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Abstract

This dissertation comprises three self-contained essays examining household behavior by means of the collective framework. In particular, it takes into account the resources invested in children's welfare and adult household members' decisions to participate in the labor market. The collective approach assumes that multi-person households do not act as a single unit, but that each member has his or her own preferences and that household allocations are the result of a Pareto efficient bargaining process between them. Therefore, resources invested in children's welfare would depend not only on the household budget set but also on the parents' individual preferences and their relative position in decision making. Additionally, if household members' wages have an influence on bargaining positions, even if they are not actually working, then any variation in the potential wage will change household allocations.

Chapter 1 estimates a structural collective model of household labor supply with children and home production for a sample of Mexican families. The framework of Blundell, Chiappori, and Meghir (2005) is used to address the manner in which household allocations are affected by the intra-household decision-making process when both parents care for their children's welfare, particularly their education. In households with characteristics at the sample average, more household resources are directed toward children's education when the balance of bargaining power tips from mothers to fathers. Moreover, in spite of mothers having a larger estimated marginal willingness to pay for resources associated with children's utility, more expenditures and time would be dedicated to children when fathers' bargaining power increases exogenously, and less when it decreases. These results draw attention to the design of targeting strategies which assume that mothers care more for children than fathers; these may be less effective in some cases than if they had been focused on increasing fathers' power.

Chapter 2 extends the collective model of household behavior to consider public consumption, like expenditures on children, together with the possibility of non-participation in the labor market of one partner of the adult couple. This model argues that structural elements of the decision process, such as individual preferences and the intra-household distribution rule of non-public expenditure, can be identified by observing the labor supply for each individual and total expenditure on the public good. The identification rests on the existence of a variable that affects household behavior only through its impact on the decision process, i.e., a distribution factor, and the existence and uniqueness of a reservation wage for each household member at which both members are indifferent to whether a member participates or not. This setting provides a conceptual framework for addressing issues related to the impact of the potential wage of a non-participating member on household allocations and the targeting of specific benefits or taxes.

Finally, chapter 3 presents an empirical application of the collective household labor supply model of the previous chapter. A simultaneous model for female participation, hours worked by the couple, and expenditures on children is postulated and the set of parametric restrictions imposed by the collective rationality is derived. A subsample of the same database employed in chapter 1 is used to estimate this model and test the restrictions implies it. The collective rationality is not rejected by the data. The collective model is estimated using a sample of Mexican nuclear families in which the male partner works, and no evidence is found that empowering mothers is more beneficial to children than empowering fathers in terms of their marginal willingness to pay. Expenditures on children and the male labor supply also change significantly with variation in the female wage, even when the woman is not working. However, parameters related to the auxiliary assumptions of the continuity of the male labor supply, and expenditures on children along the female's participation frontier, are not precisely estimated.

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Chapter 1

Parental Investment in Children's Education

1.1 Introduction

Does a change in the decision process between parents affect resources directed to children's welfare? A better understanding of the intra-household decision-making process could give us an idea of how differences in preferences between household members affect some individual outcomes, particularly those relevant to the welfare of specific individuals, such as the human capital of children. In order to answer this question, this study proposes a structural collective model of household labor supply with children and home production for a sample of Mexican families. Examination of the decision process and of the production of children's utility shows no evidence that mothers care more for their children than fathers in households with characteristics at the sample average. Indeed, there is a larger increase in the resources directed to children's utility when the change in relative wage favors the father instead of the mother. Furthermore, there is an increase in resources directed to children when the Pareto weight of the father increases via an exogenous change that does not influence the household budget constraint (and a corresponding reduction in resources accompanying an increase in Pareto weight of the mother). This suggests the need for analysis of behavior in households where children are present, and the possible role of parental bargaining to explain the choice of household outcomes, the scarcity of evidence for the decision process in Mexico, and the means by which a collective labor supply model could address a setting in which both parents care for their children.

Numerous empirical studies have found evidence of a relation between factors that may influence the distribution of power among household members and intrahousehold allocations. Changes in favor of a household member are associated with consumption patterns in benefit of his or her preferences. Control over economic resources is considered in some studies as a cause of different observed household behaviors. Examples of variables used are the recipient identity of non-labor income (Thomas 1990; Browning, Bourguignon, Chiappori, and Lechene 1994), assets (Quisumbing 1994; Quisumbing and Maluccio 2003; Thomas, Contreras, and Frankenberg 2004), and shares of income earned (Hoddinott and Haddad 1995; Bittman, England, Folbre, Sayer, and Matheson 2003). Other factors that affect the household environment and outside opportunities have been considered, including legislation and the marriage market (Gray 1998; Chiappori, Fortin, and Lacroix 2002; Park 2007), and relative skills and knowledge (Beegle, Frankenberg, and Thomas 2001; Rubalcava and Contreras 2000; Quisumbing and Maluccio 2003).

Moreover, quasi-natural experiments, such as targeted transfers and changes in welfare programs, have been used to analyze how household allocations respond to potential changes in intrahousehold decision process. Lundberg, Pollak, and Wales (1997) and Duflo (2000) provide evidence in favor of targeting strategies that could improve child welfare, the former analyzing a change in the UK child allowance policy in the late 1970s and the second a reform of the South African old-age pension program.

In general terms, in Mexico there is no evidence of household decision process that simultaneously considers changes in bargaining power distribution in favor of either parent and parental time and expenditure on children's education. Various studies have analyzed household behavior through the conditional cash transfer program known as Oportunidades (previously Progresa), which was launched by the Mexican government in 1997 with the main objective of improve the education, nutrition, and health of Mexican families with children in extreme poverty. Guided by the hypothesis that women are more willing than other household members to spend the money received in a way that benefits children, the program is designed to give a direct cash benefit to the mother (or the most senior woman in the family). The literature reports mixed results of this additional money on household consumption patterns and whether it is spent differently than others household income (in favor of a change: Attanasio and Lechene 2002; Rubalcava, Teruel, and Thomas 2009; against: Handa, Peterman, Davis, and Stampini 2009). Moreover, household monetary expense is not necessarily the sole factor used to enhance the human capital of a child. It could present an incomplete picture of household allocation to children's human capital formation, given that increases in expenditure on children may be accompanied by a reallocation of other factors dedicated to their education.

Since the recipient of Progresa/Oportunidades is always a woman in the house, it is not possible to compare what would happen if the additional money were given to a male member. However, Gutiérrez, Juárez, and Rublí (2011) study a program that gives an exogenous, non-labor additional income to both sexes. They use a pension program for elderly people resident in Mexico city (in Spanish, PAAAM: Pensión Alimentaria para Adultos Mayores) to compare the impact on children's outcomes of giving the transfer to a male or a female. The extra money is associated with greater expenditures on children between 6 and 18 years old if the recipient is a female, but it is related to a greater enrollment rate if the recipient is a male. However because the family tie with the child is not analyzed, the policy implications are limited.

Although some of the empirical evidence suggests that favorable changes in mothers' bargaining power (generally in terms of economic resources) benefit child welfare, meaning that "mothers care more for children than fathers," the lens used to interpret this type of observed behavior is crucial to policy issues. Design of policy interventions under presumptions such as "children are better off if the mother's influence in the household is increased, rather than the father's" may fail to improve child welfare if the channel by which parents' individual preferences and household decision process are reflected in children's outcomes is unknown.

With this in mind, the theoretical framework on which most of these studies are based is the "collective" approach. In contrast to the "unitary" view, in which the household maximizes a unique utility function (as if its members act as a single rational individual), the resources allocated to the well-being of a specific household member under the collective model could vary depending on both the preferences of its members and the distribution of the bargaining power between them, even when total resources are kept constant. Therefore, intra-household allocation processes could mitigate or enhance public policies involving individual welfare.

It is possible to tackle the question of household behavior when children are present assuming that both parents obtain utility (although not necessarily to the same degree) from the welfare of their offspring, meaning that it is a public good for them. However, until Blundell, Chiappori, and Meghir (2005) (hereafter, BCM), the collective approach had not adequately developed a theoretical framework to analyze households with children. The principal results under this approach have been obtained in the case of private goods, but not when public consumption was also present.

The BCM model is basically an extension of Chiappori's (1992) collective model of household labor supply where both parents care about their children's welfare and where children's utility can also be augmented by means of specific expenditures and parental time. One of the main results of the BCM model is that increases in the bargaining power of a parent result in more resources directed to children only when his or her willingness to pay for children's welfare is more sensitive to increases in his or her private consumption. Therefore, the key aspect is not that, for example, a mother cares more about the children than the father (i.e., that she has a larger willingness to pay), but that she is more willing to spend on the children when there is an increased household budget share for her private consumption.

The objective of this chapter is to examine the effects of changes in the distribution of power between parents over intra-household resource allocation directed to children's human capital, particularly education. To this end, the BCM model is identified using a particular parametrization and an estimation strategy based on Cherchye, de Rock, and Vermeulen (2010; forthcoming) (hereafter, CRV). The sample is drawn from the second wave of the Mexican Family Life Survey (MxFLS-2/ENNVIH-2, for its abbreviation in Spanish) and focused on Mexican nuclear families, with children only under 15 years of age and at least one school-age child, where both parents work. Changes in factors that influence the Pareto weights are used to assess how resources directed to children's education respond to changes in the distribution of the bargaining power.

The structure of the remainder of this chapter is as follows. Section 1.2 reviews the economic literature on the analysis of household behavior. Section 1.3 presents the BCM theoretical model. The estimation strategy applied, data set used, and empirical results are explained in section 1.4. Section 1.5 provides some final remarks.

1.2 Households in the Economic Literature

Traditionally, neo-classical consumer models are based on individual rational preference ordering over affordable consumption bundles. A first theoretical approximation to taking the household into account in economics is to transform the consumer choice problem from an individual to a household perspective. Household members' utilities and budget constraints can be systematically added, assuming either that they all have homogeneous preferences, that an altruistic household head has all the power at home, or that the weights of individuals within a household welfare function are fixed.

This type of approximation is commonly known as the “unitary” model. It has the advantage of imposing empirical restrictions on household behavior: the demand functions depend on prices and fixed income, and these functions have to satisfy the standard axioms of individual preferences. Moreover, because the fixed income of household members is aggregated, the identity of the recipient does not affect the assignment, a factor known as the “income pooling” hypothesis.

The main implications of the unitary framework -the income pooling hypothesis and the symmetry of the Slutsky matrix- have been rejected in several empirical studies. Contradicting the income pooling hypothesis, the distribution of income has been found to be significant in the allocation of family resources in the studies of Thomas (1990); Bourguignon, Browning, Chiappori, and Lechene (1993); Browning, Bourguignon, Chiappori, and Lechene (1994); Lundberg, Pollak, and Wales (1997); Fortin and Lacroix (1997); among others. The symmetry of the Slutsky matrix for demands of households with more than one member has been rejected by Browning and Meghir (1991); Fortin and Lacroix (1997); Browning and Chiappori (1998); among others.

In addition to this accumulation of empirical evidence against the unitary model, there are also concerns about the understatement of poverty and inequality levels that do not consider intra-household distribution of resources (Haddad and Kanbur 1990) and the efficacy and consequences of policies under this framework (Apps and Rees 1988; Alderman, Chiappori, Haddad, Hoddinott, and Kanbur 1995; Beninger, Bargain, Beblo, Blundell, Carrasco, Chiuri, Laisney, Lechene, Longobardi, Moreau, Myck, Ruiz-Castillo, and Vermeulen 2006).

A number of approaches seek to overcome these problems while respecting the theoretical foundations of individual preferences, attempting to explain the internal dynamics between individuals with heterogeneous preferences and the manner in which disagreements between them are resolved. Among these, a fruitful approach, based on minimal assumptions, is the “collective” model.[1] Applying the context of bargaining theory, household members with non-homogeneous preferences are trying to come to an agreement on how to assign the household gains. The outcome of this decision-making process depends on the relative bargaining power of members to “impose” their preferences.

[1] Another approach to analyzing the household decision process is the non-cooperative model. Within this framework, household members maximize their utility relative to their budget constraint, taking as given the behavior of the other members. A drawback of this approach is that in the solution found for these models it is possible to improve the welfare of one member without worsening that of other members if the resulting intrahousehold allocation is changed (see Xu 2007 and Donni and Chiappori 2011 for a more extensive discussion of non-cooperative models).

Without assuming a specific bargaining concept, the collective model assumes that the intra-household decisions are Pareto efficient (i.e., it is not possible to increase the welfare of one member without decreasing that of the others). This assumption is supported by the idea that household members can be seen as playing a repeated game where there is symmetry of information.

This setting is characterized by a weighted utilitarian household welfare function where the Pareto weights of each household member depend on prices, total household expenditure, and external factors that affect the decision process but not individual preferences or the joint budget set ("Extrahousehold Environmental Parameters," in the terminology of McElroy 1990, or "Distribution Factors," in the terminology of Browning and Chiappori 1998).[2] The idea behind the introduction of distribution factors to the collective model is to serve as an additional source of variation (different from that obtained by prices, wages, and non-labor income) to more precisely predict household behavior (Browning, Chiappori, and Weiss 2011). By considering private and public goods in the collective model, the distribution factors can serve to isolate the effect of public consumption (say, paying for childcare) on individual preferences (say, the individual trade-off between labor supply and private consumption), facilitating the recovery of individual preferences on private and public goods and the decision process (BCM). Various types of distribution factors have been used in the literature (see Table A.1), especially those associated with control over economic resources. These have included assets (current, at marriage), unearned income, targeted transfers, changes in welfare programs (such as coverage and benefits), and share of income earned by women.

One of the implications of the collective model is that it is not necessary to satisfy the income pooling hypothesis. Consequently, there is the possibility of targeting a specific household member for a public transfer, taking into account possible resource reallocations from non-beneficiaries to the beneficiary that could diminish or enhance the transfer's effectiveness. It is thus possible to propose policies that directly affect bargaining power, such as those regarding eligibility rules for training programs or rights to alimony and child support

[2] Under the axiomatic bargaining theory, household decisions represent the outcome of some bargaining process between its members (represented by Nash or Kalai-Smorodinsky solutions). Individuals compare the utility level achieved if they cooperate (the gains of living together) with the level obtained if they do not cooperate. Therefore, the non-cooperative utility of a member serves as a threat point to obtain more power in the bargaining process. Changes in distribution factors thus shift the threat points and therefore the Pareto weights. Threat points can be interpreted as the utility of being divorced or remaining single (Manser and Brown 1980; McElroy and Horney 1981), or a noncooperative equilibrium within marriage that reflects traditional gender roles and gender role expectations ("separate spheres" bargaining model of Lundberg and Pollak 1993).

for ex-wives and children of divorced parents (Alderman, Chiappori, Haddad, Hoddinott, and Kanbur 1995; Quisumbing and McClafferty 2006).

Two studies for Mexico that take into account some aspects of the collective approach are Martinelli and Parker (2008) and Bobonis (2009). In the former the authors try to isolate the substitution effect of Progresa/Oportunidades school subsidies from the effects of income and bargaining on the share of expenditures devoted to children's clothing. Bobonis tests the Pareto efficiency assumption of collective models in household decisions. He uses as changes in distribution factors the exogenous variation in women's income due to Progresa/Oportunidades and household income variation occasioned by localized rainfall shocks. The allocations of various household items are used as dependent variables, with the finding, among other effects, that income changes due to Progresa/Oportunidades have a positive effect on household budget shares for adult female and children's clothing and a negative effect on alcohol and tobacco shares, while those due to rainfall shock have the opposite effects. He interprets these findings as evidence of how women's control over a sum of money in low-resource households is used in a family-friendly way (probably responding to social pressures) while income closer to the male sphere is used in favor of the man.

1.3 Introducing Children into the Collective Approach

The presence of children generates particular dynamics that must be considered in the study of household behavior. It is possible to model children's welfare as a nonmarket good produced within the household by reallocating market and non-market resources to its production (Bourguignon 1999).

Collective models of household production assume that households obtain utility not only of market goods but also of non-market goods produced within the household. These models commonly concentrate on how the labor supply of household members is affected when agents divide their time among market activities, leisure, and domestic production.[3]

[3] Besides avoiding omitted-variable problems, the introduction of household production into the analysis has effects on the interpretation of intra-household welfare. Apps and Rees (1996; 1997) point out that if household production is not considered, low or zero levels of market labor from of one individual is interpreted as a larger leisure consumption, and his or her share of household income is interpreted as a lump-sum transfer from the other members. However, the inclusion of household production considers the division of labor between household and market production. It may be that one member specializes in domestic production; if he or she exchanges such production with the other members for market goods, there is an exchange instead of a transfer. Chiappori (1997) identifies some basic points of the collective model when household production is included.

Children's utility can be also interpreted as a form of household production. Taking the BCM model of collective labor supply with children and home production, the behavior of two-adult household with children can be modeled in the following manner. Suppose that there are two adult members in a household, where the mother is denoted by m and the father by f. Three commodities are consumed in a household: individual leisures $\left(L^m, L^f\right)$, and a Hicksian composite good C that is used for private expenditure $\left(C^m, C^f\right)$ and some public consumption (K) for the production of children's utility:

$$C = C^m + C^f + K$$

Individual time is allocated between leisure (L^i), market work (h_W^i), and production of children's welfare (h_K^i):

$$L^i + h_W^i + h_K^i = T$$

Children's welfare $\left(u^K\right)$[4] is produced using specific expenditure and parental time:

$$u^K = u^K\left(K, h_K^m, h_K^f\right)$$

The utility function of parent i depends on the consumption of his or her consumption of individual goods and children's utility (in some sense children are considered as a public consumption good to the adult household members; the latter have caring preferences about children's welfare):

$$U^i = U^i\left(L^i, C^i, u^K\right) \quad i = m, f$$

All utility functions are strictly quasiconcave and increasing, and at least twice continuously differentiable.

Wages are denoted by w^m and w^f, and the price of the Hicksian good is normalized to one. Given the household fixed income, y, the budget constraint is:

$$C^m + C^f + K = w^m h_W^m + w^f h_W^f + y$$

[4] This function has non-increasing returns and no joint production.

Assuming that the decisions made by the household are Pareto-efficient, that parents share bargaining power and children do not have any, and considering the existence of a vector of distribution factors, $\mathbf{z}$, the household allocation problem can be represented as

$$\max_{L^m,L^f,C^m,C^f,K,h_K^m,h_K^f} \lambda U^m\left(L^m,C^m,u^K\right)+(1-\lambda)\,U^f\left(L^f,C^f,u^K\right) \tag{1.1}$$

$$s.t. \quad \begin{cases} C^m+C^f+K=w^m h_W^m+w^f h_W^f+y \\ L^i+h_W^i+h_K^i=T, \quad i=m,f \\ u^K\left(K,h_K^m,h_K^f\right)=u^K \end{cases}$$

where λ and $(1-\lambda)$ are welfare weights. An interpretation of $\lambda^m=\lambda$ and $\lambda^f=(1-\lambda)$ is that they represent, respectively, the bargaining power of m and f in the intra-household allocation process, $\lambda^i \geq 0$. In a collective model, λ captures the decision process and can be a function of prices, non-labor income, and distribution factors, $\lambda\left(y,w^m,w^f,\mathbf{z}\right)$.

The efficiency assumption implies that the solution to the household problem (1.1) is equivalent to a two stage process:

1. Parents agree on:
   (a) children's utility level and the resources directed to produce it.
   (b) how to allocate the residual non-labor income among themselves.
2. Each parent separately chooses his or her private consumption, subject to his or her corresponding budget constraint.

Let $L^{i^*}\left(w^m,w^f,y,\mathbf{z}\right)$ and $C^{i^*}\left(w^m,w^f,y,\mathbf{z}\right)$, $i=m,f$, $K^*\left(w^m,w^f,y,\mathbf{z}\right)$ and $u^{K^*}\left(w^m,w^f,y,\mathbf{z}\right)$, be the solution of problem (1.1). There is a function $\phi\left(w^m,w^f,y,\mathbf{z}\right)$ such that each parent solves in the second stage:

$$\max_{L^i,C^i} U^i\left(L^i,C^i,u^{K^*}\right) \quad s.t. \quad w^iL^i+C^i=w^iT+\phi^i \tag{1.2}$$

The function $\phi(\bullet)$, known as the sharing rule, achieves the decentralization of the household problem, so ϕ^i is the fraction of total expenditure on nonpublic goods allocated to parent i. If $\phi^m\left(w^m,w^f,y,\mathbf{z}\right)=\phi\left(w^m,w^f,y,\mathbf{z}\right)$, then $\phi^f\left(w^m,w^f,y,\mathbf{z}\right)=y-$

$e^{K^*}\left(w^m, w^f, y, \mathbf{z}\right) - \phi^m\left(w^m, w^f, y, \mathbf{z}\right)$, where $e^{K^*}(\bullet)$ is the cost function of producing children's utility.

Let $V^i\left(w^i, \phi^i, u^{K*}\right)$ be the individual indirect utility of agent i conditional on u^{K^*} corresponding to the problem (1.2). The optimal choice of the sharing rule and children's utility can be obtained returning to the first stage problem:

$$\text{(a)} \quad \min_{K, h_K^m, h_K^f} \; e^K = w^m h_K^m + w^f h_K^f + K \quad s.t. \quad u^K(K, h_K^m, h_K^f) = u^K \tag{1.3}$$

$$\text{(b)} \quad \max_{\phi^m, \phi^f, u^K} \; \lambda V^m\left(w^m, \phi^m, u^K\right) + (1-\lambda) V^f\left(w^f, \phi^f, u^K\right) \quad s.t. \quad \phi^m + \phi^f + e^K = y \tag{1.4}$$

From the first-order conditions, the Samuelson condition characterizes the efficient production of children's utility:

$$\lambda \frac{\partial V^m}{\partial \phi^m} = (1-\lambda) \frac{\partial V^f}{\partial \phi^f} \qquad \Rightarrow \qquad \underbrace{\frac{\partial V^m / \partial u^K}{\partial V^m / \partial \phi^m}}_{MWP^m} + \underbrace{\frac{\partial V^f / \partial u^K}{\partial V^f / \partial \phi^f}}_{MWP^f} = \underbrace{\frac{\partial e^K}{\partial u^K}}_{MC} \tag{1.5}$$

The ratio $MWP^i = \left(\partial V^i / \partial u^K\right) / \left(\partial V^i / \partial \phi^i\right)$ is i's marginal willingness to pay for children's utility. Condition (1.5) states that the sum of parents' marginal willingness to pay must be equal to the marginal cost of the resources allocated to children's welfare (MC).

