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A SHADOW RATE NEW KEYNESIAN MODEL FOR MONETARY ECONOMIES

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Abstract

The main objective of this thesis is to develop a model that allows for both unconventional monetary policy and conventional policies. In a Zero Lower Bound (ZLB) environment, the nominal interest cannot go any lower and central banks have relied on unconventional tools like quantitative easing. To date, the literature has focused on developing the shadow rate in cashless economies. In normal times (i.e., when the ZLB is non-binding), the shadow rate corresponds to the policy rate set by the central bank. However, the shadow rate will be negative when the ZLB binds and unconventional policies are applied. The contribution of this thesis is to introduce the shadow rate concept into a monetary economy, thus allowing a wider spread in the policies implemented thanks to the role of money. The shocks under a monetary economy with unconventional policies are dissipated quicker than in a cashless economy. The main difference between a technology shock in the cashless economy and the monetary economy with unconventional policies is that the monetary economy can escape the ZLB quicker. With a preference shock or an inflationary shock in the monetary economy with unconventional policies, consumption can actually rise at the ZLB (a big difference to the results under conventional policies).

Sections

Introduction

At the beginning of the 90's the collapse of the asset price bubble In Japan caused a period of economic stagnation. In order to encourage the consumption and stimulate the economy, Japan's Central Bank set the nominal interest rate at levels close to zero. However, the economic stagnation continued and with the Zero Lower Bound (ZLB) binding the nominal interest rate, no more conventional policy could be implemented. Japan's Central Bank had to alleviate the stress of the economy with other kinds of monetary policies, they bought large amounts of government bonds, thus providing the economy with more liquidity in order to encourage more investment of the private sector (Voutsinas et al. 2011). The monetary policy implemented in Japan is the Quantitative Easing and forms part of a set of unconventional monetary policies that can be implemented when the conventional monetary policy of reducing the nominal interest rate is no longer possible. In the United States the Federal Reserves purchased substantial quantities of assets with medium and long maturities in order to keep monetary policy effective during the crisis of 2008 (Gagnon et al. 2018).

The Quantitative Easing unconventional policy has been studied in some articles, Chen et al. (2012) study the effect of large-asset purchase programs on the macroeconomic variables such as aggregate output and inflation. And in concordance with Krishnamurthy & Vissing-Jorgensen (2011), where they study the transmission mechanism of the Quantitative Easing on the interest rates, in this thesis the transmission mechanism of unconventional monetary policies will be analyzed.

This thesis considers the approach of another unconventional policy, the lending facilities. The lending facilities aim to promote liquidity in the funding markets and increase the lending to the private sector as explained in Fleming et al. (2009,2010).

The theory that studies the phenomenon of negative interest rates is known as the *shadow rate*

> The shadow rate is the federal funds rate when the ZLB is not binding; otherwise, it is negative to account for unconventional policy tools (...) A negative shadow rate is not an actual policy instrument, but rather, it can be perceived as a summary statistic for unconventional monetary policy mapped into the interest rate domain. (Wu & Zhang, 2019, p.2)

The study of the shadow rate begins with the proposal of the Shadow Rate Term Structure by Black (1995), where he establishes that the nominal interest rate will be the maximum of this shadow rate and 0. From there many studies have been carried out about the effect of the shadow rate as a summary of the unconventional policies, empirical research about this subject can be found in Wu & Xia (2016,2020). In the first paper they track the effect of their estimated shadow rate on the macroeconomic variables if the monetary policy shocks were shut down in ZLB environments. In Wu & Xia (2020) they study the effect of the negative interest rates in the case of the Swiss National Bank's interest rate of -0.75% and the European Central Bank's deposit facility rate of -0.4%.

This thesis is based on the work of Wu & Zhang (2019). Their model incorporates in the New Keynesian models the role of the shadow rate as a summary of the unconventional policies. The shadow interest rate is able to analyze the effects of both conventional and unconventional policies, when the interest rate set by the central bank is positive then the shadow rate is the same as the nominal interest rate; however, when the policy rate reaches the ZLB, the shadow rate replicates the effect of the unconventional policies and avoids the structural break of the ZLB.

A key weakness of the approach of Wu & Zhang (2019) is the assumption that the economy is cashless. In the absence of monetary aggregates, the transmission mechanism of conventional monetary policy operates solely through an aggregate demand channel: changes in the nominal interest rate affect output via changes in the real interest rate, which results in a change in inflation via the New Keynesian Phillips curve. The aim of this thesis is to consider the shadow rate in a monetary economy. In monetary economies, money demand plays a key role for equilibrium. Following Woodford (2003) and McKnight and Mihailov (2015), this thesis introduces money into the economy by allowing for the real balance effects of transactions services.

Thus, the motivation to extend the model of Wu & Zhang (2019) to monetary economies is to develop a broader model in which there are two transmission mechanisms for adverse shocks in the economy, the aggregate demand channel and the money demand channel.

Within the economy considered in this model, there exists a representative infinitely lived household, a representative infinitely lived entrepreneur who produces intermediate-goods, a continuum of monopolistically-competitive retailers that produce final goods and a central bank. The shadow rate is implemented in the ZLB environment with the unconventional policy of lending facilities, that consist of increasing the loan-to-value of the entrepreneurs in order to incentive the production via more investment. The approach of this model is to incorporate Wu & Zhang's shadow rate model in a monetary economy, and analyze the differences between the cashless and monetary models.

In addition to the development of a shadow rate New Keynesian model, in this work the model is stressed with three shocks, a preference, a factor productivity and an inflationary shocks. The importance of this is to see how the transmission mechanism of conventional and unconventional monetary policy changes in monetary economies compared to cashless economies.

Work has been done incorporating financial inter-mediation to Wu & Zhang model by considering short-term and long-term bonds, and reducing it to a system of four equation (Sims & Wu 2019). Effects and macroeconomic dynamics in a ZLB environment can be found in Borağan Aruoba et al. (2018), some related papers of effects and issues that arise in models of interest rates near the ZLB are Fernández-Villaverde et al. (2015) where they make the analysis of using non-linearities near the ZLB; and the friction analysis that happens considering the structural break of the ZLB in log-linearized models (Boneva et al. 2016).

The key findings of this model are that under the assumption of a monetary economy, the addition of the money demand transmission mechanism helps alleviate the effect and span of the shocks. In monetary economies, also the ZLB binding lasts less, and the changes of inflation and interest rates are lower, thus the monetary model is more stable. A preference shock or an inflationary shock can stimulate the monetary economy while in the cashless shadow rate model they lower consumption and aggregate output. A technology shock stimulates the aggregate output and increases consumption in both scenarios, while in a monetary economy these variables increase more than in a cashless economy, and with also a minor impact in the inflation.

The structure of the thesis is as follows. In section 1 the literature of New Keynesian models and the transition to a shadow rate scenario is explained. In section 2 the Shadow Rate New Keynesian Model for Monetary Economies is developed. In the section 3 the model is log-linearized around its steady state and the parameters of the model are established. Section 4 is dedicated to the results of model when stressed by three shocks, a preference shock, a technology shock and an inflationary shock. Finally, section 5 concludes.

