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MONETARY POLICY UNDER LIMITED ASSET MARKET PARTICIPATION AND BOUNDED RATIONALITY

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Contents

Monetary Policy Under Limited Asset Market Participation and Bounded Rationality

By Alejandro Gurrola Luna

In this thesis I analyze the consequences of incorporating bounded rationality and limited asset market participation into the popular New Keynesian model. I study extensively the implications of these features for equilibrium determinacy and the Taylor principle under four different specifications for the interest-rate rule that central banks may follow. On the one hand, limited asset market participation implies that not all agents in the economy have access to financial markets, which has important implications for monetary policy. For example, an increase in the interest rate can generate an expansionary (rather than the usually believed contractionary) effect in the economy, provided that the degree of financial market participation is sufficiently low. On the other hand, bounded rationality states that agents are not fully-rational and thus the expectations they make about the future can be inaccurate. The "less rational" agents are, the easier it is to prevent self-fulfilling expectations and determinacy is easier to achieve under the Taylor principle. I also study the dynamic effects of shocks and show how these two mechanisms affect the economy's response. Interestingly, I find that the bounded rationality does not have real implications.

1. Introduction

The popular New Keynesian (NK) textbook model is subject to several criticisms due to a set of unrealistic simplifying assumptions that it makes. In particular, the model assumes that all agents are fully-rational and that all agents have access to financial markets. This thesis aims to introduce bounded rationality and limited asset market participation into a NK framework.

Limited asset market participation

The baseline NK model assumes that all agents in the economy have access to financial markets and thus can optimally smooth consumption across time. However, in reality this assumption is difficult to defend, especially in developing countries where a significant fraction of the population does not have access to financial markets.

For example, in 2018, according to the OECD, only around 47% of the working age population in Mexico has a bank account. This evidence suggests that a large proportion of the population does not save and this “excess” consumption may be due to a poorly developed financial system, ignorance or lack of promotion of the investment opportunities available to the general public, or another behavioral characteristic such as bounded rationality as I will discuss below.

In this thesis I incorporate limited asset markets participation (LAMP) first introduced by Bilbiie (2008) into the baseline NK model. I assume that a fraction of agents have access to complete financial markets and that the remaining fraction is “constrained” in the sense that they do not have access to any sort of ownership such as shares in firms and contingent claims (thus they are not able to smooth consumption). This limitation implies that the latter fraction of agents simply consume the income they receive from working in each period.

The main implication of incorporating LAMP is that monetary policy may not work as it is usually believed. For example, an increase in the interest rate from the central bank may result in expansionary monetary policy. Formally, the sensitivity of the output gap to changes in the interest rate (the slope of the IS curve) may be negative (the usual aggregate demand logic, a higher interest rate results in a lower output) or positive (an inverted aggregate demand logic, a higher interest rate results in a higher output). And the higher the share of constrained households in the economy, the more positive the slope becomes.

This has important consequences for the design of monetary policy. In standard models, in order to prevent indeterminacy and self-fulfilling expectations, central banks should adjust the nominal interest rate to changes in inflation more than a one-to-one magnitude (the so-called Taylor principle). However, under LAMP, an inverted Taylor principle (i.e., a passive monetary policy) is required for equilibrium determinacy.

Bounded rationality

It would be naive to think that consumers, given the complexity of the decision-making process they face each period, are able to act optimally to maximize utility, Simon (1957). Moreover, there is empirical evidence that suggest that to maximize utility, agents indeed act as if they are not rational (Galí and Gertler 1999, Fuhrer and Rudebusch 2004, Lindé 2005).

To address this concept, I also incorporate behavioral factors to the baseline New Keynesian model. Following the same approach as Gabaix (2020), the behavioral factor is as follows: agents are not able to perfectly measure expected events in the future; the more into the future an event is, the less accurate agents are in measuring this event (cognitive discounting).

It is important to note that since constrained households simply consume their labor income as it is received (no saving decision), their optimization problem does not involve discounting future variables. Consequently, only unconstrained households are affected by cognitive discounting.

Mathematically, the cognitive discounting is introduced by assuming that agents use a "behavioral expectations operator" rather than a "rational expectations operator" when optimizing. That is, the future perceived by the agents in the present is biased towards the steady-state of the economy. This behavioral expectations operator underestimates the value of the rational expectation of the relevant variables.

Let $\mathbf{X}_t$ denote a state vector that in equilibrium evolves according to the transition function $\mathbf{G}(\cdot)$, that is $\mathbf{X}_{t+1} = \mathbf{G}(\mathbf{X}_t)$. Each variable depends on this state vector (for instance $r_t = r(\mathbf{X}_t)$). Linearizing the law of motion yields $\mathbf{X}_{t+1} = \mathbf{\Gamma}\mathbf{X}_t$ for some matrix $\mathbf{\Gamma}$.

However, due to the behavioral bias, agents misperceive the state vector. Agents believe that the state vector evolves according to $\mathbf{X}_{t+1} = m\mathbf{G}(\mathbf{X}_t)$, where $m \in [0, 1]$ is a cognitive discounting parameter measuring how accurate agents are with respect to the future. Therefore, linearizing again the misperceived law of motion I get $\mathbf{X}_{t+1} = m\mathbf{\Gamma}\mathbf{X}_t$. Hence the expectation of the behavioral agent is

$$\mathbb{E}_t^{BR}[X_{t+1}] = m\mathbf{\Gamma}\mathbf{X}_t.$$

Moreover, iterating the last expression forward yields the following

$$\mathbb{E}_t^{BR}[X_{t+k}] = m^k \mathbf{\Gamma} \mathbf{X}_t.$$

In general, for any variable $z(\mathbf{X}_t)$, the beliefs of the behavioral agent satisfy

$$\mathbb{E}_t^{BR}[z(\mathbf{X}_{t+k})] = m^k \mathbb{E}_t[z(\mathbf{X}_{t+k})].$$

The proof is straight-forward. Given the state vector in a particular period, any variable can be expressed (after linearizing) as $b_z' X_t$, where b_z is some factor corresponding specifically and uniquely to variable z. Then

$$\mathbb{E}_t^{BR}[z(\mathbf{X}_{t+k})] = \mathbb{E}_t^{BR}[b_z' \mathbf{X}_{t+k}] = b_z' \mathbb{E}_t^{BR}[\mathbf{X}_{t+k}] = b_z' m^k \mathbb{E}_t[\mathbf{X}_{t+k}] = m^k \mathbb{E}_t[z(\mathbf{X}_{t+k})].$$

This way, agents "discount more heavily" the future than under rational expectations. That is, the marginal rate of substitution between present consumption to future consumption is higher than otherwise. In addition, I propose a form of cognitive discounting for central banks trying to capture the idea that they cannot accurately measure expected future inflation, especially during inflationary episodes.

In this thesis I look at the determinacy properties of four commonly-employed interest rate rules by incorporating LAMP and bounded rationality. I also study the impulse response functions of the economy to exogenous shocks. Bilbiie (2008) finds that the sensitivity of aggregate demand to real interest rates depends in a non-linear way on the share of asset market participation. The lower the share, the more negative the sensitive is. And as the share increases, the sensitivity becomes positive; thus for low shares, the aggregate demand logic is as usual but this is inverted for high shares of constrained households. Incorporating bounded rationality has great implications for determinacy of the steady-state equilibrium. However, bounded rationality does not affect the aggregate demand dynamics under what I will call the rational expectations LAMP. The LAMP model is also known as a two-agent New Keynesian (TANK) model. Debortoli and Galí (2017) show that the TANK captures reasonably well the implications of a baseline heterogenous-agent New Keynesian (HANK) model regarding the effects of aggregate shocks on aggregate variables. Albonico, Paccagnini, and Tirelli (2019) find that in the European Union 39% of the population did not have access to financial markets during 1993-2012, and that the LAMP model is preferred to its representative household counterpart; probably this share is higher for most developing countries highlighting the need to let go the representative agent found in the baseline NK model.

On the behavioral side, Farhi and Werning (2017) introduce bounded rationality in the form of level-k thinking. They find that the transmission of monetary policy is mitigated the more into the future the monetary policy change takes place; this is the so-called "forward guidance puzzle". Angeletos and Huo (2018) analyze the effects of two behavioral distortions: a form of myopia or extra discounting of the future (as the present thesis), and a form of habit, or anchoring of current behavior to past behavior. They show that both of these elements may be endogenous to general equilibrium mechanisms. Angeletos, Huo, and Sastry empirically find that following any shock, forecasts appear to underreact for the first few quarters but overshoot later on; in huge contrast to rational expectations. The behavioral literature has been growing in the recent years. A caveat is the difficulty of quantifying the behavioral pattern of agents; for instance, there is no a single dominant estimate for the cognitive parameter of bounded rationality.

The main results of this thesis are as follows. Bounded rationality does not affect the slope of the dynamic IS (investment-savings) curve and does not change the threshold at which the economy passes from the "standard aggregate demand logic" (SADL) to the "inverted aggregate demand logic" (IADL) region. However, the sensitivity of aggregate demand to changes in inflation expectations does change and is reduced in absolute value. The proposed cognitive discounting of central banks shows that if they are aware of the limitation they have regarding measuring expectation, they must react to changes in inflation in a more aggressive manner than otherwise. This result in turn increases the determinacy region in the IADL economy making it easier to restore the Taylor principle even in a IADL economy under the strict forecast-based inflation targeting rule; under this interest rate rule, passive monetary policy induces determinacy, since the central bank's response to changes in inflation must be of a larger magnitude given the cognitive discounting, this may restore the Taylor principle for a larger interval of the share of constrained households. On top of that, Bilbiie (2008) shows, an hybrid Taylor rule restores the Taylor principle in a IADL economy too (in the rational expectations LAMP); incorporating bounded rationality provides a larger determinacy region where a lower output response is sufficient for the Taylor principle to produce determinacy. Additionally, under the hybrid Taylor rule there is a interval for the output response where no matter the inflation response, determinacy is never achieved and bounded rationality reduces this interval. Finally, I find that the less forward-looking agents are, the faster the economy returns to the steady-state after a monetary policy shock.

The rest of this thesis is organized as follows. Section 2 develops the NK model under LAMP and bounded rationality. Section 3 presents the solved linearized model which yields the usual two equations: the (behavioral) New Keynesian Phillips curve and the dynamic investment-saving (DIS) curve. Section 4 shows the given values for the parameters of the model used in the numerical exercises and explains why those values were chosen. Section 5 studies equilibrium determinacy under the different interest rate rules. Section 6 presents the dynamic effects of a monetary policy shock in the economy. Finally, Section 7 concludes. The appendix contains technical details such as deriving the behavioral New Keynesian Phillips curve.

2. The Model

As mentioned before, I extend a NK type model to incorporate bounded rationality and LAMP. The model is comprised of two types of households (constrained and unconstrained), a competitive final-goods producer, monopolistically competitive intermediate-goods producers, and a central bank.

The model is a closed-economy in which both types of households supply labor to intermediate good producers and purchase goods for consumption from final good producers. In addition, the unconstrained household has access to state-contingent assets available from complete financial markets, and is a shareholder of the intermediate firms making her eligible to receive profit income. In contrast, the constrained household does not have access to financial markets and does not receive this type of (profit) income. Unconstrained agents (and thus intermediate firms) are subject to the cognitive discounting.

There is a competitive final good producer that produces homogenous final goods using intermediate goods as inputs. The final good producer, in contrast to the intermediate good producer, is a price-taker.

Each intermediate good producer produces a differentiated good using (only) labor as its factor of production. In each period, a given intermediate good producer has a positive probability of being able to reset its price and a positive probability that it will not be able to adjust its price.

Finally, the monetary policy tool that the central bank uses is the interest-rate. The central bank follows an interest rate rule that may depend on a combination of the actual inflation, the next-period expected inflation, and the output-gap.

2.1. Households

There is a continuum $[0,1]$ of infinitely-lived households that maximize their expected discounted utility by choosing consumption, C_t, and labor, N_t, in every period. We assume that a proportion of households comprised in the $[0,\lambda]$ interval do not have access to financial markets, whereas households in the interval $[\lambda,1]$ have access, and we assume that there is a representative agent in each subinterval denoted by a subscript H (constrained households) and S (unconstrained households), respectively.

Constrained households do not have access to financial markets, and consequently, are not able to smooth consumption across time and thus simply consume all of their income as it is received (i.e., they have no saving decision). Let $C_{H,t}$ be consumption and $N_{H,t}$ be hours worked in period t for the representative constrained household, the nominal wage she receives is W_t and the aggregate price level is P_t. Her budget constraint is:

$$C_{H,t} = \frac{W_t}{P_t} N_{H,t}. \tag{1}$$

She then solves

$$\max_{C_{H,t},N_{H,t}} u(C_{H,t}, N_{H,t}) \quad s.t. \quad (1), \tag{2}$$

where I assume a constant relative risk aversion (CRRA)[1] instantaneous utility function of the form:

$$u(C_H, N_H) = \frac{C_H^{1-\sigma} - 1}{1-\sigma} - \omega \frac{N_H^{1+\varphi}}{1+\varphi}. \tag{3}$$

$\omega > 0$ indicates how leisure is valued relative to consumption, $\varphi > 0$ is the inverse of the labor supply elasticity, and $\sigma > 0$ is the inverse of the

[1] It is called CRRA since the coefficient of relative risk aversion $r_R(C) = -Cu''(C)/u'(C)$ is constant. Note that $u_C = C^{-\sigma}$, $u_{CC} = -\sigma C^{-\sigma-1}$, thus $r_R(C) = -C(-\sigma C^{-\sigma-1})/C^{-\sigma} = \sigma$.

intertemporal elasticity of substitution in consumption. Equivalently by substituting the budget constraint (1) into the objective function, the maximization problem can be expressed as:

$$\max_{N_{H,t}} \frac{\left(\frac{W_t}{P_t}N_{H,t}\right)^{1-\sigma} - 1}{1-\sigma} - \omega\frac{N_{H,t}^{1+\varphi}}{1+\varphi}.$$

The first order condition yields:

$$\begin{aligned} &\left(\frac{W_t}{P_t}N_{H,t}\right)^{-\sigma}\frac{W_t}{P_t} - \omega N_{H,t}^{\varphi} = 0 \\ &\iff N_{H,t}^{\sigma+\varphi} = \frac{1}{\omega}\left(\frac{W_t}{P_t}\right)^{1-\sigma}, \end{aligned} \tag{4}$$

and thus consumption of the constrained household is

$$C_{H,t} = \left(\frac{1}{\omega}\right)^{\frac{1}{\sigma+\varphi}}\left(\frac{W_t}{P_t}\right)^{\frac{1+\varphi}{\sigma+\varphi}}. \tag{5}$$

The optimization problem of unconstrained households is different since they have access to complete financial markets for state-contingent securities and thus face a different budget constraint. The period budget constraint for the representative unconstrained household is given by:

$$B_{S,t} + (\Omega_{S,t+1} - \Omega_{S,t})V_t + P_tC_{S,t} \leq Z_t + \Omega_{S,t}P_tD_t + W_tN_{S,t}, \tag{6}$$

where $B_{S,t}$ is the nominal value at end of period t of a portfolio of all state-contingent assets held, except for shares in firms, Z_t is beginning of period wealth, not including the payoff of shares, V_t is average market value at time t shares in intermediate good firms, D_t are real dividend payoffs of these shares and $\Omega_{S,t}$ are share holdings. The RHS is comprised of her initial wealth, the dividends she receives from her ownership of firms, and her labor income. The LHS is comprised of the value of a risky portfolio, the value of the change in share holdings, and nominal consumption $P_tC_{S,t}$, where P_t denotes the price-index of the final good.

I assume that both constrained and unconstrained households possess the exact same instantaneous CRRA utility function; that is, the parameters of the utility function do not change across types. Under rational expectations, the unconstrained household solves:

$$\max_{C_{S,t+j},N_{S,t+j}} \sum_{j=0}^{\infty} \beta^t \mathbb{E}_t \left[\frac{C_{S,t+j}^{1-\sigma}-1}{1-\sigma} - \omega \frac{N_{S,t+j}^{1+\varphi}}{1+\varphi} \right], \tag{7}$$

subject to (6), where $0 < \beta < 1$ is the discount factor and the operator $\mathbb{E}_t[\cdot]$ represents the expected value conditional on information up to period t.

However, we assume that the agent is subject to a behavioral bias. In particular, to a cognitive discounting outlined above in the introduction. Thus, we say that the agent is a *behavioral agent* and thus she solves

$$\max_{C_{S,t+j},N_{S,t+j}} \sum_{j=0}^{\infty} \beta^t \mathbb{E}_t^{BR} \left[\frac{C_{S,t+j}^{1-\sigma}-1}{1-\sigma} - \omega \frac{N_{S,t+j}^{1+\varphi}}{1+\varphi} \right], \tag{8}$$

subject to (6), where $\mathbb{E}_t^{BR}[\cdot]$ represents the expected value conditional on information up to period t as perceived by the behavioral agent.