From BCM Proposition 1 if the preferences of both adult members are such that both private expenditures $\left(\phi^i\right)$ and children's utility $\left(u^K\right)$ are normal (i.e., an increase in household non-labor income raises both private and public consumption), an increase in i's Pareto weight increases household expenditure on children's welfare if and only if i's MWP is more sensitive to changes in his or her share than that of the other parent $\left(\frac{\partial u^K}{\partial \phi^i} > 0 \; iff \; \frac{\partial MWP^i}{\partial \phi^i} > \frac{\partial MWP^j}{\partial \phi^j}, j \neq i\right)$. In other words, the key aspect is not the comparison between MWPs, but rather the change of MWP to a change in the shares, that is, the difference in parents' MWP response to a change in shares.

1.4 Empirical Application

1.4.1 Parametric Specification

Using the model previously presented, BCM demonstrate that it is possible to recover model structure (individual preferences and Pareto weights) from observed behavior (individual labor supplies and the resources directed to children's human capital, as functions of wages and non-labor income). In general, it is not possible to recover preferences and the decision process from a reduced-form specification, as different structural models could have generated the observed behavior. However, BCM show that identifiability requires 1) the availability of a distribution factor, or 2) that individual consumption and leisure are separable from children's welfare.

In order to identify the BCM model, a particular parametric form is used, based on CRV, that exploits the two-stage representation to derive the specification. Recurring to the second stage of the household decision process (1.2), the individual indirect utility of agent i conditional on children's welfare can be represented by means of a functional form of the price-independent generalized logarithmic (PIGLOG) class:[5]

$$V^i\left(w^i,\phi^i,u^K\right)=\frac{ln\left(w^iT+\phi^i\right)-\left(\alpha_1^i+\alpha_2^iu^K\right)lnw^i}{\left(w^i\right)^{\beta^i}},\quad i=m,f \tag{1.6}$$

Note that in this specification, the parameter α_2^i tells us first if leisure and individual consumption are separable from children's welfare. If the separability assumption is correct ($\alpha_2^i=0$), the resources allocated to children's welfare do not have a substitution effect on the consumption-leisure decision; there is only an income effect on the residual non-labor income devoted to private consumption. In this case, the distribution factors are not necessary to keep u^K at a particular level while wages and non-labor income varies; the individual preferences for private and public goods as well as the Pareto weights can be recovered from variations in wages and non-labor income, since the effect of these changes on children's utility production are captured by the sharing rule without affecting labor supplies in other ways (see subsection 2.3.2). Also, i's corresponding MWP under this specification is $MWP^i=-\alpha_2^i\ln w^i\left(w^iT+\phi^i\right)$, so according to BCM Proposition 1, a marginal change in i's Pareto weight increases the resources directed to children if and only if $\alpha_2^i\ln w^i<\alpha_2^j\ln w^j$.

The corresponding leisure demand for each individual is:

[5] In contrast to CRV, the level of children's welfare (not its logarithm) is considered.

$$L^i\left(w^i,\phi^i,u^K\right)=\left(\alpha_1^i+\alpha_2^iu^K+\beta^i\left(\ln\left(w^iT+\phi^i\right)-\left(\alpha_1^i+\alpha_2^iu^K\right)\ln w^i\right)\right)\frac{w^iT+\phi^i}{w^i} \quad (1.7)$$

The corresponding income elasticity of leisure demand is $\eta_{L^i/\left(w^iT+\phi^i\right)}=1+\beta^i\left(\frac{w^iT+\phi^i}{w^iL^i}\right)$. Therefore, if leisure is a normal good, it would be a luxury good if $\beta^i>0$ or a necessity if $-\frac{w^iL^i}{w^iT+\phi^i}<\beta^i\leq 0$.

Returning to the first stage, where the optimal choice of the sharing rule and children's welfare is obtained, it is assumed that children's utility has a CES form with constant returns:[6]

$$u^K\left(K,h_K^m,h_K^f\right)=\left[\gamma^m(h_K^m)^\rho+\gamma^f\left(h_K^f\right)^\rho+\gamma^KK^\rho\right]^{1/\rho},\quad \gamma^i>0,\ \sum\gamma^i=1,\ \rho\leq 1 \quad (1.8)$$

where the cost share of input i (proportion of total expenditure spent on children's welfare on input i) is γ^i, and the elasticity of substitution is $\sigma=\frac{1}{1-\rho}$. To take into account the number of school-age children (N_K) in children's utility production, the function $\gamma^i=\frac{\exp\left(\tilde{\gamma}_1^i+\tilde{\gamma}_2^iN_K\right)}{1+\exp(\tilde{\gamma}_1^m+\tilde{\gamma}_2^mN_K)+\exp\left(\tilde{\gamma}_1^f+\tilde{\gamma}_2^fN_K\right)}$, $\gamma^i\in(0,1)$, $(i=m,f)$ is used in the estimation process, where $\tilde{\gamma}_1^i$ and $\tilde{\gamma}_2^i$ are estimated $(i=m,f)$. This function imposes the condition $\gamma^i>0$, and γ^K is defined as the residual $\gamma^K=1-\gamma^m-\gamma^f$. The condition $\rho\leq 1$ is imposed in the estimation process by the function $\rho=1-\exp(-\tilde{\rho})$, where $\tilde{\rho}$ is estimated.

Solving problem (1.3), parents' time and expenditure allocations to children's welfare $\left(h_K^m,h_K^f,K\right)$ are given by:

$$h_K^i = \left(\frac{\gamma^i}{w^i}\right)^\sigma A^{-1/\rho}u^K,\quad i=m,f \quad (1.9)$$

$$K = \left(1-\gamma^m-\gamma^f\right)^\sigma A^{-1/\rho}u^K \quad (1.10)$$

where $A=(\gamma^m)^\sigma(w^m)^{1-\sigma}+\left(\gamma^f\right)^\sigma\left(w^f\right)^{1-\sigma}+\left(1-\gamma^m-\gamma^f\right)^\sigma$. Also, the respective expenditure function becomes $e^K=A^{1/(1-\sigma)}u^K$.

[6] The advantage of using the CES function is that it is not necessary to assume in advance the degree of substitution among resources used for the production of children's utility, parental time, and money spent on them (equal to one if the Cobb Douglas production function is used), allowing the data to determine the degree of substitutability. Initially, CRV (2010) considered a less general function, like the Cobb-Douglas. However, in their forthcoming article, they too have used a CES form.

Correspondingly, the solution to the first stage problem described in (1.4) is:

$$\begin{aligned}\phi^i &= \frac{1}{\mu}\left(\frac{\lambda^i}{(w^i)^{\beta^i}}\right) - w^i T, \quad i = m, f &(1.11)\\ u^K &= \frac{1}{A^{1/(1-\sigma)}}\left(y - \phi^m - \phi^f\right) &(1.12)\end{aligned}$$

where μ is the Lagrange multiplier of this problem and can be written as:

$$\mu = \frac{-1}{A^{1/(1-\sigma)}}\left[\lambda\left(\frac{\alpha_2^m \ln w^m}{(w^m)^{\beta^m}}\right) + (1-\lambda)\left(\frac{\alpha_2^f \ln w^f}{(w^f)^{\beta^f}}\right)\right]$$

Finally, to complete the model, we must assume a functional form for the Pareto weight of the mother. Following CRV, the form selected is:

$$\lambda\left(w^m, w^f, y, z1, z2\right) = \frac{\exp\left(\Lambda_1 + \Lambda_2\left(w^m/w^f\right) + \Lambda_3 y + \Lambda_4 z_1 + \Lambda_5 z_2\right)}{1 + \exp\left(\Lambda_1 + \Lambda_2\left(w^m/w^f\right) + \Lambda_3 y + \Lambda_4 z_1 + \Lambda_5 z_2\right)}, \ \lambda \in [0,1] \quad (1.13)$$

The resulting system consists of five equations $\left(h_K^m, h_K^f, K, L^m, L^f\right)$ as functions of $\left(w^m, w^f, y, z_1, z_2\right)$. So, substituting (1.11)-(1.13) in (1.7), (1.9) and (1.10), the specification of the system is:

$$\begin{aligned} h_K^i &= \left(\frac{\gamma^i}{w^i}\right)^{\sigma} A^{-1/\rho}\left(\frac{y + \left(w^m + w^f\right)T}{A^{1/(1-\sigma)}} + \frac{C}{D}\right), \quad i = m, f \\ K &= \left(1 - \gamma^m - \gamma^f\right)^{\sigma} A^{-1/\rho}\left(\frac{y + \left(w^m + w^f\right)T}{A^{1/(1-\sigma)}} + \frac{C}{D}\right) \\ L^i &= \left(\alpha_1^i + \alpha_2^i u^K + \beta^i\left(\ln\left(\frac{-\lambda^i\left(w^j\right)^{\beta^j} A^{1/(1-\sigma)}}{D}\right) - \left(\alpha_1^i + \alpha_2^i u^K\right)\ln w^i\right)\right) \\ &\quad \times \left(\frac{-\lambda^i\left(w^j\right)^{\beta^j} A^{1/(1-\sigma)}}{w^i D}\right), \qquad i = m, f \end{aligned} \quad (1.14)$$

where:

$$C = \lambda\left(w^f\right)^{\beta^f} + (1-\lambda)\left(w^m\right)^{\beta^m}$$

$$D \;=\; \lambda\left(w^f\right)^{\beta^f}\alpha_2^m \ln w^m + (1-\lambda)\left(w^m\right)^{\beta^m}\alpha_2^f \ln w^f$$

Note that a sufficient condition for existence of L^i is that α_2^i is negative $(i = m, f)$. This condition is imposed in the estimation process by the function $\alpha_2^i = -\exp\left(\tilde{\alpha}_2^i\right)$, where $\tilde{\alpha}_2^i$ is estimated $(i = m, f)$. If α_2^i is negative, the utility of parent i increases with the level of u^K, everything else being equal. Nonetheless, the imposition of this condition in the estimation process does not eliminate the possibility that α_2^i is not statistically different from zero.

The system is estimated using the Feasible Generalized Nonlinear Least Squares (FGNLS) estimator. Robust standard errors are used to guard against possible misspecification of the variance matrix.

1.4.2 Data

Estimation of the empirical model specified in the preceding subsection is information-demanding. It requires principally labor market data (the labor supply of individual members and their respective wage rates), the non-labor income of the household, and expenditures and time devoted to children's education. A survey that satisfies the information requirements is the Mexican Family Life Survey (MxFLS). It is a multithematic and longitudinal survey elaborated by researchers at the Universidad Iberoamericana (UIA) and the Centro de Investigación y Docencia Económicas (CIDE).

A subsample was extracted from the second wave of the MxFLS (2005-2006) containing only nuclear families with children under 15 years of age, at least one school-age child, and where both parents work. Nuclear families were used to focus on households where the decision process is concentrated in the parents, reducing the possibility of interaction with other kin within the household. Although the cutoff at age 15 may seem arbitrary, the analysis concentrated on school-age children (5-14 years) because 1) it is less likely that a child in this age range has bargaining power in household decisions, and 2) the survey only registers children's educational data in this age range.[7] This study thus captures allocation decisions for children who may be attending school through the secondary level.

[7] An interesting test of robustness would be separate the sample of households by age levels of children and see if the results are maintained. However, limiting the sample to households with children aged 14 years or less would reduce it to 158 families, and this partition would reduce it even more. Some authors, like Apps and Rees (2009), argue that a child should be considered as a decision maker. This approach may depend on a number of factors (e.g., age, education, occupation, income), exogeneity (are they "molded" by the same parents?), and the stability of children's preferences (Browning et al. 2011). In the context of the collective model, if a child has bargaining power, we would need data about a private good consumed by the

There are 8,328 households (34,873 individuals) in the MxFLS-2, of which 3,990 (48.16%) are nuclear families, and 1,380 have only children under 15 years old including at least one school-age child. A sample of 158 was chosen from the subset of families where both parents work and whose records were missing no information. The sample is relatively small; one way to enlarge it would be to include families in which a household member did not participate in the labor market.[8] A theoretical and an empirical model that consider the presence of children and non-participation are developed in chapters 2 and 3, respectively.

The time measurement unit is hours per week for those variables related to time allocations. Time devoted to children's education corresponds to helping children with studies and homework. Leisure time is computed as 112 h/week (16 h/day) less time allocated to market work and children's education. Children's educational expenses in the household include enrollment fees, exams, school supplies, uniforms, and transportation.

The measure of the wage rate is average hourly earnings (labor income/hours worked in last year). Non-labor income is average weekly household current income less labor income of parents. The distribution factors z_1 and z_2 considered in this study are age and education differences between parents, respectively. As mentioned by Quisumbing and Maluccio (2003, p. 288), in the selection of distribution factors (in their case human capital and assets at marriage), "even though they may be endogenous as a result of marriage market selection or correlation with other unobservables, they are clearly exogenous to decisions made *within* marriage."

Table 1.1 presents descriptive statistics for the sample analyzed. On average, fathers spend more time on market work and mothers on helping children with their studies. There

child in order to identify his or her respective power. We would also need data such as working time, which is collected in the MxFLS-2 survey only if the person is at least 15 years old. An exercise of this kind was done by Dauphin, El Lahga, Fortin, and Lacroix (2011) with data from children aged 16 and over.

[8] The recovery of some elements of the decision process in collective models of household labor supply has essentially been based on the observation of the household's working hours, wages and nonlabor income. In the standard unitary model, the wage of one household member does not affect the labor supply of the other member. However, wages do affect bargaining position within the household in a collective context; in this case, a change in the wage of one member can influence labor supply of his or her partner via the sharing rule of the nonlabor income.

Thus, identification is not initially possible when nonparticipation is considered. However, there have been advances in collective labor supply models that include participation decision without public consumption and household production (Donni 2003; Blundell, Chiappori, Magnac, and Meghir 2007). These studies consider the case where the working hours of the wife vary continously, but they differ about the continuity of husband's labor supply (Donni assumes that it is continuous and Blundell et al. reduce the problem to not working or working full time). In both models, at most one member decides not to participate and the participation decision relies on an explicitly postulated reservation wage accompanied by certain assumptions.

is no significant difference in wage rates. The male is on average the older of the couple, although he has less education than the female; this education difference is not surprising considering that the sample includes only families in which both parents participate in the labor market.

Table 1.1: Descriptive statistics

	Mean	Std. Dev.
Mother		
Time devoted to children's education (h/week)	2.981	4.552
Time devoted to market labor (h/week)	33.576	17.818
Wage rate (MXN per hour)	36.200	64.025
Age	34.272	5.698
Years of education	9.949	4.072
Father		
Time devoted to children's education (h/week)	1.652	2.986
Time devoted to market labor (h/week)	47.766	16.535
Wage rate (MXN per hour)	37.768	61.797
Age	36.994	7.627
Years of education	9.158	4.129
Education expenses (MXN per week)	85.563	193.294
Non-labor income (MXN per week)	257.508	389.086
Age difference w.r.t. father	-2.722	6.237
Education difference w.r.t. father	0.791	4.261
Number of school-age children	1.899	0.823

1.4.3 Results

1.4.3.1 Estimation Results

The structural collective model presented in (1.14) is highly nonlinear, so several local minima were found. The estimation results presented in Table 1.2 correspond to the minimum scaled RSS found. The estimated parameters are divided into three panels. Panel A contains the parameters related to parents' preferences (1.6). Leisure is apparently a luxury good for the father since the estimate of β^f is positive, but it is not estimated precisely for the mother; CRV (forthcoming) also found that leisure seems to be a luxury good for a sample of Dutch couples, but in their case for both parents. As is the case in CRV, because α_2^m and α_2^f are statistically different from zero, the consumption/leisure decision for both parents is nonseparable from the resources allocated to children's welfare. This means that children's welfare changes have not only an income effect but also a substitution effect on individual decisions such as labor supply. This finding can serve as an argument for including children in the estimation of a collective model, since their introduction can help avoid rejecting the hypothesis of collective rationality within a household, as mentioned by Fortin and Lacroix (1997).

Panel B contains the parameters related to children's utility production (1.8). With respect to the coefficients related to the input share γ^i, the fixed part of the share $\left(\tilde{\gamma}_1^i\right)$ is greater for the mother than for the father. In the case of the variable part $(\tilde{\gamma}_2^i)$, as the number of children increases, other things being equal, the mother's time participation increases, whereas that of the father decreases. Considering these two estimates, the average cost share of the father is thus always inferior to that of the mother in the production of children's utility. Additionally, it seems that parental time and expenses on education are imperfect complements in children's utility production (the elasticity of substitution σ, although significant, is of low magnitude). In contrast, CRV (forthcoming) found that for their sample of Dutch couples this elasticity is approximately one for an average household.

Finally, the estimated parameters of the mother's Pareto weight (1.13) are shown in Panel C. All else being equal, the mother's relative wage has a significantly positive impact on her Pareto weight (Λ_2), so an increase in her wage could give greater influence to her preferences in household allocation. An increase the mother's age difference with respect to the father (Λ_4) seems to have a positive impact on the mother's power. Non-labor income (Λ_3) and education difference (Λ_5) do not seem to have a significant impact on the mother's power in the sample studied.

Table 1.2: Estimation results

	Estimate	Robust Std. Err.
A. Parent's preferences [see (1.6)]		
Mother		
α_1^m	0.7703**	0.3550
α_2^m (Children's utility)†	-0.0027***	0.0004
$\tilde{\alpha}_2^m$	-5.9160***	0.1409
β^m	-0.0737	0.0544
Father		
α_1^f	-0.8844***	0.3076
α_2^f (Children's utility)†	-0.0033***	0.0008
$\tilde{\alpha}_2^f$	-5.7000***	0.2479
β^f	0.1577***	0.0399
B. Children's utility production [see (1.8)]		
γ^m (Mother's time)††		
$\tilde{\gamma}_1^m$	31.4583***	0.3463
$\tilde{\gamma}_2^m$	1.0956***	0.0118
γ^f (Father's time)††		
$\tilde{\gamma}_1^f$	29.5919***	0.1216
$\tilde{\gamma}_2^f$	-0.0012***	0.0000
σ (Substitution elasticity)	0.0499**	0.0242
ρ (Substitution parameter)†††	-19.0493*	9.7290
$\tilde{\rho}$	-2.9982***	0.4853
C. Pareto weight parameters [see (1.13)]		
Λ_1	-0.6434	0.5193
Λ_2 (Mother's relative wage)	0.4113***	0.0467
Λ_3 (Non-labor income)	0.0001	0.0001
Λ_4 (Age difference w.r.t. father)	0.0130*	0.0073
Λ_5 (Education difference w.r.t. father)	0.0061	0.0086

The expressions in parentheses refer to the objects that are related to the respective parameters.
* $p<0.1$, ** $p<0.05$, *** $p<0.01$
† $\alpha_2^i = -\exp(\tilde{\alpha}_2^i),\ i = m, f$
†† $\gamma^i = \frac{\exp(\tilde{\gamma}_1^i + \tilde{\gamma}_2^i N_K)}{1+\exp(\tilde{\gamma}_1^i+\tilde{\gamma}_2^i N_K)+\exp(\tilde{\gamma}_1^j+\tilde{\gamma}_2^j N_K)},\ i \neq j,\ i = m, f$
††† $\rho = 1 - \exp(-\tilde{\rho})$

1.4.3.2 Changing the Distribution of Power Between Parents: Consequences for Resources Directed Toward Children's Education

The purpose of this subsection is analyze how a change in parents' bargaining position is reflected in the intrahousehold allocation of resources, with emphasis on allocations made for children's welfare. Since one goal of this study is to identify from observed behavior how much each parent cares for his or her children in terms of the resources allocated to them, performing this analysis under a collective model that also considers public consumption seems more appropriate than assuming, like other empirical studies, that fathers do not derive utility from their children's welfare (i.e., that it is a private good for mothers). An incorrect picture of individual preferences and of the intra-household decision process can be obtained if we consider, ex ante, that only the mother is interested in investing in her children. With this in mind, Table 1.3 reports the partial derivatives of the sharing rule (columns 1 and 2) and the elasticities of some variables and functions of the model (columns 3 to 10) with respect to the factors included in the Pareto weight function. Calculations are evaluated for families with two school-age children, at the mean wage across both sexes (about MXN \$37 per hour), and sample means of the other covariates; this sets the baseline.

The impact of a marginal change in one variable on the residual non-labor income allocated after expenses to the mother and the father in children's welfare is provided in columns 1 and 2, respectively. While the non-labor income to share between parents is constant in the case of private goods only, so that there is an exact trade-off between parents' shares, this condition does not have to be fulfilled when public consumption is also considered, as the amount to share varies with the cost of producing children's utility. In this sense, a MXN \$1 increase in the mother's wage (which is equivalent to a weekly increase of MXN \$33 in her labor income, at the mean hours worked by mothers), translates into a transfer of MXN \$45 to the production of children's utility and to the father. Moreover, the transfer amounts to MXN \$6 to the father, although this effect is not precisely estimated. A MXN \$1 increase in the father's wage (equivalent to a weekly increase of MXN \$47 in his labor income), translates into a transfer of MXN \$81, of which MXN \$26 are received by the mother. The outcome that ϕ^i is decreasing in w^i is expected in the sense that it is plausible that i's wage increase does not dramatically improve i's bargaining position, so that i is not able to keep all the direct gains and to extract in addition a larger fraction of household non-labor income conditional on public expenditures. Also, these results suggest that the fathers in the sample are more concerned about their children than mothers and that they behave in a more altruistic manner toward their partners than the mothers do. The next row indicates that the effect of a MXN \$1 increase in household non-labor income on parents' shares is not statistically significant. The impact of the distribution factors on the sharing rule are reported in the next two rows. In general, their effects on the shares are imprecisely estimated at the baseline, although one year more of age difference of the mother with respect to the father induces an additional MXN $22 of income to the mothers.

Columns 3 and 4 show private consumption elasticities for mothers and fathers respectively. At the baseline, both own- and cross-wage elasticities of mothers' private consumption are positive and statistically significant (0.595 and 0.332, respectively). Fathers' own-wage elasticity is negative (-0.563) and cross-wage elasticity is positive (0.059), although neither is statistically significant. Both mothers' and fathers' private consumption elasticities with respect to non-labor income are positive (0.026 and 0.024, respectively), but only mothers' elasticity is statistically significant. These results imply that mothers benefit most from increments of household labor and non-labor income for households in the sample. Distribution factors seem to have a negligible effect on private consumption of both parents.