1 Literature review

The popular textbook version of the New Keynesian model (see, e.g., Galí, 2008) consists of three log-linear equations:

$$\hat{y}_t = \mathbb{E}_t\,\hat{y}_{t+1} - \sigma[\hat{r}_t - \mathbb{E}_t\,\hat{\pi}_{t+1}] \tag{1.1}$$

$$\hat{\pi}_t = \beta\,\mathbb{E}_t\,\hat{\pi}_{t+1} + \kappa\hat{y}_t \tag{1.2}$$

$$\hat{r}_t = \phi_r\hat{r}_{t-1} + (1-\phi_r)[\phi_y\hat{y}_t + \phi_\pi\hat{\pi}_t] \tag{1.3}$$

with the additional restriction that the nominal interest rate must be non-negative $r_t \geq 0$. Hence, in the log-linear equations, the change of the nominal interest rate can't fall further than its steady state value $\hat{r}_t \geq -r$.

Equation (1.1) is the intertemporal IS equation that relates changes in current output $\hat{y}_t$ to adjustments in the real interest rate $\hat{r}_t - \mathbb{E}_t\,\hat{\pi}_{t+1}$. Equation (1.2) is the aggregate supply equation, or New Keynesian Phillips Curve (NKPC), that gives the dynamics of inflation $\hat{\pi}_t$. Equation (1.3) is the interest-rate rule with policy inertia $\rho_r \in [0, 1)$.

In the baseline NK model, unconventional monetary policy plays no role at the ZLB. Once the ZLB hits, the policy rate $\hat{r}_t = -r$, and monetary policy is completely inactive in the system (1.1)-(1.3). Monetary policy is transmitted in this model entirely from the conventional aggregate demand channel of monetary policy. Changes in the nominal interest rate affect output from (1.1) via changes in the real interest rate, which results in a change in inflation from the NKPC (1.2).

A key assumption of the textbook NK model outlined above is that the economy is cashless. As shown by McKnight and Mihailov (2015) in a monetary economy, where money is introduced via the utility function of the agent, the NK model is now represented by the following four log-linear equation system:

$$\hat{y}_t = \mathbb{E}_t\,\hat{y}_{t+1} - \sigma[\hat{r}_t - \mathbb{E}_t\,\hat{\pi}_{t+1}] \tag{1.4}$$

$$\hat{m}_t = \eta_C\hat{y}_t - \eta_R\hat{r}_t \tag{1.5}$$

$$\hat{\pi}_t = \beta\,\mathbb{E}_t\,\hat{\pi}_{t+1} + \kappa\hat{y}_t - \kappa\chi\hat{m}_t \tag{1.6}$$

$$\hat{r}_t = \phi_r\hat{r}_{t-1} + (1-\phi_r)[\phi_y\hat{y}_t + \phi_\pi\hat{\pi}_t] \tag{1.7}$$

and the same restriction that faces the nominal interest rate $r_t \geq 0$.

In a monetary economy, money demand $\hat{m}_t$ given by equation (1.5) plays an important role. This results in an additional monetary transmission mechanism, a cost channel of monetary policy, where changes in the nominal interest rate result in changes in the demand for money (1.5), which affects the output and pricing decisions of firms, via changes in the real marginal cost of production from (1.6). With real balance effects ($\chi > 0$), money demand enters into the NKPC (1.6) as a negative cost-push shock. Thus, an increase in the nominal interest rate generates a reduction in the demand for money causing an increase in inflation.

However, the previous models ignored unconventional monetary policy when the nominal interest rate is constrained at the ZLB. Wu & Zhang (2019) added unconventional monetary policies into the cashless NK model in order to avoid the ZLB structural break and have the ability to implement efficient monetary policies even when the ZLB binds. Their model consists of the following three log-linear equations:

$$\hat{y}_t = \mathbb{E}_t\,\hat{y}_{t+1} - \sigma[\hat{s}_t - \mathbb{E}_t\,\hat{\pi}_{t+1}] \tag{1.8}$$

$$\hat{\pi}_t = \beta\,\mathbb{E}_t\,\hat{\pi}_{t+1} + \kappa\hat{y}_t \tag{1.9}$$

$$\hat{s}_t = \phi_s\hat{s}_{t-1} + (1-\phi_s)[\phi_y\hat{y}_t + \phi_\pi\hat{\pi}_t] \tag{1.10}$$

where monetary policy is applied through the nominal shadow rate $\hat{s}_t$.

It is important to note that in this system of equations there is no restriction of a ZLB, as the shadow rate can be implemented whether it binds or not.

When the nominal shadow rate is above the ZLB, monetary policy is the same as in the cashless NK model where the transmission mechanism of conventional monetary policy is governed by the aggregate demand channel. However, when the economy is constrained by the ZLB, $s_t < 0$, monetary policy is conducted by unconventional monetary policies. An example of such a policy is

Quantitative Easing, that consists of a large purchase of bonds by the central bank in order to decrease the nominal return of bonds, and this stimulates aggregate output. The unconventional policy that is going to be considered in this thesis relates to the lending facilities, that consists of an increase in the loan-to-value ratio of the private sector, encouraging them to borrow more, with the idea of increasing their investment, and thus stimulating the aggregate output.

The key findings of the Wu & Zhang (2019) shadow rate model are that the addition of unconventional monetary policies helps to avoid the structural break of the ZLB. With that change, the issues of the constraint are also evaded, and unconventional policies can help alleviate the stress of the economy in deflationary situations when the conventional policies can no longer be implemented.

The aim of this thesis is to extend the approach of Wu & Zhang (2019) to monetary economies. By introducing an additional monetary transmission mechanism into the economy, we can investigate how conventional and unconventional monetary policies are transmitted in monetary economies and check the similarities and differences against the cashless economy benchmark of Wu & Zhang.

2 A New Keynesian Model for Monetary Economies

The model economy extends the New Keynesian Money-in-the-Utility-Function (MIUF) model of McKnight & Mihailov (2015) to allow for a shadow rate. Real balance effects of transactions services are introduced by assuming that the utility function of the representative agent is non-separable between consumption and real money balances.

The structure of the model is as follows. Within the economy there exists a representative infinitely lived household, a representative infinitely lived entrepreneur who produces intermediate-goods, a continuum of monopolistically-competitive retailers that produce final goods and a central bank. Entrepreneurs are agents that maximize consumption and use the labor of the households and their own physical capital to produce intermediate-goods. Entrepreneurs can borrow from households using capital as collateral up to a constant loan-to-value ratio determined by households. Intermediate-goods are used as inputs by the retailers that produce final-goods in a monopolistically competitive market and set prices according to Calvo (2008).

The central bank implements monetary policy by setting the nominal interest rate for the economy. In this model the interest rate that it sets is the shadow rate following a contemporaneous-looking Taylor rule. In contrast to standard NK models, here the ZLB constraint doesn't bind as the shadow rate avoids the structural break with some unconventional monetary policies. The unconventional monetary policy tool we consider is the lending or funding facilities, that consists of easing the supply of loans to the private sector, by changing their loan-to-value ratio, defined as the ratio between the amount of money the entrepreneurs borrow from the households by the physical capital owned by the entrepreneurs. Increasing the loan-to-value ratio encourages lending to the entrepreneurs with the aim of stimulating the economy via higher investment.