Consider that the representative unconstrained agent reduces consumption in period t and using the resulting savings to buy a risky portfolio that produces an uncertain stream of payoffs, Z_{t+1}, $Z_{t+2}, \dots$. Since she is optimizing, the marginal utility she forgoes from reducing her consumption must equal the expected sum of the discounted marginal utilities of the future consumption the portfolio will provide her. If the value of the portfolio at t is $B_{S,t}$, in the absence of arbitrage, then it follows that:

$$\begin{aligned} u'(C_t)B_{S,t} &= \mathbb{E}_t^{BR}\left[\sum_{j=1}^{\infty} \beta^j u'(C_{t+j}) Z_{t+j}\right] \\ \iff B_{S,t} &= \mathbb{E}_t^{BR}\left[\sum_{j=1}^{\infty} \beta^j \frac{u'(C_{t+j})}{u'(C_t)} Z_{t+j}\right] \\ &:= \mathbb{E}_t^{BR}\left[\sum_{j=1}^{\infty} \Lambda_{t,t+j} Z_{t+j}\right], \end{aligned}$$

where $\Lambda_{t,t+j}$ is the stochastic discount factor. Note that under bounded rationality, I must use the behavioral expectations operator. If the portfolio only pays an uncertain payoff in the following period, it follows that:

$$B_{S,t} = \mathbb{E}_t^{BR}\left[\Lambda_{t,t+1} Z_{t+1}\right]. \tag{9}$$

Similarly, instead of considering the risky portfolio, now consider the ownership of the shares of firms that consist of price appreciation and dividends

$$V_t = \mathbb{E}_t^{BR}\left[\Lambda_{t,t+1}(V_{t+1} + P_{t+1}D_{t+1})\right]. \tag{10}$$

Iterating forward gives:

$$\begin{aligned} V_t &= \mathbb{E}_t^{BR}[\Lambda_{t,t+1}(\mathbb{E}_{t+1}[\Lambda_{t+1,t+2}(V_{t+2} + P_{t+2}D_{t+2})] + P_{t+1}D_{t+1})] \\ &= \mathbb{E}_t^{BR}[\Lambda_{t,t+2}V_{t+2}] + \mathbb{E}_t^{BR}[\Lambda_{t,t+2}P_{t+2}D_{t+2}] + \mathbb{E}_t^{BR}[\Lambda_{t,t+1}P_{t+1}D_{t+1}] \\ &\cdots \\ &= \mathbb{E}_t^{BR}[\Lambda_{t,t+k}V_{t+k}] + \sum_{j=1}^{k} \mathbb{E}_t^{BR}[\Lambda_{t,t+j}P_{t+j}D_{t+j}]. \end{aligned}$$

As k tends to infinity,

$$V_t = \sum_{j=1}^{\infty} \mathbb{E}_t^{BR}[\Lambda_{t,t+j}P_{t+j}D_{t+j}], \tag{11}$$

after imposing the following transversality condition:

$$\lim_{k\to\infty} \mathbb{E}_t^{BR}[\Lambda_{t,t+k}V_{t+k}] = 0. \tag{12}$$

Similarly for assets, the transversality condition is given by:

$$\lim_{k\to\infty} \mathbb{E}_t^{BR}[\Lambda_{t,t+k}Z_{t+k}] = 0. \tag{13}$$

Equation (9) can be expressed as

$$1 = \mathbb{E}_t^{BR}\left[\Lambda_{t,t+1}\frac{Z_{t+1}}{B_{S,t}}\right]. \tag{14}$$

Considering a riskless asset, equation (14) becomes

$$1 = \mathbb{E}_t^{BR}[\Lambda_{t,t+1}]R_t,$$

where R_t is the riskless gross short-term nominal interest rate.

Substituting the no-arbitrage conditions (9) and (10), as well as the transversality conditions (12) and (13), into the wealth dynamics equation yields the following intertemporal (or lifetime) budget constraint:

$$\sum_{j=0}^{\infty} \mathbb{E}_t^{BR}[\Lambda_{t,t+j}P_{t+j}C_{S,t+j}] \leq Z_t + V_t + \sum_{j=0}^{\infty} \mathbb{E}_t^{BR}[\Lambda_{t,t+j}W_{t+j}N_{S,t+j}]. \tag{15}$$

The Lagrangian of the optimization problem is given by:

$$\mathcal{L} = \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t^{BR} \left[\frac{C_{S,t+j}^{1-\sigma} - 1}{1-\sigma} - \omega \frac{N_{S,t+j}^{1+\varphi}}{1+\varphi} \right]$$
$$- \lambda \left[\sum_{j=0}^{\infty} \mathbb{E}_t^{BR}[\Lambda_{t,t+j} P_{t+j} C_{S,t+j}] - Z_t - V_t - \sum_{j=0}^{\infty} \mathbb{E}_t^{BR}[\Lambda_{t,t+j} W_{t+j} N_{S,t+j}] \right],$$

where the λ denotes the Lagrange multiplier. The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial C_{S,t}} = C_{S,t}^{-\sigma} - \lambda P_t = 0$$
$$\iff C_{S,t}^{-\sigma} = \lambda P_t$$
$$\iff \beta \frac{C_{S,t+1}^{-\sigma}}{C_{S,t}^{-\sigma}} = \Lambda_{t,t+1} \frac{P_{t+1}}{P_t}, \tag{16}$$
$$\frac{\partial \mathcal{L}}{\partial N_{S,t}} = -\omega N_{S,t}^{\varphi} + \lambda W_t = 0$$
$$\iff \frac{\omega N_{S,t}^{\varphi}}{C_{S,t}^{-\sigma}} = \frac{W_t}{P_t}, \tag{17}$$

and the Euler equation of the unconstrained household is

$$\frac{1}{R_t} = \beta \mathbb{E}_t^{BR} \left[\frac{C_{S,t+1}^{-\sigma}}{C_{S,t}^{-\sigma}} \frac{P_t}{P_{t+1}} \right]. \tag{18}$$

2.2. *Firms*

The final good is produced by a representative firm using CES production technology function to aggregate the continuum of intermediate goods indexed by i,

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{19}$$

where $\epsilon > 1$ is the constant elasticity of substitution between the individual goods. The final goods producer is a competitive firm that maximizes profit:

$$\max_{Y_t(i)} P_t Y_t - \int_0^1 P_t(i) Y_t(i) di, \tag{20}$$

where P_t is the overall price index of the final good. Substituting equation (19) into the final goods producer's optimization problem (20) yields:

$$\max_{Y_t(i)} P_t \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} - \int_0^1 P_t(i) Y_t(i) di,$$

and the first order condition is

$$\begin{aligned} P_t \frac{\epsilon}{\epsilon-1} \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{1}{\epsilon-1}} \frac{\epsilon-1}{\epsilon} Y_t(i)^{-\frac{1}{\epsilon}} - P_t(i) = 0 \\ \iff \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} Y_t(i)^{-1} = \left(\frac{P_t(i)}{P_t} \right)^{\epsilon} \\ \iff Y_t(i) = Y_t \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon}, \end{aligned} \tag{21}$$

which is the demand for each intermediate good. Nominal output is given by

$$P_t Y_t = \int_0^1 P_t(i) Y_t(i) di.$$

Substituting (21) into the above yields the aggregate price index:

$$\begin{aligned} P_t Y_t = \int_0^1 P_t(i) Y_t \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} di \\ \iff P_t^{1-\epsilon} = \int_0^1 P_t(i)^{1-\epsilon} di \\ \iff P_t = \left(\int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}. \end{aligned} \tag{22}$$

2.3. *Intermediate good producers*

Each intermediate good is produced by a monopolistically competitive firm indexed by i using a technology given by

$$Y_t(i) = \begin{cases} A_t N_t(i) - F & N_t(i) > F \\ 0 & N_t(i) \leq F \end{cases},$$

where A_t is a common productivity shock and F is a fixed cost assumed to be common to all firms. Cost minimization implies solving the optimization problem

$$\min_{N_t(i)} W_t N_t(i) \quad s.t. \quad Y_t(i) = A_t N_t(i) - F.$$

The Lagrangian is

$$\mathcal{L} = W_t N_t(i) + \lambda_t[Y_t(i) - A_t N_t(i) + F] = W_t N_t(i) + MC_t[Y_t(i) - A_t N_t(i) + F],$$

where I used the fact that the Lagrange multiplier represents the nominal marginal cost for the firm in period t. Note that since all firms are identical, the marginal cost is the same for all firms. The first order condition yields

$$MC_t = \frac{W_t}{A_t}. \tag{23}$$

The profit function in real terms is given by:

$$\begin{aligned}
D_t(i) &= \frac{P_t(i)}{P_t} Y_t(i) - \frac{W_t}{P_t} N_t(i) && (24)\\
&= \frac{P_t(i)}{P_t} Y_t(i) - \frac{A_t MC_t}{P_t} N_t(i)\\
&= \frac{P_t(i)}{P_t} Y_t(i) - \frac{A_t MC_t}{P_t} \frac{Y_t(i) + F}{A_t}\\
&= \left(\frac{P_t(i)}{P_t} - \frac{MC_t}{P_t}\right) Y_t(i) - \frac{MC_t}{P_t} F, && (25)
\end{aligned}$$

which aggregated over firms gives the following expression for total profits:

$$D_t = \left(1 - \frac{MC_t}{P_t}\Delta_t\right) Y_t - \frac{MC_t}{P_t} F, \tag{26}$$

where $\Delta_t \geq 1$ measures the price dispersion of intermediate goods. In the absence of pricing frictions, $\Delta_t = 1$. Δ_t is derived as follows. Substituting the demand schedule for the intermediate firms (21) into the production function and aggregating:

$$\begin{aligned}
& Y_t \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} di = A_t N_t - F\\
\iff & \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} di = \frac{A_t N_t - F}{Y_t} := \Delta_t. && (27)
\end{aligned}$$

As mentioned earlier, in this model there is price-stickiness following Calvo (1983). In each period, with probability θ, intermediate good firms keep their prices constant (i.e., keep the price of the previous period). This probability is independent across time and across firms.[2] Unconstrained households hold all the shares in the firms. Firms maximize the discounted sum of expected future nominal profits, choosing the price $P_t(i)$. Note that in the presence of this nominal friction monetary policy is non-neutral.

However, intermediate firms are also subject to the behavioral bias (we assume that, since unconstrained households are the owners of the firms, the bias is exactly the same between households and firms). Firms are not able to accurately perceive the future, thus their price-setting problem is given by:

$$\max_{P_t(i)} \sum_{j=0}^{\infty} \theta^j \mathbb{E}_t^{BR} \left[\Lambda_{t,t+j} \left(P_t(i) - MC_{t+j} \right) \left(\frac{P_t(i)}{P_{t+j}} \right)^{-\epsilon} Y_{t+j} \right],$$

after substituting the demand constraint (21). The first order condition, after some manipulation, yields

$$0 = \sum_{j=0}^{\infty} \theta^j \mathbb{E}_t^{BR} \left[\Lambda_{t,t+j} Y_{t+j} \left(\left(\frac{P_t(i)}{P_{t+j}} \right)^{-\epsilon} (1-\epsilon) + \frac{\epsilon MC_{t+j}}{P_{t+j}} \left(\frac{P_t(i)}{P_{t+j}} \right)^{-1-\epsilon} \right) \right].$$

Rearranging the above first order condition

$$\begin{aligned}
\sum_{j=0}^{\infty} \theta^j \mathbb{E}_t^{BR} \left[\Lambda_{t,t+j} Y_{t+j} \left(\frac{P_t(i)}{P_{t+j}} \right)^{-\epsilon} \right] &= \frac{\epsilon}{\epsilon-1} \sum_{j=0}^{\infty} \theta^j \mathbb{E}_t^{BR} \left[\Lambda_{t,t+j} Y_{t+j} \frac{MC_{t+j}}{P_{t+j}} \left(\frac{P_t(i)}{P_{t+j}} \right)^{-1-\epsilon} \right] \\
\iff P_t(i) &= \frac{\epsilon}{\epsilon-1} \frac{\sum_{j=0}^{\infty} \theta^j \mathbb{E}_t^{BR} \left[\Lambda_{t,t+j} Y_{t+j} \frac{MC_{t+j}}{P_{t+j}} \left(\frac{1}{P_{t+j}} \right)^{-1-\epsilon} \right]}{\sum_{j=0}^{\infty} \theta^j \mathbb{E}_t^{BR} \left[\Lambda_{t,t+j} Y_{t+j} \left(\frac{1}{P_{t+j}} \right)^{-\epsilon} \right]} \\
&= \frac{\epsilon}{\epsilon-1} \mathbb{E}_t^{BR} \left[\sum_{j=0}^{\infty} \frac{\theta^j \Lambda_{t,t+j} Y_{t+j} (P_{t+j})^{\epsilon-1}}{\sum_{k=0}^{\infty} \theta^k \mathbb{E}_t^{BR} \left[\Lambda_{t,t+k} Y_{t+k} \left(P_{t+k} \right)^{\epsilon-1} \right]} MC_{t+j} \right] \\
&:= P_t^*. \qquad (28)
\end{aligned}$$

[2]In other words, in each period, each firm faces a Bernoulli trial where the probability of "success" (being able to adjust its price) is $1-\theta$.

From the aggregate price index (22) we have

$$P_t^{1-\epsilon} = \int_0^1 P_t(i)^{1-\epsilon} di.$$

Using the fact that in equilibrium producers behave in the same manner, both the ones that are able to reset and the ones that not,

$$\begin{aligned} P_t^{1-\epsilon} &= \int_0^\theta P_{t-1}(i)^{1-\epsilon} di + \int_\theta^1 P_t^*(i)^{1-\epsilon} di \\ &= \theta P_{t-1}^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}. \end{aligned} \tag{29}$$

2.4. Monetary policy

I study instrument rules as a function of either expected inflation, current inflation, or the output gap. In addition, I assume that the monetary authority is unable to accurately predict the future. Its cognitive discounting parameter will be m^{CB} (of course is reasonable to suppose that the central bank has a better ability to predict than the average asset holder, thus $m^{CB} > m$). That is, for example,

$$\mathbb{E}_t^{BR}[\pi_{t+k}] = (m^{CB})^k \mathbb{E}_t[\pi_{t+k}].$$

Here, the cognitive parameter tries to model the inability of the central bank to properly measure expected inflation (in contrast to the asset holders cognitive parameter where its motivation is the behavioral pattern that people exhibit; no reason to say that the central bank is behavioral). Thus it tries to explain why central banks usually underestimate future inflation.[3,4]

2.5. Market clearing conditions

Market clearing in the labor market requires that

$$\begin{aligned} N_t &= \int_0^\lambda N_{H,t}(i) di + \int_\lambda^1 N_{S,t}(i) di \\ &= \lambda N_{H,t} + (1-\lambda) N_{S,t}, \end{aligned} \tag{30}$$

where the first equality follows from the two different types of households. Market clearing in the financial market requires that bonds are in zero net

[3] In 2021, inflation started to rise and many central banks underestimated the magnitude of the inflationary pressures. See Reis (2022).

[4] In an inflationary environment, central banks may be "behind the curve". See Bullard (2022).

supply

$$Z_t = 0. \tag{31}$$

Equity market clearing implies that share holdings are equally divided between the asset holders. That is,

$$\Omega_{S,t} = \frac{1}{1-\lambda} = \Omega_{S,t+1}. \tag{32}$$

Market-clearing in the financial market and the equity market imply goods market-clearing. That is, equations (31) and (32) imply:[5]

$$Y_t = C_t. \tag{33}$$

Finally, and naturally, aggregate consumption will be

$$C_t = \lambda C_{H,t} + (1-\lambda) C_{S,t}. \tag{34}$$

2.6. *Bounded rationality equilibrium*

A bounded rationality (or behavioral) equilibrium is a set of 16 endogenous variables comprised of: a sequence of prices $\{W_t, MC_t, P_t, P_t^*, \Delta_t\}$, a sequence of allocations $\{C_{H,t}, C_{S,t}, N_{H,t}, N_{S,t}, Y_t, Z_t, D_t, \Omega_{S,t}, B_{S,t}, A_t\}$, and a monetary policy $\{R_t\}$.

The allocations must satisfy the following 16 equilibrium conditions:

- The constrained household conditions; the budget constraint (1) and optimal labor supply (4).
- The unconstrained household conditions; the budget constraint (6), the optimal labor supply condition (17), and the consumption Euler equation (18).
- The aggregate version of the production function (27).
- The intermediate good producers first-order condition (23).
- Aggregate real profits of intermediate good producers (26).
- A law of motion of price dispersion derived from (27).
- Price-setting rules (28) and (29).

[5] A detailed derivation of equation (33) can be found in the appendix.

– Market clearing conditions (31) and (32).

– Aggregate labor supply (30) and aggregate consumption (34).

– An interest rate rule.

Also, the transversality conditions (12) and (13) must be satisfied.