Columns 5 and 6 present various labor supply elasticities for the mother and father respectively. At the baseline, the own-wage elasticity of mothers' labor supply is positive and statistically significant: a ten percent increase in her wage would induce an increase of approximately 1.7 percent in the hours of market labor. In contrast, a ten percent increase in her partner's wage would lead mothers to decrease their worked hours by approximately 3.1 percent. A standard labor supply model could also have accounted for mothers' observed behavior when substitution dominates income effect, but it would not necessarily have explained the effect of fathers' wages over their supply. Fathers' own-wage elasticitiy is negative but very small (-0.005) and not statistically significant. This insensibility of the male's labor supply is commonly found in the literature. His labor supply is more sensitive to a change in the mother's wage: it would be reduced by 2.8 percent if his partner's wage increases by ten percent. Overall, the labor supply of the mother is more sensitive with respect to changes in own- and cross- wage rates than the labor supply of the father.

Columns 7 to 9 report elasticities of resources directed to children's utility production. When the wage rate of one parent increases, parental time dedicated to children's education increases for both parents accompanied by an increase in related expenditure. This is expected because there is a positive relation between relative wages and bargaining power (i's Pareto weight increases with his or her relative wage; see parameter Λ_2), because both parents obtain utility from their children's welfare (parameter α_2^i is negative for both parents), and because of the complementarity of factors related to children's utility production (the magnitude of σ is relatively low). Therefore, if the bargaining power of one parent increases (via an increase in his or her wage), there would be more resources directed to children. However, contrary to the common presumption that mothers care more for children than fathers, implying that an increase in the mother's power is more beneficial to children than an increase in the father's, the wage elasticities calculated in the baseline for resources directed to children's education are higher for changes in the father's wage than for the mother's. If the mother's wage were increased by ten percent, the mother's time dedicated to children would increase by 1.7 percent (although the increase is not statistically significant) and both father's time and expenses would increase by 2.2 percent. If the father's wage increases the same amount as the mother's, the mother's time and expenses would increase by 6 percent and the father's time by 5.5 percent. From the perspective of the theoretical model, this would indicate that with an increase in the father's bargaining power equal to that of the mother, his willingness to pay for children's welfare is more responsive to increases in his resources for private consumption than hers. Consequently, the "amount" of children's utility $\left(u^K\right)$ produced inside the household would be higher with an increase in the father's wage than in the mother's. This result is shown in column 10, in which (1.12) is computed in the baseline.

Remembering that parents' care for their children is understood here as changes in resources directed to children associated with a change in the balance of intrahousehold power, this unforeseen result is the interaction of three components of the structural model: the relative weight of the parents in the household, individual preference and the technology to produce children's utility. The first component, bargaining power, implies that a beneficial change in the Pareto weight of the mother (or father) results in household allocations more according to her (or his) preferences. Although parents have different preferences, both obtain utility from their children's welfare. The final component at play, the technology used to produce children's utility, is affected by two forces in different directions: a lower use of all factors (because the factors are complementary, an increase in the price of one is associated with a lower use of all) and an increase of expenditures on children (and consequently parental time) via an increase in household labor income.

A change in non-labor income does have a positive impact on the resources directed to children. Because no significant change in parents' share for private consumption accompanies an increase in household non-labor income, more resources are directed to the production of children's utility. Indeed, a positive change of ten percent in non-labor income would induce an increase of approximately 1.3 percent of resources directed to children's utility production. Moreover, as the increase in parents' time dedicated to children is accompanied by a reduction in both labor supplies, an increase of ten percent in non-labor income would mean a decrease of aproximately 0.5 percent of both parents' time dedicated to market labor.

Finally, due to its economically negligible influence on the mother's Pareto weight in the sample studied, the distribution factors of mother's age and education difference with respect to the father do not have an impact on household market labor and resources directed to children's utility production.

Table 1.3: Partial derivatives of the sharing rule and elasticities

Variable	$\partial\phi^i / \partial\text{Variable}$		Elasticities			
			Private consumption		Market work	
	Mother (1)	Father (2)	Mother (3)	Father (4)	Mother (5)	Father (6)
Mother's wage	-45.386***	6.112	0.595***	0.059	0.169*	-0.282***
	(9.280)	(4.382)	(0.079)	(0.174)	(0.098)	(0.100)
Father's wage	26.609**	-81.261***	0.332***	-0.563	-0.308***	-0.005
	(11.999)	(11.547)	(0.071)	(2.007)	(0.102)	(0.069)
Non-labor income	0.139	-0.112	0.026**	0.024	-0.047***	-0.052**
	(0.143)	(0.111)	(0.013)	(0.051)	(0.012)	(0.022)
Age diff. w.r.t. father	22.480*	-18.113	-0.015	0.006	0.004	-0.038*
	(12.511)	(11.932)	(0.010)	(0.025)	(0.006)	(0.022)
Education diff. w.r.t. father	10.579	-8.524	0.002	-0.001	-0.001	0.005
	(14.706)	(12.501)	(0.003)	(0.004)	(0.001)	(0.007)

* p<0.1, ** p<0.05, *** p<0.01. Delta-method standard errors are in parentheses.

Table 1.3: - *Continued*

Variable	$^{\partial \phi^i}/_{\partial \text{Variable}}$		Elasticities			
			Children's utility production			
	Mother (1)	Father (2)	Mother (7)	Father (8)	Expenses (9)	u^K (10)
Mother's wage	-45.386***	6.112	0.168	0.218*	0.218*	0.190
	(9.280)	(4.382)	(0.136)	(0.124)	(0.124)	(0.138)
Father's wage	26.609**	-81.261***	0.603***	0.553***	0.603***	0.580***
	(11.999)	(11.547)	(0.123)	(0.117)	(0.123)	(0.117)
Non-labor income	0.139	-0.112	0.127***	0.127***	0.127***	0.127***
	(0.143)	(0.111)	(0.015)	(0.015)	(0.015)	(0.018)
Age diff. w.r.t. father	22.480*	-18.113	0.006	0.006	0.006	0.006
	(12.511)	(11.932)	(0.010)	(0.010)	(0.010)	(0.010)
Education diff. w.r.t. father	10.579	-8.524	-0.001	-0.001	-0.001	-0.001
	(14.706)	(12.501)	(0.002)	(0.002)	(0.002)	(0.002)

* $p<0.1$, ** $p<0.05$, *** $p<0.01$. Delta-method standard errors are in parentheses.

A change in parents' wages and household non-labor income is likely to affect the resources directed to children, simply because these modify the set of possible allocations: they have a direct impact on either preferences or the budget constraint. A clearer understanding of household behavior can be obtained if only there is a change in the distribution of power.[9] The following exercise is an analysis of the impact of exogenous changes in Pareto weights on household allocations. According to the theoretical model, a shift in the balance of power between parents could change household outcomes even when the budget set does not change. With this idea in mind, Figure 1.1 presents the effect of an exogenous change in the range of 0 to 10% in favor of one of the parents (or equivalently a change in Λ_1 of approximately 28%) on selected variables in the model. Calculations are for families with two school-age children, at the mean wage for both parents, and for sample means of the other covariates.

Changes in the mother's Pareto weight in her favor are denoted by the symbol $\bullet$ and in the father's favor by $\Diamond$. The upper left panel of the figure focuses on the change in parents' share with respect to their shares evaluated at the baseline. The mother's share responds more than the father's to changes in the distribution of power. The upper center panel shows the mother's marginal willingness to pay for children's utility standardized by the marginal cost of producing it. Therefore, both MWPs must add up to one. At the baseline, mothers have a larger willingness to pay than fathers ($MWP^f/MC > 0.5$), a situation that is maintained with shifts in their bargaining power of around 10%.

The upper right panel illustrates the percentage change in children's utility. Although mothers have a larger MWP at the baseline than fathers for resources directed to children's utility production, this feature is not translated into more resources directed to children when mothers' Pareto weight increases; there is instead a reduction in resources. Likewise, a reduction in mothers' Pareto weight is accompanied by an increase in resources directed to children. This outcome is a result of fathers' MWP being more income sensitive than that of mothers. Since both parents have the same wage at the baseline, the difference in the income sensitivity of MWPs depends on which parent has the lowest parameter $\hat{\alpha}_2^i$, with the fathers' being more negative than that of the mothers.

The lower panels focus on outcomes for both parents in the individual domain, calculated as percentage change in the three panels. The lower left panel shows private expenditures,

[9] This is the idea behind the introduction of distibution factors to the collective approach. The additional information obtained in this manner leads to greater restrictions on the set of efficient possible allocations. As an example, for any given level of household non-labor income, individual contributions can only influence household outcomes through the decision process. However, since it is not possible to adequately establish the identity of individual non-labor income in the MxFLS-2, the use of this distribution factor was not possible in this study.

the lower center panel focuses on leisure, and the lower right displays labor supply. The beneficiary of greater bargaining power enjoys an increase in both private expenditure and leisure as a consequence of the increase in his or her share, while the other member's consumption is reduced. Nevertheless, the effect of the mothers' private expenditure is larger than that of the fathers', and in the case of leisure, the fathers' effect is larger than that of the mothers. Regarding labor supply, one member's increased power is related to a reduction in their hours offered to the labor market and an increase in the hours offered by their partner.

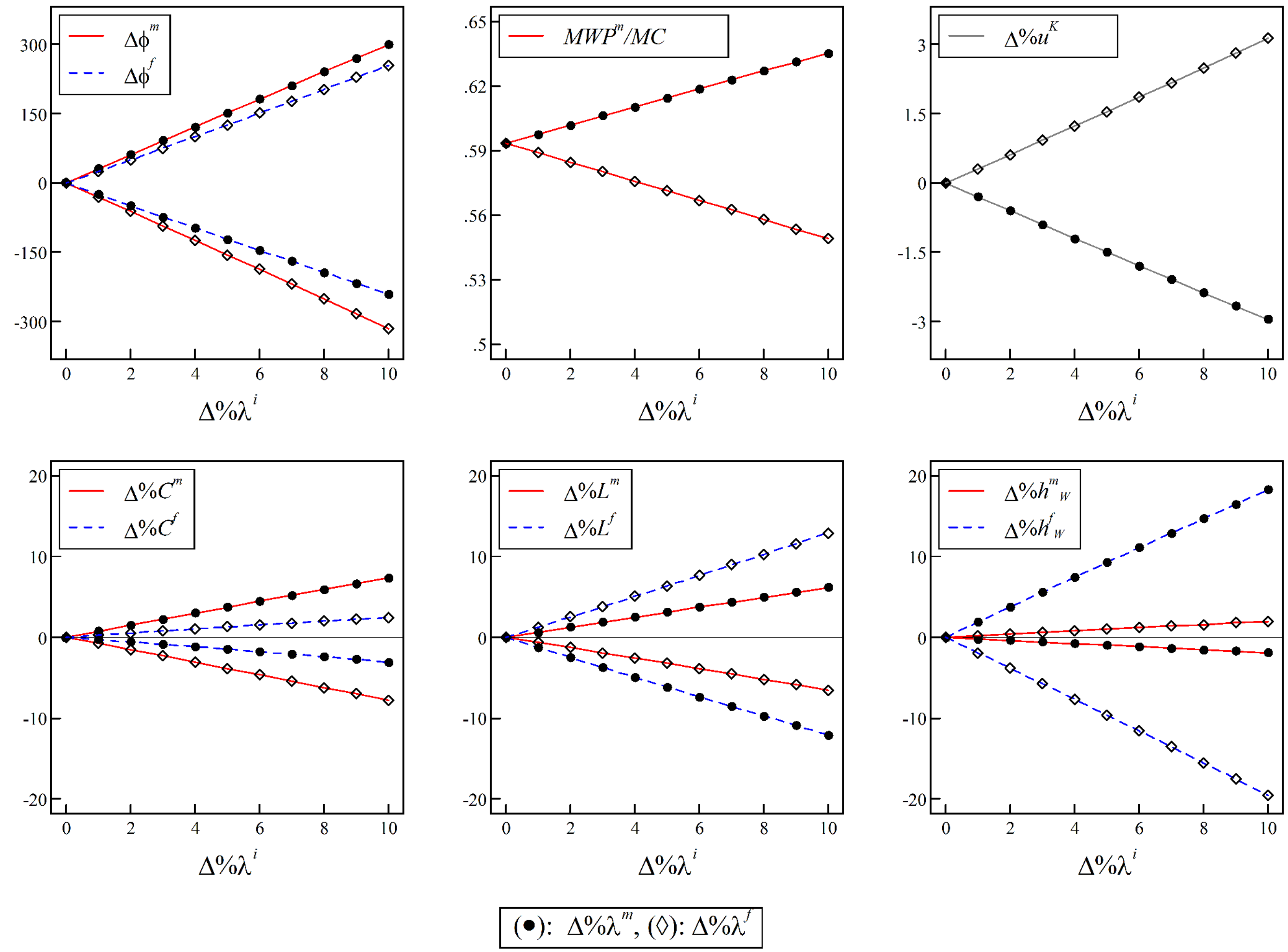

Figure 1.1: Effect of change in parents' Pareto weight

1.5 Final Remarks

A consensus has been developed that households do not behave as a singular entity, but that the behavior of each individual member should be considered as well. It is important to understand better how household decisions are made in order to enhance the effectiveness of targeting individual members.

The collective model has emerged as a more appropriate framework for analyzing household behavior. Under this approach, resources invested in children's welfare depend not only on the household budget constraint but also on the parents' individual preferences and their relative position in the decision making. This chapter employs an empirical application for the collective theoretical model of Blundell, Chiappori, and Meghir (2005). In the model, both parents derive utility from the production of children's utility by means of a public good and their time. The empirical analysis shows how household allocations respond to changes in the balance of power between the couple, particularly with respect to resources directed toward children's education.

The model is applied to a sample of Mexican nuclear families from the MxFLS-2, and focuses on couples in which both work, have only children under 15 years, at least one of school age. Based on Cherchye, de Rock, and Vermeulen (2010), the estimation strategy takes advantage of the two-stage representation of the collective model to construct a flexible functional specification for the observables.

The estimation results highlight some interesting findings for this particular sample. In the production of children's utility by means of resources used in their education, parental time and expenditure are complementary inputs, so it is not entirely possible to substitute time for expenditure or time between parents to augment children's wellbeing. Also, the distribution of bargaining power depends significantly on the mother's relative wage and age difference with respect to the father. Finally, parents' preferences are not separable from resources directed to children's welfare.

In order to analyze whether a change in the decision process between parents affects resources directed to children's welfare, some elasticities are calculated with respect to factors that influence the decision process (as represented by Pareto weights). For families with characteristics at the sample average, no evidence was found that mothers care more for their children than fathers. Other things being equal, there are more resources directed to children's education when a father has more power via his relative wage. This finding is in line with the results of Cherchye et al. (2010) and complements existing literature that has found that the fraction of household non-labor income received by each parent affects the resources directed to children (such as Thomas 1990, and Lundberg et al. 1997).

Since the set of options available for household allocations changes with wages and non-labor income, a clearer picture of the effect of a change in the balance of power between parents can be obtained if there is an exogenous change in the decision process that keeps the household budget constraint unchanged. To this end, graphical illustrations are provided of an exogenous change in mothers' Pareto weight at the baseline. Although mothers have a larger marginal willingness to pay for children's utility than fathers, it is only when fathers' power increases, and not when mothers have more power, that more resources are directed to children's utility production.

In the case of Mexico, this result complements the evidence supplied by Bobonis (2009) for a sample of rural households, where changes in female non-labor income (from the conditional cash transfer Progresa/Oportunidades) boost the share of expenditures on children's clothing and education, and where changes in household income not exclusive to the female partner (such as variations in rainfall shocks) have a lesser influence on this share. The distinct pattern found in this chapter can be partly explained by the profile of households analyzed; as Bobonis has mentioned, gender inequality in familial relations is very common in rural Mexico; social norms induce to direct most part of female partner's income to public consumption. If we consider not only rural, low-income households, the pressure of social norms on the distribution of women's private and public consumption could be weaker, especially where they have more education and participate actively in the labor market.

The results discussed here highlight the danger of assumptions in policy design about which household member cares more for children; instead, a more complete knowledge of internal dynamics between household members could improve the effectiveness of this type of targeted policy. It may be that for the particular characteristics and economic environment of families under a conditional cash transfer program such as Progresa/Oportunidades, it is appropriate to give the additional income to the mother, but if we consider the more diverse types of households analyzed in this sample, the intended policy outcomes may not be met.

Chapter 2

Children and Non-Participation in a Model of Collective Household Labor Supply

2.1 Introduction

The collective approach, unlike the unitary one, provides an adequate theoretical background for analyzing intra-household allocations: how individual preferences and the decision process can be recovered from household members' aggregate behavior. This model draws upon the idea that an increase in the decision power of one household member changes household behavior in his or her favor, even though total household resources are kept constant. In this sense, the collective approach can be used to analyze the targeting of programs. This is the case for conditional cash transfer (CCT) public programs, in which a household receives a monetary compensation for the fulfillment of certain requirements that are positively related to household welfare. The goal of many such programs is to foster the human capital of children. However, some programs give the cash transfer to a particular household member (generally the mother[1]) instead of the intended recipient (children). Therefore, the impact of the cash transfer on a child's consumption depends on how the intra-household allocation processes distribute this additional income; it could might enhance or diminish a child's consumption. For example, Bobonis (2009), using data from the evaluation of Mexican

[1] Examples of CCT programs that gives the transfer preferably to the mother (or female household head) are: Bono de Desarrollo Humano (Ecuador), Chile Solidario (Chile), Familias en Acción (Colombia), Progresa-Oportunidades (Mexico), Programa de Asistencia Familiar - PRAF (Honduras), Red de Protección Social - RPS (Nicaragua).

Progresa/Oportunidades program, found that the increase in female non-labor income from participating in the program increases the share of expenditures on children's goods. These programs could also influence other household outcomes, such as the labor status of the cash transfer receiver.

The labor supply model of Chiappori (1992) has been used extensively for empirical applications (see, for example, Fortin and Lacroix 1997, Canada; Chiappori, Fortin, and Lacroix 2002, the USA); however, this framework considers the simplest possible case of household structure (childless households with two working members), making it difficult to apply the collective approach to the broader definition of household typically found in developing countries: a two-adult household with a non-working female partner and at least one child. The contribution of this chapter is to generalize Chiappori's (1992) model to incorporate both the presence of children and the decision to participate in the labor market, providing the necessary theoretical considerations for an empirical application, to be applied to Mexican data in chapter 3. The model employs the method of Donni (2003) with the scenario of Blundell, Chiappori, and Meghir (2005), taking into account the presence of children in a household, to address the possibility of non-participation.

The proposed model generalizes the identification results of Chiappori (1992); individual preferences and the sharing rule can be recovered from observed behavior. Identification requires the knowledge of a distribution factor and the existence of a unique reservation wage for each adult household member at which both members are indifferent as to whether a member participates in the labor market or not. In contexts where children and a non-working partner are elements of a household, empirical applications of the model should increase the sample size and reduce related selection bias. Also, the model can be used to analyze the impact of CCT on expenditures on children and on household members' labor supplies.[2]

2.1.1 The Collective Approach

Given that a household's demand for goods and the labor supply of its members can be obtained from household survey data, what can be said about the structural components of the decision process that lead to this household behavior? The traditional unitary approach, with its *ad hoc* assumptions that aggregate individual preferences, considers a household as a

[2] For example, this could be an interesting form to complement Bobonis (2009)'s finding. Since in the Progresa-Oportunidades sample used there is a low female participation in the labor market, under this model can be also analyzed the impact of this CCT on female's time dedicated to the labor market.

single decision-making unit, leaving unexplained how the household reaches its agreement to allocate resources. Furthermore, the lack of a distinction between individual and household preferences in this approach is unsatisfactory from the perspective of welfare analysis.[3] The framework imposes an empirical straitjacket on public policy analysis with an individual targeting emphasis, since price changes are the only tool available for intra-household reallocations (Quisumbing and McClafferty 2006). The approach has also been criticized for a lack of empirical support of its theoretical implications, such as the consideration of total income but not its source in household consumption decisions (i.e., household members pool their income),[4] and the assumption that cross-price substitution effects are symmetric (e.g., the compensated wage changes of spouses have the same effect on each other's labor supply).[5]

Alternative approaches, such as non-cooperative and cooperative (or collective) models, have tried to take into account the multiplicity and heterogeneity of decision makers in a household.[6] On the one hand, in the absence of binding and enforceable agreements between household members, non-cooperative models have assumed that household members maximize their utility subject to an individual budget constraint and taking as given each other's behavior. However, the intra-household allocations under this framework are not necessarily Pareto efficient; if we consider deviations of the equilibrium outcome, it is possible to increase the welfare of one household member without reducing that of others. In a household context this result is not very satisfactory, since possibilities for Pareto improvements may arise from daily interaction among their members. On the other hand, the only assumption that household collective models have in common is that household decisions

[3] Neverthless, using a strategy that provides more structure to the household allocation problem or that uses more extensive data, it is possible in this framework to do welfare analysis at the individual level (Donni 2008a).

[4] For the rejection of the income pooling hypothesis see, among others, Thomas (1990); Bourguignon, Browning, Chiappori, and Lechene (1993); Browning, Bourguignon, Chiappori, and Lechene (1994); Lundberg, Pollak, and Wales (1997); and Fortin and Lacroix (1997).

[5] For the rejection of the symmetry of the Slutsky matrix see, among others, Browning and Meghir (1991); Blundell, Pashardes, and Weber (1993); Fortin and Lacroix (1997); and Browning and Chiappori (1998).

[6] Some unitary models propose ways to incorporate multiple individuals in the analysis. Samuelson (1956) considers that a household could act as one individual if its members agree on how to aggregate their preferences; as a result they choose to maximize a social welfare function. By the "rotten-kid theorem," Becker (1974; 1991) arrives at a household objective function that converges to the preferences of the "altruistic head" of the household. Suppose that a household consist of two members: an altruistic head and a selfish member. Trying to avoid retaliation from the head, the other individual would not attempt to increase his or her consumption (be rotten) at the expense of the head's consumption. However, these efforts are based on strong assumptions (Bergstrom 1989; Haddad, Hoddinott, and Alderman 1997) and their "outcomes are empirically indistinguishable from those of constrained individual utility maximization" (McElroy and Horney 1981, 333).

are Pareto efficient,[7] so it is not necessary to specify the actual process that determines the intra-household allocation on the efficiency frontier, only to assume that it exists.