Households

We assume that real money balances m_t and consumption C_t enter non-separably into the period utility function of households:

$$u(C_t, m_t, H_t) = U(C_t, m_t) - V(H_t) \tag{2.1}$$

The period utility function (2.1) has the standard assumptions for risk averse agents that is concave and strictly increasing in each variable, with the convention that real money holdings and consumption are normal goods. The period utility function is also additive in its disutility of labor supply $V(H_t)$, a function that is considered convex and increasing.

Households choose in each period t the amount of consumption C_t, labor supply H_t, real bond holdings B_t and the amount of real money balances m_t to maximize their lifetime utility function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \left(\prod_{i=1}^{t} \beta_i \right) [U(C_t, m_t) - V(H_t)] \tag{2.2}$$

where the discount factor is $0 < \beta_t < 1$ which evolves according to: $\beta_t/\beta = (\beta_{t-1}/\beta)^{\rho_\beta} \epsilon_{\beta,t}$, where $\rho_\beta \in [0,1]$ and β is the mean of the process.

Each period households face the following budget constraint:

$$C_t + m_t + B_t = \frac{R^B_{t-1} B_{t-1}}{\mathcal{T}_{t-1} \Pi_t} + \frac{m_{t-1}}{\Pi_t} + W_t H_t + T_t \tag{2.3}$$

where the gross return on the bonds holdings from period $t-1$ to t is R^B_{t-1}, $\mathcal{T}_{t-1}$ is the gross tax rate levied on the return of the bond at period t, T_t is net lump-sum taxation, W_t denotes the real wage, and the gross inflation rate is given by $\Pi_t = \frac{P_t}{P_{t-1}}$.

The first-order conditions of the household's maximization problem are:

$$U_c(C_t, m_t) = \frac{R^B_t}{\mathcal{T}_t} \mathbb{E}_t \left\{ \frac{\beta_{t+1} U_c(C_{t+1}, m_{t+1})}{\Pi_{t+1}} \right\} \tag{2.4}$$

$$W_t = \frac{V_h(H_t)}{U_c(C_t, m_t)} \tag{2.5}$$

$$\frac{R^B_t - \mathcal{T}_t}{R^B_t} = \frac{U_m(C_t, m_t)}{U_c(C_t, m_t)} \tag{2.6}$$

Equation (2.4) is the intertemporal Euler equation, (2.5) is the labor supply condition and (2.6) is the money demand equation.

Entrepreneurs

Entrepreneurs are competitive firms who produce intermediate goods using household labor and their own physical capital as inputs. These intermediate goods are sold to monopolistically-competitive retailers at price P^E_t, who produce the final goods and sell them at a price P_t to the households. The markup for the retailers is

$$X_t = \frac{P_t}{P_t^E} \tag{2.7}$$

The entrepreneurs produce the intermediate goods Y_t^E using the following production technology:

$$Y_t^E = A_t K_{t-1}^{\alpha} H_t^{1-\alpha} \tag{2.8}$$

where $0 < \alpha < 1$ is the capital share in production. The inputs of the production technology are the capital accumulated K_{t-1}, the labor supply H_t and the level of technology A_t which evolves according to: $A_t/A = (A_{t-1}/A)^{\rho_a}\epsilon_{a,t}$, where $\rho_a \in [0,1]$ and the steady-state value A is normalized to 1. Physical capital is accumulated in accordance with the following law of motion:

$$K_t = I_t + (1-\delta)K_{t-1} \tag{2.9}$$

where $\delta \in (0,1)$ is the depreciation rate of capital.

The entrepreneurs maximize their lifetime utility function by choosing each period consumption C_t^E, investment I_t, and the labor input H_t:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \gamma^t U^E(C_t^E) \tag{2.10}$$

The period budget constraint is given by:

$$\frac{Y_t^E}{X_t} + B_t = \frac{R_{t-1}^B B_{t-1}}{\mathcal{T}_{t-1}\Pi_t} + W_t H_t + I_t + C_t^E \tag{2.11}$$

The entrepreneurs borrow from the households with a loan-to-value ratio LV_t using capital as collateral, and constrained by:

$$B_t \leq LV_t\, \mathbb{E}_t \left\{ \frac{K_t \Pi_{t+1}}{R_t^B} \right\} \tag{2.12}$$

The first-order equations of the entrepreneur's maximization problem are:

$$U_c^E(C_t^E)\left(1 - \frac{LV_t\, \mathbb{E}_t\, \Pi_{t+1}}{R_t^B}\right) = \gamma\, \mathbb{E}_t \left\{ U_c^E(C_{t+1}^E)\left(\frac{\alpha Y_{t+1}^E}{X_{t+1}K_t} - \frac{LV_t}{\mathcal{T}_t} + 1 - \delta\right)\right\} \tag{2.13}$$

$$W_t = \frac{(1-\alpha)Y_t^E}{H_t X_t} \tag{2.14}$$

where (2.13) is the entrepreneur's Euler equation and (2.14) corresponds to the labor demand.

Retailers

The economy consists of a continuum of retailers $j \in [0,1]$ who produce final goods $Y_t(j)$ in a monopolistically competitive market using the inputs Y_t^E that they buy from the entrepreneurs at a price P_t^E. Their goods are sold at a price $P_t(j)$. Final goods are created according to a CES production technology:

$$Y_t = \left[\int_0^1 Y_t(j)^{\frac{(\eta-1)}{\eta}} dj\right]^{\frac{\eta}{\eta-1}} \tag{2.15}$$

where $\eta > 1$ is the elasticity of substitution for the CES aggregation. Then the demand for retail goods is

$$Y_t(j) = Y_t \left[\frac{P_t(j)}{P_t}\right]^{-\eta}$$

where the aggregate price satisfies:

$$P_t = \left[\int_0^1 P_t(j)^{1-\eta} dj\right]^{\frac{1}{1-\eta}} \tag{2.16}$$

Retail firms face Calvo-stickiness, where in each period a fraction $\psi \in (0,1)$ of firms indexes the previous-period price considering the steady-state inflation π, whereas the other fraction $1-\psi$ reoptimizes their prices. The price-setting problem is thus:

$$\max_{P_t(j)} \mathbb{E}_t \sum_{s=t}^{\infty} (\psi\beta)^{s-t} \frac{U_c(C_s, m_s)}{U_c(C_t, m_t)} \left[\frac{P_t(j)\pi^{s-t}}{P_s} - z_s\right] Y_s(j) \left[\frac{P_t(j)\pi^{s-t}}{P_s}\right]^{-\eta} \tag{2.17}$$

where $z_t = \frac{W_t}{(1-\alpha)A_t K_{t-1}^{\alpha} H_t^{-\alpha}}$ is real marginal cost at time t.
The first-order condition of the retailer's profit maximization problem is:

$$P_t(j) = \frac{\eta}{\eta-1} \frac{\mathbb{E}_t \sum_{s=t}^{\infty} (\psi\beta)^{s-t} U_c(C_s, m_s)(P_s/\pi^{s-t})^{\eta} Y_s z_s}{\mathbb{E}_t \sum_{s=t}^{\infty} (\psi\beta)^{s-t} U_c(C_s, m_s)(P_s/\pi^{s-t})^{\eta-1} Y_s} \tag{2.18}$$

