,t+s} \left(\frac{P_t^{f*}(j)}{P_{t+s}^f} \right)^{1-\varepsilon_f} C_{t+s}^f \right] = \frac{\varepsilon_f}{\varepsilon_f - 1} \mathbb{E}_t \left[\sum_{s=0}^{\infty} (\omega\beta)^s \Delta_{s,t+s} \frac{MC_{t+s}}{P_{t+s}^f} \left(\frac{P_t^{f*}(j)}{P_{t+s}^f} \right)^{-\varepsilon_f} C_{t+s}^f \right],$$

where $MC_{t+s} = mc_{t+s} P_{t+s}^f$ is the nominal marginal cost.

Since all firms who can change their prices at time t face the same problem, all firms adjusting prices will set the same price, so $P_t^{f*}(j) = P_f^{f*}$. Thus, the optimal price is:

$$P_t^{f*} = \mathcal{M} \mathbb{E}_t \sum_{s=0}^{\infty} X_{t,t+s} mc_{t+s}, \tag{1.19}$$

where

$$X_{t,t+s} = \frac{(\omega\beta)^s \Delta_{s,t+s} C_{t+s} \left(P_{t+s}^f \right)^{\varepsilon_f}}{\mathbb{E}_t \sum_{s=0}^{\infty} \Delta_{s,t+s} C_{t+s} \left(P_{t+s}^f \right)^{\varepsilon_f - 1}}$$

and $\mathcal{M} = \left(\frac{\varepsilon_f}{\varepsilon_f - 1} \right)$ is the markup of formal firms. The average price-level set by formal firms in the economy is a weighted average of the price of the firms that have changed their prices in period t and prices of firms that are unchanged:

$$P_t^f = \left[(1-\omega)(P_t^{f*})^{1-\varepsilon_f} + \omega(P_{t-1}^f)^{1-\varepsilon_f}\right]^{\frac{1}{1-\varepsilon_f}}. \tag{1.20}$$

1.5 Wage Resolution

The formal wage W_{t+1}^f is determined via Nash bargaining between a matched firm and worker and the corresponding maximization problem of their surpluses. Let $\mathscr{F}_t$ denote the value of the surplus of a match by a worker in the formal sector, $\mathscr{J}_t$ is the corresponding value of the surplus of a match by a firm, V_t is the value of the vacancy and $\mathscr{X}_t$ is the worker surplus of staying in the informal sector. If $\eta \in (0,1)$ is defined as worker bargaining power, then, the Nash bargaining problem is:

$$\begin{aligned}
&\underset{W^f}{\text{Max}} \quad (\mathscr{F}_t - \mathscr{X}_t)^{\eta}(\mathscr{J}_t - V_t)^{1-\eta} \\
&\text{subject to} \\
&\mathscr{F}_t \geq \mathscr{X}_t \\
&\mathscr{J}_t \geq V_t = 0
\end{aligned} \tag{1.21}$$

where

$$\mathscr{F}_t = W_{t+1}^f + \beta\left[\delta\mathscr{X}_{t+1} + (1-\delta)\mathscr{F}_{t+1}\right],$$

$$\mathscr{X}_t = W_{t+1}^i + \beta[\theta_t q(\theta_t)\mathscr{F}_{t+1} + (1-\theta q(\theta)\mathscr{X}_{t+1}]$$

represent the worker excess of a match ($\mathscr{F}$) and the excess of continuing on the informal sector ($\mathscr{X}$), respectively. On the other hand:

$$\mathscr{J}_t = P_t^f Y_L^f(L) - W_t^f + \beta(1-\delta)\mathscr{J}_{t+1},$$

$$V_t = -cP_t + \beta q(\theta_t)(\mathscr{J}_{t+1} - V_{t+1})$$

represent the firm's excess of a match ($\mathscr{J}$) and the excess to remain with the vacancy one more period. The ability to adapt their vacancies every period implies that firms will generate vacancies until $V = 0$, which requires:

$$\mathscr{J}_t = \frac{cP_t}{\beta q(\theta_t)}.$$

Then, the first order conditions associated with the maximization problem equation (1.21) gives:

$$\eta \mathscr{J}_t \left(\frac{\partial \mathscr{F}_t}{\partial W_t^f} \right) = (1-\eta)(\mathscr{F}_t - \mathscr{X}_t) \left(\frac{\partial \mathscr{J}_t}{\partial W_t^f} \right),$$

which implies:

$$\eta \left(\frac{P^f Y_1(L) - W^f}{1-\beta(1-\delta)} \right) = (1-\eta) \left(\frac{W^f - W^i}{1-\beta+\beta(\delta+\theta q(\theta))} \right)$$
$$\implies (1-\eta)W^f = (1-\eta)W^i + \eta\Lambda[P^f Y_L^f(L) - W^f].$$

then

$$\begin{aligned} W^f &= (1-\eta)W^i + \eta\Lambda P^f Y_L^f + \eta(1-\Lambda)W^f \\ &= (1-\eta)W^i + \eta P^f Y_L^f + \eta(\Lambda-1)(P^f Y_L^f(L) - W^f) \\ &= (1-\eta)W^i + \eta(P^f Y_L^f + cP\theta), \end{aligned}$$

where

$$\Lambda = \frac{1-\beta+\beta(\delta+\theta q(\theta))}{1-\beta(1-\delta)} = 1 + \frac{cP\theta}{\mathscr{J}(1-\beta(1-\delta))} = 1 + \frac{cP\theta}{P^f Y_L^f(L) - W^f}$$

since $\mathscr{J}\beta q(\theta) = cP$. Therefore, the formal wage given by:

$$W_{t+1}^f = (1-\eta)\mathbb{E}_t(W_{t+1}^i) + \eta\mathbb{E}_t[P_{t+1}^f Y_L^f(L_{t+1}) + cP_{t+1}\theta_{t+1}]. \tag{1.22}$$

Thus, if bargaining power is too small ($\eta \to 0$), the formal wage will be identical to the informal wage, which in equilibrium, is enough to conserve some informal workers looking for formal jobs. Otherwise, if the bargaining power is too high ($\eta \to 1$), the formal wage will be the real marginal product of formal labor plus the cost saved by the formal firm by not opening the vacancy.

It is important to note that combining equations (1.18) and (1.22), real marginal cost can be rewritten as:

$$mc_t = \left(\frac{1}{A_t^f} \right) \left[(1-\eta)\frac{W_t^i}{P_t^f} + \eta \left(Y_L^f(L_t) + \frac{cP_t\theta_t}{P_t^f} \right) \right]. \tag{1.23}$$

1.6 Central Bank

It is assumed that the central bank uses an interest-rate rule responding to inflation and output according to either:

$$\frac{1+i_t}{1+i} = \left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y}, \tag{1.24}$$

or

$$\frac{1+i_t}{1+i} = \left(\frac{\pi_t^f}{\pi^f}\right)^{\phi_\pi} \left(\frac{Y_t^f}{Y^f}\right)^{\phi_y}. \tag{1.25}$$

In equation (1.24) the central bank reacts to aggregate inflation and output (which include the informal economy), whereas in equation (1.25) central bank responds only to formal inflation and output (and ignores the informal economy when setting policy). As usual, $\phi_\pi \geq 0$ and $\phi_y \geq 0$. Also it is assumed the condition $\phi_\pi > 1$ is satisfied, so the well known Taylor Principle holds.

1.7 Market Clearing

Labor market clearing requires that:

$$N_t = L_t^i + L_t^f = \int_0^1 L_t^i(k)dk + \int_0^1 L_t^f(k)dk. \tag{1.26}$$

Goods-market clearing in each sector requires that:

$$Y_t^f = C_t^f + cV_t, \tag{1.27}$$

$$Y_t^i = C_t^i. \tag{1.28}$$

Finally, market clearing in the bond market requires:

$$B_t = 0. \tag{1.29}$$

1.8 Rational Expectations Equilibrium

A rational expectations equilibrium is defined as follows. Given the initial conditions B_0, L_0^f, the exogenous sequences of productivity shocks $\{a_t\}_{t=0}^{\infty}$ and demand shocks $\{z_t\}_{t=0}^{\infty}$, a rational expectations equilibrium consist of 19 variables: a sequence of prices $\{P_t, P_t^f, P_t^i, W_t^f, W_t^i, mc_t\}_{t=0}^{\infty}$, a sequence of allocations $\{N_t, B_t, \Psi_t, L_t^i, L_t^f, C_t, C_t^f, C_t^i, Y_t^i, Y_t^f, \theta_t, V_t\}_{t=0}^{\infty}$ and a monetary policy $\{i_t\}_{t=0}^{\infty}$ satisfying:

(i) the optimal conditions for the representative household (1.6), (1.7) and (1.8) and the transversality condition is satisfied;

(ii) the aggregate informal production function (1.15) and the informal wage-setting rule (1.16);

(iii) the aggregate formal production function (1.17), real marginal cost of formal firms (1.18) and the formal price-setting rule (1.20);

(iv) the matching function (1.2), the labor market mobility function (1.5) and the formal wage-setting rule (1.22);

(v) the aggregate price index (1.10) and the consumption demand conditions (1.11) and (1.12);

(vi) the monetary policy rule (1.24) or (1.25);

(vii) all markets clear (1.26), (1.27), (1.28) and (1.29).

1.9 Distortions

The aforementioned economy suffers form three important distortions:

(i) **Informality.** In this model we have two different kind of firms: formal and informal. Informal firms have lower productivity than formal firms so informality absorbs labor employment that can be used more efficiently in the formal sector.

(ii) **Price stickiness in the formal sector.** Price rigidity in the formal sector implies that the formal price level is sticky in the sense that it can't immediately respond to economic shocks experienced by the formal economy;

(iii) **Monopolistic competition in the formal sector.** The presence of imperfect competition in the formal sector suggests that formal firms with high monopolistic power will produce less than the optimal level of formal output and therefore, the formal output will be less than the perfect competition level of formal output.

Chapter 2

The Log-Linearized Version of the Model and Calibration

The model developed in the previous chapter is non-linear and solving this kind of systems is often difficult. To solve the model, the equilibrium conditions of the model are log-linearized around the steady state. Following Uhlig (1998, pp. 4), the principle of log-linearization is to use a Taylor approximation around the steady state to replace all equations by approximations, which are linear functions in the log-deviations of the variables. Let X_t denote the endogenous variables and X^{ss} the steady state. Log-linearization $\widehat{x_t}$ implies that:

$$\widehat{x_t} = logX_t - logX^{ss}$$

The steady state equilibrium is given in Appendix A. This chapter proceeds as follows: In section 2.1 the log-linearized model is derived and in section 2.2 the calibration of the model is discussed.

2.1 Derivation of the Log-Linearized Model

Log linearizing the definition of labor market tightness (1.1) gives:

$$\widehat{V_t} = \widehat{\theta_t} + \widehat{L_t^i}.$$

Using this result, the log-linearized version of formal goods-market clearing condition (1.27) is:

$$\widehat{Y_t^f} = \left(1 - \frac{cV}{Y^f}\right)\widehat{C_t^f} + c\left(\frac{V}{Y^f}\right)(\widehat{\theta}_t + \widehat{L_t^i}). \tag{2.1}$$

Log-linearizing the formal production function (1.17), informal production function (1.15) and informal goods-market clearing condition (1.28) yields:

$$\widehat{Y_t^f} = \widehat{L_t^f} + a_t, \tag{2.2}$$

$$\widehat{Y_t^i} = \widehat{L_t^i}, \tag{2.3}$$

$$\widehat{L_t^i} = \widehat{C_t^i}. \tag{2.4}$$

The labor market clearing condition (1.4) becomes:

$$\widehat{N_t} = \mathscr{S}\widehat{L_t^f} + (1-\mathscr{S})\widehat{L_t^i} \tag{2.5}$$

where $\mathscr{S} = \left(\frac{L^f}{N}\right) \in (0,1)$. The log-linearized versions of the households optimal conditions equations (1.6), (1.7) and (1.8) are:

$$\widehat{C_t} = \mathbb{E}_t(\widehat{C_{t+1}}) - \left(\frac{1}{\gamma}\right)\left[\tilde{i}_t - \mathbb{E}_t(\widehat{\pi_{t+1}}) - (1-\rho_z)z_t\right], \tag{2.6}$$

$$\nu\widehat{N_t} = \Phi(\widehat{w_t^i} - \gamma\widehat{C_t}) + (1-\Phi)(\vartheta\widehat{\theta}_t + \widehat{\Psi}_t), \tag{2.7}$$

$$\widehat{\Psi}_t = [1-\beta(1-\delta)]\mathbb{E}_t\left[\mu(\widehat{w_{t+1}^f} - \gamma\widehat{C_{t+1}}) + (1-\mu)\nu\widehat{N_{t+1}}\right] + \beta(1-\delta)\mathbb{E}_t(\widehat{\Psi_{t+1}}), \tag{2.8}$$

where $w_t^f = \left(\frac{W_t^f}{P_t}\right)$, $w_t^i = \left(\frac{W_t^i}{P_t}\right)$, $\Phi = \left(\frac{w^i C^{-\gamma}}{N^\nu}\right)$, $\mu = \left(\frac{w^f C^{-\gamma}}{w^f C^{-\gamma} - N^\nu}\right) > 1$, $i_t = -ln(Q_{t,t+1})$ and $\pi_{t+1} = ln\left(\frac{P_{t+1}}{P_t}\right)$ is the inflation rate.