Efficiency means that household allocations are optimal; no other consumption bundle could provide more utility for household members at the same cost. In this sense, an equivalent interpretation of Pareto efficiency is that household members initially reach an agreement on the respective amount each is allowed to spend, a "sharing rule." Then, all members independently choose their consumption, subject to their respective share. The approach does not impose a particular form on the rule; it only requires that it exists.

While assuming that efficiency of household decisions reduces the set of possible allocations, there could exist a continuum of different structural models that generate the same observable behavior (Chiappori and Ekeland 2009). It is in this sense that particular hypotheses over goods or preferences have been made within the collective framework to recover preferences and decision making from household aggregate demand. The main identification results have been made for the case where all goods consumed in a household are private (i.e., they are consumed non-jointly and exclusively by each member); where one member's consumption does not have a direct effect on another member's wellbeing; and at an interior solution for household demands. Intuitively, the quantities consumed by each member are a guide to the intra-household bargaining power distribution: the consumption of a good associated with a particular individual will be greater as his or her decision power increases.

Applying the collective framework to the case of household labor supply, the seminal collective model proposed by Chiappori (1988; 1992) allows, under certain assumptions, the recovery of some elements of the decision process from the observed labor supply of household members. Since these results are derived from the simplest possible case, empirical applications based on this model have been used as an observation unit: childless households composed of two adult members who participate in the labor market. However, estimates obtained from this type of sample could be imprecise due to small sample size and may be subject to selection biases if households with positive hours of work for their members are the only ones considered (Fortin and Lacroix 1997).

[7] This Pareto efficiency assumption can be justified if all household members are aware of the preferences and actions of the others (i.e., there is symmetric information, possibly due to proximity and durability of the household), so they can decide to cooperate to make everyone better off by means of a binding agreement. Alternatively, this agreement can emerge if the relations between household members can be represented as a repeated game. For a more detailed discussion about assuming efficiency see Browning, Chiappori, and Weiss (2011).

To properly assess the collective framework as a useful tool for welfare evaluation and policy analysis on an intra-household level, it is necessary not to limit the analysis to childless households with members who participate in the labor market. The objective of this chapter is to develop a collective theoretical framework that simultaneously takes into account the presence of children and the decision to participate in the labor market.

Although the chapter is essentially theoretical, it relies on empirically testable restrictions on household labor supply and obtaining information about aspects of the intra-household decision process that can be used for individual welfare analysis and policy evaluation. Indeed, analysis using the collective approach would have limited empirical content if its concepts could not be recovered from observed behavior. For example, the proposed model provides an adequate framework for the analysis of a social program targeted at a particular household member (say, a female member). If the policy increased women's influence in the household decision process, what would be the impact on household demand? In particular, what would the impact be on household expenditures on children? Could it be that this female "empowerment" would also lead to women's non-participation in the labor market, or could it be the case that women put so much emphasis on spending on children that they decide to work more hours when they have more power?

The chapter is organized as follows. Section 2.2 discusses the current literature on collective household labor supply models that includes the possibility of labor force participation and public consumption (like expenses on children). Section 2.3 presents the theoretical framework that incorporates both the decision to participate in the labor market and public goods consumption. Using Donni's (2003) approach, the possibility of non-participation is introduced in the framework of Blundell, Chiappori, and Meghir (2005), where parents care about children's welfare. When children are present in a household, the main conclusions of Chiappori (1992) can be extended: individual preferences and the (conditional) sharing rule can be recovered if one or both of the couple works. Furthermore, individual labor supplies have to satisfy certain testable restrictions. Some final remarks are made in section 2.4.

2.2 Labor Market Participation and Public Goods

One application that has been analyzed to a great extent in the collective literature is the household member's supply of positive hours of work with private consumption. However, this setting is too simple to describe other household compositions and dynamics that are observed in real life. When labor market participation and public goods are considered under a collective framework, there are certain aspects to take into account. First, the non-participation decision in the labor market may have an influence on outcomes even for individuals who are not directly affected by this decision. If a member's threat point involves participation in the labor market (e.g., because a woman's or man's participation involves credible outside options for her or him), (potential) wages could affect bargaining positions within a household. This result is the opposite of the one obtained within the unitary model, where potential wages of non-working members are independent of household allocations (only wages of working members matter, due to their effect on budget opportunities). Second, children are likely to be an important source of preference interdependence between parents, since it is reasonable to think that both parents could derive utility from their children's well-being (although not necessary to the same degree). Furthermore, the presence of children could generate non-separabilities in parents' commodity demand and labor supply (child care, say, may affect the tradeoff between consumption and labor force participation and hours of work at the individual level). Finally, household production could assume more relevance when children are present.

Some advances have been made in the literature to include the possibility of participation and public consumption in the collective model (i.e., goods from which both spouses derive utility, such as the amount spent on children, consumed jointly and non-exclusively by each member) but in separate branches. Donni (2003) and Blundell, Chiappori, Magnac, and Meghir (2007) have constructed theoretical frameworks to consider non-participation in the labor market. The former assumes that in a two-adult household, both members can freely choose their working hours, while the latter assume that one of the members can only decide whether to participate or not.

Donni's work extends the results of Chiappori (1988; 1992), who implicitly considered identification in the two adult members' participation set, to also take into account the case in which one of the two members does not work. To date, the only empirical application of Donni's framework has been made by Bloemen (2010, Netherlands). The work of Blundell et al. is distinct from Chiappori's model, since the choice sets are different (two positive continuous labor supplies versus one discrete and another non-negative continuous labor supply). Donni (2007) develops a model similar to that of Blundell et al.; they differ in that the former fixes the male household member's labor supply at full-time instead of allowing a choice between working full-time or not at all. Structural elements of the decision process can be identified from Donni's model if the female household member's labor supply is observed together with at least one household commodity demand.

The participation decision is included in the standard unitary model by means of a reservation wage, at which an agent is indifferent between working and not working. In translating this concept to the collective framework, the central assumption of Pareto efficiency of the household decision process requires that if one member (say, the wife) is indifferent between working and not working, the other one (say, the husband) must be indifferent as well about the participation decision of the first member (Blundell et al. 2007 called this condition the "double indifference" assumption[8]). Therefore, the participation decision in these two collective models relies on explicitly postulating a reservation wage. Individual preferences and the sharing rule can be recovered for both models.

On the other hand, Blundell, Chiappori, and Meghir (2005, hereafter BCM) introduce children into Chiappori's (1988; 1992) model, assuming that both parents care about their children's welfare (or equivalently, considering that the expenditure on their children is a public good for them). In general, the decision process cannot be recovered; a continuum of different structural models can generate the reduced form of each individual's labor supply and expenditure on children. This result is due to the fact that the level of public consumption influences the analysis of labor supply not only through an income effect but also through its impact on the individual consumption/leisure trade-off. Under this approach, identifiability of the intra-household decision-making process can thus be obtained in two cases: a) where private consumption is separable from expenditures on children, so that the consumption/leisure trade-off effect disappears; or b) by introducing a distribution factor (i.e., a variable that affects the decision process but not the individual preferences or the joint budget set, controlled for total income), allowing for expenditures on children to be kept constant. Empirical applications of the BCM model have been made by Cherchye, de Rock, and Vermeulen (forthcoming, Netherlands) and also in chapter 1 (using Mexican data).

The aim of the present chapter is to model the decision to participate in the labor market in a single collective framework that considers expenditures on children as a public good. The model simultaneously takes into account the possibility that (potential) wages affect the bargaining positions of household members, that the utility of each adult member depends on their children's wellbeing, and that individual consumption and labor supply decisions are not separable from expenditure on children. Under these assumptions, the underlying

[8] To see why, Blundell et al. used the following example. Assume that at a wage infinitesimally below the reservation wage of a husband, he is indifferent between working and not working but that his wife experiences a strict loss if he is not working. Now suppose that at the reservation wage he decides to work and he receives ε more to spend on his private consumption than initially agreed. He is better off since he is indifferent between participating or not and his consumption increases (if the goods consumed are normal). If ε is small enough, the wife is better off too, since the participation of her spouse compensates her more than the reduction in her private consumption.

structure (individual preferences and the decision process) can be recovered from observed household behavior. The model extends the results of Chiappori (1992) to a more general context that the one considered in the previous literature. An empirical application of this model appears in chapter 3.

2.3 The Framework

The model incorporates the decision to participate in the labor market in BCM's (2005) framework of household labor supply with expenditures on children. Subsection 2.3.1 presents the main assumptions of the model. Besides the assumptions of individualism and Pareto-efficiency common to the collective approach, the model assumes that both adult household members care about their own consumption (they have egoistic preferences), but that they also care about their children. Subsection 2.3.2 shows a decentralization procedure of the efficient household allocation within this context. As with the case considering only private consumption, the decision process can be represented as operating in two phases by the existence of a sharing rule conditional on the residual non-labor income after the public good purchase. Subsection 2.3.3 shows how the model determines the level of expenditures on children. Here, the framework also addresses the effect of intrahousehold redistribution of power (for example a given policy that "empowers" a specific member of the household, such as the mother) regarding household expenditures on children. Subsection 2.3.4 introduces additional assumptions to guarantee the existence of a unique reservation wage for each partner that is consistent with the Pareto-efficiency assumption. The model employs the method used by Donni (2003) to achieve this aim. Finally, subsection 2.3.5 discusses the identification of the model and the corresponding restrictions on household labor supply. Given a set of (potential) wages, non-labor income, and a distribution factor, the framework can recover individual preferences and the conditional sharing rule if one or both partners works.

2.3.1 Commodities, Preferences, and the Decision Process

The model considers the case of an adult couple ($i = m, f$) in a single period setting. Labor supply of i is denoted by h^i, with market wage w^i. Total time endowment is normalized to one and domestic production is not considered.[9] A Hicksian composite good C is consumed by the household. This good is used for private $\left(C^m, C^f\right)$ and public (K) consumption, with prices set to one (the identifiability results of the model do not require price variation). In a very general sense, the notion of public consumption should be understood here as any expenditure that increases the utility of both partners, such as expenditures on heating, electricity, housecleaning, among others. A typical example of K, by its normative implications, is the amount spent on children by the household. Non-labor income is denoted by Y.

Each spouse's utility can be written as:

$$U^i = U^i\left(1 - h^i, C^i, K\right) \quad , i = m, f$$

where U^i is strongly quasi-concave, infinitely differentiable, and strictly increasing in all its arguments. It is also assumed that $\lim_{h^i \to 1} \partial U^i/\partial h^i = \lim_{C^i \to 0} \partial U^i/\partial C^i = \lim_{K \to 0} \partial U^i/\partial K = \infty$, $i = m, f$. These conditions rule out cases where leisure, and individual and public consumption, are equal to zero; both members consume strictly positive quantities of these goods. These conditions seem reasonable since leisure is arbitrarily defined and consumption is aggregated.

It is assumed that household decisions generate Pareto-efficient outcomes, whatever the mechanism used to reach this agreement. Therefore, there is a function λ such that household allocations $\left(h^{m^*}, h^{f^*}, C^{m^*}, C^{f^*}, K^*\right)$ are the solutions to the program:

$$\max_{h^m, h^f, C^m, C^f, K} \lambda U^m\left(1 - h^m, C^m, K\right) + (1 - \lambda) U^f\left(1 - h^f, C^f, K\right) \tag{2.1}$$

$$s.t. \quad \begin{cases} C^m + C^f + K = w^m h^m + w^f h^f + Y \\ 0 \leq h^i \leq 1, \quad i = m, f \end{cases}$$

The Pareto weight λ reflects the relative power of m in the household and $(1 - \lambda)$ that of f, in the sense that a larger λ corresponds to a larger weight of m's preferences in the

[9] The model assumes implicitly that all non-market time corresponds to leisure; it does not consider the division of labor between household and market production. The seminal model of Chiappori (1992) is extended to consider domestic production by Apps and Rees (1997); Chiappori (1997); and Donni (2008b). Empirical applications have been made by Apps and Rees (1996, Australia); Donni and Matteazzi (2010b, the USA); and Rapoport, Sofer, and Solaz (2011, France), among others. A model that considers that the domestic good is public is developed and estimated with British data by Couprie (2007); and van Klaveren, van Praag, and Maassen van den Brink (2008). Under the collective models with domestic production, that of Donni and Matteazzi (2010a) is the only one that considers non-participation. A model that jointly considers non-participation, children, and household production has not been developed yet.

household allocation problem, favoring the outcomes enjoyed by m (and likewise that a smaller λ corresponds to a lesser weight of those preferences, favoring the outcomes of f). It is assumed that $\lambda \in [0,1]$ is a continuously differentiable function of wages and non-labor income, as well as at least one distribution factor z, i.e., $\lambda = \lambda\left(w^m, w^f, Y, z\right)$.

A few assumptions should be made explicit here. First, it is assumed that the bundle $\left(w^f, w^m, Y, z\right)$ varies within a compact subset $\mathcal{K}$ of $\mathbb{R}^3_+ \times \mathbb{R}$. Second, it is assumed that h^m, h^f, C, and K are observed (as functions of w^m, w^f, Y, and z), whereas the individual consumptions C^m and C^f are unobserved. In general, household surveys do not collect information about intrahousehold allocation of expenditures but about aggregate consumption C at the household level. Third, it is assumed that both partners' wages are always observed by the econometrician, even when a partner does not participate in the labor market. In practice, it is possible to calculate a potential wage for the non-participating member by means of an auxiliary equation (see subsection 3.3.3 for an example of this type of procedure for imputing wages).

2.3.2 The Conditional Sharing Rule

The solution to the household program (2.1) can be thought of as a two-stage process: 1) the couple agrees on the level of the public expenditure and how to distribute the resulting residual non-labor income between them; and 2) conditional on the outcome of the first stage, the couple decide, independently of each other, their individual consumption and labor supply. Formally, let $h^{m^*}\left(w^m, w^f, Y, z\right)$, $h^{f^*}\left(w^m, w^f, Y, z\right)$, $C^{m^*}\left(w^m, w^f, Y, z\right)$, $C^{f^*}\left(w^m, w^f, Y, z\right)$, and $K^*\left(w^m, w^f, Y, z\right)$ be the solution of program (2.1); then a function ρ^i exists such that:

$$C^{i^*}\left(w^m, w^f, Y, z\right) = \rho^i\left(w^m, w^f, Y, z\right) + w^i h^{i^*}\left(w^m, w^f, Y, z\right), \quad i = m, f$$

Here ρ^m and ρ^f characterize the *conditional sharing rule*,[10] the portion of non-labor income allocated to each member once spending on the public good has been discounted:

$$\rho^m\left(w^m, w^f, Y, z\right) + \rho^f\left(w^m, w^f, Y, z\right) = Y - K^*\left(w^m, w^f, Y, z\right)$$

[10] When private goods are considered together with public goods, the (conditional) sharing rule is implied by efficiency, but in this case it is not equivalent to efficiency for a particular level of public expenditure. The level of public consumption depends also on the allocation of private consumption and labor supply, a fact that cannot be isolated completely with the two-stage process interpretation of the household problem (see BCM 2005).

Note that ρ^i can be positive or negative; they could agree to spend beyond their non-labor income on the public good, and transfers between the two are also possible.

Fixing $K = K^*\left(w^m, w^f, Y, z\right)$, the second stage of the household program (2.1), can be represented as:

$$\max_{h^i, C^i} U^i\left(1 - h^i, C^i, K\right) \quad s.t. \quad C^i = w^i h^i + \rho^i, \qquad i = m, f \tag{2.2}$$

with $h^{i^*}\left(w^m, w^f, Y, z\right)$ and $C^{i^*}\left(w^m, w^f, Y, z\right)$ as interior solutions to the individual problem. The structure of both partners' labor supplies can be described by:

$$\begin{aligned} h^{m^*}\left(w^m, w^f, Y, z\right) &= H^m\left[w^m, \rho\left(w^m, w^f, Y, z\right)\right] \\ h^{f^*}\left(w^m, w^f, Y, z\right) &= H^f\left[w^f, Y - K - \rho\left(w^m, w^f, Y, z\right)\right] \end{aligned}$$

where $\rho = \rho^m$, and when ρ is fixed, H^m and H^f are Marshallian labor supply functions. With the idea of expressing labor supplies in terms of public expenditures (K) and maintaining the assumption that K is fixed, the following process is used. Let $\mathcal{O}$ be some open subset of $\mathcal{K}$ such that $\partial K/\partial z$ does not vanish on $\mathcal{O}$. The condition $K^*\left(w^m, w^f, Y, z\right) = K$ is used to express z as a function ζ of $\left(w^m, w^f, Y, K\right)$ by the implicit function theorem. Following from this construction, the couple's labor supplies are:

$$\tilde{h}^m\left(w^m, w^f, Y, K\right) = H^m\left[w^m, \rho\left(w^m, w^f, Y, \zeta\left(w^m, w^f, Y, K\right)\right)\right] \tag{2.3}$$

$$\tilde{h}^f\left(w^m, w^f, Y, K\right) = H^f\left[w^f, Y - K - \rho\left(w^m, w^f, Y, \zeta\left(w^m, w^f, Y, K\right)\right)\right] \tag{2.4}$$

In this way, i's labor supply is described as a function of wages, non-labor income, and a distribution factor z such that public expenditures are exactly K. Hence, the values of w^m, w^f, and Y are not constrained to assure that $K^*\left(w^m, w^f, Y, z\right) = K$; the key role of z is to guarantee that the level of public expenditure is exactly K. This structure generates testable restrictions because the same function $\rho\left(w^m, w^f, Y, z\right)$ enters each member's labor supply (see footnote 13).

2.3.3 The Determination of Public Expenditures

The Bowen-Lindahl-Samuelson condition characterizes efficiency for public good expenditures. Formally, the first-order conditions for household program (2.1), with an interior solution for individual and public consumption, give:

$$\frac{\partial U^m/\partial K}{\partial U^m/\partial C} + \frac{\partial U^f/\partial K}{\partial U^f/\partial C} = 1$$

Equivalently, this condition can be expressed in terms of individual indirect utilities. First, let $V^i\left(w^i, \rho^i, K\right)$ denote the value of the second stage of the household program (2.2) for member i:

$$V^i\left(w^i, \rho^i, K\right) = \max_{h^i, C^i} U^i\left(1 - h^i, C^i, K\right) \quad s.t. \quad C^i = w^i h^i + \rho^i, \qquad i = m, f$$

V^i is called the indirect conditional utility because it is the maximum utility that i can achieve, given his or her wage and conditional on the outcomes (ρ^i, K) of the first stage decision. Next, returning to the first stage, efficiency leads to the following program:

$$\max_{\rho^m, \rho^f, K} \lambda V^m\left(w^m, \rho^m, K\right) + (1-\lambda) V^f\left(w^f, \rho^f, K\right) \quad s.t. \quad \rho^m + \rho^f + K = Y$$

The first order conditions give:

$$\lambda \frac{\partial V^m}{\partial \rho^m} = (1-\lambda)\frac{\partial V^f}{\partial \rho^f} = \lambda \frac{\partial V^m}{\partial K} + (1-\lambda)\frac{\partial V^f}{\partial K}$$

Therefore:

$$\frac{\partial V^m/\partial K}{\partial V^m/\partial \rho^m} + \frac{\partial V^f/\partial K}{\partial V^f/\partial \rho^f} = 1 \tag{2.5}$$

The ratio $\frac{\partial V^i/\partial K}{\partial V^i/\partial \rho^i}$ is i's marginal willingness to pay (MWP) for the public good, in this case children. Thus the condition (2.5) states that individual MWPs (or Lindahl prices) must add up to the market price of expenditure on children. From BCM's Proposition 1, it is possible to state that if i's preferences are such that both public and private consumption increase with non-labor income (i.e., K and ρ^i are normal "goods", so i's MWP is decreasing in K and increasing in ρ^i), a marginal increase in i's Pareto weight increases the household's expenditure on children if and only if i's MWP is more sensitive to changes in his or her share than that of the other member. That is, a marginal increase in m's power will increase the amount spent on children if and only if m's MWP is more income sensitive that that of f, and vice versa. Because a positive transfer from one member to the other decreases the MWP for the public good of the transferer and increases the MWP of the transferee, this proposition establishes when the effect on the transferee is more than sufficient to compensate the reduction to the transferer. Hence, the key property for analyzing changes in the distribution of power within a household is not the magnitude of the MWPs (say, who cares more for children), but how the MWPs respond to changes in individual resources for private consumption.

Intuitively, empowering one household partner (say, the woman) comes with a higher fraction of household non-labor income for her. If both private and public goods are normal, she will consume more of all commodities, and, conversely, the male partner will see his share and consumption reduced. The question is when the reduction in household expenditures on the public good that comes from the male's share will be more than compensated by the increase to the female's share. The answer is when the female partner is more sensitive to changes in her share than her partner, whether she is willing to spend on children a larger fraction than her partner of the additional monetary unit that comes via her empowerment.

2.3.4 The Participation Decision

The standard unitary framework deals with the participation decision of an agent by means of the definition of a reservation wage. At this wage, the agent is indifferent between working and not working. A reasonable generalization of this definition under a collective model with two adult members is that at the reservation wage of one household member, not only that member is indifferent between working and not working, but also that the other member is indifferent (Blundell et al. 2007).

To characterize the participation decision of a household member, a procedure similar to the one used by Neary and Roberts (1980) is employed to model household behavior under rationing, or more generally of quantity constraints, which is characterized in terms of its unconstrained behavior when faced with shadow prices. The reservation wage of i (ϖ^i) is defined by:

$$\varpi^i = \frac{U^i_{h^i}\left(1, \rho^i, \bar{K}\right)}{U^i_{C^i}\left(1, \rho^i, \bar{K}\right)}$$

where the notation f_x stands for the partial derivative of function f with respect to variable x (here $f = U^i$ and $x = h^i, C^i$). This equation is the marginal rate of substitution between leisure and private consumption computed along the axis $h^i = 0$ for a given sharing rule ρ^i (and equal to C^i) and a level of public expenditures equal to $\bar{K}$.