Under Calvo price stickiness, the fraction of retailers that optimizes prices is $1-\psi$, setting an identical price of P_t*. This implies that the aggregate price index can be written as

$$(P_t)^{1-\eta} = \psi(P_{t-1})^{1-\eta} + (1-\psi)(P_t*)^{1-\eta} \tag{2.19}$$

When the labor demand (2.14) is substituted in the real marginal cost z_t yields the relation

$$z_t = \frac{1}{X_t} \tag{2.20}$$

Central Bank

Monetary policy is implemented by the central bank following a shadow rate Taylor rule:

$$\frac{S_t}{R} = \left(\frac{S_{t-1}}{R}\right)^{\phi_s} \left[\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y}\right]^{1-\phi_s} \tag{2.21}$$

where the central bank sets the nominal shadow interest rate S_t considering the previous value, with an interest rate persistence of $\phi_s \geq 0$, and $\phi_\pi, \phi_y \geq 0$ are the degrees of response to current inflation and output. R, Y, Π are the steady-state values of the nominal interest rate, output and inflation.

To make the distinction between this model and the standard NK model without unconventional policies,

$$R_t = max\{1, S_t\} \tag{2.22}$$

where the gross nominal interest rate R_t will be the maximum value between the nominal shadow rate obtained by the central banks' Taylor rule and the constant value 1, that represents the scenario when the ZLB is binding.

The model also considers that the return bonds yield is related to the gross nominal rate by the following equation:

$$R_t^B \equiv R_t RP_t \tag{2.23}$$

where as previously established, R_t^B is the gross return on the nominal asset, R_t is the gross nominal interest rate and the risk premium RP_t is the wedge between the two rates (Creal & Wu 2020).

Lending Facilities

In this model we assume that unconventional monetary policy is conducted via lending facilities. This corresponds to the extension of loans to the private sector by changing the loan-to-value ratio. This policy can be combined with a tax policy where there is a transfer from the households' gains in the interest rate income to the entrepreneurs.

When the risk premium is constant $RP_t = RP$, the central bank can use conventional monetary policies (by changing R_t) to stimulate the economy. But when the ZLB is binding, conventional monetary policies are impossible to implement and therefore unconventional policies must be adopted: changes in lending facilities can stimulate the economy by increasing both the taxation on the bond returns $\mathcal{T}_t$ and also the loan-to-value ratio LV_t. The higher loan-to-value ratio relaxes the borrowing constraint of the entrepreneurs (2.11), encouraging them to borrow more, and second, the decrease in the entrepreneurs interest rate payment, also encourages them to borrow more. With this additional liquidity, entrepreneurs raise their consumption and investment, thus increasing production and stimulating aggregate demand.

The two types of policy tools appear in the model by pairs.

- In the households' Euler equation (2.13) and budget constraint (2.11), alongside the entrepreneurs' budget constraint appears the ratio $\frac{R_t}{\mathcal{T}_t}$.
- In the borrowing constraint (2.12) and the Euler equation of the entrepreneurs (2.13) shows up the ratio $\frac{LV_t}{R_t}$.
- With a proportional increase in both variables, then the ratio of the intertemporal Euler equation (2.13) $\frac{LV_t}{R_t}$ stays constant.

Given the above, the following conditions for conventional and unconventional monetary policies can be expressed as:

$$\begin{cases} R_t = S_t, \mathcal{T}_t = 1, LV_t = LV & \text{for } S_t \geq 1 \\ R_t = 1, \mathcal{T}_t = LV_t/LV = 1/S_t & \text{for } S_t < 1 \end{cases} \tag{2.24}$$

The shadow rate summarizes both types of policies if we substitute in the model the following ratios, $R_t/\mathcal{T}_t = S_t$, $R_t/LV_t = S_t/LV$, $LV_t/\mathcal{T}_t = LV$ for every value of the shadow rate S_t.

Market Clearing and Equilibrium

The goods market clearing condition for this model is:

$$Y_t = C_t + C_t^E + I_t \tag{2.25}$$

The equilibrium is a set of sequences

$$\{C_t, C_t^E, m_t, H_t, B_t, R_t^B, S_t, R_t, \mathcal{T}_t, LV_t, \Pi_t, W_t, T_t, Y_t, K_t, I_t, X_t, P_t, P_t*\}$$

$\forall t \geq 0$ characterized by:

1. the representative household optimality conditions, (2.4)-(2.6), the budget constraint (2.3) and the definition of inflation;
2. the entrepreneurs' optimality conditions (2.13) and (2.14), and the constraints (2.11), (2.12) and the definition of X_t given by (2.7);
3. the production function (2.8), and the law of motion for capital (2.9);
4. the price-setting conditions (2.18) and (2.19);
5. the goods market clears (2.25);
6. the monetary policy rule (2.21) is fulfilled;
7. the risk-premium and ZLB equivalences are satisfied (2.22) and (2.23);
8. the unconventional policy of the lending facilities (2.24).

3 Log-Linearization and Parameterization of the Model

The log-linearized model

The model of the previous chapter is log-linearized around a zero-inflation steady state. In what follows, all hatted variables denote log deviations from the steady state. Steady state variables are denoted by an uppercase letter with no time subscripts (e.g., X).

Log-linearizing the goods market clearing condition (2.25) yields:

$$\hat{y}_t = \frac{C}{Y}\hat{c}_t + \frac{C^E}{Y}\hat{c}_t^E + \frac{I}{Y}\hat{i}_t \tag{3.1}$$

Log-linearizing equation (2.21) yields the expression for the Taylor rule:

$$\hat{s}_t = \phi_s \hat{s}_{t-1} + (1-\phi_s)[\phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t] \tag{3.2}$$

Log-linearizing the households' intertemporal Euler equation (2.4) yields the IS equation

$$\hat{c}_t = \mathbb{E}_t\, \hat{c}_{t+1} - \sigma[\hat{s}_t + \mathbb{E}_t\, \hat{\beta}_{t+1} - \mathbb{E}_t\, \hat{\pi}_{t+1} + \chi(\mathbb{E}_t\, \hat{m}_{t+1} - \hat{m}_t)] \tag{3.3}$$

where

$$\sigma \equiv -\frac{U_c}{U_{cc}C} > 0$$

is the intertemporal elasticity of substitution in consumption and

$$\chi \equiv \frac{U_{cm}m}{U_c} \geq 0$$

is the degree of non-separability of the consumption and the real balances.

Log-linearizing the money demand function (2.6) yields the LM equation

$$\hat{m}_t = \eta_C \hat{c}_t - \eta_R \hat{s}_t \tag{3.4}$$

where η_C and η_R are the consumption elasticity of money demand and the interest rate semi-elasticity of money demand respectively.