2.7. Steady-state and linearized equilibrium

In the steady-state, the equilibrium conditions are evaluated in the absence of shocks and assuming that all variables are constant. In what follows, all variables without a time subscript denote steady-state values and I normalize $A_t = A = 1$.

Since prices are fully flexible in the steady-state,

$$\begin{aligned} P &= \frac{\epsilon}{\epsilon - 1} MC \\ &= \frac{\epsilon}{\epsilon - 1} W, \end{aligned} \tag{35}$$

where the last equality follows from (23). From (27), $\Delta_t = 1$, so there is no price dispersion in the steady-state and a first-order approximation yields $\widehat{\Delta}_t = 0$; for further details, see Gali (2015, chapter 3). Also $Y = N - F = C$ and from the Euler equation, $R = \beta^{-1} > 1$.

From the optimal labor supply (17)

$$\begin{aligned} \frac{\omega N_S^{\varphi}}{C_S^{-\sigma}} = \frac{W}{P} &= \frac{\epsilon - 1}{\epsilon} \\ \iff N_S &= \left(\frac{1}{\omega} \frac{\epsilon - 1}{\epsilon} \right)^{\frac{1}{\varphi + \sigma}}. \end{aligned}$$

Define the steady-state net mark-up as $\mu = (\epsilon - 1)^{-1}$ (so that $1 + \mu = \epsilon(\epsilon - 1)^{-1}$). The share of labor income in total output is

$$\begin{aligned} \frac{W}{P}\frac{N}{Y} &= \frac{W}{P}\frac{Y + F}{Y} \\ &= \frac{\epsilon - 1}{\epsilon}\left(1 + \frac{F}{Y}\right) \\ &:= \frac{1 + F_Y}{1 + \mu}. \end{aligned}$$

F_Y is the share of the fixed cost in output. Using aggregate profits (26), profits' share in total output is

$$\begin{aligned} D_Y := \frac{D}{Y} &= 1 - \frac{MC}{P} - \frac{MC}{P}\frac{F}{Y} \\ &= 1 - \frac{\epsilon - 1}{\epsilon} - \frac{\epsilon - 1}{\epsilon} F_Y \\ &= \frac{1}{\epsilon} - \frac{F_Y}{1 + \mu} \\ &= \frac{\mu - F_Y}{1 + \mu}. \end{aligned}$$

I assume that the share of the fixed cost in steady-state output F_Y is equal to the net markup μ, so that the share of profits in the steady-state D_Y is zero. Thus, in the steady-state, unconstrained households face the same budget constraint as the constrained households, and consumption will be equal for both groups, $C_H = C_S = C = Y$; this is known as an equitable steady-state.

I log-linearize the model around this zero-inflation, non-stochastic equitable steady-state described above. The technique used for finding log-linear approximations of functions follows from Uhlig (1995). Small-case letters denote the log-deviation of a variable from its steady-state, that is, for example, $x_t = ln(X_t/X^{ss}) = ln(X_t) - ln(X^{ss})$. Deriving the approximations is not difficult, but it is tedious, so details are omitted except for the Behavioral New Keynesian Phillips Curve (BNKPC), given that its derivation differs considerably from the NKPC of the baseline model; see the appendix.

The 16 equilibrium conditions from above can be reduced, after log-linearizing, to 12 conditions, which are:

$E_1:$	*Euler equation, S*	$\mathbb{E}_t^{BR}[c_{S,t+1}] - c_{S,t} = \frac{r_t - \mathbb{E}_t^{BR}[\pi_{t+1}]}{\sigma}$
$E_2:$	*Labor supply*	$\varphi n_{S,t} = w_t - \sigma c_{S,t}$
$E_3:$	*Budget constraint, S*	$c_{S,t} = w_t + n_{S,t} + \frac{d_t}{1-\lambda}$
$E_4:$	*Labor supply, H*	$\varphi n_{H,t} = w_t - \sigma c_{H,t}$
$E_5:$	*Budget constraint, H*	$c_{H,t} = w_t + n_{H,t}$
$E_6:$	*Production function*	$y_t = (1+\mu)(n_t + a_t)$
$E_7:$	*Real marginal cost*	$mc_t = w_t - a_t$
$E_8:$	*Real profits*	$d_t = -mc_t + \frac{\mu}{1+\mu} y_t$
$E_9:$	*Behavioral Phillips curve*	$\pi_t = \beta M^f \mathbb{E}_t[\pi_{t+1}] - \psi \mu_t,\ \psi = \frac{(1-\theta)(1-\theta\beta)}{\theta},$
		$M^f = m\left(\theta + \frac{1-\beta\theta}{1-\beta\theta m}(1-\theta)\right)$
$E_{10}:$	*Labor market clearing*	$n_t = \lambda n_{H,t} + (1-\lambda) n_{S,t}$
$E_{11}:$	*Aggregate consumption*	$c_t = \lambda c_{H,t} + (1-\lambda) c_{S,t}$
$E_{12}:$	*Monetary policy*	$r_t = \phi_\pi \mathbb{E}_t^{BR}[\pi_{t+1}] + \epsilon_t = \phi_\pi m^{CB} \mathbb{E}_t[\pi_{t+1}] + \epsilon_t$

3. Model Solution

The log-linear model of the previous section can be reduced to a two-equation system which, coupled with the interest rate rule, determine the dynamics of the endogenous variables expressed in terms of inflation and the output gap. The two equations are the Behavioral New Keynesian Phillips Curve (BNKPC) and the Dynamic IS (DIS) curve:

$$\pi_t = \beta M^f \mathbb{E}_t[\pi_{t+1}] + \kappa x_t + u_t \qquad BNKPC$$

$$x_t = m\mathbb{E}_t[x_{t+1}] - \frac{\eta}{\delta\sigma}(r_t - m\mathbb{E}_t[\pi_{t+1}] - r_t^*) \qquad DIS,$$

where

$$\psi = \frac{(1-\theta)(1-\theta\beta)}{\theta}, \quad \chi = \sigma + \frac{\varphi}{1+\mu}, \quad \delta = 1 - \frac{\varphi\lambda}{(1+\mu)(1-\lambda)},$$
$$\eta = 1 + \lambda(\sigma - 1), \quad \kappa = \eta\kappa\eta^{-1}, \quad M^f = m\left(\theta + \frac{1-\beta\theta}{1-\beta\theta m}(1-\theta)\right),$$
$$r_t^* = \frac{\sigma}{\eta}\left(1 + \frac{(1-\lambda)(1-\sigma)\delta}{\chi} + \left(1 - \frac{\sigma\delta}{\chi}\right)\mu\right)(m\mathbb{E}_t[a_{t+1}] - a_t).$$

r_t^* is the natural rate of interest, i.e., the interest rate the rate that prevails under fully flexible prices.

The slope of the DIS curve

Denote the sensitivity of the output gap to changes in the real interest rate as $\gamma = -\eta/(\delta\sigma)$. Note that $\eta = 1 - \lambda + \lambda\sigma > 0$, since $\sigma > 0$, the direction of the sensitivity depends only on the sign of δ. Note that $\delta > 0$ if and only if

$$\begin{aligned} & 0 < 1 - \frac{\varphi}{1+\mu}\frac{\lambda}{1-\lambda} \\ \iff & \frac{\lambda}{1-\lambda} < \frac{1+\mu}{\varphi} \\ \iff & \lambda < \frac{1+\mu}{1+\mu+\varphi} := \lambda^*. \end{aligned} \tag{36}$$

The parameter δ is decreasing in the share of constrained households.

$$\frac{\partial\delta}{\partial\lambda} = -\frac{\varphi}{1+\mu}\frac{1}{(1-\lambda)^2} < 0.$$

If $\lambda < \lambda^*$ then $\delta > 0$ and $\gamma < 0$, thus an increase in the real interest rate would lower the output gap, that is, the increase in the interest rate results in contractionary monetary policy. Since this is the standard thinking regarding monetary policy, the region where $\delta > 0$ is called the *standard aggregate demand logic* (SADL). As λ increases closer to λ^*, δ decreases, and thus the sensitivity of aggregate demand to interest rates increases in absolute value, making policy more effective in constraining demand.

However, if $\lambda > \lambda^*$ then $\delta < 0$ and $\gamma > 0$, thus an increase in the real interest rate results in expansionary monetary policy. Note that $\delta \to -\infty$

as $\lambda \to 1$ (this limit case correspond to complete absence of financial markets). Thus the sensitivity of aggregate demand to interest rates converges to zero and therefore monetary policy is ineffective. This region was coined by Bilbiie (2008) as *inverted aggregate demand logic* (IADL).

In figure 1, I plot both regions as a function of φ, the sensitivity of wage to real income, setting the net markup, μ, to different values. One can observe that the lower the sensitivity of wages to real income, the more difficult it is to get to the IADL region. As the sensitivity gets higher, a lower number of households without access to financial markets is required.

Except for extremely high values of the net markup, μ, and for extremely low values of the sensitivity of wage to real income, φ, the IADL region is attainable. In particular, the empirical estimates for φ and μ found in the New Keynesian literature are 2 and 0.20, respectively. It can be seen from figure 1 that the IADL region not only is empirically plausible, but also can be reached under many reasonable parameterizations.

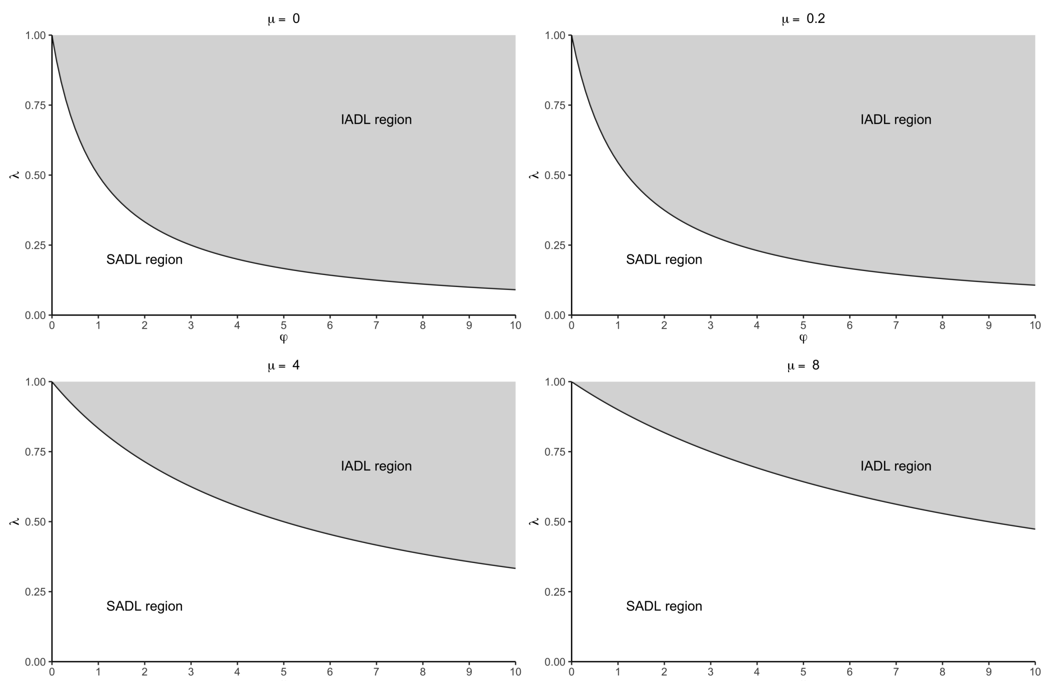

FIGURE 1. THRESHOLD SHARE OF CONSTRAINED HOUSEHOLDS AS A FUNCTION OF INVERSE LABOR SUPPLY ELASTICITY.

4. Values Used in the Numerical Exercises

According to the New Keynesian literature, in what follows unless stated otherwise, I use the following values for the structural parameters (using a quarterly parametrization):

$$\begin{aligned} \mu &= 0.20 \\ \sigma &= 2 \\ \varphi &= 2 \\ \beta &= 0.99 \\ \theta &= 0.75. \end{aligned}$$

These values are consistent with those found in the existing literature (e.g., Bilbiie, 2008). The steady-state markup, $\mu = 0.20$, implies an elasticity of substitution between the individual goods $\epsilon = 6$. It is assumed that the inverse of the intertemporal elasticity of substitution in consumption is $\sigma = 2$. The inverse of the labor supply elasticity, $\varphi = 2$, implies a Frisch elasticity of labor supply of 0.2. A usual value for the probability to adjust prices in each period is $\theta = 0.75$, which corresponds to an average price duration of one year (four quarters), and a usual value for the subjective discount factor is $\beta = 0.99$, which corresponds to an annualized return on financial assets of 4 percent.

For the cognitive parameters (which are also structural), I set

$$\begin{aligned} m &= 0.80 \\ m^{CB} &= 0.90. \end{aligned}$$

Fuhrer and Rudebusch (2004) estimate that $m \approx 0.65$; since this value is quite extreme, for illustrative purposes I use a more conservative value of 0.80 as in Gabaix (2020).[6]

It is reasonable to think that the value of the central bank's cognitive parameter varies from country to country, where, perhaps, central banks

[6] According to Galí and Gertler (1999), $\beta M^f \approx 0.80$ (and I could take $M^f \approx 0.80$ since $\beta \approx 1$). However, I omit the empirical estimate of M^f since it is a function of m, θ, and β. That is, to give a particular value for M^f, I must adjust its arguments for it to coincide. A combination for the triplete (m, β, θ) could have been $(0.80, 0.99, 1)$. However, the resulting values for β and θ would differ greatly from the New Keynesian literature.

such as the Federal Reserve and the European Central Bank, should have a greater parameter relative to central banks from developing countries. I propose a value of $m^{CB} = 0.90$. The motivation is to model the underestimation that many central banks usually make regarding inflation expectations, which is of particular interest during inflationary episodes.

Given the values of the structural parameters, the rest of the parameters are:

$$\begin{aligned} M^f &= 0.727 \\ \beta^f := \beta M^f &= 0.720 \\ \epsilon = \frac{1+\mu}{\mu} &= 6 \\ \psi = \frac{(1-\theta)(1-\theta\beta)}{\theta} &= 0.086 \\ \lambda^* = \frac{1+\mu}{1+\mu+\varphi} &= 0.375 \\ \chi = \sigma + \frac{\varphi}{1+\mu} &= 3.667. \end{aligned}$$

I omit on purpose λ (and thus δ) since it will be relevant to give it various values depending whether the economy is SADL or IADL.

5. Interest Rate Rules

In this section, I analyze the determinacy properties of various interest rate rules: strict forecast-based inflation targeting rule, hybrid Taylor rule, strict current-looking inflation targeting rule, and a current-looking Taylor rule.

In this analysis, I will refer to three different setups: rational expectations LAMP, bounded rationality LAMP, and central bank bounded rationality LAMP. Note that the first is a special case where $m = 1$, $m^{CB} = 1$, and (thus) $M^f = 1$. The second is $m < 1$ and $m^{CB} = 1$, and for the third $m < 1$ and $m^{CB} < 1$. Additionally note that the central bank cognitive discounting does not play a role under current-looking rules.

5.1. Strict forecast-based inflation targeting rule

Consider the interest rate rule

$$r_t = \phi_\pi m^{CB} \mathbb{E}_t[\pi_{t+1}] + \epsilon_t. \tag{37}$$

Substituting the interest rate rule into the DIS curve we get

$$x_t = m\mathbb{E}_t[x_{t+1}] + \frac{\eta}{\delta\sigma}(m - \phi_\pi m^{CB})\mathbb{E}_t[\pi_{t+1}] - \frac{\eta(\epsilon_t - r_t^*)}{\delta\sigma}.$$

Together with the BNKPC curve we can write a system for the vector $(x_t, \pi_t)^T$ as

$$\begin{pmatrix} 0 & \beta^f \\ m & \frac{\eta}{\delta\sigma}(m - \phi_\pi m^{CB}) \end{pmatrix} \begin{pmatrix} \mathbb{E}_t[x_{t+1}] \\ \mathbb{E}_t[\pi_{t+1}] \end{pmatrix} = \begin{pmatrix} -\kappa & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_t \\ \pi_t \end{pmatrix} + \begin{pmatrix} -u_t \\ \frac{\eta(\epsilon_t - r_t^*)}{\delta\sigma} \end{pmatrix}.$$

This can be rewritten as[7]

$$\begin{pmatrix} \mathbb{E}_t[x_{t+1}] \\ \mathbb{E}_t[\pi_{t+1}] \end{pmatrix} = \mathbf{\Gamma} \begin{pmatrix} x_t \\ \pi_t \end{pmatrix} + \mathbf{\Psi} \begin{pmatrix} \epsilon_t - r_t^* \\ u_t \end{pmatrix},$$

where

$$\mathbf{\Gamma} = \begin{pmatrix} \frac{\eta\kappa(m - \phi_\pi m^{CB})}{m\beta^f\delta\sigma} + \frac{1}{m} & -\frac{\eta(m - \phi_\pi m^{CB})}{m\beta^f\delta\sigma} \\ -\frac{\kappa}{\beta^f} & \frac{1}{\beta^f} \end{pmatrix}, \quad \mathbf{\Psi} = \begin{pmatrix} \frac{\eta}{m\delta\sigma} & \frac{\eta(m - \phi_\pi m^{CB})}{m\beta^f\delta\sigma} \\ 0 & -\frac{1}{\beta^f} \end{pmatrix}.$$

Since both inflation and output gap are forward-looking variables, determinacy requires that both eigenvalues of $\mathbf{\Gamma}$ be outside the unit circle.