To concentrate on the second stage of the household problem (2.1), particularly on labor supply decisions, public expenditures are fixed at some arbitrary level $\bar{K}$. In this way, the problem is basically reduced to that considered by Donni (2003), in which the participation decision is analyzed in a framework with only private goods. As above, let $\mathcal{O}$ be some open subset of $\mathcal{K}$ such that $\partial K/\partial z$ does not vanish on $\mathcal{O}$, and impose the condition $K^*\left(w^m, w^f, Y, z\right) = \bar{K}$, where the latter is equivalent, by the implicit function theorem, to $z = \zeta\left(w^m, w^f, Y, \bar{K}\right)$. Let $y = Y - \bar{K}$ denote the portion of non-labor income not devoted to public expenditures which could be positive or negative (labor income can also be used for public consumption). Therefore, if ϖ^i is a function of $\left(w^m, w^f, Y, \bar{K}\right)$, it can be expressed for notational simplicity as $\varpi^i\left(w^m, w^f, y\right)$. Then, i's reservation wage is implicitly defined as a function of $\left(w^m, w^f, y\right)$:

$$
\begin{aligned}
w^i &= \varpi^i\left(w^m, w^f, Y, \zeta\left(w^m, w^f, Y, \bar{K}\right)\right) \\
&= \varpi^i\left(w^m, w^f, Y, \bar{K}\right) \\
&= \varpi^i\left(w^m, w^f, y\right)
\end{aligned} \tag{2.6}
$$

Without additional assumptions, equation (2.6) could have several solutions, i.e., the uniqueness of a reservation wage for member i has to be explicitly postulated under the collective framework. Intuitively, there are two reasons to explain why there can be many wage rates for which i is indifferent between working and not working. The first comes from the assumption that the sharing rule ρ^i depends on i's wage, so there could be more than one combination of w^i and ρ^i at which i is indifferent. The second is related to the possibility that the sharing rule itself may depend on the non-participation of household members. As shown later, the existence of a well-behaved participation frontier is needed to recover the decision process when one member of the couple does not participate in the labor market. A sufficient condition to obtain a unique reservation wage (fixed point) for each member is to define that the function ϖ^i is a contraction mapping.

Assumption R. *For any $\left(w^{m^*}, w^{f^*}, y\right)$ and $\left(w^{m^o}, w^{f^o}, y\right) \in \mathbb{R}^2_+ \times \mathbb{R}$, preferences and the sharing rule are such that there is some non-negative real number $r < 1$ for which the* *following condition is satisfied:*

$$\max_{i=m,f}\left[\left|\varpi^{i}\left(w^{m^{*}},w^{f^{*}},y\right)-\varpi^{i}\left(w^{m^{o}},w^{f^{o}},y\right)\right|\right]\leq r\max_{i=m,f}\left(\left|w^{i^{*}}-w^{i^{o}}\right|\right)$$

Two remarks can be made at this point. First, this condition does not affect the level of public expenditure; z varies to guarantee that public expenditure is exactly $\bar{K}$. Consequently, the distribution factor allows that w^m, w^f, and Y –and thus also ϖ^i– can vary freely, whereas K is kept constant. Second, the assumption only holds in the neighborhood of the participation frontier; in the interior of other household participation sets the allocation of additional income stemming from the participation of one member could be more complex.

In essence, Assumption R restricts the impact on both individual shares (and hence individual consumption) of a change in one household member's wage. This amounts to assuming that the Pareto weights are smooth functions of both wages and non-labor income, and therefore that the smoothness of the individual utilities is preserved at the participation frontier of each individual.[11]

Assumption R is not expected to be very restrictive and it simplifies the analysis by not having to use more restrictive fixed point theorems to ensure the existence of a well-behaved participation frontier. Under this assumption, the system of equations ϖ^m and ϖ^f is a contraction with respect to w^m and w^f for any y. Using the Banach contraction principle,[12] two corollaries of this assumption are:

1. For any y, the functions ϖ^m and ϖ^f have a unique fixed point. Then, there exists a unique pair of wages, $\hat{w}^m(y)$ and $\hat{w}^f(y)$, such that both adult members are indifferent between working and not working.

[11] In order to understand in greater detail the intuition behind Assumption R, the effect on m's private consumption will be analyzed at m's participation frontier first when there is an infinitesimal increase in m's wage, and second when there is an infinitesimal increase in f's wage. When m's wage increases, the magnitude of the increase in m's private consumption depends on whether m is participating or not. When m is not participating, an increase in m's wage probably has a positive impact on m's bargaining power, and both m's reservation wage and consumption share increase. When m is participating, an increase in m's wage also has a positive effect on household income, and m's consumption share increases more.

When f's wage increases, the effect on m's private consumption depends also on whether f is participating or not. When f is not participating, the increase in f's wage reduce m's bargaining power. Since the sharing rule reflects the distribution of power between household members, if individual leisure is a normal good, it is expected that the decrease of m's share is associated with a reduction in m's reservation wage. When f is participating, an increase in f's wage also has a positive effect on household income, which may compensate m's share for the increase in f's bargaining power.

Then, the condition that the difference in m's reservation wage can not be greater in absolute value than the initial increase in m (f)'s wage is satisfied when m's consumption share responds less, in absolute value, to changes on m (f)'s wage when m (f) is not participating than when m (f) is participating.

[12] See Green and Heller (1981) for a definition of contraction and of the Banach contraction principle (contraction mapping theorem).

2. For any w^j $(j \neq i)$ and y, each ϖ^i has a unique fixed point with respect to w^i. Then, there exists a function $\gamma^i(w^j, y)$ such that member i participates in the labor market if and only if $w^i > \gamma^i(w^j, y)$, $i = m, f$.

Considering the possible interactions of household members' participation decision, four connected sets can then be defined:

- Participation set (P): The set of $\left(w^m, w^f, y\right)$ is such that both household members choose to work.
- f's non-participation set (N^f): The set of $\left(w^m, w^f, y\right)$ is such that f chooses not to work and m chooses to work.
- m's non-participation set (N^m): The inverse of N^f.
- Non-participation set (N): The set of $\left(w^m, w^f, y\right)$ is such that both household members choose not to work.

2.3.5 Identification

This section discusses the empirical restrictions on each household member's labor supply implied by the collective setting with children and non-participation. Also, it shows that is possible to recover the structural model (preferences and the sharing rule) simply by observing the labor supplies and the household expenditure on children.

The non-participation set N is not taken into account in identifying individual utilities and the decision process, given the lack of information for this purpose (if the hours of work for both partners are zero, the sharing rule within the household cannot be deduced from the labor supply of both individuals, so individual utilities cannot be recovered). Therefore, it is assumed that at least one of the partners' supplies is an interior solution to (2.1). The following theorem establishes the identification and testability results.

Theorem 1. *Let $\left(\tilde{h}^m, \tilde{h}^f\right)$ be a pair of labor supplies, satisfying the regularity conditions listed in Lemmas 1-3 (below). Under Assumption R:*

1. *Both labor supplies have to satisfy some testable restrictions in the form of partial equations on the participation set P.*

2. *Individual preferences and the sharing rule are identified up to some additive constant $D\left(\bar{K}\right)$ when at least one of the partners works. Moreover, for each choice of $D\left(\bar{K}\right)$, preferences are exactly identified.*

The proof of this theorem is developed in the next subsections. First, subsection 2.3.5.1 identifies the sharing rule in the participation set in which both household members choose to work (P). The knowledge of the two labor supplies in the set P allows recovery of ρ simply by applying a theorem from Chiappori (1992). Next, subsection 2.3.5.2 identifies ρ in the set in which one of the couple does not work (N^f and N^m). The recovery of ρ on the set P can be extended to the set in which one of the couple does not work by the knowledge of the sharing rule along the participation frontier.

2.3.5.1 Identification in the Partners' Participation Set

This case considers only a positive labor supply for both adults. This is the only situation implicitly considered by BCM (2005). For any $\left(w^m, w^f, y\right) \in P$ such that $\tilde{h}_y^m \cdot \tilde{h}_y^f \neq 0$, the following definitions are introduced:

$$A\left(w^m, w^f, y\right) = \frac{\tilde{h}_{w^f}^m\left(w^m, w^f, y\right)}{\tilde{h}_y^m\left(w^m, w^f, y\right)}, \qquad B\left(w^m, w^f, y\right) = \frac{\tilde{h}_{w^m}^f\left(w^m, w^f, y\right)}{\tilde{h}_y^f\left(w^m, w^f, y\right)}$$

Note that A and B are indeed the marginal rates of substitution of the sharing rule $\left(\frac{\rho_{w^f}}{\rho_y} = \frac{\tilde{h}_{w^f}^m\left(w^m,w^f,y\right)}{\tilde{h}_y^m\left(w^m,w^f,y\right)} \text{ and } \frac{\rho_{w^m}}{\rho_y} = \frac{\tilde{h}_{w^m}^f\left(w^m,w^f,y\right)}{\tilde{h}_y^f\left(w^m,w^f,y\right)}\right)$, which can be identified in terms of the observable labor supplies of m and f.

Lemma 1. *It is assumed that $\tilde{h}_y^m \cdot \tilde{h}_y^f \neq 0$, and $AB_y - B_{w^f} \neq BA_y - A_{w^m}$ for any $\left(w^m, w^f, y\right) \in P$. Then for any given $\bar{K}$, the individual preferences and the sharing rule are identified on P up to an increasing function of $\bar{K}$.*

Proof. See Lemma 1 in BCM (2005) and proposition 4 in Chiappori (1992). □

The sketch of the proof is as follows. The idea under a collective framework is that the labor supply of spouse i is affected by changes either in the non-labor income or in j's wage by means of their effects on the sharing rule. Therefore, from (2.3) and (2.4) it is possible to obtain a system of two partial differential equations in ρ:

$$\rho_{w^f} - A\rho_y = 0 \quad \text{and } \rho_{w^m} - B\rho_y = -B$$

The indifference surfaces of i's share can be derived in the space (w^j, y) from noting that if there is a simultaneous change in non-labor income and in j's wage that maintain i's labor supply at the same level, then i's share also remains constant. In addition, j's share can be derived from the fact that both shares must add up to the non-labor income devoted to non-public consumption. The system of partial differential equations can be solved if it is differentiated again and if the symmetry of cross-partial derivatives is taken into account.[13]

The sharing rule and couples' preferences have to be adjusted to consider the presence of public expenditures. For the sharing rule ρ and the pair of utilities U^m and U^f there exists a constant $D\left(\bar{K}\right)$ such that, for all $\left(w^m, w^f, y\right) \in P$

$$\begin{aligned}
\tilde{\rho}\left(w^m, w^f, y\right) &= \rho\left(w^m, w^f, y\right) + D\left(\bar{K}\right) \\
\tilde{U}^m\left(h^m, C^m, \bar{K}\right) &= g^m\left[U^m\left(h^m, C^m - D\left(\bar{K}\right), \bar{K}\right), \bar{K}\right] \\
\tilde{U}^f\left(h^f, C^f, \bar{K}\right) &= g^f\left[U^f\left(h^f, C^f + D\left(\bar{K}\right), \bar{K}\right), \bar{K}\right]
\end{aligned}$$

where g^m and g^f are twice continuously differentiable mappings, increasing in their first argument. The functions $\tilde{U}^i$ and U^i are different, although impossible to distinguish solely from observation of labor supplies,[14] but once $D\left(\bar{K}\right)$ has been chosen, $\tilde{U}^i$ and g^i coincide up to an increasing function of $\bar{K}$.

2.3.5.2 Identification When One Member of the Couple Does Not Participate

In the case where only one of the adult household members works ($w^i > \gamma^i\left(w^j, y\right)$ and $w^j \leq \gamma^j\left(w^i, y\right)$), the observation of i's labor supply characterizes the sharing rule on the set

[13] The solution consists of partial derivatives of the sharing rule that can be deduced from observed labor supplies. Assuming that $AB_y - B_{w^f} \neq BA_y - A_{w^m}$, let $\alpha = \left(1 - \frac{BA_y - A_{w^m}}{AB_y - B_{w^f}}\right)^{-1}$ and $\beta = 1 - \alpha$. The partial derivatives are given by $\rho_y = \alpha$, $\rho_{w^f} = A\alpha$, and $\rho_{w^m} = B\left(\alpha - 1\right) = -B\beta$. In words, α (β) is the share of marginal non-labor income not devoted to public expenditures received by m (f).

[14] The intuition in the case of member m is the following. Switching from ρ and U^m to $\tilde{\rho}$ and $\tilde{U}^m$ affects: 1) the budget constraint of m, with a vertical translation of magnitude $D\left(\bar{K}\right)$; 2) all of m's indifference curves are also shifted downward by $D\left(\bar{K}\right)$, so m's labor supply does not change. Because m's consumption, C^m, cannot be observed, (ρ, U^m) is empirically indistinguishable from $\left(\tilde{\rho}, \tilde{U}^m\right)$.

N^j. In addition, the values of the partial derivatives of the sharing rule are identified on j's frontier by Lemma 1, providing boundary conditions for the identification of the sharing rule on N^j. Indeed, by continuity of $\tilde{h}^i$ and ρ,[15] the recovery of the sharing rule on P can be extended to the frontier between P and N^j if w^j approaches the participation frontier $\gamma^j(w^i, y)$.

To better understand the technique employed, the participation set N^f in which member m works and f does not (i.e., $w^f \leq \gamma^f(w^m, y)$) is initially considered. For any $(w^m, w^f, y) \in int(N^f)$ such that $\tilde{h}^m_y \neq 0$, it is defined that:

$$A\left(w^m, w^f, y\right) = \frac{\tilde{h}^m_{w^f}\left(w^m, w^f, y\right)}{\tilde{h}^m_y\left(w^m, w^f, y\right)}$$

Along f's participation frontier, for any set I^f of (w^m, y) such that $w^m \geq \hat{w}^m(y)$, the following definition is made by a continuity argument, if $\lim_{w^f \uparrow \gamma^f} \tilde{h}^m_y \neq 0$:

$$a\left(w^m, y\right) = A\left(w^m, \gamma^f\left(w^m, y\right), y\right)$$

Lemma 2. *It is assumed that $\lim_{w^f \uparrow \gamma^f} \tilde{h}^m_y \neq 0$ and $1 + a \cdot \gamma^f_y \neq 0$ for any $(w^m, y) \in I^f$ and $\tilde{h}^m_y \neq 0$ for any $(w^m, w^f, y) \in int(N^f)$. Then the sharing rule is identified on N^f up to some additive constant $D(\bar{K})$.*

Proof. The same technique used by Donni (2003) can be applied; the only adjustment that must be made is that the additive constant is indexed by the level of public expenditures. From Lemma 1, it is known that ρ must satisfy the partial differential equation

$$\rho_{w^f} - A\rho_y = 0 \tag{2.7}$$

which characterizes the sharing rule on N^f. Additionally, the sharing rule along the participation frontier $\left(w^f - \gamma^f\left(w^m, y\right) = 0\right)$ gives a boundary condition for the partial differential equation. From standard theorems in partial differential equations theory, the identification of the sharing rule (up to an additive constant) is achieved if the following condition is fulfilled. First, (2.7) can be written as $\nabla \boldsymbol{\rho} \mathbf{u} = 0$, where $\nabla \boldsymbol{\rho}$ denotes the gradient

[15] Although $\tilde{h}^m$, $\tilde{h}^f$, and ρ are generally nondifferentiable along the participation frontiers, it can be shown that couples' labor supplies and the sharing rule are infinitely differentiable in all their arguments on P, $int(N^f)$, and $int(N^m)$ (for an appropiate proof of this result see Theorem A.3 of Magnus and Neudecker 2007, 163).

of ρ and $\mathbf{u}$ is the vector $(0, 1, -A)$. Now, the condition is that $\mathbf{u}$ is not tangent to f's participation frontier. The intuition behind this condition is the following: (2.7) defines the indifference surfaces of the sharing rule (the values of w^f, w^m, and y that keep constant the sharing rule at some level) that pass through f's participation frontier. Since $\bigtriangledown\boldsymbol{\rho}$ is a vector normal to surfaces of constant ρ, and $\mathbf{u}$ indicates the direction in which the sharing rule is constant, (2.7) states that $\mathbf{u}$ is everywhere perpendicular to $\bigtriangledown\boldsymbol{\rho}$. Therefore, $\mathbf{u}$ is a vector that is tangent to the surfaces of constant ρ at every point and, in particular, is a tangent vector to the surface in the participation frontier of f. Given that, on the frontier, A coincides with a, this condition states that, for all $(w^m, y) \in I^f$:

$$1 + a \cdot \gamma_y^f \neq 0$$

If this condition is fulfilled on the frontier, then the partial differential equation (2.7) together with the boundary condition defines ρ up to an additive constant, $D\left(\bar{K}\right)$, in the context analyzed. $\square$

Now, the participation set N^m in which only member f works (i.e., $w^m \leq \gamma^m\left(w^f, y\right)$) is considered. The approach is the same as that for N^f. For any $\left(w^m, w^f, y\right) \in int\left(N^m\right)$ such that $\tilde{h}_y^f \neq 0$, it is defined that:

$$B\left(w^m, w^f, y\right) = \frac{\tilde{h}_{w^m}^f\left(w^m, w^f, y\right)}{\tilde{h}_y^f\left(w^m, w^f, y\right)}$$

Along m's participation frontier, for any set I^m of $\left(w^f, y\right)$ such that $w^f \geq \hat{w}^f(y)$, the following definition is made by a continuity argument, if $\lim_{w^m \uparrow \gamma^m} \tilde{h}_y^f \neq 0$:

$$b\left(w^f, y\right) = B\left(\gamma^m\left(w^f, y\right), w^f, y\right)$$

Lemma 3. *It is assumed that* $\lim_{w^m \uparrow \gamma^m} \tilde{h}_y^f \neq 0$ *and* $1 + b \cdot \gamma_y^m \neq 0$ *for any* $\left(w^f, y\right) \in I^m$ *and* $\tilde{h}_y^f \neq 0$ *for any* $\left(w^m, w^f, y\right) \in int\left(N^m\right)$*. Then the sharing rule is identified on* N^m *up to some additive constant* $D\left(\bar{K}\right)$*.*

Proof. As above, using the partial differential equation

$$\rho_{w^m} - B\rho_y = -B$$

and the boundary condition $w^m - \gamma^m\left(w^f, y\right) = 0$. $\square$

2.4 Final Remarks

The richness of collective models comes from the opportunities the framework provides for considering the theoretical foundations of how individuals share resources within a basic unit of analysis in an intragroup decision-making process such as a household. In this sense, the approach could serve as an empirical tool for understanding intrahousehold allocations, particularly when evaluating policies with a targeting purpose. However, the literature on identifying the structural elements of household behavior in a more general case than private consumption with interior solutions is relatively recent. In particular, the literature has provided some results based on the separate consideration of the presence of children and non-working individuals within a household.

This chapter extends Chiappori's (1992) model of collective labor supply to bring together the decision to participate in the labor market and expenditures on public goods, such as expenditures on children. The chapter unites in a single framework the works of Blundell, Chiappori, and Meghir (2005) for children and Donni (2003) for non-participation. The model generates testable restrictions on household labor supply behavior. In particular, labor supply functions have to satisfy certain structural conditions in the form of partial differential equations. Moreover, the model can recover individual preferences and the sharing rule from the simple observation of adult members' labor supply and expenditure on children. Identifiability when at least one of the partners works requires i) the knowledge of a distribution factor to control for the effect of public consumption on the optimal individual choice of consumption and labor supply; and ii) the explicit postulation of a unique reservation wage to identify the structure in the non-participation sets of each household member.

Two topics for future research are the consideration of household production and empirical application of the model. Wefare comparisons at the individual level can be biased if household production is not taken into account. For example, the specialization of a woman in domestic activities is interpreted as an increase in her individual leisure consumption; her share of household non-labor income is interpreted as a lump-sum transfer from her partner instead of the exchange of her domestic production for market goods. Also, the stochastic specification of the model has to take into account that wages are not observed for non-participants, and that both the labor supply of the participating member and the sharing rule have to be continuous at the participation frontier of the other member. The data necessary for future applications of the model can generally be obtained from household income and expenditure surveys. This type of survey includes information on household composition, income sources, the labor status of individual members, and expenditures on children (for example, education, food, and health care). Considering the issues discussed here, the model is estimated using a sample of Mexican nuclear families in chapter 3.

Chapter 3

Collective Household Labor Supply: An Empirical Approach with Children and Non-Participation

3.1 Introduction

This chapter expands on the empirical literature of collective household labor supply by implementing the model proposed in chapter 2, which simultaneously takes into account the presence of children and the decision to participate in the labor market. The individual preferences of the couple as well as the rule governing the sharing of household resources conditional on expenditures on children are recovered from estimates of adult household members' labor supply and those expenditures. The chapter also investigates whether expenditures on children and male labor supply depend on the woman's wage even when she is not working. Mexican data for nuclear families in which the male partner works are used to estimate this model and test the restrictions implied by it.

As in other developing countries, Mexico's female labor force participation is still at a very low level (Arceo and Campos 2010). However, this low participation does not necessarily imply that women's preferences are not taken into account in household resource allocations. If (potential) wages affect bargaining positions within a household, then any variation in the wage of a female household member will modify household behavior even if she does not work.

There is little empirical literature on collective household labor supply behavior that separately considers the presence of children (Cherchye, de Rock, and Vermeulen forthcoming and chapter 1 of this dissertation) and the decision to participate in the labor market (Blundell, Chiappori, Magnac, and Meghir 2007; Bloemen 2010), and there is no empirical literature that considers the two issues simultaneously. In this chapter a first attempt is made to fill this gap with an empirical implementation of the theoretical model presented in chapter 2. This chapter recovers the rule governing the sharing of household resources conditional on the level of expenditures on children from estimates of a system of equations comprising the woman's participation, the couple's labor supplies, and expenditures on children. Despite rejection of the supplementary assumption of continuity of both the male's labor supply and the sharing rule, the parameter restrictions that are imposed by the collective rationality are not rejected.

The chapter is organized as follows. Section 3.2 briefly discusses the model developed in chapter 2, which includes the possibility of labor force participation and public consumption (like expenses on children). Section 3.3 proposes how to specify the model parametrically, section 3.4 shows the data set used, and section 3.5 presents empirical results. Some final remarks are presented in section 3.6.

3.2 Theoretical Framework

This chapter implements the model presented in chapter 2 of collective household labor supply which, based on the models of Donni (2003) and Blundell, Chiappori, and Meghir (2005), introduces the possibility of non-participation in a framework where parents care about children's welfare. In this section we discuss the main features of the model, which was discussed in detail in chapter 2.