Log-linearizing the labor demand equation (2.14) and using the entrepreneurs' budget constraint (2.11) yields the equation

$$C^E \hat{c}_t^E = \frac{\alpha Y}{X}[\hat{y}_t - \hat{x}_t] + B\hat{b}_t - I\hat{i}_t - R^B B[\hat{s}_{t-1} + \hat{b}_{t-1} - \hat{\pi}_t] \tag{3.5}$$

Log-linearizing the borrowing constraint (2.12) when it's binding yields the equation

$$\hat{b}_t = \mathbb{E}_t\,\hat{\pi}_{t+1} + \hat{k}_t - \hat{s}_t \tag{3.6}$$

Log-linearizing the entrepreneurs' intertemporal Euler equation (2.13) yields the equation

$$0 = \left(1 - \frac{LV}{R^B}\right)(\hat{c}_t^E - \mathbb{E}_t\,\hat{c}_{t+1}^E) + \frac{LV}{R^B}(\mathbb{E}_t\,\hat{\pi}_{t+1} - \hat{s}_t) + \frac{\alpha\gamma Y}{XK}(\mathbb{E}_t\,\hat{y}_{t+1} - \mathbb{E}_t\,\hat{x}_{t+1} - \hat{k}_t) \tag{3.7}$$

Log-linearizing labor supply (2.5) and using the production technology (2.8) yields the equation

$$\hat{y}_t = \frac{(1+\omega)}{\alpha+\omega}(\hat{a}_t + \alpha\hat{k}_{t-1}) - \frac{1-\alpha}{\alpha+\omega}[\hat{x}_t + \sigma^{-1}\hat{c}_t - \chi\hat{m}_t] \tag{3.8}$$

where

$$\omega \equiv \frac{V_{hh}H}{V_h} > 0$$

is the output elasticity of real marginal cost

Log-linearizing the price-setting equations (2.18) and (2.18) yields the New Keynesian Phillips Curve

$$\hat{\pi}_t = \beta\,\mathbb{E}_t\,\hat{\pi}_{t+1} - \lambda\hat{x}_t + u_t \tag{3.9}$$

where u_t, the residual of the Phillips curve, corresponds to an AR(1) inflation shock with mean 0.

Log-linearizing the law of motion for capital (2.9) yields the equation

$$\hat{k}_t = (1-\delta)\hat{k}_{t-1} + \delta\hat{i}_t \tag{3.10}$$

Log-linearizing the policy conditions of the lending facilities (2.24) yields

$$\begin{cases} \hat{r}_t = \hat{s}_t, \hat{\tau}_t = \hat{lv}_t = 0 & \text{for } s_t \geq 0 \\ \hat{r}_t = -s, \hat{\tau}_t = \hat{lv}_t = -(\hat{s}_t + s) & \text{for } s_t < 0 \end{cases} \tag{3.11}$$

The log-linearized version of the preference shock is

$$\hat{\beta}_t = \rho_\beta \hat{\beta}_{t-1} + \epsilon_{\beta,t} \tag{3.12}$$

where ρ_β is the autocorrelation parameter.

The log-linearized version of the technology shock is

$$\hat{a}_t = \rho_a \hat{a}_{t-1} + \epsilon_{a,t} \tag{3.13}$$

where ρ_a is the autocorrelation parameter.

The log-linearized version of the inflationary shock is

$$u_t = \rho_\pi u_{t-1} + \epsilon_{a,t} \tag{3.14}$$

where ρ_a is the autocorrelation parameter.

The above equations (3.1)-(3.14) characterize the complete log-linearized shadow rate NK model for monetary economies.

With this system of equations developed for monetary economies it is possible to return to a cashless economy, after setting $\chi = 0$.

As well this model can also be used to compare the shadow rate New Keynesian model for monetary economies with the standard NK model that ignores unconventional monetary policies. To achieve this, the policy rate $\hat{r}_t$ is used instead of the shadow rate $\hat{s}_t$ in equations (3.3)-(3.10), with the goods market clearing condition (3.1) and the ZLB constraint

$$\hat{r}_t = \begin{cases} \hat{s}_t & \text{if } s_t \geq 0 \\ -s & \text{if } s_t < 0 \end{cases} \tag{3.15}$$

The model uses the policy rate $\hat{r}_t$ in all its equations except the Taylor rule, that still follows the shadow rate model and keeps track of the shadow rate $\hat{s}_t$ dynamics. The changes in the policy rate are identical to the shadow rate dynamics when the nominal shadow rate is positive. The change in the nominal policy rate $\hat{r}_t$ when the nominal shadow rate s_t is negative, remains at the negative steady state value for the shadow rate, $-s$, which implies that the nominal policy rate $r_t = 0$.

Parameterization

Table 1 summarizes the parameter values that are used to conduct the impulse response analysis.

Table 1: Benchmark parameters values in the quantitative model

Parameter	Description	Value
β	Discount factor of households	0.99
γ	Discount factor of entrepreneurs	0.98
σ	Intertemporal elasticity of substitution in consumption	1
ω	Output elasticity of (real) marginal cost	0.47
ψ	Degree of price stickiness	0.5
χ	Degree of non-separability of utility function	$\chi = 0$ or 0.02
δ	Capital depreciation rate	0.03
α	Capital share in production	0.3
η_C	Consumption elasticity of money demand	1
η_R	Interest rate semi-elasticity of money demand	28
ϕ_s	Degree of interest-rate smoothing	0.00
ϕ_y	Interest rate response to output	0.33
ϕ_π	Interest rate response to inflation	0.9
X	Steady-state gross markup	1.05
LV	Loan-to-value ratio for entrepreneurs	0.89
rp	Risk premium	1.009
ρ_β	Autocorrelation of preference shock	0.8
ρ_a	Autocorrelation of technology shock	0.90
ρ_π	Autocorrelation of inflationary shock	0.59

Most of the parameters values are taken from Iacoviello (2005) and Wu & Zhang (2019). We set the discount factor $\beta = 0.99$ that is a common value in the literature and the discount factor for entrepreneurs $\gamma = 0.98$ as a proxy for the firm's internal rate of return. We set the degree of price stickiness $\psi = 0.5$, which implies that prices are fixed on average for two quarters.

We set the capital depreciation rate $\delta = 0.03$; $X = 1.05$ represents a markup price of 20% by the retailers; and the capital share in production $\alpha = 0.3$, following the monetary business cycle literature (e.g., Christiano et al. 1995).

The policy parameters for the Taylor rule are calibrated for smoothing purposes for the shock responses. Specifically, we set the degree of interest-rate smoothing $\phi_s = 0.00$, the response to output $\phi_y = 0.33$ and response to inflation $\phi_\pi = 0.90$. We set the loan-to-value ratio for entrepreneurs $LV = 0.89$ following Wu & Zhang (2019).