The necessary and sufficient conditions for determinacy are as follows[8]: Case 1: $det(\mathbf{\Gamma}) > 1$, $det(\mathbf{\Gamma}) - tr(\mathbf{\Gamma}) > -1$, and $det(\mathbf{\Gamma}) + tr(\mathbf{\Gamma}) > -1$; Case 2: $det(\mathbf{\Gamma}) < -1$, $det(\mathbf{\Gamma}) - tr(\mathbf{\Gamma}) < -1$, and $det(\mathbf{\Gamma}) + tr(\mathbf{\Gamma}) < -1$. The second case is ruled out due to sign restrictions. For the first case

Condition (A1.1) $det(\mathbf{\Gamma}) > 1$

$$det(\mathbf{\Gamma}) = \frac{1}{m\beta^f} > 1.$$

Condition (A1.2) $det(\mathbf{\Gamma}) - tr(\mathbf{\Gamma}) > -1$

$$\frac{\phi_\pi m^{CB} - m}{\delta} > -\frac{\sigma(1 - \beta^f)(1 - m)}{\eta\kappa}.$$

Condition (A1.3) $det(\mathbf{\Gamma}) + tr(\mathbf{\Gamma}) > -1$

$$\frac{\sigma(1 + \beta^f)(1 + m)}{\eta\kappa} > \frac{\phi_\pi m^{CB} - m}{\delta}.$$

[7]See the appendix for a full derivation under interest rate (37). The derivation is omitted for the following rules since the procedure is vastly similar.

[8]See Woodford (2003).

Strict forecast-based inflation targeting rule in a SADL economy

When $\delta > 0$, noting that condition (A1.1) always holds and combining (A1.2) and (A1.3), the necessary and sufficient condition for determinacy can be expressed as:

$$m - \frac{\delta\sigma(1-\beta^f)(1-m)}{\eta\kappa} < \phi_\pi m^{CB} < m + \frac{\delta\sigma(1+\beta^f)(1+m)}{\eta\kappa}.$$

Look at the lower bound first. First, under rational expectations for all participants, $m = m^{CB} = 1$. Thus the lower bound is given by the Taylor principle ($\phi_\pi > 1$). Next, in the behavioral setup, the lower bound is less than one if and only if

$$m - \frac{\delta\sigma(1-\beta^f)(1-m)}{\eta\kappa} < m^{CB}.$$

This condition will always hold since by assumption $m^{CB} > m$ and the second term of the LHS is positive (negative considering the minus). So for standard parameter values, the Taylor principle is still needed for determinacy.

How important is the cognitive parameter m^{CB} of the central bank?. Given the bounded rationality of asset holders and intermediate goods producers, we could have $\phi_\pi < 1$, thus determinacy can arise under a passive monetary policy stance. However, incorporating bounded rationality for the central bank, reduces the region where passive monetary policy is allowed. We conclude that if the central bank is aware of the downward bias it has regarding measuring future inflation, its response to changes in inflation must be of a larger magnitude than otherwise.

Now, for the upper bound, under rational expectations we get $1+2\delta\sigma(1+\beta)/(\eta\kappa)$. The upper bound under rational expectations is larger if and only if

$$\frac{\delta\sigma\left[(1+\beta^f)(1+m) - 2(1+\beta)\right]}{\eta\kappa} < 1 - m,$$

which always holds. Thus the lower bound of the determinacy region is getting lower at the expense of lowering the upper bound too.[9]

[9] A word of caution. Look at the lower bound of ϕ_π. In figure 2, I will plot ϕ_π as

Strict forecast-based inflation targeting rule in a IADL economy

When $\delta < 0$, the necessary and sufficient condition is

$$m + \frac{\delta\sigma(1+\beta^f)(1+m)}{\eta\kappa} < \phi_\pi m^{CB} < m - \frac{\delta\sigma(1-\beta^f)(1-m)}{\eta\kappa}.$$

Look at the upper bound first. It is greater than one if and only if

$$m - \frac{\delta\sigma(1-\beta^f)(1-m)}{\eta\kappa} > m^{CB}.$$

By inspection, we can see that this condition holds except for values close to λ^* (and $\lambda > \lambda^*$). Therefore, even in the IADL region, there is a threshold at which the Taylor principle still holds. This is different from a rational expectations where only a passive monetary policy can induce determinacy.

This threshold gets larger as the share of constrained households increases. Incorporating the central bank's cognitive discounting induces an even larger region where the Taylor principle holds (the logic, is as before, the central bank must act more aggressively to changes in inflation).

For the lower bound, it is larger under rational expectations than under bounded rationality if and only if

$$\frac{\delta\sigma\left[(1+\beta^f)(1+m) - 2(1+\beta)\right]}{\eta\kappa} < 1 - m$$

which always hold. Thus in the IADL region, both the upper bound gets larger (generally) and the lower bound gets lower.

Figure 2 shows the determinacy region for ϕ_π as a (inverse) function of the share of constrained households λ under the different setups. It is clear that, once we incorporate bounded rationality into the model, the determinacy

a (inverse) function of the share of constrained households λ. Let $a_L := \sigma(1-\beta^f)(1-m)/(\eta\kappa)$. Then the lower bound looks like $m - a_L\delta < \phi_\pi m^{CB}$. Recall that $\delta = 1 - \frac{\varphi}{1+\mu}\frac{\lambda}{1-\lambda}$ and let $b = \frac{\varphi}{1+\mu}$. Rearranging the above inequality, $\lambda < \frac{a_L - m + m^{CB}\phi_\pi}{ba_L + a_L - m + m^{CB}\phi_\pi}$. The RHS is a function of ϕ_π that has a vertical asymptote at $\phi_\pi = \frac{m-(1+b)a_L}{m^{CB}}$. Thus, if this vertical asymptote is at $\phi_\pi > 0$, then the determinacy region changes, and a determinacy region arises for small values of θ_π in a SADL economy.

region becomes considerably larger. Also note that central bank bounded rationality increases the region in a IADL economy, but decreases it in a SADL economy; this is illustrated by the dashed lines in figure 2. Figure 3 shows comparative statistics for the cognitive parameter m.

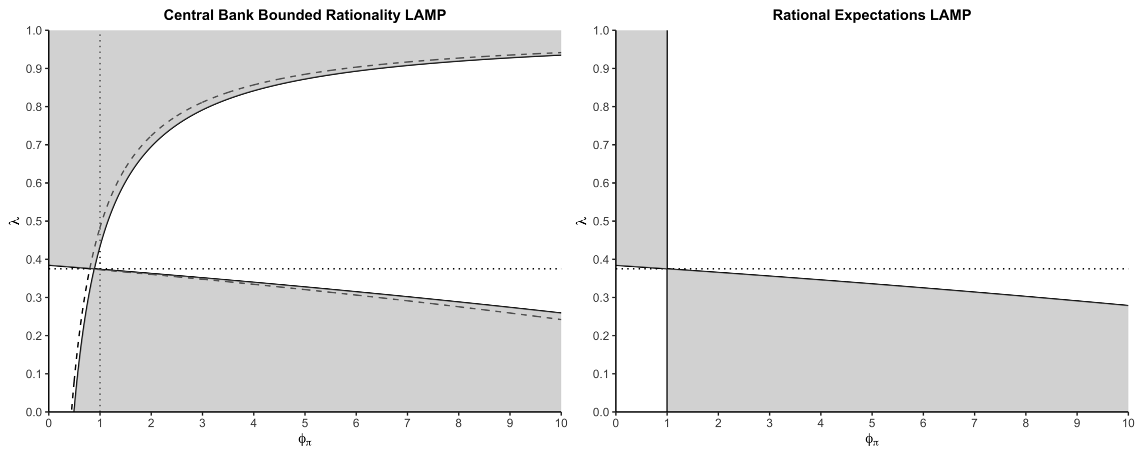

Figure 2. Determinacy region under interest rate rule (37) for interest rate sensitivity to changes in inflation as a function of the share of constrained households considering different setups.

Note: Filled regions indicate determinacy regions. The dashed curves show the determinacy region under $m^{CB} = 1$. The horizontal-dotted line represents $\lambda^* = 0.375$.

It can be seen that as m decreases, in the SADL region, both the lower and upper bounds are getting smaller, and in the IADL region, the upper bound gets larger. That is, the less rational households are, the larger the determinacy region where passive monetary policy can arise.

5.2. *Hybrid Taylor rule*

Now I incorporate an output stabilization objective into the policy rule as

$$r_t = \phi_\pi m^{CB}\mathbb{E}_t[\pi_{t+1}] + \phi_x x_t + \epsilon_t. \tag{38}$$

Substituting the interest rate rule into the IS curve and considering the BNKPC curve we can write a system for the vector $(x_t, \pi_t)^T$ as

$$\begin{pmatrix} 0 & \beta^f \\ m & \frac{\eta(m-\phi_\pi m^{CB})}{\delta\sigma} \end{pmatrix} \begin{pmatrix} \mathbb{E}_t[x_{t+1}] \\ \mathbb{E}_t[\pi_{t+1}] \end{pmatrix} = \begin{pmatrix} -\kappa & 1 \\ 1+\frac{\eta\phi_x}{\delta\sigma} & 0 \end{pmatrix} \begin{pmatrix} x_t \\ \pi_t \end{pmatrix} + \begin{pmatrix} -u_t \\ \frac{\eta(\epsilon_t - r_t^*)}{\delta\sigma} \end{pmatrix}.$$

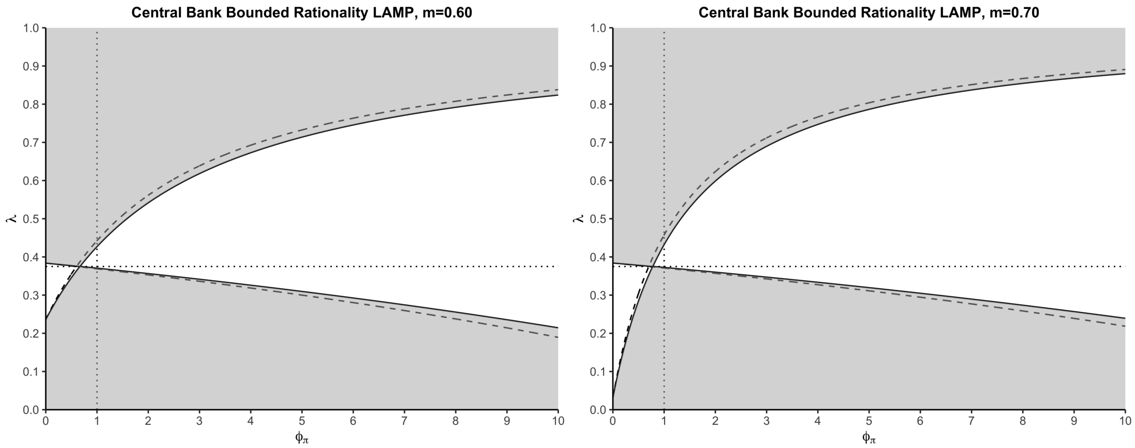

FIGURE 3. DETERMINACY REGION SENSITIVITY ANALYSIS FOR THE COGNITIVE PARAMETER, m, UNDER INTEREST RATE RULE (37).

Note: Filled regions indicate determinacy regions. The dashed curves show the determinacy region under $m^{CB} = 1$. The horizontal-dotted line represents $\lambda^* = 0.375$.

This can be rewritten as

$$\begin{pmatrix} \mathbb{E}_t[x_{t+1}] \\ \mathbb{E}_t[\pi_{t+1}] \end{pmatrix} = \mathbf{\Gamma} \begin{pmatrix} x_t \\ \pi_t \end{pmatrix} + \mathbf{\Psi} \begin{pmatrix} \epsilon_t - r_t^* \\ u_t \end{pmatrix},$$

where

$$\mathbf{\Gamma} = \begin{pmatrix} \frac{1}{m} - \frac{\eta}{m\delta\sigma}\left(\frac{\kappa(\phi_\pi m^{CB} - m)}{\beta^f} - \phi_x\right) & \frac{\eta(\phi_\pi m^{CB} - m)}{m\beta^f\delta\sigma} \\ -\frac{\kappa}{\beta^f} & \frac{1}{\beta^f} \end{pmatrix}, \quad \mathbf{\Psi} = \begin{pmatrix} \frac{\eta}{m\delta\sigma} & \frac{\eta(m - \phi_\pi m^{CB})}{m\beta^f\delta\sigma} \\ 0 & -\frac{1}{\beta^f} \end{pmatrix}.$$

Using the same method as before, we obtain the following necessary and sufficient conditions for an equilibrium to exist. Note that with the dual objective rule, the second case of conditions cannot be discarded.

Condition (B1.1), $det(\mathbf{\Gamma}) > 1$

$$\frac{\phi_x}{\delta} > -\frac{\sigma(1 - m\beta^f)}{\eta}.$$

Condition (B1.2), $det(\mathbf{\Gamma}) - tr(\mathbf{\Gamma}) > -1$

$$\frac{1}{\delta}\left(\kappa(\phi_\pi m^{CB} - m) + (1 - \beta^f)\phi_x\right) > -\frac{\sigma(1 - \beta^f)(1 - m)}{\eta}.$$

Condition (B1.3), $det(\mathbf{\Gamma}) + tr(\mathbf{\Gamma}) > -1$

$$\frac{\sigma(1+\beta^f)(1+m)}{\eta} > \frac{1}{\delta}\left(\kappa(\phi_\pi m^{CB} - m) - (1+\beta^f)\phi_x\right).$$

Condition (B2.1), $det(\mathbf{\Gamma}) < -1$

$$\frac{\phi_x}{\delta} < -\frac{\sigma(1+m\beta^f)}{\eta}.$$

Condition (B2.2), $det(\mathbf{\Gamma}) - tr(\mathbf{\Gamma}) < -1$

$$\frac{1}{\delta}\left(\kappa(\phi_\pi m^{CB} - m) + (1-\beta^f)\phi_x\right) < -\frac{\sigma(1-\beta^f)(1-m)}{\eta}.$$

Condition (B2.3), $det(\mathbf{\Gamma}) + tr(\mathbf{\Gamma}) < -1$

$$\frac{\sigma(1+\beta^f)(1+m)}{\eta} < \frac{1}{\delta}\left(\kappa(\phi_\pi m^{CB} - m) - (1+\beta^f)\phi_x\right).$$

Hybrid Taylor rule in a SADL economy

When $\delta > 0$, the second set of necessary conditions are ruled out due to sign restrictions. Thus we only focus in the first case. Condition (B1.1) is non-binding, and combining conditions (B1.2) and (B1.3), the necessary and sufficient condition for determinacy can be expressed as:

$$m - \frac{\delta\sigma(1-\beta^f)(1-m)}{\eta\kappa} - \frac{1-\beta^f}{\kappa}\phi_x < \phi_\pi m^{CB} < m + \frac{\delta\sigma(1+\beta^f)(1+m)}{\eta\kappa} + \frac{1+\beta^f}{\kappa}\phi_x.$$

Take the first side of the above inequality condition and rewrite it as

$$Z < \phi_\pi + Q\phi_x,$$

where

$$Z = \frac{m}{m^{CB}} - \frac{\delta\sigma(1-\beta^f)(1-m)}{\eta\kappa m^{CB}}, \quad Q = \frac{1-\beta^f}{\kappa m^{CB}}.$$

Q is the long-run inflation elasticity of output. Under rational expectations, $Z = 1$, and the above conditions is called the generalized Taylor principle. With a policy response to output, determinacy can now be achieved under a passive policy response to inflation ($\phi_\pi < 1$), provided the output response coefficient ϕ_x is set sufficiently large. Under rational expectations, output targeting is more powerful, and the region of indeterminacy will be smaller, the greater is the degree of price stickiness.

Figure 4 shows that for any $\phi_x > 0$, there is a region where the standard Taylor principle holds. Again, as in the previous interest rate rule, the benefits of incorporating bounded rationality are considerable. Central bank cognitive discounting makes the lower bound of the region tighter. However, since the Taylor principle is guaranteed, this is perceived as positive, since it is reducing the region where passive monetary policy (regarding inflation) is permitted. Additionally, the upper bound is less tight.