In a household composed of an adult couple and their offspring, it is assumed that: i) (potential) wages could affect bargaining positions of the couple; ii) the utility of each adult member depends on his or her individual consumption and the children's well-being; and iii) the decision about how much is spent on children affects the trade-off between consumption and labor supply at the individual level. Under these assumptions, chapter 2 shows that the underlying structure of the model (individual preferences and the decision process) can be recovered from observed behavior. Identifiability is feasible if we know a distribution factor (i.e., a variable that affects the decision process but not the individual preferences or the joint budget set), and there is a unique reservation wage for each adult household member at which both members are indifferent as to whether a member participates in the labor market or not.

Formally, let $i = m,\ f$ denote, respectively, the household's male and female members. The labor supply of i is denoted by h^i (with $0 \leq h^i \leq 1$), with market wage w^i. There is a Hicksian good whose price is set to unity that is used for private expenditures (C^i) and for public expenditures –in this chapter, expenditures on children, (K). Let Y and z denote, respectively, household non-labor income and a distribution factor.[1] It is assumed that h^m, h^f, and K are observed (as functions of w^m, w^f, Y, and z), whereas, as is standard in the literature on the collective model, the distribution of private consumption within the couple is not.

The basic idea of the collective approach lies in the assumption that intrahousehold decisions lead to Pareto-efficient outcomes, so it is not necessary to make additional assumptions about the decision process. In the case of private and public goods, the efficiency assumption implies the existence of a (conditional) sharing rule; the portion of non-labor income allocated to each member once spending on the public good has been discounted. In this case, the household decision process can be thought of as a two-stage process. In the first stage, the couple agrees on the level of the public expenditure and how to distribute the resulting residual non-labor income between them. To be more concrete, let $K^*\left(w^m, w^f, Y, z\right)$ and $\phi^i\left(w^m, w^f, Y, z\right)$ denote, respectively, the optimal choice of expenditures on children and i's share of residual non-labor income, with $\phi^m\left(w^m, w^f, Y, z\right) + \phi^f\left(w^m, w^f, Y, z\right) = Y - K^*\left(w^m, w^f, Y, z\right)$. The Bowen-Lindahl-Samuelson condition characterizes the efficiency for public expenditures; individual marginal willingness to pay for expenditures on children must add up to its market price ($MWP^m + MWP^f = 1$, where $MWP^i = V^i_K / V^i_{\phi^i}$, V^i is the indirect utility of member i, and the notation f_x stands for the partial derivative of function f with respect to variable x). Then, in the second stage the couple decide, independently from one another, their individual consumption and labor supply. Fixing $K = K^*\left(w^m, w^f, Y, z\right)$, the labor supply and consumption functions, $h^{i^*}\left(w^m, w^f, Y, z\right)$ and $C^{i^*}\left(w^m, w^f, Y, z\right)$, solve as:

$$\max_{h^i, C^i} U^i\left(1 - h^i, C^i, K\right) \quad s.t. \quad C^i = w^i h^i + \phi^i, \qquad i = m, f$$

In what follows, with the idea of focusing on the information contained in the couple's labor supplies, public expenditures are fixed to some arbitrary level $\bar{K}$. Using the condition $K^*\left(w^m, w^f, Y, z\right) = \bar{K}$, the distribution factor can be expressed, by the implicit function

[1] The distribution factors are defined as variables that affect the distribution of the bargaining power between household members, but that do not have any direct influence on the individuals' preferences and the household budget set (after controlling for total income). For a more detailed discussion of distribution factors, see section 1.2; for examples of some that have been used in the literature, see Table A.1.

theorem, as a function ζ of $\left(w^m, w^f, Y, \bar{K}\right)$. Following from this construction, the couples' structural labor supplies are:

$$\tilde{h}^m\left(w^m, w^f, Y, K\right) = H^m\left[w^m, \phi\left(w^m, w^f, Y, \zeta\left(w^m, w^f, Y, \bar{K}\right)\right)\right] \tag{3.1}$$

$$\tilde{h}^f\left(w^m, w^f, Y, K\right) = H^f\left[w^f, Y - \bar{K} - \phi\left(w^m, w^f, Y, \zeta\left(w^m, w^f, Y, \bar{K}\right)\right)\right] \tag{3.2}$$

where $\phi = \phi^m$. In this way, i's labor supply is described as a function of wages, non-labor income, and a distribution factor z such that public expenditures are exactly $\bar{K}$. Hence, the values of w^m, w^f, and Y are not constrained to assure that $K^*\left(w^m, w^f, Y, z\right) = \bar{K}$; the key role of z is to guarantee that the level of public expenditures is exactly $\bar{K}$. From (3.1) and (3.2) it follows that j's wage rate enters i's labor supply function only through the sharing rule. In this way, the problem is basically reduced to that considered by Donni (2003), in which the participation decision is analyzed in a framework with only private goods.

Blundell et al. (2005) show that when both partners work, the knowledge of the two labor supply functions (3.1) and (3.2) allows recovery of the conditional sharing rule and the individual utilities of the couple. Using a theorem in Chiappori (1992), the sharing rule can be recovered by solving a system of first- and second-order order partial differentials of the two labor supply functions conditional on the public-good expenditure.[2]

As in the standard unitary framework, the definition of a reservation wage can be used in the collective framework to address the decision to participate in the labor market. However, because the wage of one partner affects the labor supply of the other via the sharing rule, the sharing rule itself could also depend on the non-participation decision of the couple. Therefore, under the collective approach, the uniqueness of a reservation wage has to be explicitly postulated in order to recover the decision process when one member of the couple does not participate in the labor market. The reservation wage of i (ϖ^i) is defined by:

$$\varpi^i = \frac{U^i_{h^i}\left(1, \phi^i, \bar{K}\right)}{U^i_{C^i}\left(1, \phi^i, \bar{K}\right)}$$

This equation is the marginal rate of substitution between leisure and private consumption computed along the axis $h^i = 0$ for a given sharing rule ϕ^i (and equal to

[2] Define $D = \tilde{h}^m_{w^f}/\tilde{h}^m_y$, $E = \tilde{h}^f_{w^m}/\tilde{h}^f_y$. Assuming that $DE_y - E_{w^f} \neq ED_y - D_{w^m}$, let $\varkappa = \left(1 - \frac{ED_y - D_{w^m}}{DE_y - E_{w^f}}\right)^{-1}$ and $\beta = 1 - \varkappa$. The partial derivatives of the sharing rule with respect to non-labor income and wages are given by $\phi_y = \varkappa$, $\phi_{w^m} = E\left(\varkappa - 1\right) = -E\beta$, and $\phi_{w^f} = D\varkappa$.

C^i) and a level of public expenditures equal to $\bar{K}$. Let $y = Y - \bar{K}$ denotes the portion of the household's non-labor income devoted to private expenditures. Therefore, if ϖ^i is a function of $\left(w^m, w^f, Y, \bar{K}\right)$, it can be expressed for notational simplicity as $\varpi^i\left(w^m, w^f, y\right)$. A sufficient condition to obtain a unique reservation wage for each member is to define the function ϖ^i as a contraction mapping (assumption R in subsection 2.3.4). The recovering of the sharing rule when both partners work can be extended to the case in which one of the couple does not work by knowledge of the rule along the participation frontier. The sharing rule when i chooses not to work and $j \neq i$ chooses to work is defined by the partial differential equation of j's labor supply in ϕ and the fact that both the sharing rule and j's labor supply are continuous along i's participation frontier.[3]

3.3 Parametric Specification

This section proposes an empirical implementation of the model developed in chapter 2. Specific functional forms and simplifying assumptions have been chosen to give a simple but realistic illustration of the model.

3.3.1 Preferences, Labor Supply, Expenditures on Children, and the Sharing Rule

For the illustration of the collective model with expenditures on children and non-participation, it is important to have some relatively simple parametric specification in mind. When both partners work, their individual structural labor supply functions can be specified as:

$$h^m = \psi_0 + \psi_1 \phi^m + \psi_2 \ln w^m + \psi_3 K \tag{3.3}$$

$$h^f = \gamma_0 + \gamma_1 \phi^f + \gamma_2 \ln w^f + \gamma_3 K \tag{3.4}$$

This kind of semi-log specification is popular in empirical work (Blundell, MaCurdy, and Meghir 2007), and also it has already been used in the collective empirical literature

[3] When only the male partner works, the partial differential equation of his labor supply in ϕ is determined by $\phi_{w^f} - D\phi_y = 0$.

(Chiappori, Fortin, and Lacroix 2002). The fact that equations (3.3) and (3.4) are linear in parameters eases the estimation process. The underlying indirect utility functions of the labor supply functions (3.3) and (3.4) are of the Stern (1986) type:

$$V^m\left(w^m, \phi^m\right) = \left(\frac{\exp(\psi_1 w^m)}{\psi_1}\right)\left(\psi_0 + \psi_1\phi^m + \psi_2 \ln w^m + \psi_3 K\right) - \frac{\psi_2}{\psi_1}\int_{\infty}^{\psi_1 w^m} \frac{\exp(t)}{t} dt$$

and

$$V^f\left(w^f, \phi^f\right) = \left(\frac{\exp(\gamma_1 w^f)}{\gamma_1}\right)\left(\gamma_0 + \gamma_1\phi^f + \gamma_2 \ln w^f + \gamma_3 K\right) - \frac{\gamma_2}{\gamma_1}\int_{\infty}^{\gamma_1 w^f} \frac{\exp(t)}{t} dt$$

Applying Roy's identity to each of these indirect utility functions yields the individual labor supply system (3.3) and (3.4). In this specification, K appears non-separably in the utility function of both members.[4] Note that the efficiency condition for public-good expenditures ($MWP^m + MWP^f = 1$) implies the following restriction in parameters:

$$\frac{\gamma_3 - \gamma_1}{\gamma_1} = -\frac{\psi_3}{\psi_1} \tag{3.5}$$

As in Chiappori et al. (2002), the sharing rule is specified as:[5]

$$\begin{aligned} \phi &= \alpha_0 + \alpha_1 Y + \alpha_2 \ln w^m + \alpha_3 \ln w^f + \alpha_4 \ln w^m \ln w^f + \alpha_5 z \qquad (3.6) \\ &= \boldsymbol{\alpha}'\mathbf{W} \end{aligned}$$

From the definition of the sharing rule, the expenditure on children has to satisfy the identity $K = Y - \left(\phi^m + \phi^f\right)$, so that the reduced form is specified as:

$$K = c_0 + c_1 Y + c_2 \ln w^m + c_3 \ln w^f + c_4 \ln w^m \ln w^f + c_5 z \tag{3.7}$$

[4] A similar specification has been used by Conway (1997) to analyze the effect of income taxation and government spending on both sexes' labor supply behavior.

[5] The interaction between log wage rates is included in the specification of the sharing rule because the identifiability of the sharing rule depends on the first and second derivatives of both partners' labor supply functions; the second-order cross-partial derivatives with respect to wages do not vanish.

$$= \mathbf{c}'\mathbf{W}$$

Inserting the sharing rule (3.6) in the structural labor supply functions (3.3) and (3.4), the reduced functions are:

$$\begin{aligned} h^m &= a_0 + a_1 Y + a_2 \ln w^m + a_3 \ln w^f + a_4 \ln w^m \ln w^f + a_5 z & (3.8)\\ &= \mathbf{a}'\mathbf{W} \\ h^f &= b_0 + b_1 Y + b_2 \ln w^m + b_3 \ln w^f + b_4 \ln w^m \ln w^f + b_5 z & (3.9)\\ &= \mathbf{b}'\mathbf{W} \end{aligned}$$

3.3.2 Restrictions of the Model

With the intention to focus on labor supplies, the level of public expenditures is fixed to $K\left(w^m, w^f, Y, z\right) = \bar{K}$. Hence, using the change in variable $y = Y - \bar{K}$ and rearranging equation (3.7), the distribution factor can be expressed as:

$$z = \frac{1}{c_5}\left[\left(1 - c_1\right)\bar{K} - c_0 - c_1 y - c_2 \ln w^m - c_3 \ln w^f - c_4 \ln w^m \ln w^f\right] \tag{3.10}$$

Therefore, using (3.10), the reduced labor supply functions (3.8) and (3.9) can be written also as:

$$\begin{aligned} h^m &= A_0 + A_1 y + A_2 \ln w^m + A_3 \ln w^f + A_4 \ln w^m \ln w^f + A_5 \bar{K} & (3.11)\\ h^f &= B_0 + B_1 y + B_2 \ln w^m + B_3 \ln w^f + B_4 \ln w^m \ln w^f + B_5 \bar{K} & (3.12) \end{aligned}$$

The relation between the parameters of the equations (3.8)-(3.9) and the parameters of (3.11) and (3.12) is shown in Table 3.1.

Using equations (3.11) and (3.12), the conditional sharing rule when both partners work, in terms of the household non-labor income devoted to private expenditures and wages, is characterized by the partial derivatives (see footnote 2):

Table 3.1: Relation between parameters of the reduced labor supply functions

Member m (3.11)	Member f (3.12)
$A_0 = a_0 - \frac{a_5 c_0}{c_5}$	$B_0 = b_0 - \frac{b_5 c_0}{c_5}$
$A_1 = a_1 - \frac{a_5 c_1}{c_5}$	$B_1 = b_1 - \frac{b_5 c_1}{c_5}$
$A_2 = a_2 - \frac{a_5 c_2}{c_5}$	$B_2 = b_2 - \frac{b_5 c_2}{c_5}$
$A_3 = a_3 - \frac{a_5 c_3}{c_5}$	$B_3 = b_3 - \frac{b_5 c_3}{c_5}$
$A_4 = a_4 - \frac{a_5 c_4}{c_5}$	$B_4 = b_4 - \frac{b_5 c_4}{c_5}$
$A_5 = a_1 + \frac{a_5(1-c_1)}{c_5}$	$B_5 = b_1 + \frac{b5(1-c_1)}{c_5}$

$$
\begin{aligned}
\phi_y &= \frac{A_1 B_4}{A_1 B_4 - B_1 A_4} \\
\phi_{w^m} &= \frac{A_4 B_2 + A_4 B_4 \ln w^f}{A_1 B_4 w^m - B_1 A_4 w^m} \\
\phi_{w^f} &= \frac{A_3 B_4 + A_4 B_4 \ln w^m}{A_1 B_4 w^f - B_1 A_4 w^f}
\end{aligned}
$$

Solving this system of differential equations, the conditional sharing rule recovered is:

$$\phi = \tilde{\alpha}_0 + \tilde{\alpha}_1 y + \tilde{\alpha}_2 \ln w^m + \tilde{\alpha}_3 \ln w^f + \tilde{\alpha}_4 \ln w^m \ln w^f \tag{3.13}$$

Table 3.2 shows the parameters of the sharing rule (3.6) and its conditional version (3.13) in terms of the reduced labor suppply functions (3.8-3.9, 3.11-3.12), with $\Delta = (a_1 c_5 - a_5 c_1)(b_4 c_5 - b_5 c_4) - (a_4 c_5 - a_5 c_4)(b_1 c_5 - b_5 c_1)$ and $\tilde{\alpha}_0$ as an unknown constant.

Besides the parameter constraints from the efficiency condition for public-good expenditures (3.5), the collective rationality implies the restrictions:

$$\frac{a_4}{b_4} = \frac{a_5}{b_5} \tag{3.14}$$

$$\frac{a_1 + \frac{a_5}{c_5}(1 - c_1)}{b_1 + \frac{b_5}{c_5}(1 - c_1)} = 1 \tag{3.15}$$

The restriction (3.14) imposes testable cross-equation restrictions in the couple's labor supply functions; under a collective approach and with the chosen functional form of the

Table 3.2: Parameters of the sharing rule

Parameter	Reduced labor suppply functions (3.8-3.9)	(3.11-3.12)
Sharing rule (3.6)		
α_0	$\tilde{\alpha}_0 - \frac{c_0(a_1c_5-a_5c_1)(b_4c_5-b_5c_4)}{\Delta}$	$\tilde{\alpha}_0 - \frac{c_0A_1B_4}{A_1B_4-B_1A_4}$
α_1	$\frac{(1-c_1)(a_1c_5-a_5c_1)(b_4c_5-b_5c_4)}{\Delta}$	$\frac{(1-c_1)A_1B_4}{A_1B_4-B_1A_4}$
α_2	$\frac{(a_4c_5-a_5c_4)(b_2c_5-c_2b_5)-c_2(a_1c_5-a_5c_1)(b_4c_5-b_5c_4)}{\Delta}$	$\frac{A_4B_2-c_2A_1B_4}{A_1B_4-B_1A_4}$
α_3	$\frac{(a_3c_5-a_5c_3)(b_4c_5-b_5c_4)-c_3(a_1c_5-a_5c_1)(b_4c_5-b_5c_4)}{\Delta}$	$\frac{A_3B_4-c_3A_1B_4}{A_1B_4-B_1A_4}$
α_4	$\frac{(a_4c_5-a_5c_4)(b_4c_5-b_5c_4)-c_4(a_1c_5-a_5c_1)(b_4c_5-b_5c_4)}{\Delta}$	$\frac{A_4B_4-c_4A_1B_4}{A_1B_4-B_1A_4}$
α_5	$\frac{-c_5(a_1c_5-a_5c_1)(b_4c_5-b_5c_4)}{\Delta}$	$\frac{-c_5A_1B_4}{A_1B_4-B_1A_4}$
Cond. sharing rule (3.13)		
$\tilde{\alpha}_1$	$\frac{(a_1c_5-a_5c_1)(b_4c_5-b_5c_4)}{\Delta}$	$\frac{A_1B_4}{A_1B_4-B_1A_4}$
$\tilde{\alpha}_2$	$\frac{(a_4c_5-a_5c_4)(b_2c_5-b_5c_2)}{\Delta}$	$\frac{A_4B_2}{A_1B_4-B_1A_4}$
$\tilde{\alpha}_3$	$\frac{(a_3c_5-a_5c_3)(b_4c_5-b_5c_4)}{\Delta}$	$\frac{A_3B_4}{A_1B_4-B_1A_4}$
$\tilde{\alpha}_4$	$\frac{(a_4c_5-a_5c_4)(b_4c_5-b_5c_4)}{\Delta}$	$\frac{A_4B_4}{A_1B_4-B_1A_4}$

labor supply functions, it is required that the ratio of the marginal effects of the interaction between log wage rates has to be equal to the corresponding ratio of the marginal effects of the distribution factor on labor supplies. This restriction stems from the fact that the cross term and the distribution factor enter the labor supply functions only through the sharing rule.

The restriction (3.15) relates the ratio of the marginal effects of the expenditures on children (K) on each partner's labor supply functions. The marginal effect of K is the sum of two terms. The first (a_1 and b_1) is the marginal effect that corresponds to the individual preferences via a change in the household's non-labor income. The second term $[a_5/c_5\ (1-c_1)$ and $b_5/c_5\ (1-c_1)]$ is the marginal change of K on the sharing rule via the distribution factor. Therefore, changes in the expenditures on children only impact individual labor supply functions through income effects, the impact for both partners being equal.

Finally, the parameters of the structural labor supplies (3.3) and (3.4) can be expressed in terms of the parameters of their reduced form (Table 3.3).

Table 3.3: Parameters of the structural labor supply functions

	Structural labor supply function	
τ	m's (3.3): ψ_τ	f's (3.4): γ_τ
0	$A_0 - c_0 A_1 + \frac{A_1 B_5 - A_5 B_1}{B_1 - B_5}\left(\tilde{\alpha}_0 - \frac{c_0 A_1 B_4}{A_1 B_4 - A_4 B_1}\right)$	$B_0 - c_0 B_1 + \frac{A_1 B_5 - A_5 B_1}{A_1 - A_5}\left(\tilde{\alpha}_0 - \frac{c_0 A_4 B_1}{(A_1 B_4 - A_4 B_1)}\right)$
1	$\frac{B_1 A_5 - A_1 B_5}{B_1 - B_5}$	$\frac{A_1 B_5 - B_1 A_5}{A_1 - A_5}$
2	$A_2 + B_2 \frac{A_5 - A_1}{B_1 - B_5}$	$B_3 + A_3 \frac{B_5 - B_1}{A_1 - A_5}$
3	A_5	B_5

If the female partner does not work, there is a regime switch in the male partner's labor supply and the sharing rule, and the parameters change:

$$\begin{aligned} h^m &= \breve{a}_0 + \breve{a}_1 Y + \breve{a}_2 \ln w^m + \breve{a}_3 \ln w^f + \breve{a}_4 \ln w^m \ln w^f + \breve{a}_5 z && (3.16) \\ &= \breve{\mathbf{a}}'\mathbf{W} \\ \phi &= \breve{\alpha}_0 + \breve{\alpha}_1 Y + \breve{\alpha}_2 \ln w^m + \breve{\alpha}_3 \ln w^f + \breve{\alpha}_4 \ln w^m \ln w^f + \breve{\alpha}_5 z && (3.17) \\ &= \breve{\boldsymbol{\alpha}}'\mathbf{W} \end{aligned}$$

To identify the decision process, the model imposes the restrictions that both the male's labor supply function and the sharing rule have to be continuous along the female's participation frontier:

$$\begin{aligned} \breve{\mathbf{a}}'\mathbf{W} &= \mathbf{a}'\mathbf{W} + \mathbf{s}\cdot(\mathbf{b}'\mathbf{W}) && (3.18) \\ \breve{\boldsymbol{\alpha}}'\mathbf{W} &= \boldsymbol{\alpha}'\mathbf{W} + \mathbf{r}\cdot(\mathbf{b}'\mathbf{W}) && (3.19) \end{aligned}$$

Using the partial differential equation of the male's labor supply in ϕ, a relation between s and r is obtained when the female partner does not work:

$$\frac{\tilde{\alpha}_3 + rB_3 + (\tilde{\alpha}_4 + rB_4)\ln w_m}{(\tilde{\alpha}_1 + rB_1)\, w_f} = \frac{A_3 + sB_3 + (A_4 + sB_4)\ln w_m}{(A_1 + sB_1)\, w_f}$$

Using the equalities of the parameters of the sharing rule (3.13) shown in Table 3.2, the relation $r = \frac{sB_4}{\Delta}$ is obtained.