The remaining benchmark parameters are taken from Woodford (2003), McKnight & Mihailov (2015) and the papers that they quote. We set the output elasticity of (real) marginal cost $\omega = 0.47$, and the intertemporal elasticity of substitution in consumption is $\sigma = 1$. We set the interest rate semi-elasticity of money demand $\eta_R = 28$ following Kurozumi (2006), and the consumption elasticity of money demand $\eta_c = 1$, in concordance with Mankiw & Summers (1986). Ireland (2004) and Andrés et al. (2006) estimate the degree of non-separability of the utility function $\chi = 0.02$ for monetary economies, and $\chi = 0$ for cashless economies.

The nominal risk premium is 1.005, matches an average 2% annualized risk premium, in a zero-inflation scenario this risk premium yields an annualized policy rate of nearly 2%. Finally, we set the autocorrelation of the technology shock $\rho_a = 0.90$, the autocorrelation of the preference shock $\rho_\beta = 0.80$ and the autocorrelation of the inflation shock $\rho_\pi = 0.59$.

4 Results

In this section there will be compared the results of a technology shock, an inflationary shock and a preference shock in two different versions of the model: the standard model, where the ZLB doesn't apply, and the shadow rate model, under both the cashless economy benchmark and the monetary economy.

To analyze the effect of the unconventional policies, first we induce the economy to a ZLB environment in any scenario of the 3 shocks. When in a ZLB environment, conventional monetary policies are impossible to carry out, as the policy rate is bound, in this section the transmission mechanisms of the shadow rate will be studied.

In the case to contrast the cashless and monetary economies, the main difference stands in the absence of a cost channel of monetary policy in the cashless economies. For considering a cashless economy, the parameter of non-separability degree of the utility function is set to 0, ($\chi = 0$), and the money demand channel is excluded from the model.

Conventional Monetary Policy

We start by conducting an impulse response analysis under conventional monetary policies. In order to replicate the standard New Keynesian models from the model developed in this thesis, the shadow rate $\hat{s}_t$ is substituted by the nominal interest rate $\hat{r}_t$ in all the equations above. With exception of the Taylor rule (3.2). The ZLB equation (3.14) is added to the Taylor rule in order to set the nominal interest rate. Unconventional policies are not considered for this model.

Preference Shock

Figure 1 depicts the impulse responses under a positive 1% preference shock. This shock will increase temporally the discount factor of the households, giving more value to future consumption, hence investing more in the present.

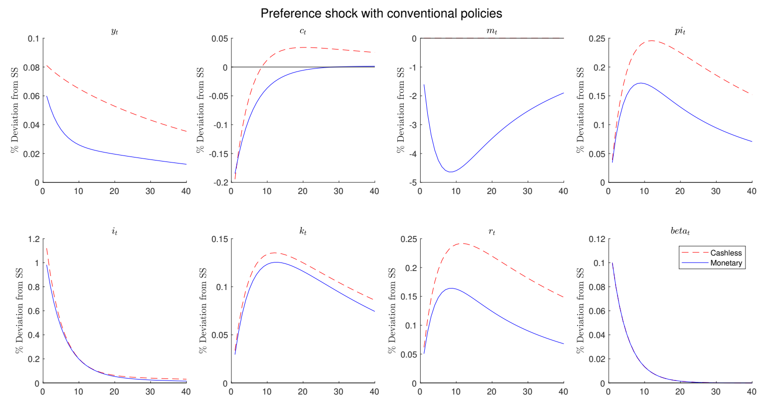

Figure 1: Impulse responses under a positive 1% preference shock with conventional monetary policies

With a 1% positive shock to preferences, households decide to consume 0.2% less and invest 1% more in monetary economies and 1.1% more in the cashless economies. As the shock is temporary, the nominal interest rate increases to higher aggregate demand increase above their steady-state to reduce investment and return to the steady-state, in cashless economies the interest rate goes up to 0.25%, while in the monetary economies it reaches a deviation from its steady state of 0.15%.

A positive preference shock results in both an increase in output (for the intertemporal IS equation) and inflation (from the NKPC). As a result, the nominal interest rate rises in response to the positive preference shock. However, since the rise in both output and inflation is relatively lower in the monetary economy, the nominal interest rate responds less. Why? In addition to the aggregate demand channel, there exists a cost channel of monetary policy. The increase in the nominal interest rate generates a reduction in the demand for money m_t, which affects the output and pricing decisions of firms, via changes in the real marginal cost of production from the NKPC.

In monetary economies there is another effect, investment increases and consumption is reduced, this also leads to an increase of the aggregate output. This increase is less than in a cashless economy, but it returns faster to its steady state, being more stable with the shocks. This effect is caused by the money demand transmission mechanism, where the reduction of the consumption and the increase in the interest rates, the real money balances are reduced via the

LM equation (3.4), and this affects the output and the real marginal cost. These two effects enter the NKPC (3.9) reducing the expectations of inflation and as seen in the graph, also reducing the impact in the inflation and then in the nominal rate. Thus, in a monetary economy the shock is dissipated faster via the money demand transmission mechanism.

Technology Shock

The next shock we consider is a positive 1% change in technology. The results of the deviations from the steady state of the macro variables are represented in Figure 2.

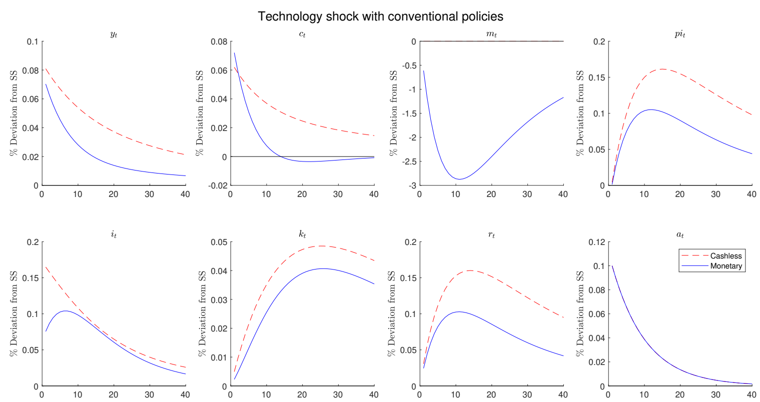

Figure 2: Impulse responses under a positive 1% technology shock with conventional monetary policies

Under a positive technology shock, the aggregate output increases, and as the productivity is also higher, the firms are willing to invest more, in the cashless economy they invest around 0.15% more and in a monetary economy they reach an increase of 0.1%. With the increase in the aggregate output, consumption is also increased.

To mitigate the effect of the shock, the central bank sets a higher nominal interest rate that affects aggregate output by the aggregate demand transmission mechanism. As the aggregate production is affected by the real interest rate, a higher nominal interest rate will promote consumption, greater consumption stimulates more the economy but also affects the inflation by the NKPC (3.9).

In the monetary economy, the money demand transmission mechanism helps alleviate faster the effect of the shock. With an increase in the interest rate, the money demand decreases, this reduction affect aggregate output and real marginal cost, and this effect is also noticed in the NKPC. Hence in a monetary economy, the technology shock has less impact and can be dissipated faster, with a lesser increase in the nominal rate.