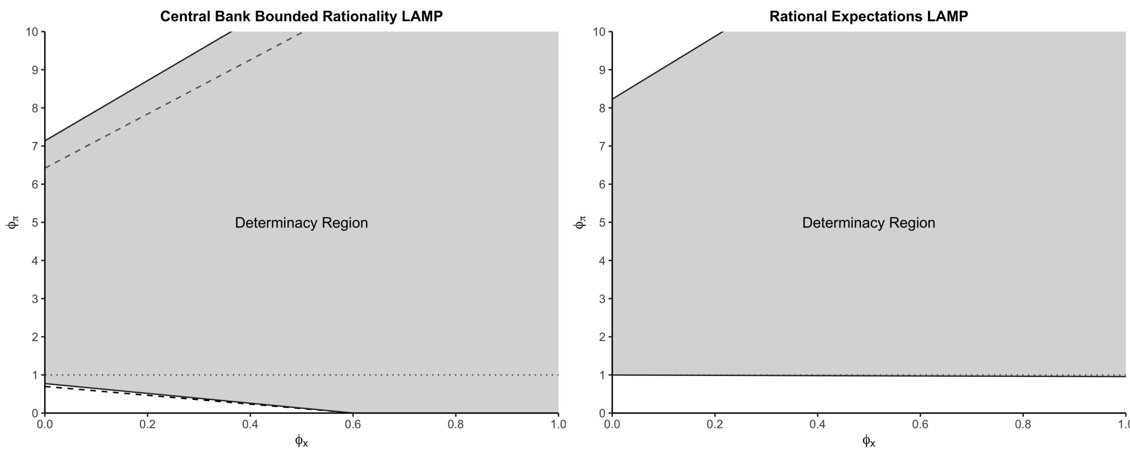

Figure 4. Determinacy region under interest rate rule (38) for interest rate sensitivity to changes in inflation and to changes in output in a SADL economy where $\lambda = 0.30$ considering different setups.

Note: The dashed curves show the determinacy region under $m^{CB} = 1$.

Hybrid Taylor rule in a IADL economy

When $\delta < 0$, we can get both cases for the necessary and sufficient conditions. In the first case, condition (B1.1) yields $\phi_x < -\delta\sigma(1 - m\beta^f)/\eta$, and combining (B1.2) and (B1.3), the necessary and sufficient condition for determinacy can be expressed as:

$$m + \frac{\delta\sigma(1+\beta^f)(1+m)}{\eta\kappa} + \frac{1+\beta^f}{\kappa}\phi_x < \phi_\pi m^{CB} < m - \frac{\delta\sigma(1-\beta^f)(1-m)}{\eta\kappa} - \frac{1-\beta^f}{\kappa}\phi_x.$$

In the second case, condition (B2.1) yields $-\delta\sigma(1 + m\beta^f)/\eta < \phi_x$, and combining (B2.2) and (B2.3) the necessary and sufficient condition is given by

$$m - \frac{\delta\sigma(1-\beta^f)(1-m)}{\eta\kappa} - \frac{1-\beta^f}{\kappa}\phi_x < \phi_\pi m^{CB} < m + \frac{\delta\sigma(1+\beta^f)(1+m)}{\eta\kappa} + \frac{1+\beta^f}{\kappa}\phi_x.$$

Figure 5 shows the determinacy regions in both cases. Note that bounded rationality effectively enlarges the determinacy region. Let's see if we can generalize this last result for any parametrization. Consider for a moment $m^{CB} = 1$ and focus on the second set of conditions. Comparing the upper bound, condition (B2.3), this condition is smaller under rational expectations than under bounded rationality if and only if

$$\phi_x < \frac{\delta\sigma((1+\beta^f)(1+m) - 2(1+\beta))}{\eta(\beta-\beta^f)} - \frac{(1-m)\kappa}{\beta-\beta^f}.$$

That is, for low values of ϕ_π, bounded rationality enlarges the determinacy region. Incorporating bounded rationality for the central bank increases this region further. Finally, it can be shown that the lower bound is always larger under rational expectations than under bounded rationality.

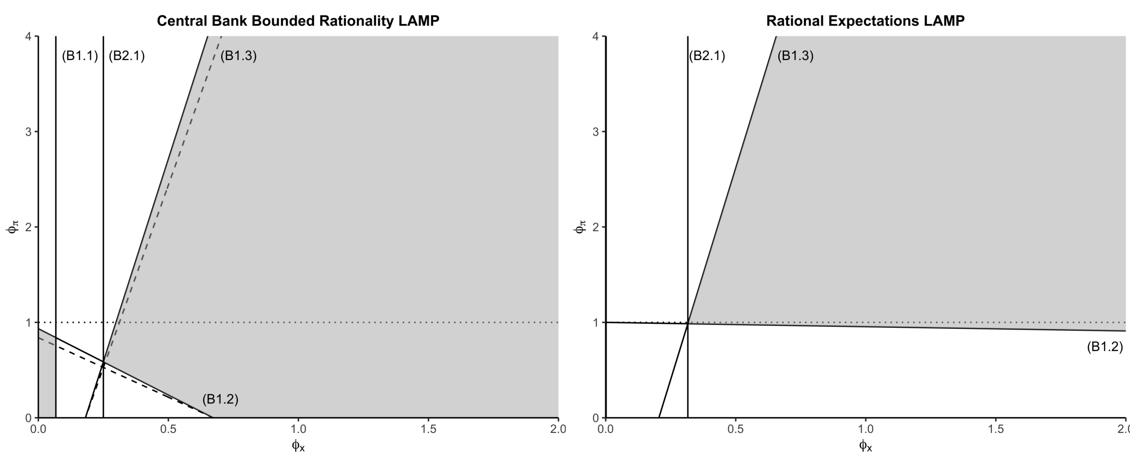

FIGURE 5. DETERMINACY REGION UNDER INTEREST RATE RULE (38) FOR INTEREST RATE SENSITIVITY TO CHANGES IN INFLATION AND TO CHANGES IN OUTPUT IN A IADL ECONOMY WHERE $\lambda = 0.40$ UNDER DIFFERENT SETUPS.

Note: The small region on the left corresponds to the region determined by conditions B1.1, B1.2, and B1.3. The dashed curves show the determinacy region under $m^{CB} = 1$. Filled regions indicate determinacy.

Now, I plot the same figure setting $\lambda = 0.45$ (note that the economy is still IADL since $\lambda > \lambda^* = 0.375$). The region from the first set of conditions gets larger, and the Taylor principle holds for low values of ϕ_x. For the second set of conditions, the Taylor principle is satisfied provided that the output response coefficient is high enough. And the higher the share of constrained households, the more aggressive the central bank must be regarding the output gap, in order to maintain the Taylor principle.

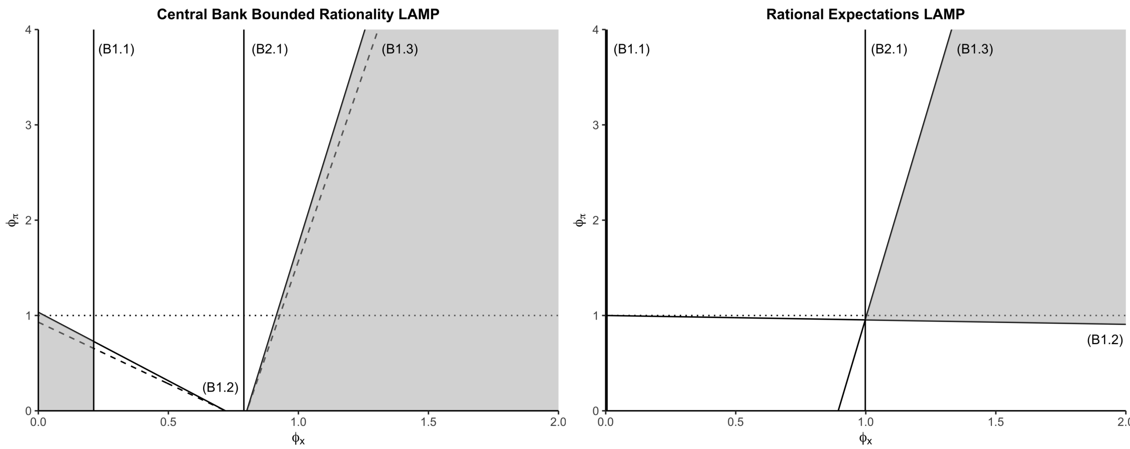

FIGURE 6. DETERMINACY REGION UNDER INTEREST RATE RULE (38) FOR INTEREST RATE SENSITIVITY TO CHANGES IN INFLATION AND TO CHANGES IN OUTPUT IN A IADL ECONOMY WHERE $\lambda = 0.45$ UNDER DIFFERENT SETUPS.

Note: The small region on the left corresponds to the region determined by conditions B1.1, B1.2, and B1.3. The dashed curves show the determinacy region under $m^{CB} = 1$. Filled regions indicate determinacy.

Finally, note that there is a region for ϕ_x where determinacy cannot be achieved regardless of the value of ϕ_π. This region is the interval

$$\left(-\delta\sigma(1 - m\beta^f)/\eta, -\delta\sigma(1 + m\beta^f)/\eta\right),$$

and its length is $-2\delta\sigma m\beta^f/\eta$ (the distance between the two vertical in the previous figure). Since $\partial\delta/\partial\lambda < 0$, the size of the interval increases as the share of non-asset holders increases. In the rational expectations LAMP, the interval turns to $(-\delta\sigma(1-\beta), -\delta\sigma(1+\beta))$ which length is $2\delta\sigma\beta$. Since $\beta > m\beta^f$, bounded rationality reduces the size of the region where determinacy cannot be achieved.

5.3. Strict current-looking inflation targeting rule

Consider the interest rate rule

$$r_t = \phi_\pi \pi_t + \epsilon_t. \tag{39}$$

Substituting the interest rate rule into the DIS curve and considering the BNKPC curve we can write a system for the vector $(x_t, \pi_t)^T$ as

$$\begin{pmatrix} 0 & \beta^f \\ m & \frac{m\eta}{\delta\sigma} \end{pmatrix} \begin{pmatrix} \mathbb{E}_t[x_{t+1}] \\ \mathbb{E}_t[\pi_{t+1}] \end{pmatrix} = \begin{pmatrix} -\kappa & 1 \\ 1 & \frac{\eta\phi_\pi}{\delta\sigma} \end{pmatrix} \begin{pmatrix} x_t \\ \pi_t \end{pmatrix} + \begin{pmatrix} -u_t \\ \frac{\eta(\epsilon_t - r_t^*)}{\delta\sigma} \end{pmatrix}.$$

This can be rewritten as

$$\begin{pmatrix} \mathbb{E}_t[x_{t+1}] \\ \mathbb{E}_t[\pi_{t+1}] \end{pmatrix} = \mathbf{\Gamma} \begin{pmatrix} x_t \\ \pi_t \end{pmatrix} + \mathbf{\Psi} \begin{pmatrix} \epsilon_t - r_t^* \\ u_t \end{pmatrix},$$

where

$$\mathbf{\Gamma} = \begin{pmatrix} \frac{\eta\kappa}{\beta^f\delta\sigma} + \frac{1}{m} & -\frac{\eta}{\beta^f\delta\sigma} + \frac{\eta\phi_\pi}{m\delta\sigma} \\ -\frac{\kappa}{\beta^f} & \frac{1}{\beta^f} \end{pmatrix}, \quad \mathbf{\Psi} = \begin{pmatrix} \frac{\eta}{m\delta\sigma} & \frac{\eta}{\beta^f\delta\sigma} \\ 0 & -\frac{1}{\beta^f} \end{pmatrix}.$$

Using the same method as before, we obtain the following necessary and sufficient conditions for an equilibrium to exist.

Condition (C1.1) $det(\mathbf{\Gamma}) > 1$

$$\frac{\phi_\pi}{\delta} > -\frac{\sigma(1 - m\beta^f)}{\eta\kappa}.$$

Condition (C1.2) $det(\mathbf{\Gamma}) - tr(\mathbf{\Gamma}) > -1$

$$\frac{\phi_\pi - m}{\delta} > -\frac{\sigma(1 - \beta^f)(1 - m)}{\eta\kappa}.$$

Condition (C1.3) $det(\mathbf{\Gamma}) + tr(\mathbf{\Gamma}) > -1$

$$\frac{\phi_\pi + m}{\delta} > -\frac{\sigma(1 + \beta^f)(1 + m)}{\eta\kappa}.$$

Condition (C2.1), $det(\mathbf{\Gamma}) < -1$

$$\frac{\phi_\pi}{\delta} < -\frac{\sigma(1 + m\beta^f)}{\eta\kappa}.$$

Condition (C2.2), $det(\mathbf{\Gamma}) - tr(\mathbf{\Gamma}) < -1$

$$\frac{\phi_\pi - m}{\delta} < -\frac{\sigma(1-\beta^f)(1-m)}{\eta\kappa}.$$

Condition (C2.3), $det(\mathbf{\Gamma}) + tr(\mathbf{\Gamma}) < -1$

$$\frac{\phi_\pi + m}{\delta} < -\frac{\sigma(1+\beta^f)(1+m)}{\eta\kappa}.$$

Strict current-looking inflation targeting rule in SADL economy

When $\delta > 0$, the second set of conditions is ruled out. For the first set

$$\phi_\pi > -\frac{\delta\sigma(1-m\beta^f)}{\eta\kappa} := \alpha_{11}$$
$$\phi_\pi > m - \frac{\delta\sigma(1-\beta^f)(1-m)}{\eta\kappa} := \alpha_{12}$$
$$\phi_\pi > -m - \frac{\delta\sigma(1+\beta^f)(1+m)}{\eta\kappa} := \alpha_{13}.$$

There is no upper bound on ϕ_π and, noting that, $\max\{\alpha_{11}, \alpha_{12}, \alpha_{13}\} = \alpha_{12}$, the lower bound is the same as under the strict forecast-based inflation targeting rule setting $m^{CB} = 1$ (as expected, the central bank's bias does not affect at all when the rule does not take into account future periods). The determinacy region in the SADL economy is

$$\phi_\pi \in \left(m - \frac{\delta\sigma(1-\beta^f)(1-m)}{\kappa}, \infty\right).$$

Note that the Taylor principle holds for any $\lambda < \lambda^*$.

Strict current-looking inflation targeting rule in a IADL economy

When $\delta < 0$, the first case of the conditions are

$$\phi_\pi < -\frac{\delta\sigma(1-m\beta^f)}{\kappa} := \alpha_{21}$$
$$\phi_\pi < m - \frac{\delta\sigma(1-\beta^f)(1-m)}{\kappa} := \alpha_{22}$$
$$\phi_\pi < -m - \frac{\delta\sigma(1+\beta^f)(1+m)}{\kappa} := \alpha_{23}.$$

For the second case of the conditions

$$\alpha_{31} := -\frac{\delta\sigma(1+m\beta^f)}{\kappa} < \phi_\pi$$
$$\alpha_{32} := m - \frac{\delta\sigma(1-\beta^f)(1-m)}{\kappa} < \phi_\pi$$
$$\alpha_{33} := -m - \frac{\delta\sigma(1+\beta^f)(1+m)}{\kappa} < \phi_\pi.$$

Therefore, the determinacy region is given by

$$(0, min\{\alpha_{21}, \alpha_{22}, \alpha_{23}\}) \cup (max\{\alpha_{31}, \alpha_{32}, \alpha_{33}\}, \infty).$$

Figure 7 shows the determinacy region under bounded rationality and rational expectations. By inspection, as λ approaches λ^* from the right, determinacy arises under a passive monetary policy stance. However, this is the case only for a very small interval of the share of constrained households. Additionally, let $l := min\{\alpha_{21}, \alpha_{22}, \alpha_{23}\}$ and $u := max\{\alpha_{31}, \alpha_{32}, \alpha_{33}\}$. Since $\frac{\partial\delta}{\partial\lambda} < 0$, both l and u are increasing in the share of non-asset holders λ (each argument of l and u are increasing). Again, the benefits from incorporating bounded rationality are considerable.

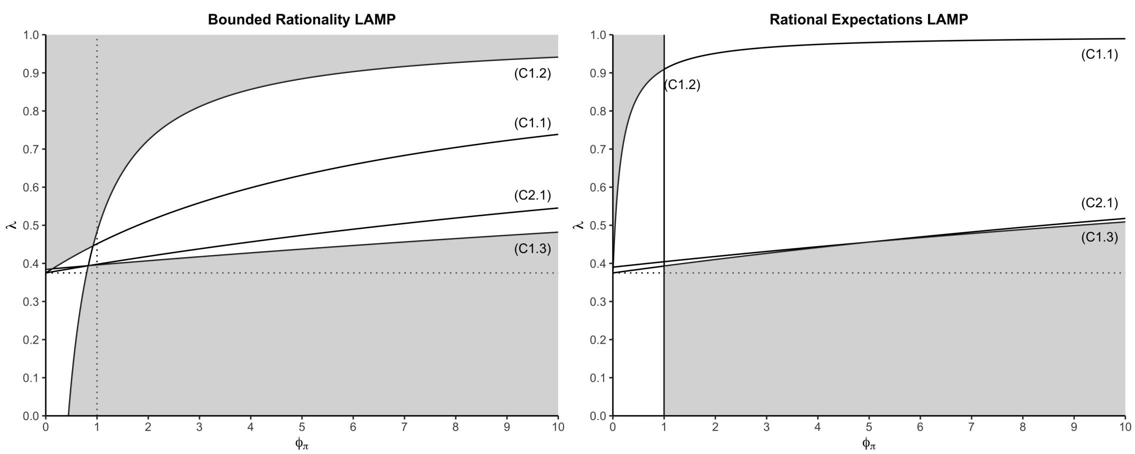

Figure 7. Determinacy region under interest rate rule (39) for interest rate sensitivity to changes in inflation as a function of the share of constrained households under different setups.