3.3.3 Stochastic Specification and the Likelihood Function

For household t, starting from equations (3.7-3.9) and (3.16), the complete system of equations to estimate, by maximum likelihood, can be described as:

$$
\begin{aligned}
K_t &= \mathbf{c}'\mathbf{W}_t + \mathbf{\Gamma}_K \mathbf{X}_{tK} + \varepsilon_{tK} \\
h_t^f &= \begin{cases} h_t^{f*} = \mathbf{b}'\mathbf{W}_t + \mathbf{\Gamma}_f \mathbf{X}_{tf} + \varepsilon_{tf} & \text{if } h_t^{f*} > 0 \\ 0 & \text{if } h_t^{f*} \leq 0 \end{cases} \\
h_t^m &= \begin{cases} h_{tp}^m = \mathbf{a}'\mathbf{W}_t + \mathbf{\Gamma}_m \mathbf{X}_{tm} + \varepsilon_{tp} & \text{if } h_t^{f*} > 0 \\ h_{tnp}^m = \mathbf{a}'\mathbf{W}_t + \mathbf{\Gamma}_m \mathbf{X}_{tm} + s\cdot \left(\mathbf{b}'\mathbf{W}_t + \mathbf{\Gamma}_f \mathbf{X}_{tf}\right) + \varepsilon_{tnp} & \text{if } h_t^{f*} \leq 0 \end{cases}
\end{aligned} \tag{3.20}
$$

where $\mathbf{X}_{tl}$ is a vector of exogenous variables. The approach adopted to allow stochastic terms on the right-hand side of these equations is to add an error term to each equation, where the vector of errors $(\varepsilon_{tp}, \varepsilon_{tnp}, \varepsilon_{tf}, \varepsilon_{tK})'$ follows a joint normal distribution with a covariance matrix:

$$
\mathbf{\Sigma} = \begin{bmatrix} \sigma_p^2 & \sigma_p \sigma_{np} \rho_{p,np} & \sigma_p \sigma_f \rho_{p,f} & \sigma_p \sigma_K \rho_{p,K} \\ \sigma_p \sigma_{np} \rho_{p,np} & \sigma_{np}^2 & \sigma_{np} \sigma_f \rho_{np,f} & \sigma_{np} \sigma_K \rho_{np,K} \\ \sigma_p \sigma_f \rho_{p,f} & \sigma_{np} \sigma_f \rho_{np,f} & \sigma_f^2 & \sigma_f \sigma_K \rho_{f,K} \\ \sigma_p \sigma_K \rho_{p,K} & \sigma_{np} \sigma_K \rho_{np,K} & \sigma_f \sigma_K \rho_{f,K} & \sigma_K^2 \end{bmatrix} \tag{3.21}
$$

The stochastic model is a type 4 Tobit model (Amemiya 1985) or switching regression model (Maddala 1983), with simultaneity. The log-likelihood function of the econometric model is:

$$
\ln L = \sum_{t=1}^{T} \left\{ \begin{array}{l} \ln\left(\frac{1}{\sigma_K}\phi\left(S_{tK}\right)\right) + I_t \left[\ln\left(\frac{1}{\sigma_{z_p}}\phi\left(S_{tp}\right)\right) + \ln\left(\frac{1}{\sigma_{z_f}}\phi\left(S_{tf}\right)\right)\right] \\ + \left(1 - I_t\right)\left[\ln\left(\frac{1}{\sigma_{z_{np}}}\phi\left(S_{tnp}\right)\right) + \ln\left(1 - \Phi\left(\eta_{tf}\right)\right)\right] \end{array} \right\}
$$

where

$$
\begin{aligned}
I_t &= \begin{cases} 1 & \text{if } \quad h_t^{f*} > 0 \\ 0 & \text{if } \quad h_t^{f*} \leq 0 \end{cases} \\
S_{tK} &= \frac{K_t - \mathbf{c}'\mathbf{W}_t}{\sigma_K}
\end{aligned}
$$

$$
\begin{aligned}
S_{tp} &= \frac{\left[h_{tp}^{m} - \mathbf{a}'\mathbf{W}_t\right] - \sigma_p \left[\left(\frac{\rho_{p,f}-\rho_{p,K}\rho_{f,K}}{1-\rho_{f,K}^2}\right)\left(\frac{h_t^f-\mathbf{b}'\mathbf{W}_t}{\sigma_f}\right) + \left(\frac{\rho_{p,K}-\rho_{p,f}\rho_{f,K}}{1-\rho_{f,K}^2}\right)\left(\frac{K_t-\mathbf{c}'\mathbf{W}_t}{\sigma_K}\right)\right]}{\sigma_p\sqrt{1-\left(\frac{\rho_{p,f}-\rho_{p,K}\rho_{f,K}}{1-\rho_{f,K}^2}\right)\rho_{p,f} - \left(\frac{\rho_{p,K}-\rho_{p,f}\rho_{f,K}}{1-\rho_{f,K}^2}\right)\rho_{p,K}}} \\
\sigma_{z_p} &= \sigma_p\sqrt{1-\left(\frac{\rho_{p,f}-\rho_{p,K}\rho_{f,K}}{1-\rho_{f,K}^2}\right)\rho_{12} - \left(\frac{\rho_{p,K}-\rho_{p,f}\rho_{f,K}}{1-\rho_{f,K}^2}\right)\rho_{p,K}} \\
S_{tf} &= \frac{\left[h_t^f - \mathbf{b}'\mathbf{W}_t\right] - \sigma_f\rho_{f,K}\left(\frac{K_t-\mathbf{c}'\mathbf{W}_t}{\sigma_K}\right)}{\sigma_f\sqrt{1-\rho_{f,K}^2}} \\
\sigma_{z_f} &= \sigma_f\sqrt{1-\rho_{f,K}^2} \\
S_{tnp} &= \frac{\left[\frac{h_{tnp}^m-(\mathbf{a}'\mathbf{W}_t+s\cdot(\mathbf{b}'\mathbf{W}_t))}{\sigma_{np}}\right] - \rho_{np,K}\left(\frac{K_t-\mathbf{c}'\mathbf{W}_t}{\sigma_K}\right)}{\sqrt{1-\rho_{np,K}^2}} \\
\sigma_{z_{np}} &= \sigma_{np}\sqrt{1-\rho_{npK}^2} \\
\eta_{tf} &= \frac{\left(\frac{\mathbf{b}'\mathbf{W}_t}{\sigma_f}\right) + \rho_{np,f}\left(\frac{h_{tnp}^m-(\mathbf{a}'\mathbf{W}_t+s\cdot(\mathbf{b}'\mathbf{W}_t))}{\sigma_{np}}\right) + \left(\rho_{f,K}-\rho_{np,f}\rho_{np,K}\right)\left(\frac{K_t-\mathbf{c}'\mathbf{W}_t}{\sigma_K}\right)}{\sqrt{\left(1-\rho_{f,K}^2\right)\left(1-\frac{\left(\rho_{np,f}-\rho_{np,K}\rho_{f,K}\right)^2}{\left(1-\rho_{np,K}^2\right)\left(1-\rho_{f,K}^2\right)}\right)}}
\end{aligned}
$$

Up to this point it has been assumed that both partners' wages are always observed, even if someone is not working. In order to estimate the model (3.20-3.21), the following equations are specified for women:

$$
\begin{aligned}
\ln w_t^f &= \boldsymbol{\pi}'\mathbf{X}_{tw^f} + u_t && (3.22) \\
\ln w_t^m \ln w_t^f &= \boldsymbol{\psi}'\mathbf{X}_{tw^f w^m} + u_t && (3.23)
\end{aligned}
$$

Based on Wooldridge (2002), for non-working women the empirical analysis uses a Tobit selection procedure for imputing both a wage rate and the interaction between the couple's wage rates, taking into account the simultaneity between expenditures on children and the couple's labor decisions. First, using all of the sample, a standard Tobit of h_t^f on all the exogenous variables is estimated:

$$
h_t^f = b_0 + b_1 Y + b_2 \ln w^m + \tilde{\boldsymbol{\pi}}'\mathbf{X}_{tw^f} + \tilde{\boldsymbol{\psi}}'\mathbf{X}_{tw^f w^m} + b_5 z + \boldsymbol{\Gamma}_f\mathbf{X}_{tf} + \boldsymbol{\Gamma}_K\mathbf{X}_{tK} + v_t
$$

Thus, all variables that determine the female's log wage rate are included as well as the cross product of the couple's log wage rates and expenditures on children. Then, using observations for which $h_t^f > 0$, equations (3.22) and (3.23) are estimated including the residuals $\hat{v}_t$ from the previous step as a covariate. In this parametric approach, the female's log wage rate and the cross product of the couple's log wage rate equations are identified from the exclusion of household non-labor income, the distribution factor, the male partner's age and education, a second-order polynomial in the number of children in the household under 15, and a dummy variable for the number of children under five. To identify the effect of the woman's log wage rate and the cross product of the couple's log wage rate on the woman's labor supply, it is necessary that $\mathbf{X}_{tw^f}$ and $\mathbf{X}_{tw^f w^m}$ each contain at least one variable not in $\mathbf{X}_{tf}$ and $\mathbf{X}_{tK}$. The chosen variables for $\mathbf{X}_{tf}$ are the cross product of the woman's age and education (see, e.g., Mroz 1987), and the unemployment rate by state and by year-quarter of the first survey visit to the household as a means of accounting for local labor market conditions. For $\mathbf{X}_{tw^f w^m}$, the male partner's log wage, the same variables considered for $\mathbf{X}_{tf}$, and the interaction between them. The choice of instruments was based on the discussion in Wooldridge (2002) of identification in simultaneous equations models that are nonlinear in endogenous variables, particularly models with interactions between exogenous variables (here, $\ln w_t^m$) and endogenous variables (here, $\ln w_t^f$).

Finally, the fitted values of $\ln w^f$ and $\ln w^m \ln w^f$ are calculated, correcting for selection bias $\left(\widehat{\ln w_t^f} = \hat{\boldsymbol{\pi}}'\mathbf{X}_{tw^f}, \ln \widehat{w^m \ln} w^f = \hat{\boldsymbol{\psi}}'\mathbf{X}_{tw^f w^m}\right)$.

3.4 Data

A survey that satisfies the data requirements for applying the model is the Mexican Family Life Survey (MxFLS/ENNVIH for its abbreviation in Spanish). It is a multi-thematic and longitudinal survey, elaborated by researchers of the Universidad Iberoamericana (UIA) and the Centro de Investigación y Docencia Económicas (CIDE).

From the original sample (8,328 households), a subsample is first extracted from the second wave (2005-2006) that includes nuclear families only with children under 15 years of age (1,921 households, 48.15% of nuclear families). The reason for using only nuclear families is to focus on households where the decision process is centralized in the parents, reducing the possibility of interaction with other kin within the household. Also, the analysis is focused on children under 15 because a child of this age is not likely to have bargaining power in household decisions.

Next, the sample is restricted to couples living together, where both the male and the female are less than 60 years of age. The sample excludes households where a member is unemployed (the choice between working or not has to be freely-made, to avoid misinterpretation of the findings), self-employed (to avoid problems in measuring labor income), or working without remuneration. Households where the male partner is not employed are dropped because their number is negligible in the subsample. These selection rules and the exclusion of households with missing and outlier data leave us with a total of 1,002 households. The information on wage rates and working hours of both partners is used, as well as information on women with missing wage rates. Expenditures on children include education (enrollment fees, exams, school supplies, uniforms, and transportation), clothes and shoes, toys, and clothes and items for babies. Non-labor income is the annual household current income minus the couple's labor incomes.

Table 3.4 reports descriptive statistics of the final sample. In this sample, only 18 percent of women participate in the labor market (180 of 1,002 women). The low female participation rate represents a challenge to the model estimation since the procedure for imputing potential wages to all women in the sample is based upon the information from working women. The mean annual number of working hours is 308 for all women in the sample and 2,408 for men. However, working women have on average an hourly wage rate higher than men (MXN $43 versus $29). Using the procedure described in the previous section, the female's log wage rate and the interaction between the couple's log wage rates are then replaced for all observations by their respective fitted values (see Table B.1). There is no significant difference in years of education (approximately eight years). The mean age for women is about two years less than that of men.

In the collective framework, the intrahousehold decision process depends on a range of variables that reflects the household environment. These variables, also known as distribution factors, leave the individual preferences and the joint budget set unchanged and can only shift the distribution of power. The sex ratio is a distribution factor that is a proxy for the situation in the couple-matching market; it reflects the couple's outside opportunities and can influence ultimately the final allocation of resources. It has been used by Chiappori, Fortin, and Lacroix (2002) and Park (2007), among others. In theory, a higher sex ratio (denoting a smaller percentage of women on the couple-matching market) improves the female's bargaining position; if the relationship dissolves, she has a higher probability of finding a new partner than he does, so he is willing to concede to her a larger share of the gains of living in a couple in order to avoid an end to the relationship. Following Park (2007), two kinds of sex ratio variables at the state level are constructed using the microdata sample data from the *Conteo de Población y Vivienda* of 2005. The age-to-age sex ratio is the number of men of the same age as the male partner of each household over the corresponding number of women. A 2-year-band sex ratio is also calculated; this ratio uses the weighted sum of women who are at most two years younger than the male partner of the household.[6] This alternative distribution factor is probably a better summary measure of the couple-matching market, since in the sample there is on average a difference of two years in the couple's ages.

Table 3.4: Descriptive statistics

	Mean	Std. Dev.
Woman		
Not employed (percentage)	82.04	
Employed (percentage)	17.96	
Working hours per year	307.71	762.41
Wage rate (MXN per hour)	42.81	88.98
Age	30.15	6.46
Years of education	8.50	3.74
Man		
Working hours per year	2,407.66	880.61
Wage rate (MXN per hour)	28.30	43.68
Age	32.70	7.02
Years of education	8.69	3.93
Expenditures on children (MXN per year)	4,105.25	6,362.91
Non-labor income (MXN per year)	9,822.41	15,686.06
Number of children under 15 years	2.15	1.02
Children under 5 years (percentage)	62.77	
Sex ratio:		
Age-to-age	0.90	0.07
2-year-band	0.88	0.07
Number of observations	1,002	

[6] The 2-year-band sex ratio is based on the assumption that a man and a woman aged 15 years or older can be a couple with an equal chance if the man is between zero and two years older than the woman. Then, the 2-year-band sex ratio for age x is defined as $M_x \Big/ \sum_{\tau=0}^{2} q_{x-\tau}^{x} I\left(x-\tau \geq 15\right) F_{x-\tau}$ where $q_{x-\tau}^{x} = M_x \Big/ \sum_{l=0}^{2} I\left(x-\tau+l \geq 15\right) M_{x-\tau+l}$, M_ϱ is the number of ϱ-year-old men, F_ϱ is the number of ϱ-year-old women, and $I\left(\cdot\right)$ the indicator function. The weight, $q_{x-\tau}^{x}$, is the probability that an x-year-old man is matched to a woman who is younger than him by τ years.

3.5 Estimation Results

Tables 3.5-3.7 and B.2 show the parameter estimates of the unrestricted model (3.20-3.21), which assumes that the male's labor supply function is continuous along the female's participation frontier, and its associated collective version, which imposes the restrictions (3.14-3.15) in the estimation process (the constrained parameters are identified with the symbol ††). Two versions are estimated, one using the age-to-age sex ratio variable as a distribution factor (denoted by (1)), and the other using the 2-year-band sex ratio variable (denoted by (2)). Using the set of log-likelihood values for each model (see Table B.2) it is possible to construct likelihood-ratio statistics to test the collective restrictions (3.14-3.15). In the version employing the age-to-age (2-year-band) sex ratio, the test statistic of 1.79 (4.55) is to be compared with the critical value of $\chi^2_{0.05}(2) = 5.99$. The collective model is not rejected for the two sex ratio versions even at the 1% level. This finding is consistent with the hypothesis that the presence of children in a household generates non-separabilities in individual consumption, since other documents that have not explicitly considered this aspect have usually rejected the collective rationality when they analyze a household with children (see Fortin and Lacroix 1997; Donni 2007). In general terms, the two constraints, (3.14) and (3.15), imposed on the parameters by the collective model do not appear to be very restrictive, since their implied values are relatively close to their unconstrained counterparts, an observation consistent with the likelihood-ratio test of collective rationality. However, only 32 parameters of the unrestricted version with the age-to-age sex ratio, out of 65, are statistically significant at the 10% level (31 for the version with the 2-year-band ratio). For the collective model, only 30 (age-to-age) and 28 (2-year-band) are significant at the same level. Although the effect of some important variables is quite precisely measured, this limited number of significant parameters can be explained, at least partially, by the small size of the sample.

Table 3.5 presents the estimates of the parameters of expenditures on children. The magnitudes of the coefficients are very similar in the unrestricted and the collective versions. The marginal effect of a change in the male's wage rate on the expenditures on children is $\left(c_2 + c_4 \ln w^f\right)/w^m$, so for all specifications and everything else being equal, an increase in the male's wage rate implies an increase in the money spent on children if the female's wage is more than MXN \$8 [that is if $w^f > \exp\left(-c_2/c_4\right)$], which is the case for the large majority of the sample. For example, in both versions of the unrestricted model, at the mean wage rate of both parents, a MXN \$1 increase in the male's wage (equivalent to an annual increase of MXN \$2,408 in labor income at the mean hours worked by men) increases the annual expenditure on children by approximately MXN \$61. The marginal effect of the female's wage rate is determined by $\left(c_3 + c_4 \ln w^m\right) / w^f$. The marginal effect of the female's wage rate is positive if the male's wage is larger than MXN \$23 using the age-to-age sex ratio as distribution factor, and \$26 with the 2-year-band (that is if $w^m > \exp\left(-c_3/c_4\right)$); it is positive for just over half of the sample. In the unrestricted model with the age-to-age sex ratio as distribution factor and at the mean wage rate of both parents, a MXN \$1 increase in the mother's wage (equivalent to an annual increase of MXN \$308 in her labor income, at the mean hours worked by women) increases the annual expenditure on children by approximately MXN \$5 (approximately \$2 with the 2-year-band). The non-labor income seems not to be statistically significant at conventional levels.

The age-to-age sex ratio has a negative and statistically significant effect on expenditures on children; for example, a one-standard deviation increase in the age-to-age sex ratio (0.07 points) reduces the annual money spent on children by approximately MXN \$646 in the unrestricted model. Because an increase in the sex ratio is related to an increase in the bargaining power of the female partner (and a corresponding decrease in that of the male partner), this result suggests initially that fathers care more for their children than mothers (under the proposed specification, the adequate indicator of parents' preferences regarding children is their marginal willingness to pay, whose estimated values are shown later in this chapter). These results thus reject the implication of the unitary approach that no distribution factor is associated with intra-household allocations.

Most parameter estimates of the control variables are statistically significant at conventional levels. As expected, the presence of a larger number of children under 15 increases the expenditure on them. However, if a child under five is present, if all else is equal, the expenditures are reduced. Children under five contribute to higher expenditures through the count of total children, but an autonomous correction is made since there are no school expenditures for them and thus the total measure of expenditures on children tends to be smaller. Parents' education has a positive effect on the expenditures on children, especially the female's; while an additional year in the male's education increases the annual money spent on children by approximately MXN \$215, that same factor in the female's education increases the expenditure by MXN \$300.

Table 3.5: Parameter Estimates. Expenditures on Children [Model (3.20-3.21)]

	Unrestricted Model		Collective Model †	
	(1)	(2)	(1)	(2)
$\ln w^m$	-2,110.244***	-2,052.694***	-2,107.689***	-2,052.502***
	(755.282)	(761.974)	(755.251)	(761.966)
$\ln w^f$	-3,218.229**	-3,282.515**	-3,227.715**	-3,303.747**
	(1,570.866)	(1,586.456)	(1,570.793)	(1,586.447)
$\ln w^m \ln w^f$	1,023.365***	1,012.113***	1,023.439***	1,013.046***
	(278.762)	(281.249)	(278.765)	(281.280)
Non-labor income (††)	0.020	0.019	0.020	0.019
	(0.013)	(0.013)	(0.013)	(0.013)
Sex ratio (††):				
Age-to-age	-9,224.334**	—	-8,948.663**	—
	(3,831.198)		(3,805.978)	
2-year-band	—	-5,914.022	—	-5,343.813
		(4,218.697)		(4,203.801)
Female's education	303.542***	300.054***	303.477***	300.087***
	(68.441)	(68.209)	(68.439)	(68.209)
Female's age	102.993*	100.598*	102.779*	100.317*
	(54.265)	(54.496)	(54.262)	(54.492)
Male's education	214.400***	217.284***	214.513***	217.274***
	(58.844)	(58.949)	(58.842)	(58.950)
Male's age	-36.331	-48.868	-36.631	-48.690
	(43.559)	(43.455)	(43.555)	(43.452)
N. of children < 15	1,584.207**	1,623.509**	1,587.529**	1,625.201**
	(681.113)	(682.118)	(681.137)	(682.180)
N. of children < 15 squared	-177.249	-181.418	-177.746	-181.634
	(126.852)	(127.079)	(126.863)	(127.091)
Children < 5	-1,292.749***	-1,300.210***	-1,293.948***	-1,300.883***
	(448.442)	(449.293)	(448.448)	(449.314)
Intercept	11,112.689*	8,774.182	10,912.489*	8,349.874
	(6,521.690)	(6,728.217)	(6,514.149)	(6,725.028)
Region dummies	Yes	Yes	Yes	Yes

Note. * p<0.1, ** p<0.05, *** p<0.01. Standard errors in parentheses. The regions are: North, Capital, Gulf, Pacific, South, Central-North, and Central.
† Restrictions (3.14-3.15) are imposed in the estimation process.
†† Parameter constrained in the estimation process by imposing the restrictions (3.14-3.15).

Table 3.6 shows the estimates of the parameters of the reduced female household member's labor supply function. The own-wage effect of female labor supply is determined by $\left(b_3 + b_4 \ln w^m\right) / w^f$. This is positive at male hourly wage rates inferior to MXN \$9 but the negative backward bending effect dominates for higher male wage rates. Therefore, if the husband earns more than MXN \$9, a higher potential wage for the woman does not result in a greater labor supply for her; only if the man earns less than MXN \$9 is the wife inclined to work more hours. The cross-wage effect of female labor supply, $\left(b_2 + b_4 \ln w^f\right) / w^m$, is positive for female wage rates less than MXN \$62 in the model with the age-to-age sex ratio as a distribution factor (and for rates less than MXN \$59 using the 2-year-band). Thus, for the most relevant female wage range, all other factors being equal, women who participate work more if the husband has a higher wage, but for those women who do not work, the probability of starting to participate increases with the wage of their partner. In sum, the own-wage income effect tends to dominate the substitution effect for very small values of the male wage rate, while a woman tends to increase her working hours upon a wage increase of her partner within a wide range of her own wage rate.