Inflationary Shock

The final shock to be considered is a positive 1% inflationary shock. This affects directly the aggregate demand as the real interest rate is modified. The results are in Figure 3.

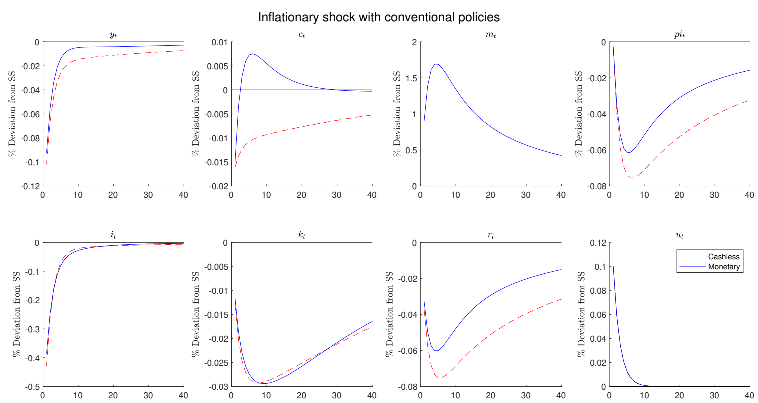

Figure 3: Impulse responses under a positive 1% inflationary shock with conventional monetary policies

With a 1% inflationary shock, the increase in prices will reduce the consumption and the marginal cost, this effect enters the NKPC, lowering actual inflation. With lower inflation, the nominal interest rate is set lower via the Taylor Rule.

In a cashless economy, a reduction in the nominal interest rates affect the real rate and this reduces the aggregate output. With less aggregate output, the inflation also is reduced via the NKPC.

In a monetary economy, the transmission mechanism of the money demand helps stimulate the economy faster. With lower interest rates, the money demand increases the real money holdings. That excess of liquidity stimulates the consumption in subsequent periods and then help stimulate the economy by the money demand transmission mechanism.

It is clear that the additional transmission mechanism in the monetary economies helps to alleviate the impact and duration of the shocks. With less change from the nominal interest rate is possible to return to the steady state of aggregate output and all the other variables, without having a large deviation from the steady state of the inflation.

Unconventional Monetary Policy

In this section we will contrast the unconventional monetary policies under the benchmark cashless model of Wu & Zhang (2019) against the Shadow Rate New Keynesian model for monetary economies implemented in this thesis.

Preference Shock

In a shadow rate model when there is a ZLB for the nominal interest rate and a set of unconventional policies when the ZLB binds, the effects of a positive 1% preference shock are shown in Figure 4.

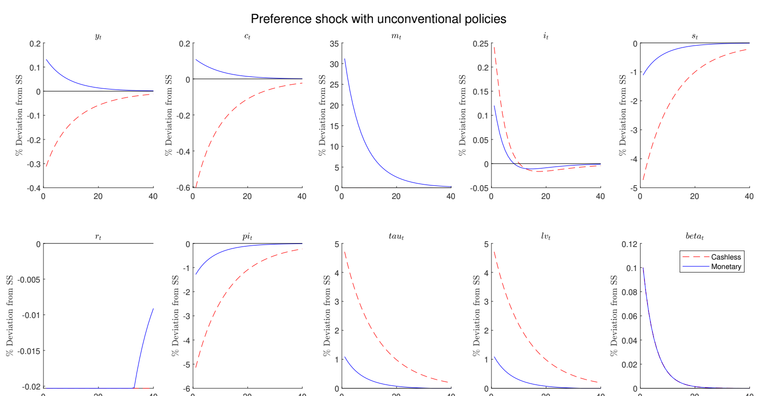

Figure 4: Impulse responses under a positive 1% preference shock with unconventional monetary policies

In response to a positive preference shock, in the cashless economy this will reduce actual consumption by 6% and increase investment by 2.5%. In order to return to the steady state, the central bank will lower their nominal interest rate, but it will fall farther than the ZLB, so the unconventional policies are implemented.

The effect of the unconventional policies and the ZLB is replicated with the shadow rate that is implemented in the economy. As the ZLB binds since the first period, the unconventional policy of the lending is implemented, the loan-to value is increased 5% in the cashless economy and 1% in the monetary economy. This monetary policy incentive the investment and then the aggregate demand transmission channel of the monetary policy is the one that helps mitigate the effect of the preference shock.

As the ZLB is binding, the shadow rate replicates the effect of the lending facilities in both economies. By period 36, the monetary economy has been stimulated enough to bypass the ZLB constraint, then the shadow rate's value is the same as the nominal interest rate.

In a monetary economy, in addition to the aggregate demand transmission mechanism, the money demand transmission mechanism is the one that helps mitigate the effect quicker. With the real money balances in the economy, there is a large increase in the money demand due to the reduction in the shadow rate and less amount of investment than in a cashless economy, this amount of real money balances helps to increase the future consumption, without lowering actual consumption. The increase in the investment and the consumption helps to stimulate the economy and then with both transmission mechanisms it is possible to escape from the ZLB by the period 36, while in the cashless economy the nominal interest rate keeps binding.

The key difference is that with unconventional policies both output and inflation fall in cashless economies rather than rise as with conventional policies. For the monetary economy, consumption can actually rise at the ZLB (a big difference to the results under conventional policies).

Technology shock

The second shock that is considered for the shadow rate models is a 1% technology shock. Where the increase in the factors' productivity will increase the aggregate production. The effects of this shock are in Figure 5.

For both economies this technology shock generates a greater production, this reduces the prices and the marginal cost, with that the consumption grows. The transmission mechanism for the monetary policy changes at this point.

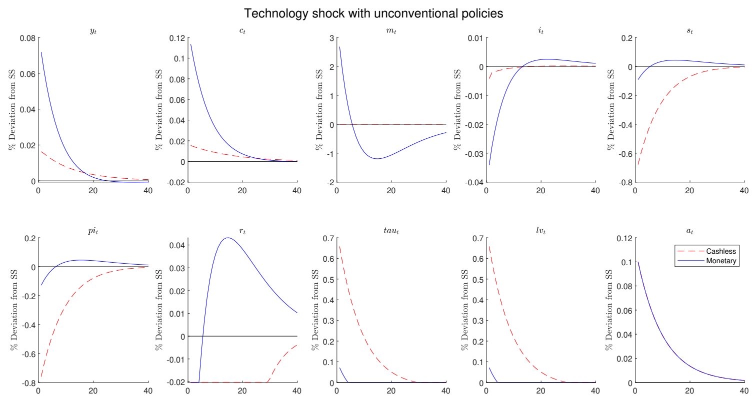

Figure 5: Impulse responses under a positive 1% technology shock with unconventional monetary policies

In a cashless economy, in order to return to the steady state, the nominal interest rate falls beyond the ZLB, when the unconventional policies are implemented by the central bank. The reduction in the interest rate reduces investment, thus the aggregate output is reduced and then starts to return to the steady state. The reduction of the prices implies deflation, that reduces the real rate, and in the case without unconventional policies and the ZLB binding, then the real rate will far beyond zero.