Log-linearizing the consumption demand equations (1.12) and (1.11) and the aggregate price index (1.10) yields:

$$\widehat{C_t^f} = \widehat{C_t} + \tau(\widehat{P}_t - \widehat{P_t^f}), \tag{2.9}$$

$$\widehat{C_t^i} = \widehat{C_t} + \tau(\widehat{P}_t - \widehat{P_t^i}), \tag{2.10}$$

$$\widehat{\pi}_t = \Xi\widehat{\pi_t^f} + (1-\Xi)\widehat{\pi_t^i}, \tag{2.11}$$

where $\Xi = \chi\left(\frac{P^f}{P}\right)^{1-\tau}$.

Log-linearizing the law of motion for formal labor (1.5), the informal wage rate (1.16) and the formal wage rate (1.22) gives:

$$\widehat{L_{t+1}^f} = \delta\left(\widehat{L_t^i} + \vartheta\widehat{\theta}_t\right) + (1-\delta)\widehat{L_t^f}. \tag{2.12}$$

$$\widehat{w_t^i} = \widehat{P_t^i} - \widehat{P}_t. \tag{2.13}$$

$$\widehat{w_{t+1}^f} = (1-\eta)\zeta\widehat{w_{t+1}^i} + (1-(1-\eta)\zeta)\left[(1-\varpi)\left(\widehat{Y_{t+1}^f} - \widehat{L_{t+1}^f} + \widehat{P_{t+1}^f} - \widehat{P_{t+1}}\right) + \varpi\widehat{\theta_{t+1}}\right], \tag{2.14}$$

with $\zeta = \left(\frac{w^i}{w^f}\right)$ and $\varpi = \left(\frac{c\theta}{\left(\frac{P^f}{P}\right)\left(\frac{Y_t^f}{L_t^f}\right)+c\theta}\right)$.

From equation (1.23), real marginal cost can be expressed as:

$$\widehat{mc_t} = \Upsilon_1(\widehat{w_t^i} + \widehat{P}_t - \widehat{P_t^f}) + \Upsilon_2(\widehat{Y_t^f} - \widehat{L_t^f}) + (1-\Upsilon_1-\Upsilon_2)(\widehat{\theta}_t + \widehat{P}_t - \widehat{P_t^f}) - a_t, \tag{2.15}$$

where $\Upsilon_1 = \left(\frac{(1-\eta)w^i P}{P^f mc}\right)$ and $\Upsilon_2 = \left(\frac{\eta Y^f}{L^f mc}\right)$.

Combining the log-linearized versions of (1.19) and (1.20) gives the New Keynesian Phillips Curve (NKPC) for the formal sector:

$$\widehat{\pi_t^f} = \lambda\widehat{mc_t} + \beta\mathbb{E}_t\left[\widehat{\pi_{t+1}^f}\right], \tag{2.16}$$

where $\lambda = \left[\frac{(1-\omega)(1-\omega\beta)}{\omega}\right]$.

Finally, the log-linearized versions of the interest-rate rules (1.24) or (1.25) are:

$$\tilde{i}_t = \phi_\pi\widehat{\pi}_t + \phi_y\widehat{Y}_t, \tag{2.17}$$

or

$$\tilde{i}_t = \phi_\pi \widehat{\pi_t^f} + \phi_y \widehat{Y_t^f}. \tag{2.18}$$

The complete log-linearized system of equations is summarized as follows:

$$\widehat{Y_t^f} = \left(1 - \frac{cV}{Y^f}\right)\widehat{C_t^f} + c\left(\frac{V}{Y^f}\right)(\widehat{\theta}_t + \widehat{L_t^i}), \tag{2.19}$$

$$\widehat{Y_t^f} = \widehat{L_t^f} + a_t, \tag{2.20}$$

$$\widehat{Y_t^i} = \widehat{L_t^i}, \tag{2.21}$$

$$\widehat{L_t^i} = \widehat{C_t^i}, \tag{2.22}$$

$$\widehat{N_t} = \mathscr{S}\widehat{L_t^f} + (1 - \mathscr{S})\widehat{L_t^i}, \tag{2.23}$$

$$\widehat{C_t} = \mathbb{E}_t(\widehat{C_{t+1}}) - \left(\frac{1}{\gamma}\right)\left[\tilde{i}_t - \mathbb{E}_t(\widehat{\pi_{t+1}}) - (1-\rho_z)z_t\right], \tag{2.24}$$

$$\nu\widehat{N_t} = \Phi(\widehat{w_t^i} - \gamma\widehat{C_t}) + (1-\Phi)(\vartheta\widehat{\theta}_t + \widehat{\Psi}_t), \tag{2.25}$$

$$\widehat{\Psi}_t = [1 - \beta(1-\delta)]\mathbb{E}_t\left[\mu(\widehat{w_{t+1}^f} - \gamma\widehat{C_{t+1}}) + (1-\mu)\nu\widehat{N_{t+1}}\right] + \beta(1-\delta)\mathbb{E}_t(\widehat{\Psi_{t+1}}), \tag{2.26}$$

$$\widehat{C_t^f} = \widehat{C_t} + \tau(\widehat{P}_t - \widehat{P_t^f}), \tag{2.27}$$

$$\widehat{C_t^i} = \widehat{C_t} + \tau(\widehat{P}_t - \widehat{P_t^i}), \tag{2.28}$$

$$\widehat{\pi}_t = \Xi\widehat{\pi_t^f} + (1-\Xi)\widehat{\pi_t^i}, \tag{2.29}$$

$$\widehat{L_{t+1}^f} = \delta\left(\widehat{L_t^i} + \vartheta\widehat{\theta}_t\right) + (1-\delta)\widehat{L_t^f}, \tag{2.30}$$

$$\widehat{w_t^i} = \widehat{P_t^i} - \widehat{P}_t, \tag{2.31}$$

$$\widehat{w_{t+1}^f} = (1-\eta)\zeta\widehat{w_{t+1}^i} + (1-(1-\eta)\zeta)\left[(1-\varpi)\left(\widehat{Y_{t+1}^f} - \widehat{L_{t+1}^f} + \widehat{P_{t+1}^f} - \widehat{P_{t+1}}\right) + \varpi\widehat{\theta_{t+1}}\right], \tag{2.32}$$

$$\widehat{mc_t} = \Upsilon_1(\widehat{w_t^i} + \widehat{P}_t - \widehat{P_t^f}) + \Upsilon_2(\widehat{Y_t^f} - \widehat{L_t^f}) + (1 - \Upsilon_1 - \Upsilon_2)(\widehat{\theta}_t + \widehat{P}_t - \widehat{P_t^f}) - a_t, \tag{2.33}$$

$$\widehat{\pi_t^f} = \lambda\widehat{mc_t} + \beta\mathbb{E}_t\left[\widehat{\pi_{t+1}^f}\right], \tag{2.34}$$

$$\tilde{i}_t = \phi_\pi\widehat{\pi}_t + \phi_y\widehat{Y}_t \qquad \text{or,} \tag{2.35}$$

$$\tilde{i}_t = \phi_\pi\widehat{\pi_t^f} + \phi_y\widehat{Y_t^f}, \tag{2.36}$$

and two types of exogenous shocks:

Demand Shock:

$$z_t = \rho_z z_{t-1} + \epsilon_t^z \tag{2.37}$$

Technology Shock in the formal sector:

$$a_t = \rho_a a_{t-1} + \epsilon_t^a. \tag{2.38}$$

2.2 Calibration

The quantitative results are obtained using the parameter calibrations summarized in Table 2.1. The unit of time is one quarter. Following Galí (2015), we assume that $\beta = 0.99$ and $\omega = 0.75$ which are value commonly used in the New Keynesian literature for the discount factor and the degree of price stickiness. Similarly, this literature usually assumes that the households utility function is logarithmic $\gamma = 1$ and sets $\nu = 1$ which implies a Frisch elasticity of labor supply of 1.

Observe that the average mark-up for formal firms can be expressed as $\varepsilon_f = \frac{\mathcal{M}}{\mathcal{M}-1}$. Galí et al. (2001) argue that an appropriate empirical value for the Euro area and U.S. economy is $\mathcal{M} = 1.1$ which implies a elasticity of substitution between formal goods of $\varepsilon_f = 11$.[4] Then, ε_f is set to 11 which implies a mark-up of 10 percent in the formal sector.

There is a little empirical evidence to guide us for values of the consumption preference for formal goods, the elasticity of substitution between formal and informal goods and the formal worker bargaining strength in emerging economies. Batini et al. (2011) is one of the few who study plausible values for emerging economies. They set $\chi = 0.63$ and $\tau = 1.5$ arguing that this values are consist with emerging economy behavior. However, McKnight and De la O (2016) found that using $\chi = 0.83$, $\tau = 1.8$ and $\eta = 0.6$ their model replicates several aspects in the steady state for the Mexican economy. This calibration uses the values proposed by McKnight and De la O (2016) given that this work focuses on Mexico.

Petrongolo and Pissarides (2001) summarized the empirical evidence and estimation issues about the matching function and found that a plausible range for the empirical ϑ is 0.5 to 0.7 and a common value for the matching coefficient of 1. Therefore we set $\vartheta = 0.5$ and $M = 1$. According to Bosch and Esteban-Pretel (2012), $\delta = 0.03$ is consistent with the empirical estimation of the job destruction rate for emerging economies. The real vacancy cost is set to $c = 0.2$ following the empirical estimates of Albrecht et al. (2009). Fernandez and Meza (2015) found that, in steady state, the total factor productivity ratio among formal and informal sectors is 2.19, so it is used $A^f/A^i = 2$.

[4] Cespedes et al. (2005) using the Generalized Method of Moments confirms this result for Chilean Economy.

Table 2.1: Calibration

Parameter	**Description**	**Value**
β	Subjective discount factor	0.99
γ	Intertemporal elasticity of substitution (log utility)	1
ν	Frisch elasticity (labor supply elasticity)	1
ε^f	Elasticity of substitution between formal goods	11
χ	Consumption preference for formal goods	0.83
τ	Elasticity of substitution among formal and informal goods	1.8
ϑ	Matching elasticity	0.5
M	Matching coefficient	1
δ	Job destruction rate	0.03
c	Real vacancy cost	0.2
η	Formal worker bargaining power	0.6
$\frac{A^f}{A^i}$	Productivity ratio	2
ω	Index of price stickiness	0.75
ϕ_π	Central bank degree of response to inflation	3
ϕ_y	Central bank degree of response to output	0.125
ρ_a	Persistence of productivity shock	0.9
σ_a	Standard deviation of productivity shock	1 SD
ρ_z	Persistence of demand shock	0.7
σ_z	Standard deviation of demand shock	1 SD

Finally, as discused by Galí (2015), a good approximation for the interest rate parameters used in the standard New Keynesian literature are $\phi_\pi = 1.5$ $\phi_y = 0.125$. These are parameters proposed by Taylor (1993) as an approximation to the Fed's behavior in recent years. Woodford (2001) shows that these numerical values satisfy the Taylor Principle and therefore leads to determinacy in traditional New Keynesian models. As a benchmark, chapter 3 studies two versions of the model. The first version is the formal-only model where we shut-down the informal sector and, therefore the model collapses to the standard New Keynesian model. The second version is the informal model which is an extension of the New Keynesian model allowing informality. In the formal-only model, the Taylor Principle leads to determinacy. This result change in the informal model since the Taylor Principle don't ensures a unique stationary solution. Then, the baseline calibrated coefficients are $\phi_y = 0.125$, a commonly value used in the New Keynesian literature and $\phi_\pi = 3$, a rather larger value than the proposed in the standard literature but necessary for avoid indeterminacy in the presence of informality. Finally, it is assumed a strong persistence of productivity shocks, $\rho_a = 0.9$ and a moderate persistence of demand shocks, $\rho_z = 0.7$. One standard deviation for both shocks is assumed.

Chapter 3

Optimal Simple Rules and Welfare

Optimal monetary policy is often defined as a set of contingents plans under commitment that maximizes the representative households utility function subject to the competitive equilibrium conditions.[5] Sometimes this contingent plans are functions of several unobservable variables so, according to Zoltán et al. (2010), from a practical perspective is better to use optimal simple rules as an approximation of the welfare-maximizing policy.