Note: Filled regions indicate determinacy regions. The horizontal-dotted line represents $\lambda^* = 0.375$.

Figure 8 shows comparative statistics for the cognitive parameter m. The regions are effectively enlarged, reinforcing the idea that, the less forward-looking agents are, the easier it is to achieve determinacy.

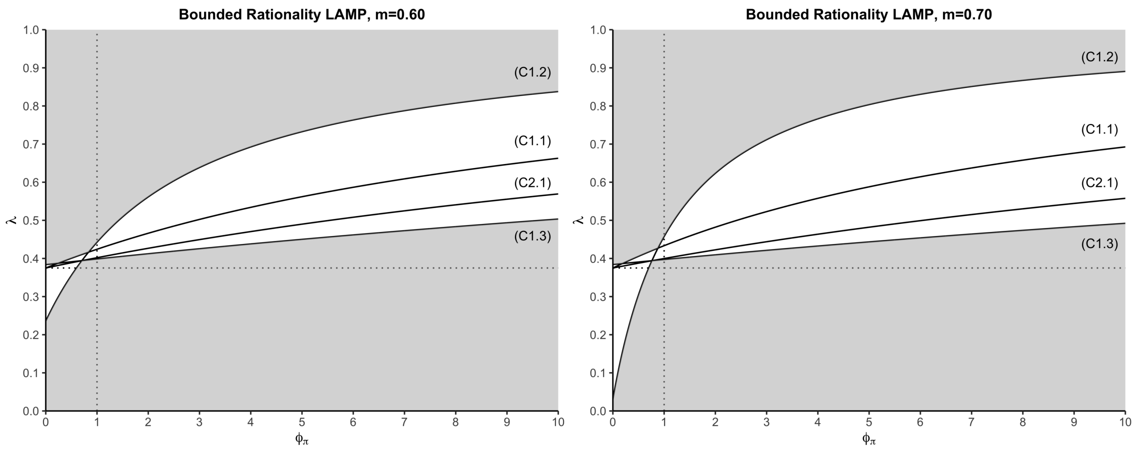

FIGURE 8. DETERMINACY REGION SENSITIVITY ANALYSIS FOR THE COGNITIVE PARAMETER, m, UNDER INTEREST RATE RULE (39).

Note: Filled regions indicate determinacy regions. The horizontal-dotted line represents $\lambda^* = 0.375$.

5.4. *Current-looking Taylor rule*

Consider the interest rate rule

$$r_t = \phi_\pi \pi_t + \phi_x x_t + \epsilon_t. \tag{40}$$

Substituting the interest rate rule into the DIS curve and combining with the BNKPC curve we can write a system for the vector $(x_t, \pi_t)^T$ as

$$\begin{pmatrix} 0 & \beta^f \\ m & \frac{m\eta}{\delta\sigma} \end{pmatrix} \begin{pmatrix} \mathbb{E}_t[x_{t+1}] \\ \mathbb{E}_t[\pi_{t+1}] \end{pmatrix} = \begin{pmatrix} -\kappa & 1 \\ 1 + \frac{\eta\phi_x}{\delta\sigma} & \frac{\eta\phi_\pi}{\delta\sigma} \end{pmatrix} \begin{pmatrix} x_t \\ \pi_t \end{pmatrix} + \begin{pmatrix} -u_t \\ \frac{\eta(\epsilon_t - r_t^*)}{\delta\sigma} \end{pmatrix}.$$

This can be rewritten as

$$\begin{pmatrix} \mathbb{E}_t[x_{t+1}] \\ \mathbb{E}_t[\pi_{t+1}] \end{pmatrix} = \mathbf{\Gamma} \begin{pmatrix} x_t \\ \pi_t \end{pmatrix} + \mathbf{\Psi} \begin{pmatrix} \epsilon_t - r_t^* \\ u_t \end{pmatrix},$$

where

$$\mathbf{\Gamma} = \begin{pmatrix} \frac{\eta\kappa}{\beta^f\delta\sigma} + \frac{1}{m} + \frac{\eta\phi_x}{m\delta\sigma} & -\frac{\eta}{m\beta^f\delta\sigma} + \frac{\eta\phi_\pi}{m\delta\sigma} \\ -\frac{\kappa}{\beta^f} & \frac{1}{\beta^f} \end{pmatrix}, \quad \mathbf{\Psi} = \begin{pmatrix} \frac{\eta}{m\delta\sigma} & \frac{\eta}{\beta^f\delta\sigma} \\ 0 & -\frac{1}{\beta^f} \end{pmatrix}.$$

Using the same method as before, the following necessary and sufficient conditions for determinacy are

Condition (D1.1) $det(\mathbf{\Gamma}) > 1$

$$\frac{\phi_x + \kappa\phi_\pi}{\delta} > -\frac{\sigma(1 - m\beta^f)}{\eta}.$$

Condition (D1.2) $det(\mathbf{\Gamma}) - tr(\mathbf{\Gamma}) > -1$

$$\frac{\phi_x(1-\beta^f) + \kappa(\phi_\pi - m)}{\delta} > -\frac{\sigma(1-\beta^f)(1-m)}{\eta}.$$

Condition (D1.3) $det(\mathbf{\Gamma}) + tr(\mathbf{\Gamma}) > -1$

$$\frac{\phi_x(1+\beta^f) + \kappa(\phi_\pi + m)}{\delta} > -\frac{\sigma(1+\beta^f)(1+m)}{\eta}.$$

Condition (D2.1), $det(\mathbf{\Gamma}) < -1$

$$\frac{\phi_x + \kappa\phi_\pi}{\delta} < -\frac{\sigma(1 + m\beta^f)}{\eta}.$$

Condition (D2.2), $det(\mathbf{\Gamma}) - tr(\mathbf{\Gamma}) < -1$

$$\frac{\phi_x(1-\beta^f) + \kappa(\phi_\pi - m)}{\delta} < -\frac{\sigma(1-\beta^f)(1-m)}{\eta}.$$

Condition (D2.3), $det(\mathbf{\Gamma}) + tr(\mathbf{\Gamma}) < -1$

$$\frac{\phi_x(1+\beta^f) + \kappa(\phi_\pi + m)}{\delta} < -\frac{\sigma(1+\beta^f)(1+m)}{\eta}.$$

Current-looking Taylor rule in a SADL economy

When $\delta > 0$, the second set of conditions is ruled out, and for the first case I get

$$\begin{aligned}
-\frac{\delta\sigma(1-m\beta^f)}{\eta\kappa} &< \phi_\pi + \frac{1}{\kappa}\phi_x \\
m - \frac{\delta\sigma(1-\beta^f)(1-m)}{\eta\kappa} &< \phi_\pi + \frac{1-\beta^f}{\kappa}\phi_x \\
-m - \frac{\delta\sigma(1+\beta^f)(1+m)}{\eta\kappa} &< \phi_\pi + \frac{1+\beta^f}{\kappa}\phi_x.
\end{aligned}$$

These conditions can be rearranged to obtain the generalized Taylor condition, and will coincide with the Hybrid Taylor rule after setting $m^{CB} = 1$ and $m = 1$. In the plane (ϕ_x, ϕ_π), the largest intercept from the inequalities is from the second (the only greater than 0) and the smallest slope in absolute value is from the second too. I.e., the second condition is above the other two for any $\phi_x > 0$ and therefore is the condition most likely to bind. Thus there is determinacy if and only if

$$m - \frac{\delta\sigma(1-\beta^f)(1-m)}{\eta\kappa} - \frac{1-\beta^f}{\kappa}\phi_x < \phi_\pi.$$

The determinacy region is plotted in figure 9.

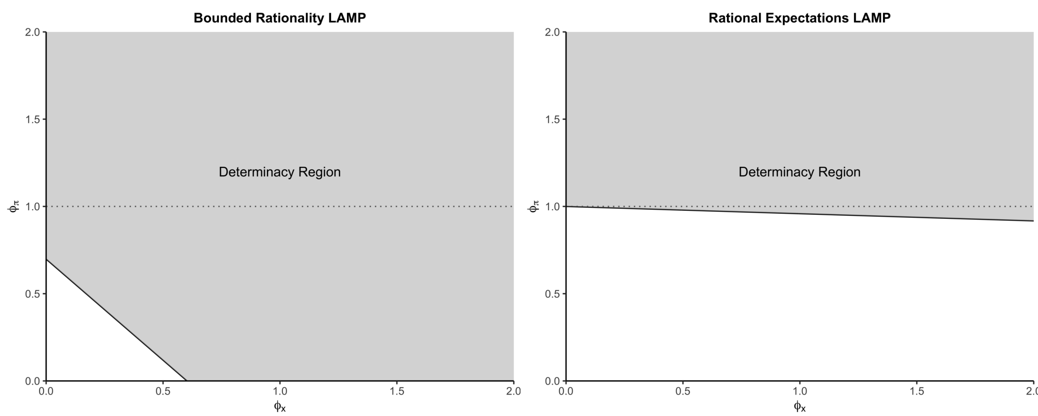

FIGURE 9. DETERMINACY REGION UNDER INTEREST RATE RULE (40) FOR INTEREST RATE SENSITIVITY TO CHANGES IN INFLATION AND TO CHANGES IN OUTPUT IN A SADL ECONOMY WHERE $\lambda = 0.30$.

CURRENT-LOOKING TAYLOR RULE IN A IADL ECONOMY

When $\delta < 0$, the first case of the conditions are

$$\phi_\pi + \frac{1}{\kappa}\phi_x < -\frac{\delta\sigma(1-m\beta^f)}{\eta\kappa}$$
$$\phi_\pi + \frac{1-\beta^f}{\kappa}\phi_x < m - \frac{\delta\sigma(1-\beta^f)(1-m)}{\eta\kappa}$$
$$\phi_\pi + \frac{1+\beta^f}{\kappa}\phi_x < -m - \frac{\delta\sigma(1+\beta^f)(1+m)}{\eta\kappa}.$$

It can be shown that condition (D1.1) is the most binding in this case.

For the second case of the conditions

$$
\begin{aligned}
-\frac{\delta\sigma(1+m\beta^f)}{\eta\kappa} &< \phi_\pi + \frac{1}{\kappa}\phi_x \\
m - \frac{\delta\sigma(1-\beta^f)(1-m)}{\eta\kappa} &< \phi_\pi + \frac{1-\beta^f}{\kappa}\phi_x \\
-m - \frac{\delta\sigma(1+\beta^f)(1+m)}{\eta\kappa} &< \phi_\pi + \frac{1+\beta^f}{\kappa}\phi_x.
\end{aligned}
$$

Figures 10 and 11 plot the determinacy region under both set of conditions for $\lambda = 0.40$ and $\lambda = 0.50$, respectively. The region for the first set of conditions is minuscule. The Taylor principle is needed for determinacy, provided the output response is set sufficiently large. And the higher the share of constrained households, the larger the response to changes in the output gap must be for determinacy under the the Taylor principle.

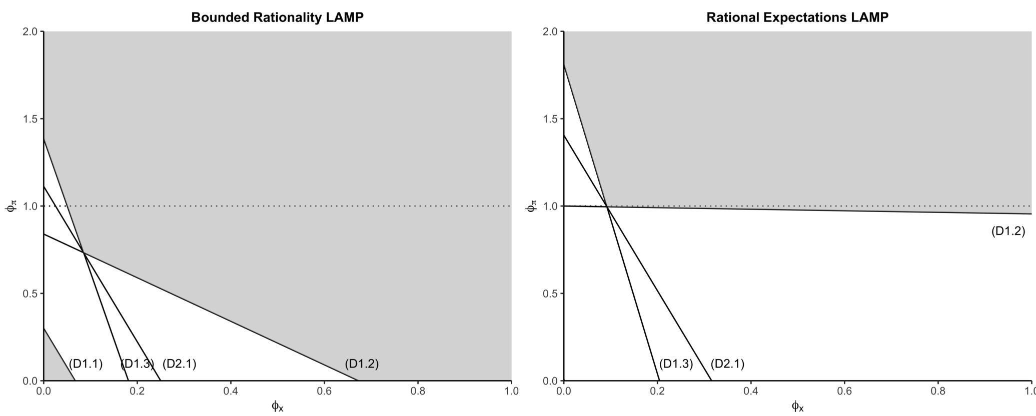

FIGURE 10. DETERMINACY REGION UNDER INTEREST RATE RULE (40) FOR INTEREST RATE SENSITIVITY TO CHANGES IN INFLATION AND TO CHANGES IN OUTPUT IN A IADL ECONOMY WHERE $\lambda = 0.40$.

Note: Filled regions indicate determinacy regions.

6. Model Dynamics

In this section, I illustrate the dynamic effects of a contractionary monetary policy shock on the output gap, on inflation, and on the nominal and real interest rates. I assume that the interest rate shock ϵ_t found on the interest rate rules considered above follows an autoregressive process, $AR(1)$.

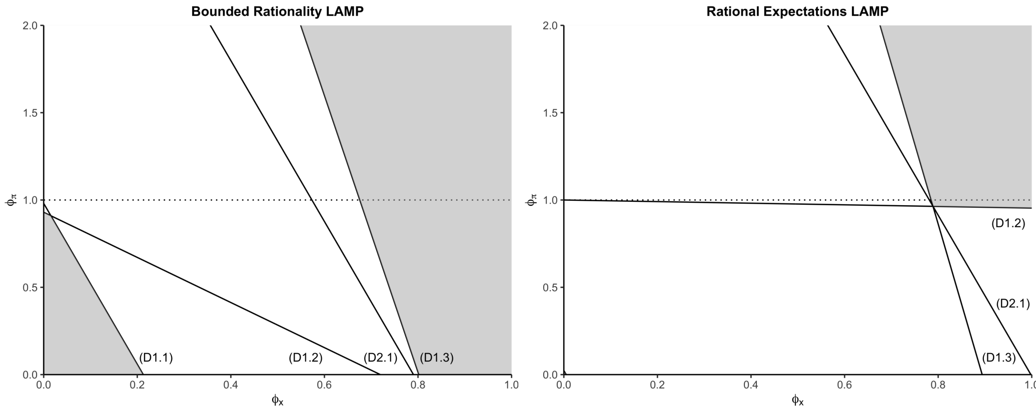

FIGURE 11. DETERMINACY REGION UNDER INTEREST RATE RULE (40) FOR INTEREST RATE SENSITIVITY TO CHANGES IN INFLATION AND TO CHANGES IN OUTPUT IN A IADL ECONOMY WHERE $\lambda = 0.45$.

Note: Filled regions indicate determinacy regions.

I approach the analysis considering the strict forecast-based inflation targeting rule.

Consider the interest rate rule (37) and set $\phi_\pi = 1.50$. First, look at the SADL case, where determinacy is achieved, and consider different values for the cognitive discounting parameter m. As expected, the contractionary monetary policy reduces the output gap and reduces inflation. Note that as m approaches to zero, unconstrained agents are not at all forward-looking, so they will only care about the present. Figure 12 plots the impulse response for $\lambda = 0.30$ (SADL) economy. Note that the larger m is, the faster the economy gets back to the steady state.

As λ increases and the IADL region is reached, there might not be determinacy. That is, the policy rule induces indeterminacy of equilibrium and may destabilize the macroeconomy which can result in large reductions in the welfare of the economy. In this case, to do comparative statistics I will employ a method for solving and estimating linear expectations models that exhibit indeterminacy following Farmer, Khramov, and Nicolò (2015).

Figure 13 plots the impulse response for $\lambda = 0.40$ (IADL economy), where $\phi_\pi = 1.50$ induces indeterminacy. It can be seen that the contractionary monetary policy shock increases the output gap contrary to the usual aggregate demand logic; recall that this is a IADL economy. Note that the less rational agents are, the greater the expectations-adjustment is. In this case the contractionary monetary policy shock causes inflation to decline below its steady state causing the nominal interest rates to decline in response. This dampens the effect monetary policy shocks have on output (the decrease in output is smaller in the presence of indeterminacy).