In a monetary economy the technology shock also impulses the aggregate output, reducing marginal cost and with that also reducing the inflation by the NKPC (3.9), and this will lead to a reduction in the nominal interest rate until it reaches the ZLB. However, with the additional money demand transmission mechanism, the reduction of the interest rates leads to an increase in the money demand, this incentive output and inflation. With this cross-effect, the impact of the technology shock in the deflation is lower, with the excess liquidity, more consumption in the economy is made with a trade-off of less investment.

With the additional transmission mechanism, the differences between the cashless economy model and the shadow rate New Keynesian model for monetary economies are that a technology shock reduces the deflation in the economy, as well that stimulating more the economy with a quicker return to the steady-state. Consumption increments and investment decrements are higher at the moment of the shock, but tend faster to the steady state.

The most important result in this section is that in presence of a technology shock, the money demand transmission mechanism makes more efficient the monetary policies and let the economy to escape from the ZLB constraint.

Inflationary shock

The last shock corresponds to a negative 0.5% inflationary shock. The results are in Figure 6

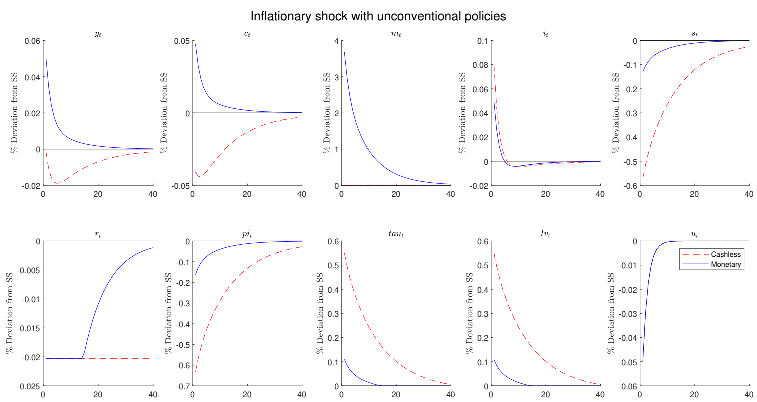

Figure 6: Impulse responses under a negative 0.5% inflationary shock with unconventional monetary policies

With a 0.5% negative inflationary shock, the effect in the NKPC lowers present inflation, this enters in the Taylor Rule and the central bank sets a lower shadow rate. The shadow rate is below the ZLB, so the nominal interest rate binds.

With the ZLB binding, the monetary policy implemented is the lending facilities, where in a cashless economy the loan-to-value lv_t increase 5% and in a monetary economy it increases 1%, This higher loan-to-value encourages investment in the private sector, and this stimulates aggregate output. The aggregate demand transmission channel is reached with the unconventional policies in both economies.

In the monetary economy the ZLB binds for 16 periods, in this time the shadow rate replicates the unconventional policy that was implemented. After period 16, the nominal interest rate escapes the ZLB and then the shadow rate mirrors the value of the nominal interest rate. The cashless economy doesn't avoid the ZLB bind, thus the shadow rate replicates the effect of the lending facilities.

For the monetary economy, with the additional transmission mechanism of the monetary policy, the lower shadow rate encourages the money demand to increase. This increase affects the output and pricing decisions of firms, increasing the marginal cost that enters the NKPC and decrease inflation.

The key difference is that with unconventional policies, for the monetary economy, consumption can actually rise at the ZLB.

5 Conclusions

In this thesis we have developed a Shadow Rate New Keynesian model that incorporates the unconventional policies central banks can implement under a ZLB environment into a monetary economy. There is a set of unconventional policies that can be implemented in order to keep efficient the monetary policies, the unconventional policy used in this model are the lending facilities, where the loan-to-value ratio of the private sector is increased in order to encourage the lending and the investment. Such unconventional policies are used when the nominal interest rate is bound by the ZLB, and the shadow rate can simulate the effect of this unconventional policies in our model.

In cashless economies the transmission mechanism of monetary policy is via the aggregate demand channel. In this transmission mechanism, changes in the nominal interest rate affect output from the inter-temporal IS equation via changes in the real interest rate, which results in changes in inflation via the NKPC.

In monetary economies there is an additional transmission mechanism of monetary policy, where the reduction of the consumption and the increase in the interest rates reduce the demand for money via the LM equation, and this affects the output and the real marginal cost. With the two transmission mechanisms in a monetary economy, the monetary policies can dissipate quicker the effect of adverse shocks.

With the real money balances in the economy, there is a large increase in the money demand due to the reduction in the shadow rate and less amount of investment than in a cashless economy, this amount of real money balances helps to increase the future consumption, without lowering actual consumption.

For preference shocks the key difference is that with unconventional policies both output and inflation fall in cashless economies rather than rise as with conventional policies. For the monetary economy, consumption can actually rise at the ZLB (a big difference to the results under conventional policies).

The technology shock is dissipated quicker in monetary economies, with a greater stimulus to the consumption and aggregate output, as well as a less deflation than in a cashless economy. In this monetary economy the monetary policies are more efficient and the economy can escape the ZLB quicker.

With a negative inflationary shock and the incorporation of unconventional policies, the differences between a cashless economy and a monetary economy are that with the additional transmission mechanism, consumption can actually rise at the ZLB and the economy can escape form the ZLB bind in less periods. In the monetary economy with a less change in the shadow rate than in a cashless economy, the economy can be stimulated and return to the steady state, dissipating quicker the inflationary shock. Less decrease in the shadow rate implies less use of the unconventional policies.

The monetary economy with unconventional policies developed in this thesis gives alternative response to shocks that could affect the economy. Where the two transmission mechanisms of the monetary policies and the incorporation of unconventional policies help to alleviate the stress of this shocks without having a structural break due to the ZLB constraint.

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Appendix

Appendix A. Steady States

- The Law of motion yields the investment to capital ratio.

$$\frac{I}{K} = \delta$$

- The market clearing condition establishes that

$$\frac{I}{Y} = 1 - \frac{C}{Y} - \frac{C^E}{Y}$$

- With the previous assumption of the value of the steady-state for gross tax of income to be 1, the Intertemporal Euler Equation of the Households gives us the steady-state private borrowing rate.

$$R^B = \frac{1}{\beta}$$

- The Taylor Rule and the definition of gross return of the assets yield the following steady state for the shadow rate and at the same time for the policy rate.

$$S = R = \frac{R^B}{RP}$$

- With the First-order conditions, the budget constraint and the borrowing constraint of the entrepreneurs it yields the debt-to-output and the investment-to-output ratios:

$$\delta\frac{B}{Y} = \beta LV\frac{I}{Y}$$

$$\frac{I}{Y} = \frac{1}{X}\left[\frac{\gamma\alpha\delta}{1 - LV(\beta - \gamma) - \gamma(1 - \delta)}\right]$$

- The household's budget constraint with the previous steady states, yield the consumption-to-output ratio of the households:

$$\frac{C}{Y} = \frac{X - \alpha}{X} + \frac{B}{Y}\left(\frac{1 - \beta}{\beta}\right)$$