Usually, a quadratic approximation of welfare is assumed and monetary policy is designed based on evaluations of which rule attains for the minimum value social welfare loss function. As a result, social welfare usually depends on inflation variance and the relevant output gap. A big problem arises when there is incomplete information about potential output. Informality represents a big economic distortion which complicates the knowledge of the output gap. Yet, even when it is possible to find an analytical expression for the output gap, given that the informality analysis is conducted for a two sector economy, such expressions could contain output levels for both sectors, as shown by Mattesini and Rossi (2009). Given that the estimates for the output size of the informal economy could be biased by measurement error problems, it may not be possible to obtain a good approximation for the output gap in the presence of informality and such uncertainty about the true output gap would distort any analysis on the performance of Taylor rules.[6]

Even when is possible to find the output gap, and therefore the relevant quadratic approximation of welfare, as Batini et al. (2011) did, welfare could depend on informal variables like informal inflation and employment. As Blanchard and Galí (2010) have shown under labor market frictions, the degree of labor market tightness is relevant for welfare function. Since in this model there is no unemployment, labor market tightness defines the movements of formal workers to

[5] See Khan et al. (2003) and Woodford (2003) for a detailed discussion of this problem.
[6] A full discussion of this point is given by Ehrmann and Smets (2003).

the informal sector. Again, it is not possible to know this variable with any certainty. Then given the nature of the informal sector and labor market tightness, central banks cannot observe accurately of the dynamics of informal markets, which complicates the design of monetary policy.

The approach adopted here to circumvent these problems is that we assume that the central bank is only interested in stabilizing output (and inflation) and not the output gap. This approach is plausible since central banks can observe output and inflation variations and they know their preferences about the weights of each variable. In this simplest case, the welfare loss function depends on output and inflation variances with respect to their steady state values. Informality is an important distortion considered in the model and, therefore, is assumed that central bank faces two options when designing monetary policy: reacting to formal-only measures or taking informality into account and reacts to aggregate measures. Therefore, the central bank aims to minimize the next quadratic loss function when reacting to aggregate measures:

$$\mathbb{L}(\widehat{\pi}_t, \widehat{Y}_t) = \frac{1}{2}\left[\widehat{\pi}_t^{\,2} + \xi_y \widehat{Y}_t^{\,2}\right], \tag{3.1}$$

and conducts the monetary policy using an interest rate rule of the form:

$$\tilde{i}_t = \phi_\pi \widehat{\pi}_t + \phi_y \widehat{Y}_t. \tag{3.2}$$

When the central bank reacts to formal-only measures the relevant quadratic loss function is:

$$\mathbb{L}(\widehat{\pi_t^f}, \widehat{Y_t^f}) = \frac{1}{2}\left[\widehat{\pi_t^f}^{\,2} + \xi_y \widehat{Y_t^f}^{\,2}\right], \tag{3.3}$$

and the monetary policy is conducted using an interest rate rule of the form:

$$\tilde{i}_t = \phi_\pi \widehat{\pi_t^f} + \phi_y \widehat{Y_t^f}. \tag{3.4}$$

In the existing literature there are two ways to measure the central bank degree of output responsiveness ξ_y. The traditional approach is deriving analytically the ξ_y value that will depend upon deep parameters of the model. Since the model is quite complex, a closed form solution is not attainable.[7] Another way to proceed, which this work follows, is using estimated values for ξ_y. Recently, McKnight et al. (2016) using Bayesian estimation have found that this value is around 0.6 for Mexican economy, consequently we set $\xi_Y = 0.6$ in order to keep things simple. In the following sections, two experiments are studied. Section 3.1 performs an evaluation of social welfare loss ranking based on different optimal simple rules for the benchmark case when the

[7] Bandaogo (2015) propose to use numerical simulations to ease the problem.

economy is completely formal. Next, these results are contrasted when the economy is distorted by informality. In section 3.2 the social welfare loss ranking is calculated but considering the case when central bank reacts to aggregate inflation and output measures and when the central bank responds to formal-only inflation and output measures.

3.1 The Benchmark Case: The Formal-Only Model

The model without informality is a simplified version of the standard closed economy New Keynesian model presented in Galí (2015).[8] When designing monetary policy in this benchmark case a key assumption is made with respect to the standard New Keynesian model. It is assumed that central bank's relevant welfare loss equation is given by (3.1) and the monetary policy is conducted via the interest rate equation (3.2). In this context, formal-only and aggregate measures of inflation and output are the same concept.

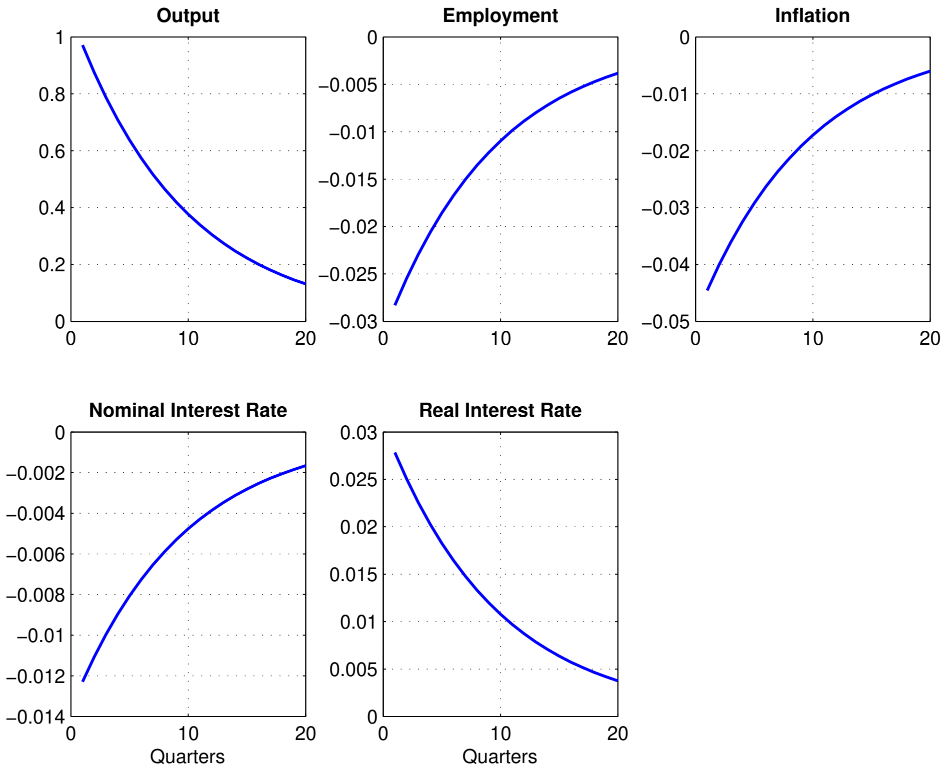
Galí (2015) has shown that in this simplified New Keynesian model it is easy to find the natural level of output (defined as the corresponding level of output when prices are fully flexible), and therefore the output gap. In this model, the central bank's welfare function depends on inflation variability and the output gap. Appendix B presents the relevant results of this model calibrated with the numerical values proposed in Table 2.1. Then, while the central bank reacts inflation deviations and the output gap in the model presented in Appendix B, the results presented in this chapter assumes the central bank reacts to inflation and output deviations respect their steady state value.

Figure 3.1 displays the dynamic responses of the relevant macroeconomic variables due to two different shocks. Panel A shows the effects of a technology shock in formal sector (this shock is thought as an increase in a_t). Since technology shock implies an increase in productivity, output rises but the demand for labor is lower since less workers are necessary for production. The increase of output reduces the nominal interest rate but this movement is not enough to prevent the real interest rate rising.[9] Nominal interest rate falls but aggregate inflation falls more drastically and this is driving up the real interest rate. Nominal interest rate is function of both aggregate inflation and aggregate output. Since output is going up and inflation is falling, the nominal interest rate falls.

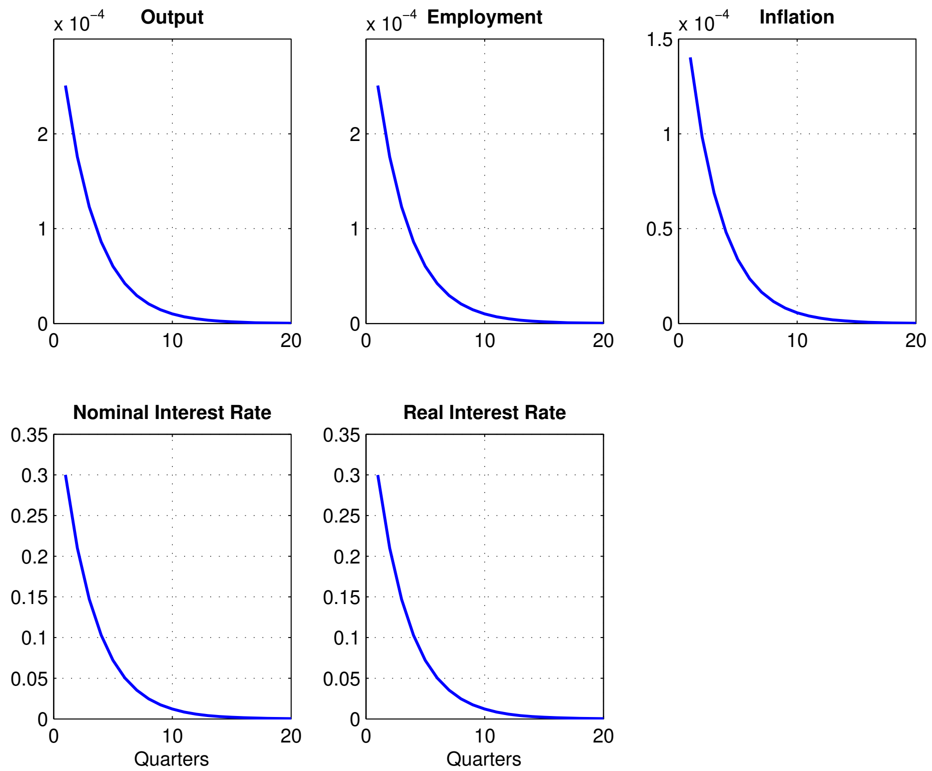
[8] Since here is assumed a cashless economy and a linear production function for firms.

[9] In the standard New Keynesian model presented in Appendix B, when the central bank reacts to output-gap deviations, a productivity shock indeed pushs-down the real interest rate since a more drastic reduction in the nominal interest rate is needed.

Figure 3.1: Benchmark Case. IRF's to Different Shocks



(a) Technology Shock



(b) Demand Shock

Panel B shows the dynamics of a positive discount rate shock, or demand shock. This shock can be interpreted as households become more impatient respect to present utility relative to future utility. The rise in z_t leads to a expansion in real interest rate, output, employment, inflation and nominal interest rate. Since technology is not changing, the increase in output leads an increase in the demand for labor. The nominal interest rate rises due the increase in inflation and output but the inflation rise is lower than the rise in nominal interest rate which causes an increase in the real interest rate. Overall, the results shown in Figure 3.1 are consistent with the New Keynesian model presented in Appendix B.

Table 3.1 reports the standard deviations of relevant variables for both productivity and demand shocks. Three different value for the interest-rate response to inflation ϕ_π and for the interest-rate response to output ϕ_y are selected for the Taylor rule (3.2) in order to evaluate their performance in terms of social welfare loss. As is discussed in Section 4, the presence of informality implies that the Taylor Principle is not a necessary condition to guarantee determinacy. Thus, bigger values for ϕ_π must be used in order to compare the welfare rankings across the different versions of the model.

Panel A column 1 displays the results for the benchmark parameter values described in Table 2.1 for a technological shock. Column 2 shows that a bigger nominal interest-rate rule response to inflation and a zero response to output reduces inflation volatility, output volatility remains unchanged and employment volatility is reduced. However, as we increase the nominal interest-rate response to output (column 3), output and inflation volatility rises but employment volatility falls. Thus, when technology shocks are the source of business cycle fluctuations, a very aggressive anti-inflationary policy with no response to output will reduce the volatility of inflation and output. However, if we are only interested in employment volatility, then an aggressive anti-inflationary and a moderate output stabilization monetary policy is required. However, in terms of welfare loss, a bigger nominal interest-rate rule response to inflation and output reduces welfare.

Panel B displays the results for a demand shock. Column 2 shows that if the policy rule does not react to output $\phi_y = 0$, a larger policy response to inflation is required to reduce output and employment volatility. This result is unaffected if we increase the nominal interest-rate rule response to output. Thus, when the business cycle is driven by a demand shock, then a strong stabilization of inflation and output is required. Nevertheless, bigger nominal interest-rate rule response to inflation and a zero response to output attains the lower welfare loss which is a opposite pattern that shown with the productivity shock.