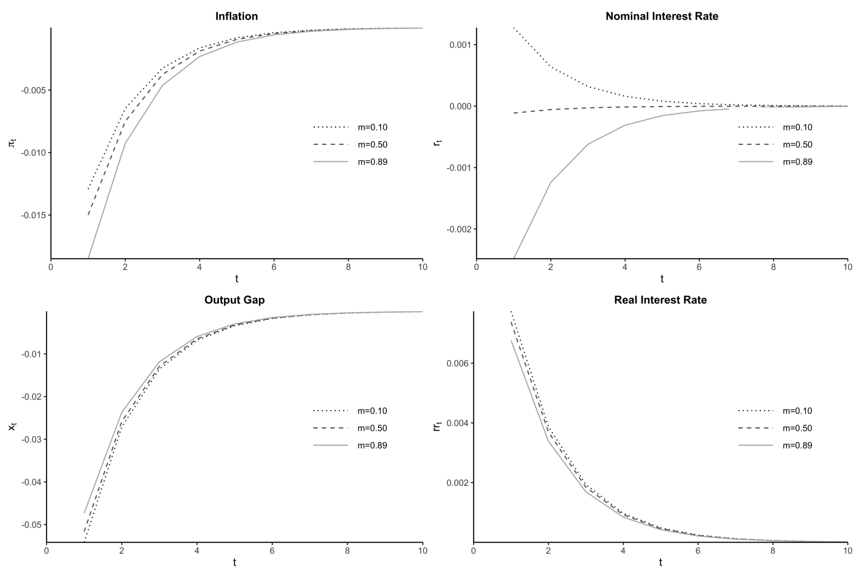

FIGURE 12. DYNAMIC RESPONSES TO A MONETARY POLICY SHOCK UNDER INTEREST RATE RULE (37) IN A SADL ECONOMY WHERE $\lambda = 0.30$ AND $\phi_\pi = 1.50$.

What happens if the share of constrained households rises? Figure 14 plots the impulse response for $\lambda = 0.50$ (still a IADL economy). The movements are similar to those found in figure 13. In response to the monetary policy shock, either inflation or the real interest rate must rise (if one variable is going in the opposite direction, the increase should dominate the decrease). Figure 14 shows that the real interest rate increase and inflation decreases. Consumption of unconstrained households decreases due to the real rate decrease, which implies a leftward shift of labor supply, and hence a increase in wage and decrease in hours. Consumption of constrained households increases since the wage is increasing, and hence aggregate demand rises, making the output gap positive.

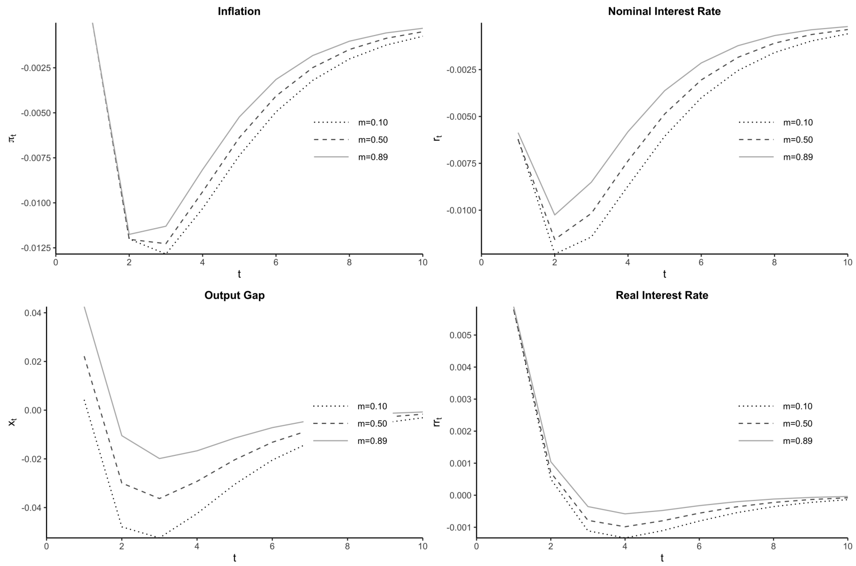

FIGURE 13. DYNAMIC RESPONSES TO A MONETARY POLICY SHOCK UNDER INTEREST RATE RULE (37) IN A IADL ECONOMY WHERE $\lambda = 0.40$ AND $\phi_\pi = 1.50$.

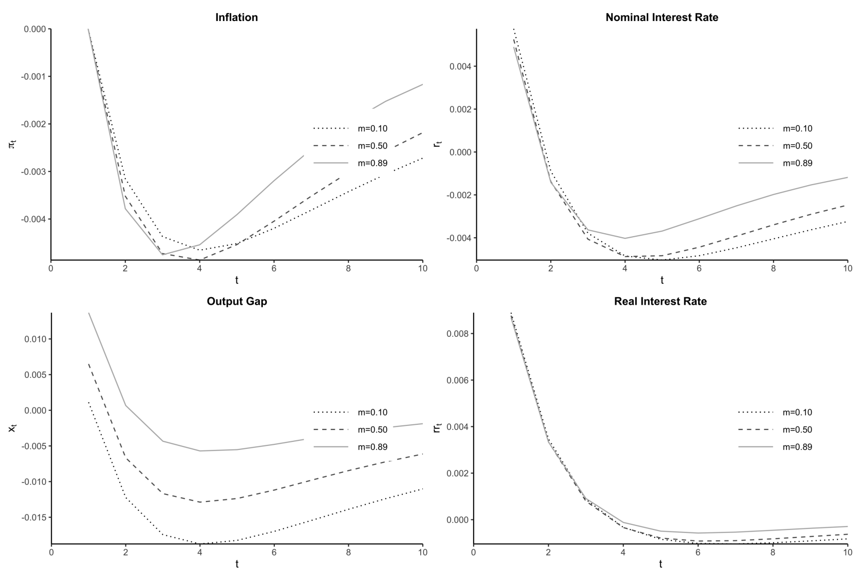

FIGURE 14. DYNAMIC RESPONSES TO A MONETARY POLICY SHOCK UNDER INTEREST RATE RULE (37) IN A IADL ECONOMY WHERE $\lambda = 0.50$ AND $\phi_\pi = 1.50$.

7. Conclusion

In this thesis I study extensively the dynamics of the baseline NK model augmented for bounded rationality and limited asset market participation as well as the determinacy properties of interest rate rules. Both bounded rationality and LAMP are relevant for developing countries such as Mexico.

For a high share of constrained households, LAMP changes aggregate demand sensitivity to interest rates in an inverted manner, that is, an increase in the interest rate increases aggregate demand. Thus the central bank must be careful in setting monetary policy since it may induce the opposite effect it is looking for. Interestingly, incorporating bounded rationality does not affect the slope of the DIS curve and does not change the threshold at which the economy passes from the SADL to the IADL region. However, the sensitivity of aggregate demand to changes in inflation expectations does change and is reduced in absolute value.

In addition, I propose a cognitive discounting parameter for the central bank trying to capture the inability of the authority to properly measure expected future inflation. It is shown that if the central bank is aware of this limitation, it must act to changes in inflation in a more aggressive manner than otherwise. I find that central bank reduces determinacy region in a SADL economy but increases it in a IADL economy. This is seen as positive since the main issues such as a passive monetary policy arise only in the IADL region, thus we can sacrifice some space in the SADL region to benefit the IADL region. This may restore the Taylor principle even in a IADL economy under the strict forecast-based inflation targeting rule. The hybrid Taylor rule restores the Taylor principle in a IADL economy too (in the rational expectations LAMP). Incorporating bounded rationality "fixes" the IADL region further, by providing a larger determinacy region where a lower output response is sufficient for the Taylor principle to produce determinacy. Additionally, under the hybrid Taylor rule there is a interval for the output response where no matter the inflation response, determinacy is never achieved; bounded rationality reduces this interval. Finally, I find that the less forward-looking agents are, the faster the economy returns to the steady-state after a monetary policy shock.

8. Appendixes

A. Derivation of Equation (33)

Substituting (31) and (32) into the asset holder's budget constraint:

$$P_t C_{S,t} = \frac{1}{1-\lambda} P_t D_t + W_t N_{S,t}$$
$$\iff C_{S,t} = \frac{1}{1-\lambda} D_t + \frac{W_t N_{S,t}}{P_t}.$$

Substituting both of the budget constraints, total consumption is

$$\begin{aligned} C_t &= \lambda C_{H,t} + (1-\lambda) C_{S,t} \\ &= \lambda \frac{W_t N_{H,t}}{P_t} + (1-\lambda)\left(\frac{1}{1-\lambda} D_t + \frac{W_t N_{S,t}}{P_t}\right). \end{aligned} \tag{A1}$$

From (24), aggregating profits yields

$$D_t = \int_0^1 D_t(i)di = \int_0^1 \left(\frac{P_t(i)}{P_t} Y_t(i) - \frac{W_t}{P_t} N_t(i)\right) di.$$

Substituting the demand for the intermediate good (21) into the above

$$\begin{aligned} D_t &= \int_0^1 \left(\frac{P_t(i)}{P_t}\left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t - \frac{W_t}{P_t} N_t(i)\right) di \\ &= \int_0^1 \left(\left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon} Y_t - \frac{W_t}{P_t} N_t(i)\right) di \\ &= \left(\frac{1}{P_t}\right)^{1-\epsilon} Y_t \int_0^1 P_t(i)^{1-\epsilon} di - \frac{W_t}{P_t} \int_0^1 N_t(i) di \\ &= Y_t - \frac{W_t}{P_t}(\lambda N_{H,t} + (1-\lambda) N_{S,t}). \end{aligned} \tag{A2}$$

Substituting (A2) into (A1)

$$\begin{aligned} C_t &= \lambda \frac{W_t N_{H,t}}{P_t} + (1-\lambda)\left(\frac{1}{1-\lambda}\left(Y_t - \frac{W_t}{P_t}(\lambda N_{H,t} + (1-\lambda) N_{S,t})\right) + \frac{W_t N_{S,t}}{P_t}\right) \\ &= \lambda \frac{W_t N_{H,t}}{P_t} + Y_t - \frac{W_t}{P_t}(\lambda N_{H,t} + (1-\lambda) N_{S,t}) + (1-\lambda)\frac{W_t N_{S,t}}{P_t} \\ &= Y_t. \end{aligned}$$

B. The Behavioral New Keynesian Phillips Curve

From (28),

$$P_t^* = \frac{\epsilon}{\epsilon - 1}\mathbb{E}_t^{BR}\left[\sum_{j=0}^{\infty}\frac{\theta^j \Lambda_{t,t+j}Y_{t+j}(P_{t+j})^{\epsilon-1}}{\sum_{k=0}^{\infty}\theta^k \mathbb{E}_t^{BR}\left[\Lambda_{t,t+k}Y_{t+k}\left(P_{t+k}\right)^{\epsilon-1}\right]}MC_{t+j}\right]$$

$$\iff \frac{P_t^*}{P_t}\sum_{k=0}^{\infty}\theta^k \mathbb{E}_t^{BR}\left[\Lambda_{t,t+k}Y_{t+k}\left(P_{t+k}\right)^{\epsilon-1}\right] = \frac{\epsilon}{\epsilon-1}\frac{1}{P_t}\sum_{j=0}^{\infty}\theta^j \mathbb{E}_t^{BR}\left[\Lambda_{t,t+j}Y_{t+j}(P_{t+j})^{\epsilon-1}MC_{t+j}\right].$$

Log-linearizing around the zero-inflation steady-state:

$$\sum_{k=0}^{\infty}(\theta)^k \Lambda_k^{ss}Y^{ss}\left(P^{ss}\right)^{\epsilon-1}\left(1 + p_t^* - p_t + \mathbb{E}_t^{BR}[ln(\Lambda_{t,t+k})] + \mathbb{E}_t^{BR}[y_{t+k}] + (\epsilon-1)\mathbb{E}_t^{BR}[p_{t+k}]\right)$$
$$= \sum_{j=0}^{\infty}(\theta)^j \Lambda_j^{ss}Y^{ss}(P^{ss})^{\epsilon-1}\left(1 - p_t + \mathbb{E}_t^{BR}[ln(\Lambda_{t,t+j})] + \mathbb{E}_t^{BR}[y_{t+j}] + (\epsilon-1)\mathbb{E}_t^{BR}[p_{t+j}] + \mathbb{E}_t^{BR}[mc_{t+j}]\right)$$

$$\iff \sum_{k=0}^{\infty}(\theta)^k \Lambda_k^{ss}Y^{ss}\left(P^{ss}\right)^{\epsilon-1}p_t^* = \sum_{j=0}^{\infty}(\theta)^j \Lambda_j^{ss}Y^{ss}(P^{ss})^{\epsilon-1}\mathbb{E}_t^{BR}[mc_{t+j}]$$
$$\iff p_t^*\sum_{k=0}^{\infty}(\theta\beta)^k = \sum_{j=0}^{\infty}(\theta\beta)^j \mathbb{E}_t^{BR}[mc_{t+j}]$$
$$\iff p_t^*\frac{1}{1-\theta\beta} = \sum_{j=0}^{\infty}(\theta\beta m)^j \mathbb{E}_t[mc_{t+j}]$$
$$\iff p_t^* = (1-\theta\beta)\sum_{j=0}^{\infty}(\theta\beta m)^j \mathbb{E}_t[mc_{t+j}].$$

Therefore,

$$
\begin{aligned}
p_t^* - p_t &= (1-\theta\beta)\sum_{j=0}^{\infty}\rho^j \mathbb{E}_t[\pi_{t+1} + \cdots + \pi_{t+j} - \mu_{t+j}] \\
&= (1-\theta\beta)\mathbb{E}_t\left[H_t - \sum_{j=0}^{\infty}\rho^j\mu_{t+j}\right] \\
&= (1-\theta\beta)\sum_{j=0}^{\infty}\rho^j\mathbb{E}_t\left[\frac{1}{1-\rho}\pi_{t+j}\mathbb{I}_{j>0} - \mu_{t+j}\right] \\
&= \sum_{j=0}^{\infty}\rho^j\mathbb{E}_t\left[\frac{1-\theta\beta}{1-\rho}\pi_{t+j}\mathbb{I}_{j>0} - (1-\theta\beta)\mu_{t+j}\right]. \qquad \text{(B1)}
\end{aligned}
$$

Denote H_t as

$$
H_t = \sum_{j=1}^{\infty}\rho^j\sum_{k=1}^{j}\pi_{t+k} = \sum_{k=1}^{\infty}\pi_{t+k}\sum_{j=k}^{\infty}\rho^j = \frac{1}{1-\rho}\sum_{k=1}^{\infty}\pi_{t+k}\rho^k = \frac{1}{1-\rho}\sum_{k=0}^{\infty}\pi_{t+k}\rho^k\mathbb{I}_{k>0}.
$$

From the aggregate price level

$$
\begin{aligned}
p_t &= \theta p_{t-1} + (1-\theta)p_t^* \\
\iff p_t - p_{t-1} &= (1-\theta)(p_t^* - p_{t-1}) \\
\iff \pi_t &= (1-\theta)(p_t^* - p_t + p_t - p_{t-1}) = (1-\theta)(p_t^* - p_t + \pi_t) \\
\iff \pi_t &= \frac{1-\theta}{\theta}(p_t^* - p_t). \qquad \text{(B2)}
\end{aligned}
$$

Substituting (B1) into (B2)

$$
\pi_t = \frac{1-\theta}{\theta}\sum_{j=0}^{\infty}\rho^j\mathbb{E}_t\left[\frac{1-\theta\beta}{1-\rho}\pi_{t+j}\mathbb{I}_{j>0} - (1-\theta\beta)\mu_{t+j}\right].
$$

Let F be a forward operator define as $Fy_t = y_{t+1}$. Then, for example,

$$
\sum_{k=0}^{\infty}\rho^k y_{t+k} = \sum_{k=0}^{\infty}\rho^k F^k y_t = y_t\sum_{k=0}^{\infty}\rho^k F^k = \frac{y_t}{1-\rho F}.
$$

Now I get

$$\begin{aligned}\pi_t &= \frac{1-\theta}{\theta}\sum_{j=0}^{\infty}\rho^j\mathbb{E}_t\left[\frac{1-\theta\beta}{1-\rho}\pi_{t+j}\mathbb{I}_{j>0} - (1-\theta\beta)\mu_{t+j}\right]\\ &= \frac{1-\theta}{\theta}\mathbb{E}_t\left[\frac{1-\theta\beta}{1-\rho}\frac{\rho F}{1-\rho F}\pi_t - (1-\theta\beta)\frac{1}{1-\rho F}\mu_t\right]\\ &= \frac{1-\theta}{\theta}\mathbb{E}_t\left[\frac{1}{1-\rho F}\left(\frac{1-\theta\beta}{1-\rho}\rho F\pi_t - (1-\theta\beta)\mu_t\right)\right].\end{aligned}$$

Multiplying by $(1-\rho F)$ yields

$$\begin{aligned}\pi_t &= \left(1+\frac{1-\theta}{\theta}\frac{1-\theta\beta}{1-\rho}\right)\rho\mathbb{E}_t\left[F\pi_t\right] - \frac{(1-\theta)(1-\theta\beta)}{\theta}\mu_t\\ &= \beta m\left(\theta+\frac{(1-\theta)(1-\theta\beta)}{1-\beta\theta m}\right)\mathbb{E}_t\left[\pi_{t+1}\right] - \frac{(1-\theta)(1-\theta\beta)}{\theta}\mu_t\\ :&= \beta M^f\mathbb{E}_t[\pi_{t+1}] - \psi\mu_t.\end{aligned}$$