The parameter of the sex ratio variable in the couple's reduced labor supply functions is the result of two effects, one an effect of the sharing rule and the other of the expenditures on children (see the couple's structural labor supply functions (3.3) and (3.4)). Interestingly, the effect of both sex ratios on the female's labor supply is positive, but imprecisely determined, in both the unrestricted and collective model. In the collective version, the magnitude of both sex ratios is smaller and better determined: the age-to-age sex ratio parameter passes from a p-value of 60% in the unrestricted model to 21% in the collective one, while the corresponding value for the 2-year-band falls from 53% to 33%.

With respect to the control variables, the female household member's age and education have a significantly positive effect on her labor supply. As expected, an increase in the number of children, other factors being equal, is accompanied by a decrease in her number of hours worked; the presence of a pre-school child also reduces the number of hours worked.

Table 3.6: Parameter Estimates. Female Labor Supply [Model (3.20-3.21)]

	Unrestricted Model		Collective Model †	
	(1)	(2)	(1)	(2)
$\ln w^m$	1,089.380**	1,105.518**	1,096.719**	1,105.610**
	(450.590)	(455.006)	(436.139)	(441.196)
$\ln w^f$	574.838	589.798	617.201	630.653
	(875.231)	(881.747)	(859.036)	(865.680)
$\ln w^m \ln w^f$ (††)	-264.210*	-270.585*	-267.814*	-270.757*
	(151.476)	(152.873)	(146.559)	(148.305)
Non-labor income (††)	0.008	0.007	0.008	0.008
	(0.006)	(0.006)	(0.006)	(0.006)
Sex ratio (††):				
Age-to-age	1,187.142	—	121.586	—
	(2,261.005)		(97.847)	
2-year-band	—	1,585.192	—	72.249
		(2,518.236)		(74.347)
Female's education	210.729***	209.617***	209.857***	209.338***
	(39.431)	(39.240)	(39.300)	(39.106)
Female's age	95.593***	95.413***	96.436***	96.679***
	(33.292)	(33.419)	(33.249)	(33.326)
Male's education	4.616	4.477	5.157	5.322
	(34.418)	(34.452)	(34.394)	(34.390)
Male's age	-24.030	-21.364	-23.447	-23.349
	(26.560)	(26.694)	(26.525)	(26.533)
N. of children < 15	-355.098***	-357.618***	-357.286***	-357.594***
	(129.531)	(129.528)	(129.411)	(129.376)
Children < 5	-578.354**	-574.808**	-581.987**	-582.713**
	(267.455)	(267.615)	(267.518)	(267.409)
Intercept	-8,138.800**	-8,559.303**	-7,371.339**	-7,373.822**
	(3,929.801)	(4,075.067)	(3,438.628)	(3,480.545)
Region dummies	Yes	Yes	Yes	Yes

Note. * p<0.1, ** p<0.05, *** p<0.01. Standard errors in parentheses. The regions are: North, Capital, Gulf, Pacific, South, Central-North, and Central.

† Restrictions (3.14-3.15) are imposed in the estimation process.

†† Parameter constrained in the estimation process by imposing the restrictions (3.14-3.15).

Table 3.7 reports the estimates of the parameters of the reduced male labor supply function. In a working couple, the own-wage effect of the labor supply, $\left(a_2 + a_4 \ln w^f\right)/w^m$, is always negative and the cross-wage effect, $\left(a_3 + a_4 \ln w^m\right)/w^f$, is positive for a wide range of male wage rates. The former indicates a backward bending of the male labor supply, and the latter suggests that men tend to increase working hours upon a wage increase of their partner. Evidence of this male labor supply behavior has been also found for the Netherlands by Bloemen (2010) and Kapteyn, Kooreman, and van Soest (1990) when male and female labor supply is estimated simultaneously.

Comparing the unrestricted model with the collective one, there is a change of sign in the effect of both sex ratios on the male labor supply; it passes from a positive effect to a negative one. The constraints (3.14) and (3.15) imposed by the collective model seem to be restrictive regarding the influence of distribution factors on the male's hours worked. Nevertheless, only the unrestricted model with the 2-year-band sex ratio as a distribution factor is statistically significant at the 5% level. Within the control variables, only the male's education is significant (with a positive sign for all estimated versions) in the male's labor supply.

The parameter estimate of s, associated to (3.18), that determines the assumption of a regime switch in the male's labor supply and its continuity along the female participation frontier, is negative but not estimated precisely. Bloemen (2010), under a similar logic of the parametric specification for a sample of all the possible combinations of working and non-working partners, has found for a sample of Dutch couples that the corresponding parameter for a working husband and a non-working wife is statistically significant, whereas the parameter associated to a working wife and a non-working husband is not significantly different from zero. This unsatisfactory result does not constitute a rejection of the collective approach but instead a rejection of the auxiliary assumptions of a continuous regime switch of the male labor supply function due to a change in the female's participation decision. Female non-participation in the labor market affects the working hours of her partner via her potential wage and the correlation between them ($\rho_{np,f} \approx -0.53$, see Table B.2), but a non-working female partner does not involve a continuous shift in the male labor supply. The reason for the rejection of a regime switch may be that the female reservation wage tends to show little variation and is only captured by the correlation coeficient.

Table 3.7: Parameter Estimates. Male Labor Supply [Model (3.20-3.21)]

	Unrestricted Model		Collective Model †	
	(1)	(2)	(1)	(2)
$\ln w^m$	-182.662	-174.075	-185.539	-185.434
	(142.057)	(143.793)	(141.054)	(142.222)
$\ln w^f$	492.879**	517.573**	466.796**	467.926**
	(221.783)	(223.809)	(220.338)	(222.300)
$\ln w^m \ln w^f$ (††)	-65.346	-68.001	-63.438	-63.478
	(45.570)	(46.151)	(45.206)	(45.613)
Non-labor income (††)	-0.002	-0.002	-0.003	-0.003
	(0.002)	(0.002)	(0.002)	(0.002)
Sex ratio (††):				
Age-to-age	-631.514	—	28.800	—
	(530.699)		(39.170)	
2-year-band	—	-1,164.464**	—	16.938
		(586.188)		(25.351)
Female's education	15.906	16.632	15.410	15.690
	(17.084)	(17.090)	(16.984)	(16.929)
Female's age	12.218	12.831	11.666	11.755
	(10.046)	(10.087)	(10.063)	(10.094)
Male's education	19.426**	19.736**	19.503**	19.486**
	(8.080)	(8.069)	(8.090)	(8.091)
Male's age	-7.494	-8.715	-8.159	-8.124
	(6.242)	(6.160)	(6.209)	(6.206)
N. of children < 15	-12.051	-12.624	-9.483	-9.406
	(37.455)	(37.684)	(37.466)	(37.492)
Children < 5	-67.505	-70.096	-65.831	-65.657
	(76.937)	(76.875)	(77.083)	(77.120)
Intercept	1,948.241*	2,321.786**	1,490.660	1,485.768
	(1,112.535)	(1,152.092)	(1,007.380)	(1,011.289)
s	-0.043	-0.046	-0.041	-0.040
	(0.083)	(0.084)	(0.083)	(0.083)
Region dummies	Yes	Yes	Yes	Yes

Note. * p<0.1, ** p<0.05, *** p<0.01. Standard errors in parentheses. The regions are: North, Capital, Gulf, Pacific, South, Central-North, and Central.

† Restrictions (3.14-3.15) are imposed in the estimation process.

†† Parameter constrained in the estimation process by imposing the restrictions (3.14-3.15).

With respect to the nuisance parameters (Table B.2), all the standard deviations of the dependent variables are estimated precisely. Additionally, the only correlations that are statistically significant at the 10% level are those between the female's participation equation and the male's labor supply when she does not work (negative), and the female's participation equation and the expenditures on children (positive). These findings suggest that unobserved variables that influence women's decision to participate in the labor market are negatively correlated with those that similarly influence men's hours worked, and positively correlated with money spent on children.

Given the empirical framework of section 3.3, it is possible to recover the parameters of the conditional sharing rule (3.6) and (3.13) when both partners work, as well as the parameter r in (3.19) that allows a regime switch in the sharing rule if the female partner does not work and its continuity along the female's participation frontier. The parameter estimates are presented in Table 3.8. Here also the version employing the age-to-age sex ratio variable as a distribution factor is denoted by (1), and the one using the 2-year-band sex ratio variable is denoted by (2). The parameters turn out to be not very precisely estimated; the most significant parameter is the one related to non-labor income (both the total in specification (3.6) and the one that discounts the expenditures on children in specification (3.13)), with a p-value of approximately 10.3%. The parameter of non-labor income is around 0.57, indicating that couples seem to share their non-labor income such that 57% goes to man and the remaining 43% to the woman.

The marginal effect of the male and female wage rate on the sharing rule (3.6) is $\left(\alpha_2 + \alpha_4 \ln w^f\right)/w^m$ and $\left(\alpha_3 + \alpha_4 \ln w^m\right)/w^f$, respectively. The marginal effect on the specification (3.13) is $\left(\tilde{\alpha}_2 + \tilde{\alpha}_4 \ln w^f\right)/w^m$ and $\left(\tilde{\alpha}_3 + \tilde{\alpha}_4 \ln w^m\right)/w^f$. The estimated parameters of the sharing rule using the age-to-age sex ratio thus imply that, as long as the female's hourly wage is less than approximately MXN \$67, all other factors being equal, the female partner benefits, in terms of a non-labor income transfer, from an increase in the male's wage (and for rates less than approximately MXN \$74 with the 2-year-band). The female's share also benefits from increases in her wage within a wide range of the male's wage rate. By way of illustration, the parameter estimates of the conditional sharing rule equation (3.13), with the level of expenditures on children fixed, indicate that in the collective model with the age-to-age sex ratio variable as distribution factor and at the mean wage rate of both parents, a MXN \$1 increase in the male's wage (equivalent to an annual increase of MXN \$2,408 in his labor income, at the mean hours worked by men) induces him to transfer an additional MXN \$214 to the female partner. Also, an extra MXN \$1,367 will be transferred to the female partner when her wage increases MXN \$1 (equivalent to an annual increase

Table 3.8: Parameter Estimates of the Sharing Rule

	Collective Model	
	(1)	(2)
Sharing rule (3.6)		
α_1 (Y)	0.561	0.565
	(0.345)	(0.346)
α_2 $(\ln w^m)$	-56,771.396	-57,038.836
	(50,343.536)	(50,809.781)
α_3 $(\ln w^f)$	-102,744.125	-103,492.947
	(85,950.793)	(86,834.318)
α_4 $(\ln w^m \ln w^f)$	13,196.633	13,301.720
	(10,110.434)	(10,246.393)
α_5 (z)	5,126.260	3,077.755
	(3,861.181)	(3,113.048)
Conditional Sharing Rule (3.13)		
$\tilde{\alpha}_1$ (y)	0.573	0.576
	(0.352)	(0.353)
$\tilde{\alpha}_2$ $(\ln w^m)$	-57,978.791	-58,220.969
	(50,030.373)	(50,502.335)
$\tilde{\alpha}_3$ $(\ln w^f)$	-104,593.129	-105,395.732
	(86,306.474)	(87,197.639)
$\tilde{\alpha}_4$ $(\ln w^m \ln w^f)$	13,782.912	13,885.181
	(10,041.201)	(10,176.163)
r	1.161e-07	3.253e-07
	(2.376e-07)	(6.699e-07)

of MXN \$308 in her labor income, at the mean hours worked by women). Hence, at the mean wage rate of both parents, part of the male's gain in labor income is transferred to his partner, whereas the female's wage increase dramatically improves her bargaining position; she is able to keep the direct gains and in addition extract a larger portion of household non-labor income devoted to private expenditures.

The parameter estimate of r, associated to (3.19), that determines the assumption of a regime switch in the sharing rule and its continuity along the female's participation frontier, is not significantly different from zero; the previously estimated values of the sharing rule's parameters are maintained when the female partner does not work. In Bloemen (2010), the corresponding parameter for a working woman with a non-working husband is also not significantly different from zero. Although the non-participation of a female partner would have reduced overall household resources, it does not imply a shift in the resources toward her. For the sample used, the female's bargaining power does not seem to be affected by her non-participation in the labor market. Nevertheless, the male partner's share decreases if the wage rate of his partner increases, regardless of her labor status. The wage rate of a non-working woman may still function as a threat point.

The reason that the male labor supply and the sharing rule of a working man and his non-working female partner is not significantly different from the male labor supply and the sharing rule of a working couple may be that reservation wages of women tend to be very low and show little variation in the sample used. In this scenario, there is a negligible reduction in overall resources for the household when the woman is not working, so there is no visible response in the male partner's hours worked or in the distribution of household non-labor income.

Using the estimates of the parameters of the expenditures on children and the reduced form labor supply equations from Tables 3.5-3.7, the parameters of the structural individual labor supply functions (3.3) and (3.4) can be computed using the expressions in Table 3.3. It should first be observed that, in general terms, the parameters in Table 3.9 are not estimated precisely. The small sample size, together with the low variation in the potential wage, can explain part of this result. Nevertheless, if the marginal willingness to pay for expenditures on children is calculated for each member ($MWP^m = \psi_3/\psi_1$ and $MWP^f = \gamma_3/\gamma_1$), the male partner seems to care more for the children than the female: an increase of MXN \$1 in the male's share, ϕ^m, is associated with an increase of MXN \$1.3 in the money spent on children; a corresponding increase in the female's share is associated with a reduction of MXN \$0.3. Using the same database but considering only working couples and including home production, chapter 1 also found that when time and expenditure on children's education is evaluated, fathers care more than mothers.

Table 3.9: Parameter Estimates of the Structural Labor Supply Functions (3.3) and (3.4)

	Collective Model	
	(1)	(2)
Male labor supply function (3.3)		
ψ_1 (ϕ^m)	-0.004	-0.004
	(0.004)	(0.004)
ψ_2 ($\ln w^m$)	-445.322	-444.640
	(446.214)	(502.561)
ψ_3 (K)	-0.006	-0.006
	(0.005)	(0.006)
Female labor supply function (3.4)		
γ_1 (ϕ^f)	0.018	0.019
	(0.020)	(0.022)
γ_2 ($\ln w^f$)	-1,353.463	-1,365.233
	(3,361.797)	(3,845.730)
γ_3 (K)	-0.006	-0.006
	(0.014)	(0.018)
Marginal Willingness to Pay		
Male	1.310*	1.306
	(0.756)	(0.958)
Female	-0.310	-0.306
	(0.756)	(0.958)

3.6 Final Remarks

This chapter has specified an empirical model of collective household labor supply, based on the theoretical model of chapter 2, that jointly considers the non-participation of one partner of the household couple and the presence of children. As a basis for the model, it specifies each partner's labor supply function, based on individual preferences, as a linear function of their own log wage rate, the sharing rule, and expenditures on children. Also, the sharing rule and expenditures on children functions are defined as a linear function of individuals' and the cross product of the couple's log wage rates, household non-labor income, and a distribution factor.

The chapter provides empirical evidence on the relevance of factors that influence the couple's bargaining positions, such as the female's potential wage rate and the state-level sex ratio, and through these factors the household resource allocations. The stochastic specification consists of the estimation, by full-information maximum likelihood, of the couple's reduced labor supply functions by a type 4 Tobit (or switching regression) model simultaneously with the child expenditure function. The two sex ratios considered are the age-to-age and 2-year-band state-level sex ratios. The empirical analysis is based on the couple's labor supplies and expenditures on children of Mexican nuclear families drawn from the 2005-2006 wave of the MxFLS. Unconstrained and constrained versions of the model are estimated.

The estimated parameters satisfy the conditions imposed by the proposed collective labor supply model. Previous studies that included a household group with the presence of more than one child or pre-school children (such as Fortin and Lacroix 1997; Donni 2007) have generally rejected the restrictions implied by the collective rationality. As in chapter 1, there is no evidence here that empowering mothers is more beneficial to the children than empowering fathers; indeed, there is a larger increase in expenditure on children if their fathers, rather than mothers, are empowered. Cherchye, de Rock, and Vermeulen (forthcoming) have also found this unanticipated behavior in a sample of Dutch couples.

Another important finding is that expenditures on children and male labor supply vary significantly with the female wage even when the woman is not working. Nevertheless, the auxiliary assumptions of a continuous regime switch on the male labor supply and sharing rule functions to a change on the female participation decision are rejected; the difference between the labor supply and sharing rule functions of a working man and his non-working partner and the corresponding functions of a working couple are not statistically significant.

The reservation wages of non-working female partners may be relatively low and without sufficient fluctuation.

A more particular formulation consists of the use of a closed form for the female's shadow wage rate and thus accounts for rationing in the woman's hours worked. Introducing this wage into the male's labor supply function, the latter is continuous everywhere. Additionally, one can assume that the sharing rule is the same without considering the female's labor participation change.

The lack of precision of the sharing rule actually indicates avenues for further empirical exploration. For instance, although the sample of households of working couple without offspring was enlarged by including households with a non-working female partner and children under 15 years of age, the imprecision of some parameters may still be due to the small sample size. In particular, the female's potential wage rate has been estimated using information from only 18% of households, the percentage corresponding to that of working women in the sample.[7] Also, because extended families are common in developing countries, it would be desirable to extend the model to include the possibility of a household with more than two persons with bargaining power. In this case, it would be necessary to have a private good that was consumed by each member with power and a distribution factor that affected the distribution of power for each of those members.

[7] For future research, a possible means of obtaining greater precision would be to estimate the wage equation using a larger sample, like the Encuesta Nacional de Ocupación y Empleo (ENOE). This sample would have to include exactly the same variables used in the wage equation for a similiar group of women.

Appendix A

Some Distribution Factors Used in the Literature

Table A.1: Some distribution factors used in the literature

Distribution factors	Articles
Assets	
Current	Doss (1996); Beegle, Frankenberg, and Thomas (2001)
At marriage	Quisumbing and Maluccio (2003); Thomas, Contreras, and Frankenberg (2004)
Inherited	Quisumbing (1994)
Nonlabor income	Schultz (1990); Thomas (1990); Rubalcava and Contreras (2000)
Targeted transfers and changes in welfare programs	Lundberg, Pollak, and Wales (1997); Adato, de la Brière, Mindek, and Quisumbing (2000); Duflo (2000); Attanasio and Lechene (2002); Myck, Bargain, Beblo, Beninger, Blundell, Carrasco, Chiuri, Laisney, Lechene, Longobardi, Moreau, Ruiz-Castillo, and Vermeulen (2006); Handa, Peterman, Davis, and Stampini (2009); Rubalcava, Teruel, and Thomas (2009)
Sex ratio	Angrist (2002); Chiappori, Fortin, and Lacroix (2002); Grossbard-Shechtman and Neuman (2003); Park (2007)
Divorce law	Gray (1998); Chiappori, Fortin, and Lacroix (2002)
Abortion law	Oreffice (2007)
Gender-specific public policies	Folbre (1997)
Human capital differences (partners, parents of partners)	Beegle, Frankenberg, and Thomas (2001); Rubalcava and Contreras (2000); Quisumbing and Maluccio (2003)
Social status	Beegle, Frankenberg, and Thomas (2001)
Domestic violence	Bloch and Rao (2002)

Appendix B

Further Empirical Results

Table B.1: Parameter Estimates. Female's log wage rate (3.22) and the cross product of couple's log wage rate (3.23)

	$\ln w^f$		$\ln w^m \ln w^f$	
	(1)	(2)	(1)	(2)
Residuals female's participation equation	-0.000***	-0.000***	-0.001***	-0.001***
	(0.000)	(0.000)	(0.000)	(0.000)
Female's education	-0.024	-0.024	-0.174	-0.173
	(0.107)	(0.106)	(0.330)	(0.329)
Female's age	-0.036	-0.036	-0.142	-0.142
	(0.035)	(0.035)	(0.108)	(0.108)
Female's education × age	0.001	0.001	-0.010	-0.010
	(0.003)	(0.003)	(0.011)	(0.011)
Unemployment rate by state	0.236**	0.236**	1.312	1.323
	(0.104)	(0.104)	(0.804)	(0.803)
$\ln w^m$	—	—	1.745*	1.767*
			(0.942)	(0.941)
$\ln w^m$ × Female's education × age	—	—	0.005***	0.005***
			(0.002)	(0.002)
$\ln w^m$ × Unemployment rate by state	—	—	-0.080	-0.083
			(0.180)	(0.179)
Intercept	3.883***	3.907***	7.523	7.479
	(1.413)	(1.410)	(5.377)	(5.363)
Region dummies	Yes	Yes	Yes	Yes

Note. * p<0.1, ** p<0.05, *** p<0.01. Standard errors in parentheses. The regions are: North, Capital, Gulf, Pacific, South, Central-North, and Central.

Table B.2: Parameter Estimates. Other Parameters [Model (3.20-3.21)]

	Unrestricted Model		Collective Model †	
	(1)	(2)	(1)	(2)
Std. Devs. and Corr. Coeffs.				
σ_p	798.817***	801.662***	796.530***	796.604***
	(44.682)	(44.994)	(44.418)	(44.426)
σ_{np}	848.225***	845.188***	849.784***	850.002***
	(27.838)	(27.752)	(27.913)	(27.923)
σ_f	2,423.854***	2,424.039***	2,424.987***	2,424.993***
	(152.166)	(152.208)	(152.252)	(152.254)
σ_K	5,892.423***	5,903.727***	5,892.357***	5,903.810***
	(131.837)	(132.085)	(131.833)	(132.094)
$\rho_{p,f}$	0.139	0.141	0.135	0.136
	(0.147)	(0.147)	(0.147)	(0.147)
$\rho_{p,K}$	0.081	0.082	0.083	0.082
	(0.052)	(0.052)	(0.052)	(0.052)
$\rho_{np,f}$	-0.527***	-0.521***	-0.529***	-0.530***
	(0.101)	(0.102)	(0.100)	(0.100)
$\rho_{np,K}$	0.050	0.050	0.049	0.049
	(0.041)	(0.041)	(0.041)	(0.041)
$\rho_{f,K}$	0.104***	0.104***	0.104***	0.103***
	(0.035)	(0.035)	(0.035)	(0.035)
Log-likelihood function	-20,138.027	-20,138.624	-20,138.922	-20,140.897

Note. * p<0.1, ** p<0.05, *** p<0.01. Standard errors in parentheses.
† Restrictions (3.14-3.15) are imposed in the estimation process.

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