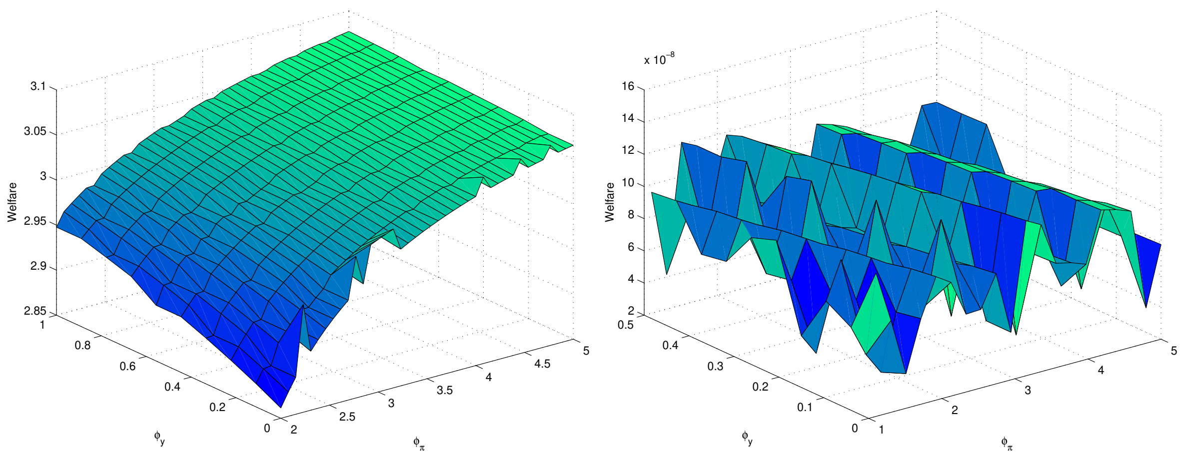
We can vary the nominal interest-rate rule coefficients in order to display the different welfare loss values associated to different values for ϕ_π and ϕ_y.[10] These results are shown in Figure 3.2. As Panel A shows, lower values for the nominal interest-rate rule response to output and inflation attain improvements in social loss welfare. Panel B does not allow for any conclusion.

The aforementioned results could be contrasted with the standard New Keynesian model with where the welfare loss function depends on the output gap. First, as Appendix B shows, in the standard New Keynesian model there is a trade-off between output stabilization and inflation stabilization when the economy is hit by a technological shock. Table 3.1 shows that considering the output instead of the output gap in the welfare function, this result is holds. Second, in the standard New Keynesian economy a bigger ϕ_π reduces the welfare loss.However, when output enters the welfare loss function this result can be reversed since improvements in welfare are obtained under a lower inflation response coefficient.

Table 3.1: Benchmark Case. Evaluation of simple rules

	Panel A. Technology Shock			*Panel B. Demand Shock*		
ϕ_π	3	5	5	3	5	5
ϕ_y	0.125	0	0.175	0.125	0	0.175
$\sigma(Y)$	2.2292	2.2592	2.2601	0.0004	0.0003	0.0003
$\sigma(\pi)$	0.1023	0.0551	0.0537	0.0002	0.0002	0.0002
$\sigma(N)$	0.0649	0.0350	0.0341	0.0004	0.0003	0.0003
$\mathbb{L}$	**2.9921**	**3.0654**	**3.0677**	**1.14E-07**	**7.88E-08**	**7.60E-08**

Figure 3.2: Benchmark Case. Welfare Loss



(a) Technology Shock (b) Demand Shock

[10]The Welfare Loss graphs were made by a `Matlab` loop using different values for ϕ_π and ϕ_y. The `osr` command in `Dynare` was used and this allows us to calculate the welfare function values and then plot them.

3.2 The Economy under Informality

This section considers the economy with informality. Two cases are studied simultaneously: when the monetary policy interest-rate rule reacts to aggregate inflation and aggregate output and when the monetary policy interest-rate rule reacts to formal-only inflation and formal-only output. Figure 3.3 shows the effects of a technology shock experienced by formal firms. Panel A exhibits that when central bank reacts to aggregate economic measures exist a kind of sector substitution effect. An increase in formal productivity increase formal output and consumption but also pushs-up formal employment, rising the incentives for informal workers of moving to formal sector. Then, informal employment is lower reducing informal output and then lowering the informal consumption. Aggregate inflation is pushed up, so the central bank raises nominal interest rate.

As can be seen, the formal productivity shock has a positive aggregate effect on output, consumption and employment. Formal and aggregate employment have a non-standard behavior. The productivity increase in the formal sector has a positive effect on aggregate output but don't reduce the aggregate employment because the employment reduction in informal sector is offset by an employment rise in formal sector. Thus, the negative correlation between output and employment disappears. However, reacting to formal-only inflation and output measures leads to different dynamics, as in Panel B shows. An improvement in formal productivity increases formal output and consumption but reduces formal employment. Lower demand for formal labor implies a movement of formal workers to informal employment so, informal output and consumption rises by the total flexibility of informal labor market to absorb the excess of formal workers. Despite of this phenomena, the aggregate consumption, output and employment are bigger.

What can be learned from the dynamics of this exogenous shock is that depending on which economic measure the central bank is reacting, there are different labor sector dynamics. When the policy rule reacts to formal-only inflation and output, positive productivity shocks result in a flow of formal workers to the informal sector, the opposite is the case when the policy rule responds to aggregate inflation and output. However, in both cases the effect on aggregate output, consumption and employment is positive. Then, in aggregate terms, the positive formal productivity shock implies a bigger aggregate employment and then, the negative relation shown in standard New Keynesian models between productivity shocks and employment disappears.[11]

[11]This result is partially consistent with previous findings for Mexico. Boz et al. (2011) expands the model developed by Aguiar and Gopinath (2007) considering the case in which agents cannot perfectly distinguish between permanent and transitory productivity shocks. They calibrate their model for Mexico and found that under perfect information, a positive productivity shock causes a fall in labor. This implies a negative correlation between output and employment. However, Fernandez and Meza (2011) have shown that, under certain circumstances, a productivity shock could be procyclical under the model of Aguiar and Gopinath (2007).

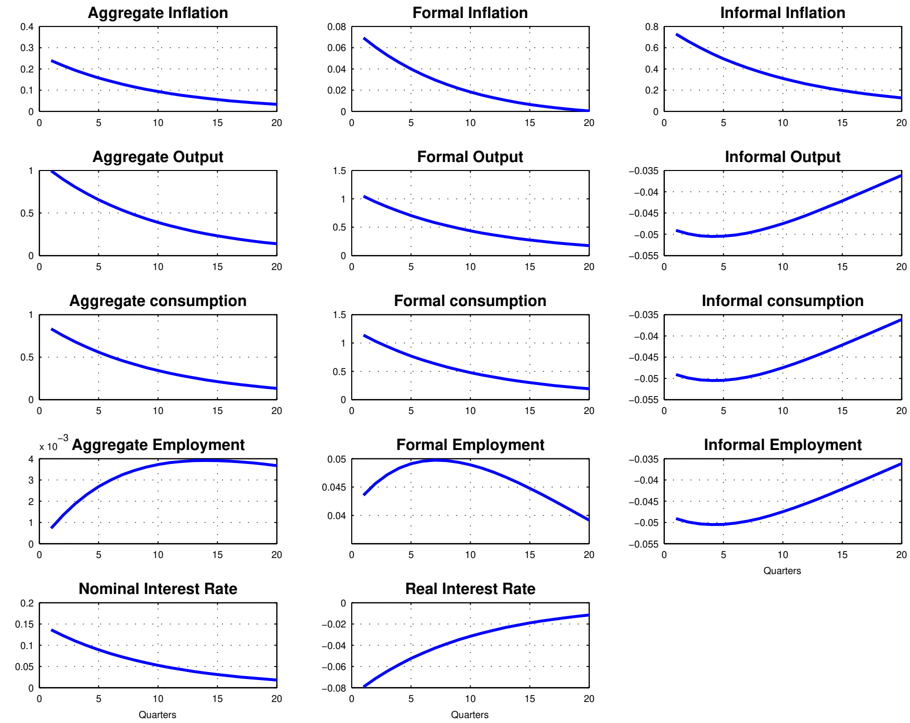
The key difference of both dynamics seems to be the nominal interest rate. If the interest-rate rule focus on aggregate output and inflation, the fall in real interest rate stimulate formal consumption by a bigger amount relative to the case when the nominal interest-rate rule reacts to formal-only inflation and output which stimulates aggregate inflation. Thus, the real interest rate falls due to the large rise in aggregate inflation. However, if the interest-rate rule focuses on formal-only output and formal-only inflation, demand for formal goods is lower so the real interest rate has not fallen enough and then, for goods market clearing, formal inflation falls. Thus, rather to produce more formal goods, indeed, the economy needs to produce less formal goods and this shifts resources to the informal sector.

Another important point is the quantitative effects for both cases. It seems to be the case that when the nominal interest-rate rule responds to aggregate inflation and aggregate output, formal inflation, informal inflation, formal output, formal consumption, aggregate inflation, aggregate consumption and nominal interest rate are more volatile. Nevertheless, when the nominal interest-rate rule responds to formal-only inflation and formal-only output, aggregate employment, formal employment, informal employment, informal consumption and informal output are more volatile. Then, there are volatility differences for both case and this could be explained by a different propagation effects driven by the inclusion of informality.

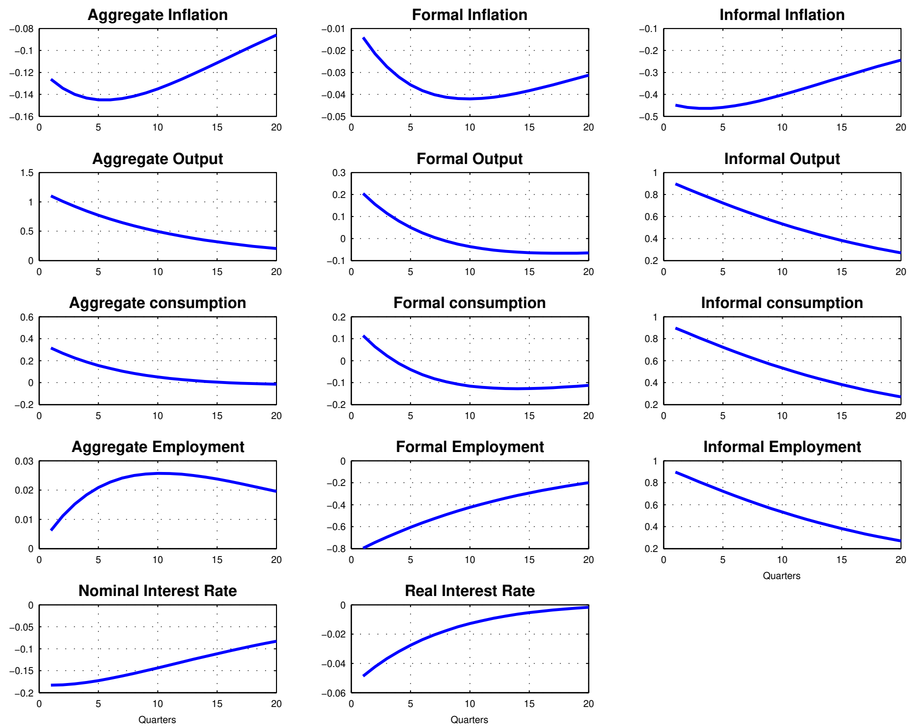
Figure 3.4 displays the impulse response functions of a positive demand shock. Panel A shows the case when central bank responds to aggregate inflation and aggregate output. A positive demand shock raise aggregate consumption via the Euler equation. Given that the model assumes a strong preference for formal goods, formal consumption goes up and informal consumption is lower. Higher formal consumption encourage formal output and formal employment but this reduces informal output and informal employment since the incentives for informal workers to join the formal labor market have risen. Finally, since aggregate inflation has growth, nominal interest rate must be raised.

Panel B shows that reacting to formal-only inflation and output gives the same qualitative dynamics of Panel A. The dynamics of this shock confirms the labor markets sector substitution. However, there are quantitative differences for both kind of nominal interest-rate rules. When the nominal interest-rate rule is responding to aggregate output and aggregate inflation the key variables of the model seem to be more volatile than when the nominal interest-rate rule reacts to formal-only output and formal-only inflation. The quantitative pattern shown by Panel B is closer to the benchmark model, then, nominal interest-rate rule responding to aggregate output and aggregate inflation produces more volatile propagation effects than in the benchmark model.