C. Model Solution

Since hours of constrained households are constant, $n_{H,t} = 0$, i.e., their consumption is the real wage, $c_{H,t} = w_t$. Total labor supply (from the labor market clearing condition) is

$$\begin{aligned}n_t &= \lambda n_{H,t} + (1-\lambda)n_{S,t}\\ &= (1-\lambda)n_{S,t}.\end{aligned}$$

From aggregate consumption,

$$
\begin{aligned}
c_{S,t} &= c_t - \lambda(c_{H,t} - c_{S,t}) \\
&= y_t - \lambda(w_t - c_{S,t}) \\
&= y_t - \lambda\left(\frac{\varphi}{1-\lambda}n_t + \sigma c_{S,t}\right) + \lambda c_{S,t} \\
&= y_t - \frac{\varphi\lambda}{1-\lambda}n_t - \lambda(\sigma-1)c_{S,t} \\
\iff c_{S,t}(1+\lambda(\sigma-1)) &= y_t - \frac{\varphi\lambda}{1-\lambda}n_t \\
&= y_t - \frac{\varphi\lambda}{1-\lambda}\left(\frac{y_t}{1+\mu} - a_t\right) \\
&= \left(1 - \frac{\varphi\lambda}{(1-\lambda)(1+\mu)}\right)y_t + \frac{\varphi\lambda}{1-\lambda}a_t \\
&:= \delta y_t + (1-\delta)(1+\mu)a_t \\
\iff c_{S,t} &= (\delta y_t + (1-\delta)(1+\mu)a_t)\eta^{-1}, \qquad \text{(C1)}
\end{aligned}
$$

where $\eta = 1+\lambda(\sigma-1)$. Substituting into the Euler equation of asset holders

$$m\mathbb{E}_t[(\delta y_{t+1} + (1-\delta)(1+\mu)a_{t+1})\eta^{-1}] - (\delta y_t + (1-\delta)(1+\mu)a_t)\eta^{-1} = \frac{r_t - m\mathbb{E}_t\left[\pi_{t+1}\right]}{\sigma}$$

(C2)

$$\iff y_t = m\mathbb{E}_t[y_{t+1}] - \frac{\eta}{\delta\sigma}(r_t - m\mathbb{E}_t[\pi_{t+1}]) - \frac{(1+\mu)(1-\delta)}{\delta}(a_t - m\mathbb{E}_t[a_{t+1}]).$$

Substituting equation (C1), and noting that $n_t = (1-\lambda)n_{S,t}$ and $n_t = y_t/(1+\mu) - a_t$, thus $n_{S,t} = (y_t/(1+\mu) - a_t)/(1-\lambda)$, into the labor supply equation of asset holders

$$\begin{aligned}
w_t &= \sigma c_{S,t} + \varphi n_{S,t} \\
&= (\delta y_t + (1-\delta)(1+\mu)a_t)\sigma\eta^{-1} + \frac{\varphi}{1-\lambda}\left(\frac{y_t}{1+\mu} - a_t\right) \\
&= \left[\delta\sigma\eta^{-1} + \frac{\varphi}{(1-\lambda)(1+\mu)}\right] y_t - \left[\frac{\varphi}{1-\lambda} + (1-\delta)(1+\mu)\sigma\eta^{-1}\right] a_t \\
&= \left[\left(1 - \frac{\varphi\lambda}{(1-\lambda)(1+\mu)}\right)\sigma\eta^{-1} + \frac{\varphi}{(1-\lambda)(1+\mu)}\right] y_t - \left[\frac{\varphi}{1-\lambda} - \frac{\varphi\lambda\sigma\eta^{-1}}{1-\lambda}\right] a_t \\
&= \left[\sigma\eta^{-1} + \frac{\varphi}{(1-\lambda)(1+\mu)}\left(1 - \lambda\sigma\eta^{-1}\right)\right] y_t - \frac{\varphi(1-\lambda\sigma\eta^{-1})}{1-\lambda} a_t \\
&= \left[\sigma\eta^{-1} + \frac{\varphi}{(1-\lambda)(1+\mu)}(1-\lambda)\eta^{-1}\right] y_t - \frac{\varphi(1-\lambda)\eta^{-1}}{1-\lambda} a_t \\
&= \eta^{-1}\left[\sigma + \frac{\varphi}{1+\mu}\right] y_t - \varphi\eta^{-1} a_t
\end{aligned}$$

(C3)
$$= \eta^{-1}(\chi y_t - \varphi a_t),$$

where $\chi = \sigma + \varphi/(1+\mu)$. Substituting equation (C3) into the real marginal cost equation

$$\begin{aligned}
mc_t &= w_t - a_t \\
&= \eta^{-1}(\chi y_t - \varphi a_t) - a_t \\
&= \eta^{-1}\chi y_t - (1+\varphi\eta^{-1})a_t. \qquad \text{(C4)}
\end{aligned}$$

Finally, substituting equation (C4) into the BNKPC

$$\begin{aligned}
\pi_t &= \beta M^f \mathbb{E}_t[\pi_{t+1}] + \psi mc_t \\
&= \beta M^f \mathbb{E}_t[\pi_{t+1}] + \psi(\eta^{-1}\chi y_t - (1+\varphi\eta^{-1})a_t) \\
&= \beta M^f \mathbb{E}_t[\pi_{t+1}] + \eta\kappa\eta^{-1} y_t - \psi(1+\varphi\eta^{-1})a_t \\
&:= \beta M^f \mathbb{E}_t[\pi_{t+1}] + \kappa y_t - \psi(1+\varphi\eta^{-1})a_t \qquad \text{(C5)}
\end{aligned}$$

Let $x_t = y_t - y_t^*$ denote the output gap (difference between actual output and natural output). Natural levels of all variables are those occurring under fully-flexible prices, and hence inflation is zero and real marginal cost and markup are constant. From (C5), the natural output level is

$$y_t^* = \frac{\psi(1+\varphi\eta^{-1})}{\kappa} a_t = \frac{\psi(1+\varphi\eta^{-1})}{\eta\kappa\eta^{-1}} a_t = \frac{1+\varphi\eta^{-1}}{\chi\eta^{-1}} a_t = \frac{\eta+\varphi}{\chi} a_t.$$

So marginal cost is related to the output gap by

$$\begin{aligned} mc_t &= \eta^{-1}\chi y_t - (1+\varphi\eta^{-1})a_t \\ &= \eta^{-1}\chi(y_t - y_t^*) - (1+\varphi\eta^{-1})a_t + \eta^{-1}\chi y_t^* \\ &= \eta^{-1}\chi x_t - (1+\varphi\eta^{-1})a_t + \eta^{-1}\chi\frac{\eta+\varphi}{\chi}a_t \\ &= \eta^{-1}\chi x_t - (1+\varphi\eta^{-1})a_t + \eta^{-1}(\eta+\varphi)a_t \\ &= \eta^{-1}\chi x_t - (1+\varphi\eta^{-1})a_t + (1+\varphi\eta^{-1})a_t \\ &= \eta^{-1}\chi x_t. \end{aligned}$$

However, I introduce cost-push shocks as $\tilde{u}_t = \psi^{-1}u_t$, thus I redefine the marginal cost as

$$mc_t := \eta^{-1}\chi x_t + \frac{u_t}{\psi}. \tag{C6}$$

Substituting into the BNKPC (C5), we get its expression as a function of the output gap,

$$\begin{aligned} \pi_t &= \beta M^f \mathbb{E}_t[\pi_{t+1}] + \psi mc_t \\ &= \beta M^f \mathbb{E}_t[\pi_{t+1}] + \psi\left(\eta^{-1}\chi x_t + \frac{u_t}{\psi}\right) \\ &= \beta M^f \mathbb{E}_t[\pi_{t+1}] + \kappa x_t + u_t. \end{aligned} \tag{C7}$$

Evaluating the IS curve under flexible prices, we get from (C2)

$$\begin{aligned} x_t &= m\mathbb{E}_t[x_{t+1}] - \frac{\eta}{\delta\sigma}(r_t - m\mathbb{E}_t[\pi_{t+1}]) - \frac{(1+\mu)(1-\delta)}{\delta}(a_t - m\mathbb{E}_t[a_{t+1}]) + m\mathbb{E}_t[y_{t+1}^*] - y_t^* \\ &= m\mathbb{E}_t[x_{t+1}] - \frac{\eta}{\delta\sigma}(r_t - m\mathbb{E}_t[\pi_{t+1}]) - \frac{(1+\mu)(1-\delta)}{\delta}(a_t - m\mathbb{E}_t[a_{t+1}]) - \frac{\eta+\varphi}{\chi}(a_t - m\mathbb{E}_t[a_{t+1}]) \\ &= m\mathbb{E}_t[x_{t+1}] - \frac{\eta}{\delta\sigma}(r_t - m\mathbb{E}_t[\pi_{t+1}]) - \left[\frac{(1+\mu)(1-\delta)}{\delta} + \frac{\eta+\varphi}{\chi}\right](a_t - m\mathbb{E}_t[a_{t+1}]). \end{aligned} \tag{C8}$$

Let

$$
\begin{aligned}
c &:= \frac{(1+\mu)(1-\delta)}{\delta} + \frac{\eta+\varphi}{\chi} \\
&= \frac{(1+\mu)(1-\delta)\chi + (\eta+\varphi)\delta}{\delta\chi} \\
&= \frac{(1+\mu)\chi + (\eta+\varphi-(1+\mu)\chi)\delta}{\delta\chi} \\
&= \frac{(1+\mu)\chi + (\eta+\varphi-((1+\mu)\sigma+\varphi))\delta}{\delta\chi} \\
&= \frac{(1+\mu)\chi + (\eta+\varphi-(1+\mu)\sigma-\varphi)\delta}{\delta\chi} \\
&= \frac{(1+\mu)\chi + (\eta-(1+\mu)\sigma)\delta}{\delta\chi} \\
&= \frac{\chi + (\eta-\sigma)\delta + (\chi-\sigma\delta)\mu}{\delta\chi} \\
&= \frac{1}{\delta}\left(1 + \frac{(\eta-\sigma)\delta}{\chi} + \frac{(\chi-\sigma\delta)\mu}{\chi}\right) \\
&= \frac{1}{\delta}\left(1 + \frac{(1-\lambda)(1-\sigma)\delta}{\chi} + \left(1-\frac{\sigma\delta}{\chi}\right)\mu\right).
\end{aligned}
$$

Now, let r_t^* denote the natural interest rate. It is given by

$$
r_t^* := \frac{\sigma}{\eta}\left(1 + \frac{(1-\lambda)(1-\sigma)\delta}{\chi} + \left(1-\frac{\sigma\delta}{\chi}\right)\mu\right)(m\mathbb{E}_t[a_{t+1}] - a_t).
$$

Thus, substituting the above into equation (C8)

$$
\begin{aligned}
x_t &= m\mathbb{E}_t[x_{t+1}] - \frac{\eta}{\delta\sigma}(r_t - m\mathbb{E}_t[\pi_{t+1}]) - \frac{1}{\delta}\left(1 + \frac{(1-\lambda)(1-\sigma)\delta}{\chi} + \left(1-\frac{\sigma\delta}{\chi}\right)\mu\right)(a_t - m\mathbb{E}_t[a_{t+1}]) \\
&= m\mathbb{E}_t[x_{t+1}] - \frac{\eta}{\delta\sigma}(r_t - m\mathbb{E}_t[\pi_{t+1}]) + \frac{\eta}{\delta\sigma}\frac{\sigma}{\eta}\left(1 + \frac{(1-\lambda)(1-\sigma)\delta}{\chi} + \left(1-\frac{\sigma\delta}{\chi}\right)\mu\right)(m\mathbb{E}_t[a_{t+1}] - a_t) \\
&= m\mathbb{E}_t[x_{t+1}] - \frac{\eta}{\delta\sigma}(r_t - m\mathbb{E}_t[\pi_{t+1}]) + \frac{\eta}{\delta\sigma}r_t^* \\
&= m\mathbb{E}_t[x_{t+1}] - \frac{\eta}{\delta\sigma}(r_t - m\mathbb{E}_t[\pi_{t+1}] - r_t^*).
\end{aligned}
\tag{C9}
$$

Equations (C7) and (C9) correspond to the BNKPC and IS curve, respectively.

D. Interest Rate Rules

The two-equation system under interest rate rule (30) is

$$\begin{pmatrix} 0 & \beta^f \\ m & \frac{\eta}{\delta\sigma}(m-\phi_\pi m^{CB}) \end{pmatrix} \begin{pmatrix} \mathbb{E}_t[x_{t+1}] \\ \mathbb{E}_t[\pi_{t+1}] \end{pmatrix} = \begin{pmatrix} -\kappa & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_t \\ \pi_t \end{pmatrix} + \begin{pmatrix} -u_t \\ \frac{\eta(\epsilon_t - r_t^*)}{\delta\sigma} \end{pmatrix}.$$

The inverse of the first matrix is given by

$$\begin{pmatrix} 0 & \beta^f \\ m & \frac{\eta}{\delta\sigma}(m-\phi_\pi m^{CB}) \end{pmatrix}^{-1} = -\frac{1}{m\beta^f} \begin{pmatrix} \frac{\eta}{\delta\sigma}(m-\phi_\pi m^{CB}) & -\beta^f \\ -m & 0 \end{pmatrix} = \begin{pmatrix} -\frac{\eta(m-\phi_\pi m^{CB})}{m\beta^f\delta\sigma} & \frac{1}{m} \\ \frac{1}{\beta^f} & 0 \end{pmatrix}.$$

Multiplying both sides by the inverse of the matrix

$$\begin{pmatrix} -\frac{\eta(m-\phi_\pi m^{CB})}{m\beta^f\delta\sigma} & \frac{1}{m} \\ \frac{1}{\beta^f} & 0 \end{pmatrix} \begin{pmatrix} -\kappa & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \frac{\eta\kappa(m-\phi_\pi m^{CB})}{m\beta^f\delta\sigma} + \frac{1}{m} & -\frac{\eta(m-\phi_\pi m^{CB})}{m\beta^f\delta\sigma} \\ -\frac{\kappa}{\beta^f} & \frac{1}{\beta^f} \end{pmatrix} := \mathbf{\Gamma},$$

and

$$\begin{pmatrix} -\frac{\eta(m-\phi_\pi m^{CB})}{m\beta^f\delta\sigma} & \frac{1}{m} \\ \frac{1}{\beta^f} & 0 \end{pmatrix} \begin{pmatrix} -u_t \\ \frac{\eta(\epsilon_t - r_t^*)}{\delta\sigma} \end{pmatrix} = \begin{pmatrix} \frac{\eta u_t(m-\phi_\pi m^{CB})}{m\beta^f\delta\sigma} + \frac{\eta(\epsilon_t - r_t^*)}{m\delta\sigma} \\ -\frac{u_t}{\beta^f} \end{pmatrix} = \begin{pmatrix} \frac{\eta}{m\delta\sigma} & \frac{\eta(m-\phi_\pi m^{CB})}{m\beta^f\delta\sigma} \\ 0 & -\frac{1}{\beta^f} \end{pmatrix} \begin{pmatrix} \epsilon_t - r_t^* \\ u_t \end{pmatrix}$$

$$:= \mathbf{\Psi} \begin{pmatrix} \epsilon_t - r_t^* \\ u_t \end{pmatrix}.$$

Therefore

$$\begin{pmatrix} \mathbb{E}_t[x_{t+1}] \\ \mathbb{E}_t[\pi_{t+1}] \end{pmatrix} = \mathbf{\Gamma} \begin{pmatrix} x_t \\ \pi_t \end{pmatrix} + \mathbf{\Psi} \begin{pmatrix} \epsilon_t - r_t^* \\ u_t \end{pmatrix}.$$

And it follows that

$$\begin{aligned} det(\mathbf{\Gamma}) &= \frac{1}{m\beta^f} \\ tr(\mathbf{\Gamma}) &= \frac{1}{m} + \frac{1}{\beta^f} - \frac{\eta\kappa(\phi_\pi m^{CB} - m)}{m\beta^f\delta\sigma} \\ det(\mathbf{\Gamma}) - tr(\mathbf{\Gamma}) &= \frac{1}{m\beta^f}\left(1 - m - \beta^f + \frac{\eta\kappa(\phi_\pi m^{CB} - m)}{\delta\sigma}\right) \\ det(\mathbf{\Gamma}) + tr(\mathbf{\Gamma}) &= \frac{1}{m\beta^f}\left(1 + m + \beta^f - \frac{\eta\kappa(\phi_\pi m^{CB} - m)}{\delta\sigma}\right). \end{aligned}$$

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