Figure 3.3: Informal Economy. Effects of a Technology Shock

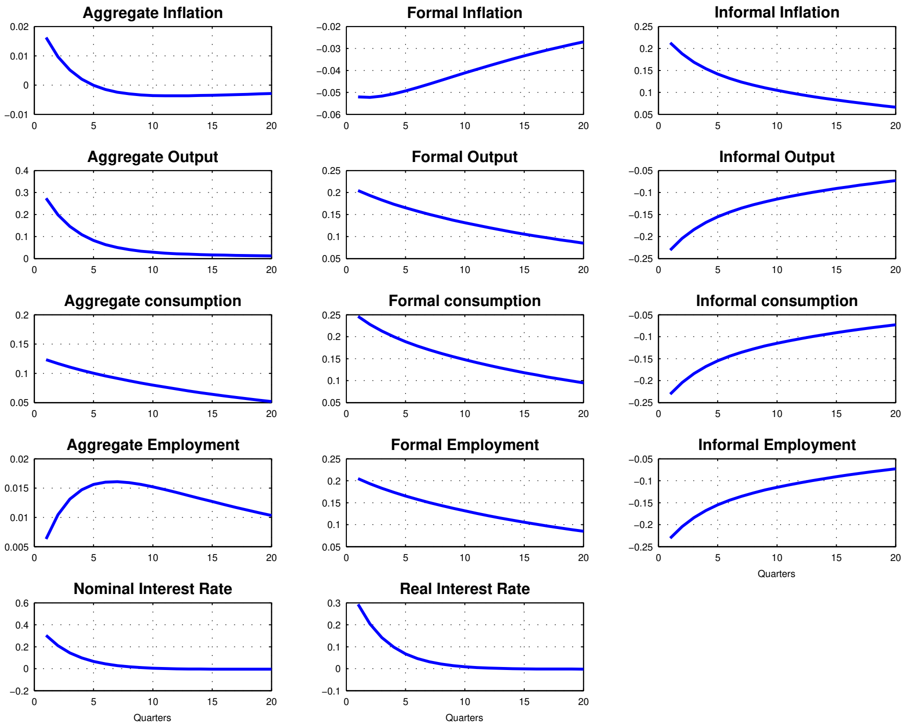


(a) Taylor rule with aggregate inflation and aggregate output

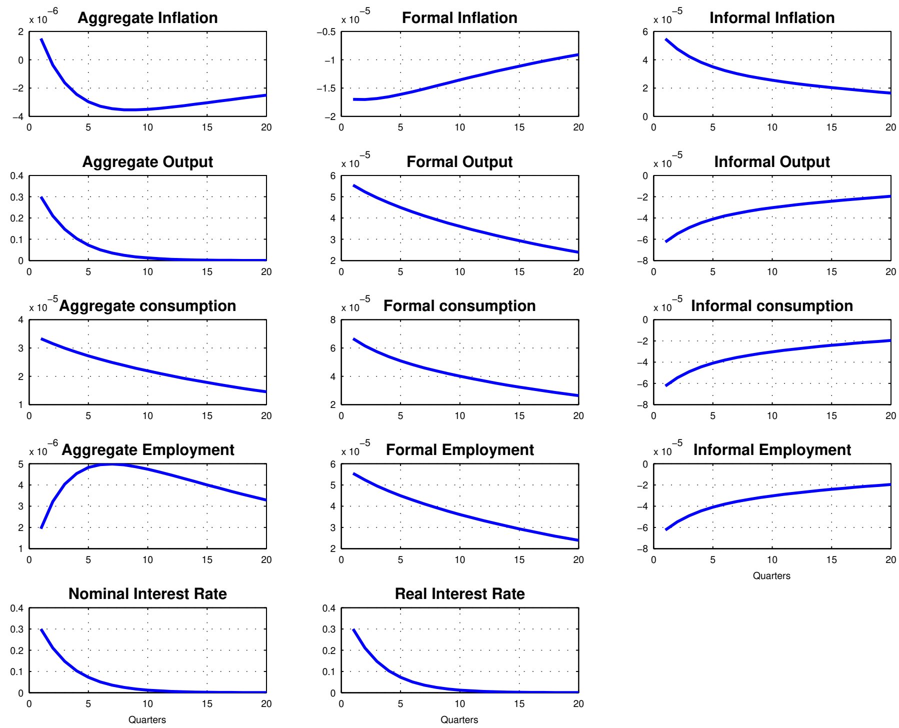


(b) Taylor rule with formal-only inflation and formal-only output

Figure 3.4: Informal Economy. Effects of a Demand Shock



(a) Taylor rule with aggregate inflation and aggregate output



(b) Taylor rule with formal-only inflation and formal-only output

Table 3.2: Informal Case. Evaluation of Simple Rules

Panel A. Technology Shock						
	Reacting to Aggregate Measures			*Reacting to Formal-only Measures*		
ϕ_π	3	5	5	3	5	5
ϕ_y	0.125	0	0.175	0.125	0	0.175
$\sigma(\pi)$	0.5513	0.5501	0.5500	0.5741	0.5720	0.5871
$\sigma(\pi^f)$	0.1386	0.1402	0.1402	0.1535	0.1516	0.1656
$\sigma(\pi^i)$	1.7644	1.7530	1.7528	1.8144	1.8124	1.8254
$\sigma(Y)$	2.2917	2.2923	2.2923	2.7087	2.7073	2.7178
$\sigma(Y^f)$	2.5009	2.4900	2.4898	0.4349	0.4360	0.4279
$\sigma(Y^i)$	0.2483	0.2371	0.2370	2.6482	2.6477	2.6500
$\sigma(C)$	1.9723	1.9656	1.9655	0.5759	0.5759	0.5766
$\sigma(C^f)$	2.7310	2.7180	2.7178	0.5945	0.5958	0.5855
$\sigma(C^i)$	0.2483	0.2371	0.2370	2.6482	2.6477	2.6500
$\sigma(N)$	0.0210	0.0203	0.0203	0.0966	0.0957	0.1029
$\sigma(L^f)$	0.2569	0.2460	0.2459	2.2060	2.2069	2.1980
$\sigma(L^i)$	0.2483	0.2371	0.2370	2.6482	2.6477	2.6500
$\mathbb{L}$	**3.4550**	**3.4553**	**3.4553**	**0.1370**	**0.1370**	**0.1373**
Panel B. Demand Shock						
	Reacting to Aggregate Measures			*Reacting to Formal-only Measures*		
ϕ_π	3	5	5	3	5	5
ϕ_y	0.125	0	0.175	0.125	0	0.175
$\sigma(\pi)$	0.0420	0.0272	0.0268	0.0005	0.0005	0.1457
$\sigma(\pi^f)$	0.2116	0.2213	0.2183	0.0002	0.0002	0.0411
$\sigma(\pi^i)$	0.7362	0.6428	0.6298	0.0014	0.0012	0.4724
$\sigma(Y)$	0.4033	0.4104	0.4106	0.4209	0.4206	0.7932
$\sigma(Y^f)$	0.8104	0.7470	0.7334	0.0003	0.0003	0.0515
$\sigma(Y^i)$	0.7729	0.6996	0.6865	0.0016	0.0011	0.5898
$\sigma(C)$	0.4920	0.4530	0.4448	0.0002	0.0002	0.0288
$\sigma(C^f)$	0.9306	0.8526	0.8370	0.0007	0.0006	0.2103
$\sigma(C^i)$	0.7729	0.6996	0.6865	0.0016	0.0011	0.5898
$\sigma(N)$	0.0740	0.0749	0.0738	0.0006	0.0004	0.2613
$\sigma(L^f)$	0.8104	0.7470	0.7334	0.0003	0.0003	0.0515
$\sigma(L^i)$	0.7729	0.6996	0.6865	0.0016	0.0011	0.5898
$\mathbb{L}$	**0.0994**	**0.1018**	**0.1019**	**8.16E-05**	**1.18E-07**	**3.28E-03**

Therefore, based on the previous results, can be stated that economies with a large informal sector behave differently to formal only-economies. These different dynamics should matter for designing optimal monetary policy. Table 3.2 displays different values for nominal interest-rate rule responses to output and inflation with their implications for the variances of the key variables of the model and their associated welfare loss values. Panel A the results when the business cycle is driven by a technological shock. When the nominal interest-rule reacts to aggregate output and aggregate inflation, an aggressive anti-inflationary monetary policy with a moderate output stabilization ($\phi_\pi = 5$ and $\phi_y = 0.175$) reduces the variances of the key variables of the model.[12]

However, when the nominal interest-rate rule responds to formal-only inflation and formal-only output, an aggressive anti-inflationary monetary policy with zero response to output ($\phi_\pi = 5$ and $\phi_y = 0$) gives lower variances for the key variables of the model. In terms of welfare, if the nominal interest-rate rule reacts less to aggregate inflation and aggregate output ($\phi_\pi = 3$ and $\phi_y = 0.125$) the lower welfare loss is attained.[13] However, when central bank is reacting to formal-only inflation and formal-only output an aggressive anti-inflationary monetary policy with zero response to output or an aggressive anti-inflationary monetary policy with a moderate output stabilization give the same welfare loss.

Now consider the case when the economy is hit by a demand shock shown in Panel B. Consider the case when the nominal interest-rate rule reacts to aggregate inflation and aggregate output. In general, an aggressive anti-inflationary monetary policy with a moderate output stabilization performs lower variances and lower welfare loss. Nevertheless, when the nominal interest-rate rule is reacting to formal-only inflation and formal-only output, an aggressive anti-inflationary monetary policy with zero response to output reduces the variances of the key variables and reduces the welfare loss.

Besides the previous findings, it is possible to extract the following facts. First, the trade-off between output stabilization and inflation stabilization seems to hold: in general, a bigger values for ϕ_π reduces aggregate inflation volatility but increases aggregate output volatility. Second, a bigger value for ϕ_π increases the welfare loss. This results are consistent with those found in Section 3.1. Third, when the origin of fluctuations arises from a technology shock in formal sector, in general, the central bank achieves a lower volatility in relevant macroeconomic variables if reacts to aggregate economic measures, but, when the source of variation is a demand shock, reacting to formal-only measures attains a lower volatility in relevant variables. Fourth and more important result: when central bank reacts to formal-only inflation and formal-only output the welfare loss is lower than reacting to aggregate inflation and aggregate output.

3.3 Robustness and Sensitivity Analysis

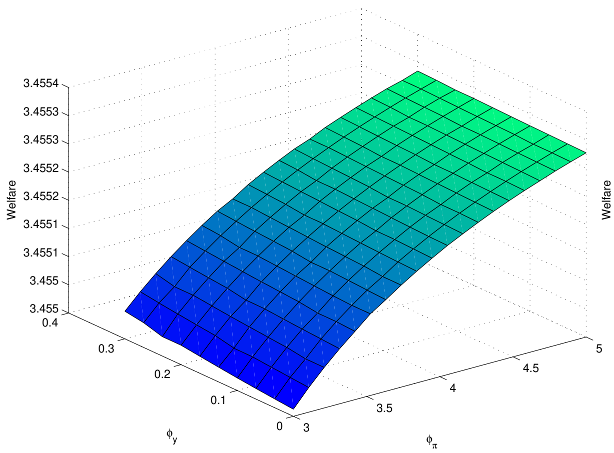
This section studies the robustness and sensitivity of some previous results. Figure 3.5 plots the welfare loss values for both shocks changing the ϕ_π and ϕ_y parameter values of the feedback interest rate rule. For the selected intervals of ϕ_π and ϕ_y, it can be seen that reacting to aggregate inflation and aggregate output represent a higher social welfare loss for both shocks.

[12] Overall, the smaller variances are given by the nominal interest-rate rule with $\phi_\pi = 5$ and $\phi_y = 0.175$.

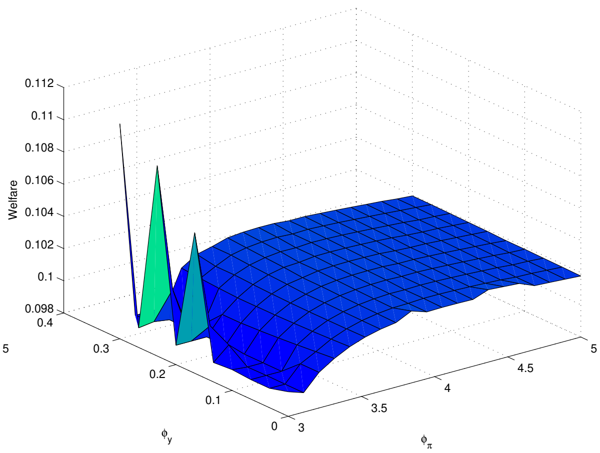
[13] This pattern is consistent with the benchmark model.

Then, a lower welfare loss is achieved when the central bank reacts to formal-only inflation and formal-only output than when reacts to aggregate inflation and aggregate output with different values for the ϕ_π and ϕ_y parameters.

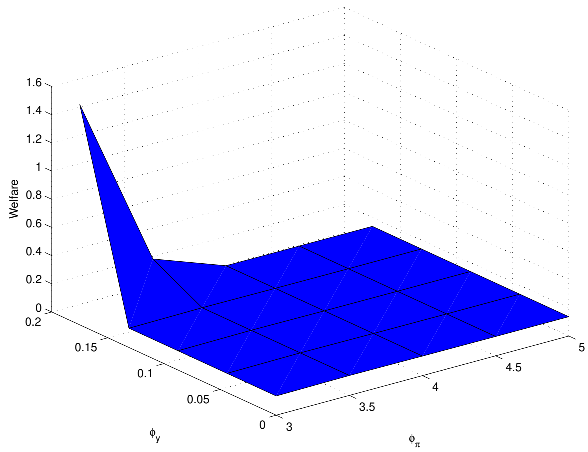
Figure 3.5: Informal Case. Welfare Loss



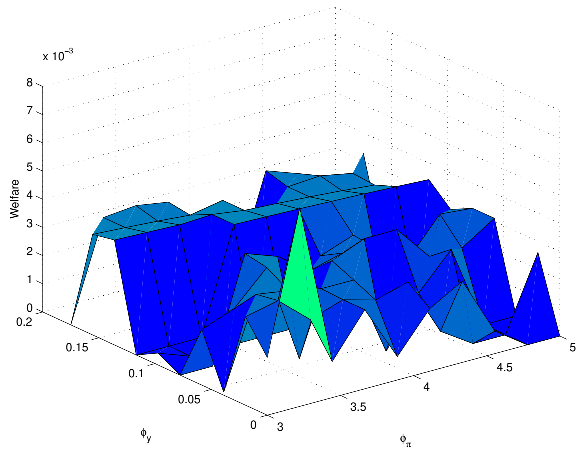
(a) Technology Shock (Agg. M.)



(b) Demand Shock (Agg. M.)



(c) Technology Shock (Form. M.)



(d) Demand Shock (Form. M.)

A sensitivity analysis is performed for the following four deep parameter: the job destruction rate δ, the degree of price stickiness for formal firms ω, the formal workers bargaining strength η and the consumption preference for formal goods χ. The results are reported in Table 3.3. First, consider an increase of δ from 0.03 to 0.15. As the formal job destruction rate increases less formal jobs are available. In the case of productivity shocks, a bigger δ improves welfare when reacting to aggregate inflation and output measures raising the importance of the informal channel as the formal employment duration is reduced but, welfare worse off when reacting to formal-only inflation and output measures because in this case the informality channel is neglected in welfare loss function. If the economy is affected by a demand shock then a higher δ improves welfare for both possible elections of central bank magnifying the informality channel.

Table 3.3: Sensitivity Analysis and Welfare Loss

Panel A. Aggregate measures		
	Technology shock	*Demand shock*
Job Destruction Rate:		
$\delta = 0.03$	3.455	0.099
$\delta = 0.15$	3.390	0.098
Index of price stickiness:		
$\omega = 0.50$	4.899	0.045
$\omega = 0.75$	3.455	0.099
Formal worker bargaining power:		
$\eta = 0.60$	3.455	0.099
$\eta = 0.90$	1.181	0.117
Consumption preference for formal goods:		
$\chi = 0.83$	3.455	0.099
$\chi = 0.99$	1.063	0.008
Panel B. Formal-only measures		
	Technology shock	*Demand shock*
Job Destruction Rate:		
$\delta = 0.03$	0.137	8.16E-05
$\delta = 0.15$	1.353	7.99E-08
Index of price stickiness:		
$\omega = 0.50$	0.396	7.79E-08
$\omega = 0.75$	0.137	8.16E-05
Formal worker bargaining power:		
$\eta = 0.60$	0.137	8.16E-05
$\eta = 0.90$	0.213	6.39E-08
Consumption preference for formal goods:		
$\chi = 0.83$	0.137	8.16E-05
$\chi = 0.99$	0.170	3.17E-04

Second, consider a decrease of ω from 0.75 to 0.50. In the productivity shock environment, as formal prices become less stickier welfare reduces decreasing the importance of formal-informal sector substitution mechanism on aggregate economy. This result holds for both central bank's possible choices. The less stickier formal prices effect on welfare when a demand hits the economy on welfare is opposite. In this case, formal market is pushed to depend less on informal sector production which increase welfare. Third, consider an increase of η from 0.60 to 0.90 and consider a technology shock. A bigger negotiation power by formal workers increases welfare when central bank reacts to aggregate economy because informal workers have more incentives to join the formal market labor, but again, as informality channel is neglected when central bank reacts to formal-only economy, in this case welfare is lower. With a demand shock the result is the opposite.

Fourth, consider an increase of χ from 0.85 to 0.99. Overall, a higher consumption preference for formal goods for both shocks leads a welfare improvement. Finally, Table 3.3 confirms that reacting to aggregate economic measures represents a higher welfare loss for both shocks even when deep structural parameters are changing. Then, the result found in Section 3.2 are robust and not sensitive.

Therefore, the main conclusions of this chapter are as following. First, the informal sector helps the formal sector to adjust to shocks and this can result in welfare gains provided the interest-rate rule reacts to formal-only inflation and output. In this case, the informal sector appears to buffer the formal economy with respect to the shocks. This result suggests that simple, formal-only Taylor rules can do a good job in terms of welfare under informality and such rules are easily implementable without the need for the central bank to accurately measure the informal sector.

Chapter 4

Equilibrium Determinacy under Informality

In the previous chapter, the impulse-response and welfare analysis were conducted for a set of parameters that assume a unique stationary solution. In this section we investigate under what conditions can multiple solutions exists (i.e. indeterminacy). Woodford (2001) show that in standard New Keynesian model, policy rules with $\phi_\pi, \phi_y \geq 0$, the system has a unique stationary solution if and only if this conditions holds:

$$\phi_\pi + \left(\frac{1-\beta}{\kappa}\right)\phi_y > 1 \qquad\qquad \kappa \geq 0, \tag{4.1}$$

Equation (4.1) is the well known Taylor Principle and says that a Taylor-type feedback rule will lead to a unique stationary solution only if an increase in the inflation rate by κ percent is followed by a nominal interest rate increment by *more than* κ percent. Woodford (2001) shows that the coefficients values related with the classical Taylor rule, $\phi_\pi = 1.5$ and $\phi_y = 0.5$ ($\phi_y = 0.5/4$ in quarters), necessary satisfy (4.1).

This chapter shows that under informality, depending on the selected parameter values the model could lead an indeterminacy issue. Under informality, in much of the cases, the Taylor Principle is not enough to guarantee determinacy: higher values for the inflation coefficient of the feedback interest-rate are usually needed to get a unique stable solution path. This result is consistent with Schmitt-Grohé and Uribe (2007)[14]. They found the size of inflation coefficient in the interest-rate rule plays a minor role for welfare but has an important role in the equilibrium determinacy. Thus, in their model, bigger values for the size of inflation coefficient serves mainly for ensuring the uniqueness of rational expectations equilibrium. However, we find that when

[14] In their model, the effects of informality on determinacy are not considered.

central bank reacts to aggregate inflation and aggregate output exists regions where the Taylor Principle is not required to drive determinacy.

Nevertheless, in general terms, we find the following policy trade-off. While informality can improve the performance of optimal simple rules in terms of welfare, depending on the calibration it could lead problems of indeterminacy. With high inflation values, the model shows a bigger welfare loss but with lower inflation values the analysis cannot even be conducted because the multiple equilibria issues. Thus, economies with large informal sectors must target inflation more aggressive, because otherwise the probability of getting welfare-reducing multiple equilibrium is higher. Then, central banks must take into account this potential policy trade-off when designing monetary policy.

4.1 Determinacy Analysis

This section documents what are the implications of informality on determinacy of the rational expectations equilibrium. Figure 4.1 displays the determinacy regions for different versions of the model using different configurations for ϕ_π and ϕ_y[15]. Panel A considers the economy without informality as described in Section 3.1. It is shown that in order to guarantee determinacy, the central bank must react to any inflation deviation raising ϕ_π more than proportionately, which implies that for determinacy the Taylor Principle must holds.

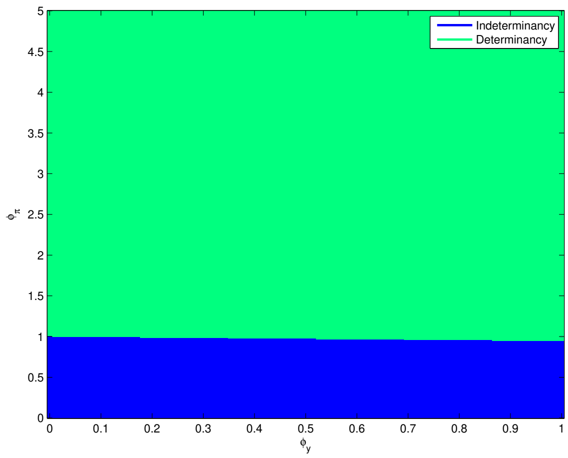
Pane B considers the economy with informality under the case when central bank responds to aggregate inflation and aggregate output. It is shown that, in general, bigger values for ϕ_y amplifies the indeterminacy regions. However, there are certain values for $0 \leq \phi_\pi < 1$ that lead determinacy. In particular, with $0 \leq \phi_\pi < 1$ and $\phi'_\pi in[0.45, 1]$ the model has a unique stationary solution. This result is opposite to the standard condition for determinacy in New Keynesian models. Thus, for certain regions, the Taylor Principle is not required for determinacy. Panel C shows the informal case when the central bank responds to formal-only. In this case, central bank needs to use an aggressive anti-inflationary monetary policy to get determinacy. Opposite with Panel B, a higher values for ϕ_y decrease the indeterminacy regions.

The next result studied in this Chapter is the sensitivity of the previous finding. Then, the relation between changes in deep structural parameters and determinacy is studied. Figures 4.2 and 4.3 display the relevant results. For both figures, left column shows the case when central bank reacts to aggregate inflation and aggregate output while right column shows the case when central bank reacts to formal-only inflation and formal-only output. From Figures 4.2 and 4.3 the following conclusions can be stated. First, an increase in the job destruction rate

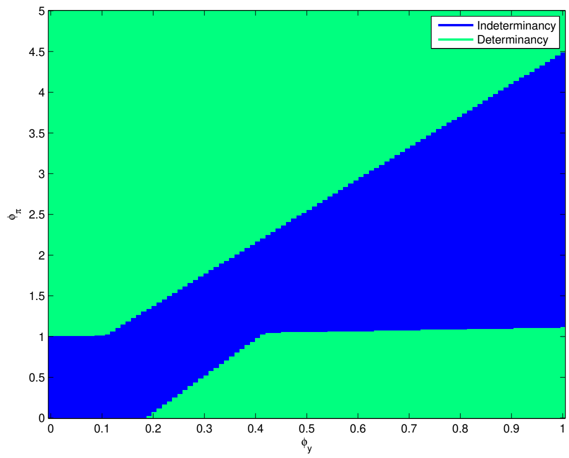
[15] The determinacy graphs were constructed in a `Matlab` external file using a loop for different parameters values. This allows us to know for which parameter configurations the model was determined.

δ reduces the indeterminacy regions. Second, a decrease in the degree of price stickiness reduces the indeterminacy areas. Third, an increase in the formal worker bargaining power reduce the indeterminacy regions. Finally, an increase of consumption preference for formal goods increase the indeterminacy areas when the central bank is reacting to aggregate inflation and aggregate output but reduces the indeterminacy regions when the central bank is reacting to formal-only inflation and formal-only output. In general, the Taylor Principle is not a necessary condition to guarantee determinacy as can be seen from Figures 4.2 and 4.3. In most cases, $\phi_\pi > 1$ don't ensure determinacy. Additional evidence supporting this result is given in Appendix C.

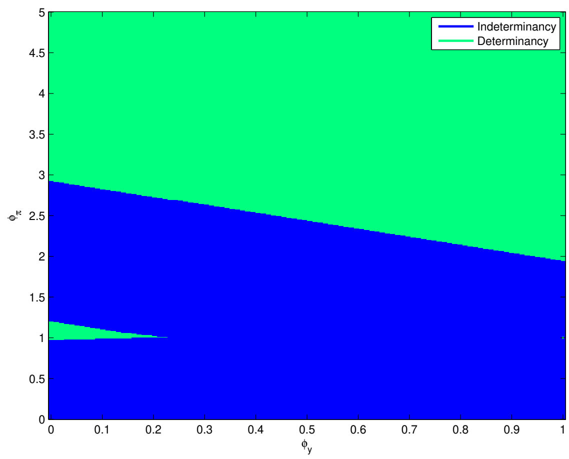
Figure 4.1: Determinacy Regions



(a) Benchmark Case

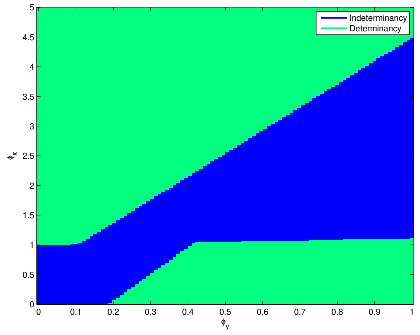


(b) Informal Case (Reacting to Agg. M.)

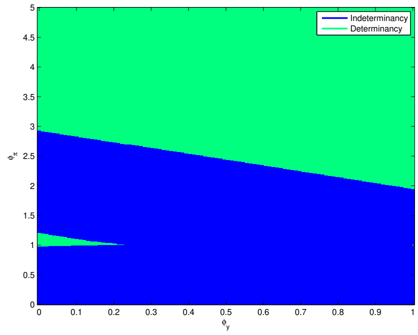


(c) Informal Case (Reacting to For. M.)

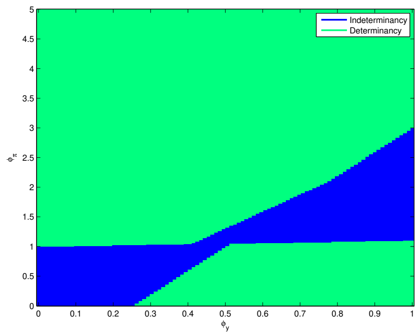
Figure 4.2: Sensitivity Analysis



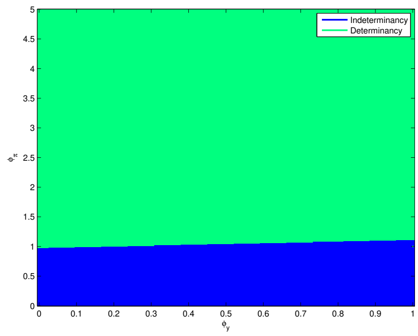
(a) $\delta = 0.03$ (Agg. M.)



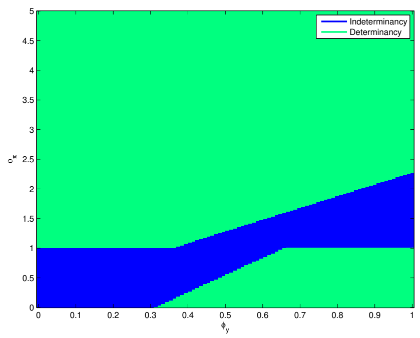
(b) $\delta = 0.03$ (For. M.)



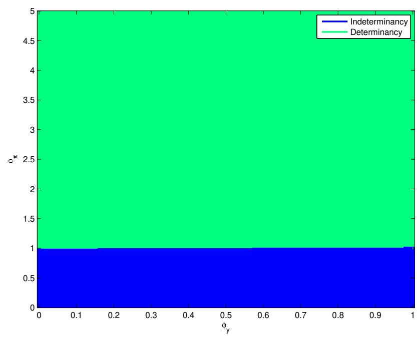
(c) $\delta = 0.15$ (Agg. M.)



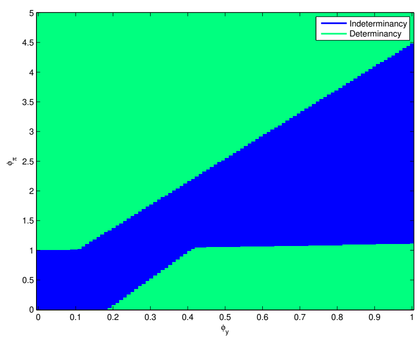
(d) $\delta = 0.15$ (For. M.)



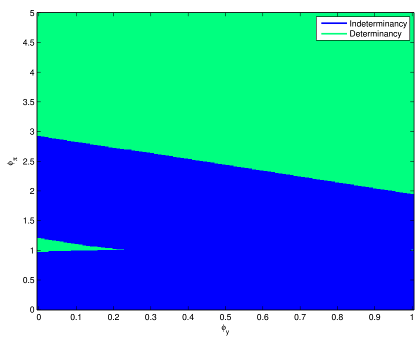
(e) $\omega = 0.50$ (Agg. M.)



(f) $\omega = 0.50$ (For. M.)

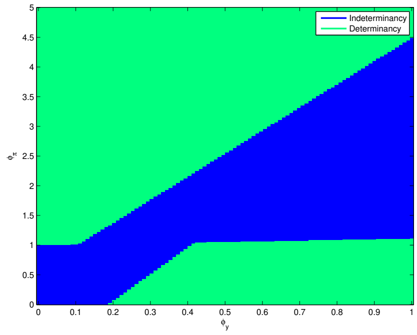


(g) $\omega = 0.75$ (Agg. M.)

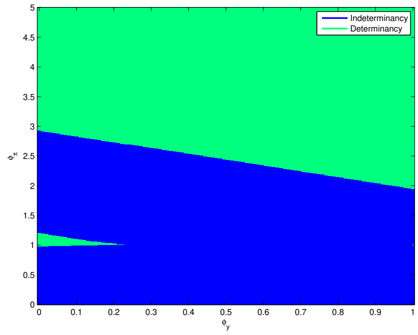


(h) $\omega = 0.75$ (For. M.)

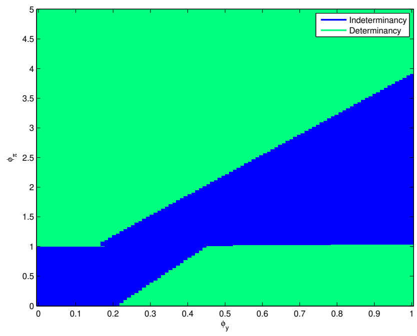
Figure 4.3: Sensitivity Analysis (cont.)



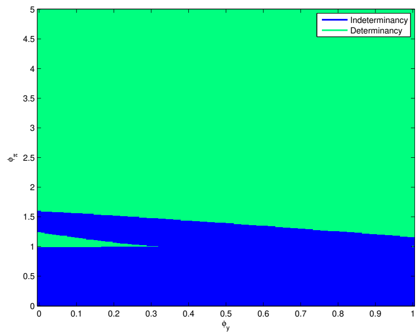
(a) $\eta = 0.60$ (Agg. M.)



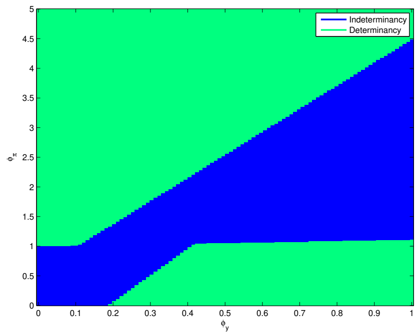
(b) $\eta = 0.60$ (For. M.)



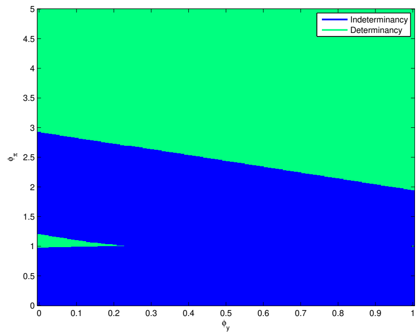
(c) $\eta = 0.90$ (Agg. M.)



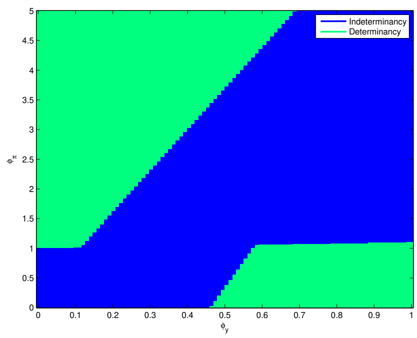
(d) $\eta = 0.90$ (For. M.)



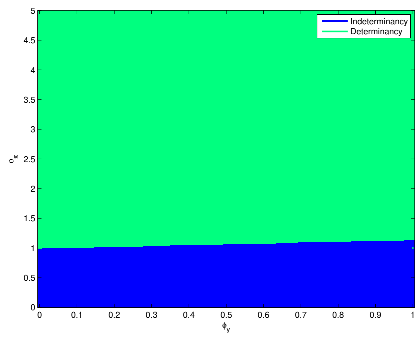
(e) $\chi = 0.83$ (Agg. M.)



(f) $\chi = 0.83$ (For. M.)



(g) $\chi = 0.99$ (Agg. M.)



(h) $\chi = 0.99$ (For. M.)

Conclusions

In this thesis we have derived a two-sector closed-economy New Keynesian model with formal labor market frictions in order to study the effects of informality for the design of optimal monetary policy. It is shown that when the central bank reacts to formal-only inflation and output there is a transition from the formal sector to the informal sector. The opposite transition is found to exist under a policy rule that reacts to aggregate inflation and output. We also find that a positive formal productivity shock implies a larger aggregate employment response, so there is a pro-cyclical effect of aggregate output on aggregate employment. In terms of welfare, a larger value for the inflation response coefficient generally reduces welfare. We also find that reacting to formal-only inflation and output attains a lower welfare loss than reacting to aggregate inflation and output. Another important finding is that higher values for the inflation response coefficient are usually required to obtain determinacy under a formal-only Taylor rule, whereas a passive monetary policy can achieve determinacy when aggregate inflation and output enter the interest-rate rule.

Therefore, in summary: The informal sector can helps the formal sector to adjust to shocks and this can increase welfare provided the central bank conducts monetary policy using formal-only inflation and output. In this case, the informal sector appears to buffer the formal economy with respect to shocks. This result suggests that simple formal-only Taylor rules can do a good job in terms of welfare in the presence of informality. However, we find a policy trade-off between welfare and the possibility of indeterminacy. While informality can improve the welfare performance in terms of optimal simple rules it could lead to problems of indeterminacy. With high values for the inflation response coefficient, the model shows a lower welfare loss but with lower values for the inflation response coefficient the analysis cannot be conducted because multiple equilibria arise. Thus, economies with large informal sectors must target formal inflation more aggressively, because otherwise the probability of getting welfare-reducing multiple equilibrium is higher.

Appendix A

Steady State Equilibrium

Denote the steady state value of the function x_t by x^{ss} (without time index). Observe that in steady state $P_{t+1} = P_t = P$, then, in the steady state equilibrium the inflation is always zero. In order to simplify the solution of the model we set $P = 1$. Thus, the relevant system is:

$$V = \theta L^i \tag{A.1}$$

$$N = L^i + L^f \tag{A.2}$$

$$L^f = \left(\frac{\theta q(\theta)}{\delta}\right) L^i \tag{A.3}$$

$$Q = \beta \tag{A.4}$$

$$N^\nu = \frac{W^i}{P} C^{-\gamma} + \theta q(\theta)\Psi \tag{A.5}$$

$$\Psi = \left(\frac{\beta}{1-\beta(1-\delta)}\right)\left[\frac{W^f}{P}C^{-\gamma} - N^\nu\right] \tag{A.6}$$

$$P = \left[\omega P^{f(1-\mu)} + (1-\omega)P^{i(1-\mu)}\right]^{\frac{1}{1-\mu}} \tag{A.7}$$

$$C^f = \omega\left(\frac{P^f}{P}\right) C \tag{A.8}$$

$$C^i = (1-\omega)\left(\frac{P^i}{P}\right) C \tag{A.9}$$

$$Y^i = A^i L^i = C^i \tag{A.10}$$

$$Y^f = A^f L^f \tag{A.11}$$

$$W^f = (1-\eta)W^i + \eta\left(P^f Y_L^f(L) + cP\theta\right) \tag{A.12}$$

$$W^i = A^i P^i \tag{A.13}$$

$$P^f = \mathcal{M}mc \tag{A.14}$$

$$mc = \left(\frac{1}{A^f}\right)\left[(1-\eta)\frac{W^i}{P^f} + \eta\left(Y_L^f(L) + \frac{cP\theta}{P^f}\right)\right] \tag{A.15}$$

$$C = \left(\chi^{\frac{1}{\tau}}[C^f]^{\frac{\tau-1}{\tau}} + (1-\chi)^{\frac{1}{\tau}}[C^i]^{\frac{\tau-1}{\tau}}\right)^{\frac{\tau}{\tau-1}}. \tag{A.16}$$

Appendix B

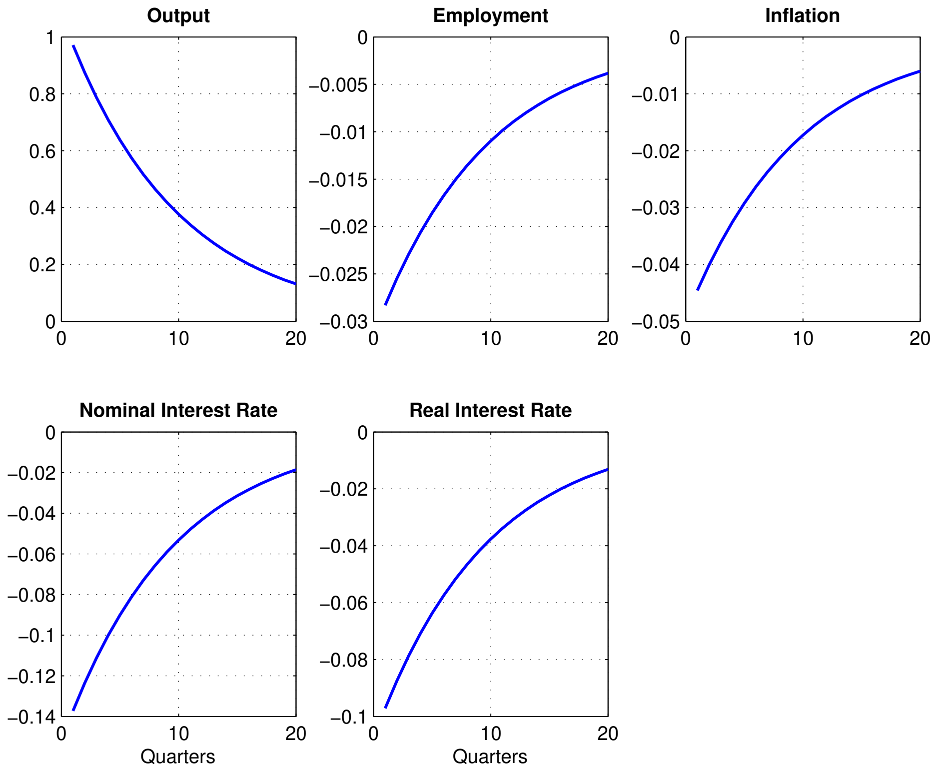
The Output Gap in a Standard New Keynesian Model

This brief appendix show the relevant results of the standard New Keynesian model in terms propagation of shock and welfare of simple feedback interest rate rules when the central bank reacts to output gap. This model introduce imperfect competition in the good markets an price stickiness environment. These key issues in the model developed in Chapter 1. As Galí (2015) have shown, in this simple model is easy to find the natural level of output (defined as the corresponding level of output when prices are fully flexible) and therefore the output gap. The model was calibrated using the numerical values given in Table 2.1.

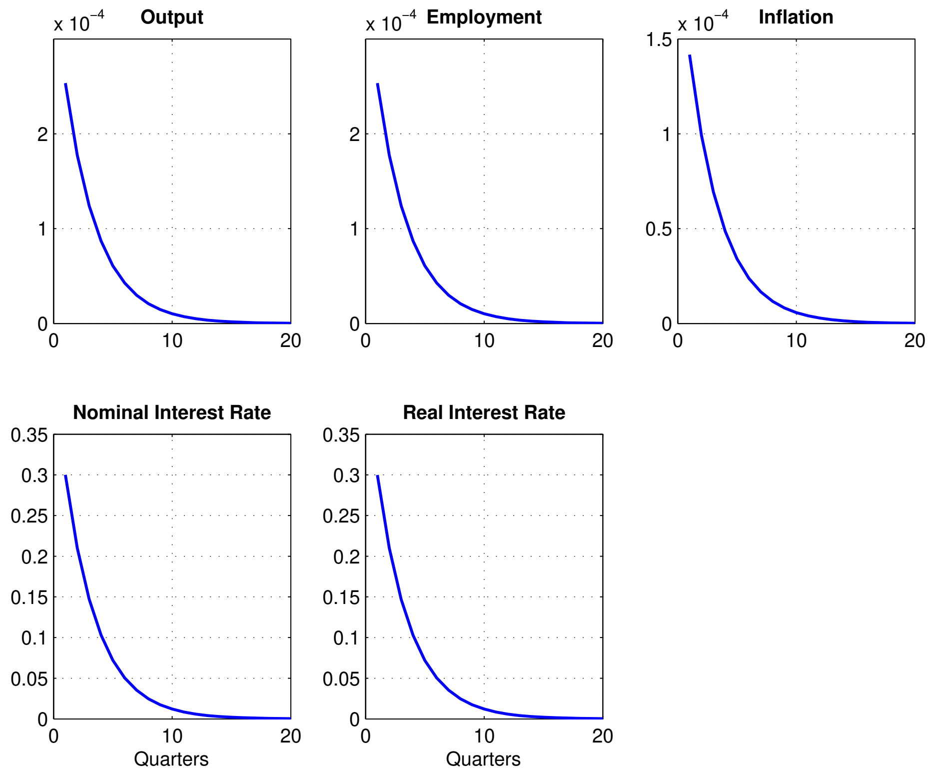
The relevant distinction between the results of this Appendix and those presented in Chapter 3 rest in the fact that here we consider that central bank reacts to inflation deviations respect the steady state and the output gap movements when conducting monetary policy. Also the central bank seeks to minimize a welfare function that depends of output gap variance and inflation variance. Figure B.1 displays the dynamics for both exogenous shocks. Panel A shows that a positive productivity shock increases output reducing the demand for labor pushing down inflation and nominal and real interest rates. Panel B exhibit that a positive discount factor shock rise the nominal interest rate which leads a expansion in output, employment, inflation and real interest rate.

Table B.1 highlights important results in the design of monetary policy. First, when as technology shock hits the economy, a trade-off emerges between output stabilization and inflation stabilization. A bigger ϕ_y reduces output and employment volatility but increases output volatility. Second, when central bank only reacts to ϕ_π and set higher value for it, then a lower welfare loss is attained. The last result is confirmed by Figure B.2.

Figure B.1: The standard New Keynesian Model. IRF's to Different Shocks



(a) Technology Shock

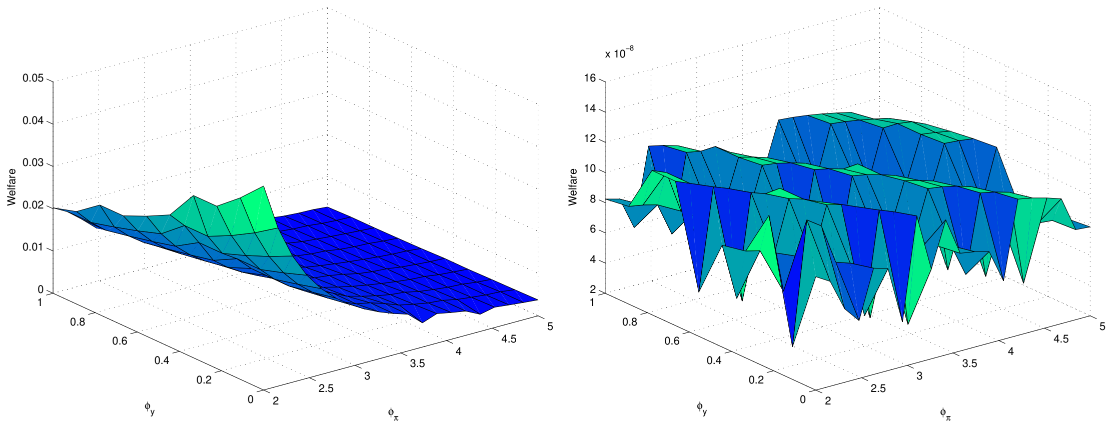


(b) Demand Shock

Table B.1: New Keynesian Model. Evaluation of simple rules

	Panel A. Technology Shock			*Panel B. Demand Shock*		
$\phi(\pi)$	3	5	5	3	5	5
$\phi(y)$	0.125	0	0.175	0.125	0	0.175
$\sigma(Y)$	2.2292	2.2592	2.2601	0.0004	0.0003	0.0003
$\sigma(\pi)$	0.1023	0.0551	0.0537	0.0002	0.0002	0.0002
$\sigma(N)$	0.0649	0.0350	0.0341	0.0004	0.0003	0.0003
$\mathbb{L}$	**0.0130**	**0.0038**	**0.0039**	**1.15E-07**	**7.89E-08**	**7.60E-08**

Figure B.2: Welfare Loss in the Standard New Keynesian Model

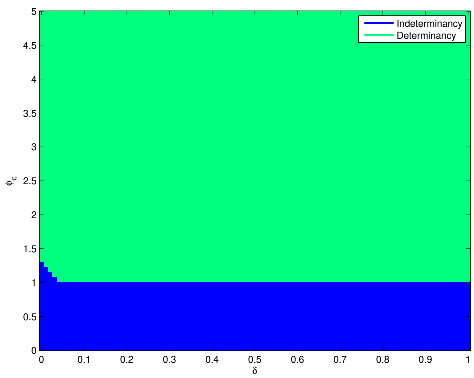


(a) Technology Shock (b) Demand Shock

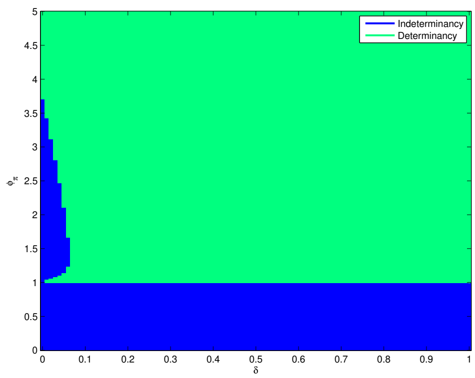
Appendix C

Robustness for Determinacy Analysis

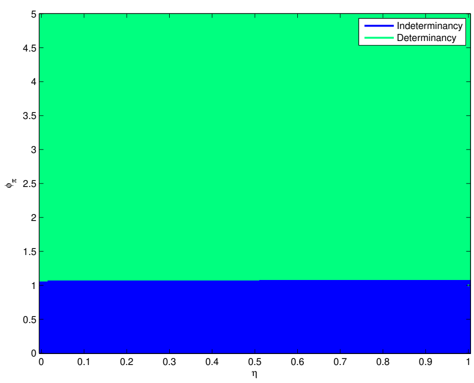
Figure C.1: Robustness



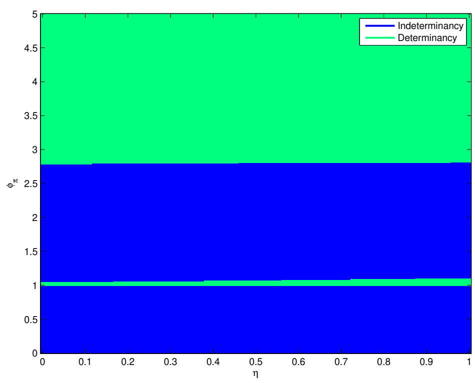
(a) δ(Agg. M.)



(b) δ(For. M.)

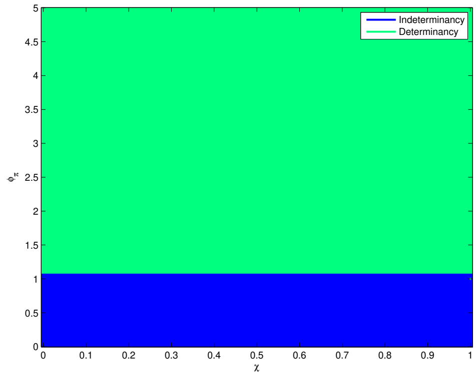


(c) η(Agg. M.)

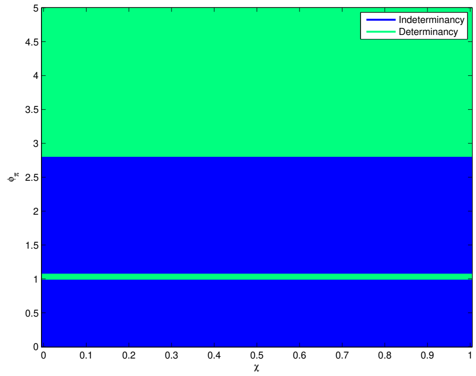


(d) η(For. M.)

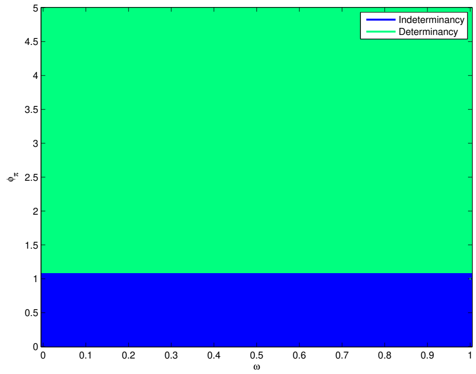
Figure C.2: Robustness (cont.)



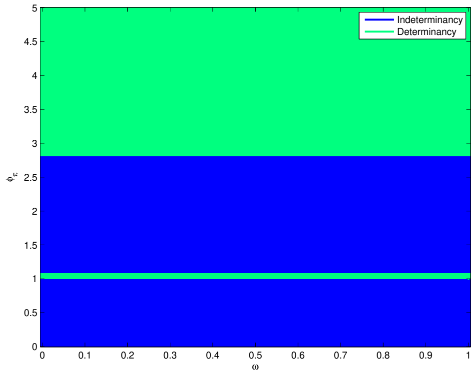
(a) χ(Agg. M.)



(b) χ(For. M.)



(c) ω(Agg. M.)



(d) ω(For. M.)

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