

# SHARING INFORMATION IN COST ASYMMETRIC LOAN MARKETS

**TESIS PRESENTADA POR** 

# SAMUEL VÁZQUEZ HERRERA

PROMOCIÓN 2020-2023

CIUDAD DE MÉXICO

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PARA OPTAR POR EL GRADO DE

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A Nelly, Sofía e Isabel, mis amores

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## **Summary**

This thesis aims to know the conditions under two competitors are willing to share its own private information and still compete for the market. Data and information has been being more relevant to compete due to digitization. Financial markets, and specially loan markets, are good examples. Private information about the borrowers' characteristics could be a competitive advantage. Regulators and antitrust offices around the world have acknowledge the importance of this information to compete in credit markets. In the economic literature we can find that asymmetric information in financial markets could drive to credit rationing, inefficiencies, or even to monopolize the market and exert market power.

We present three different models of asymmetric information between to competing lenders. The cost of granting a loan is the probability of defaulting the loan. A high-risk borrower has a greater probability a default, which make more expansive to grant her a loan. In the first chapter we propose an one-stage Bayesian game in which only one player is fully-informed about the probability of defaulting a loan by borrowers. The share of the borrowers' types is according to a Bernoulli distribution. We find three types of equilibria depending on the conditions about the probability of default, and the proportion of low and high-risk borrowers. In the first-case the equilibrium is in pure strategies, while in the other two equilibria are characterized by mixed strategies. In each case the the player with full information about the borrowers' type makes positive profits, thus this player has no incentives to share its information as competition becomes à la Bertrand and gets no profits.

In chapter 2, each of the two competitors has a different share of the market, but there is still a part of it unattended. Hence, they have partial information about the borrowers' type. In this model we have a uniform distribution of borrowers' types. Besides, customers cannot switch lenders because of some lock-in circumstances. In the equilibrium of this Bayesian game, the player with the current highest cost offers the lowest interest rate and gets the rest of the market. Nevertheless, in the presence of switching costs, the expected profit could be positive, zero or even negative. Despite this, the winner has incentives to get the competitor's information in order to increase profits or prevent loses. The loser player share its information only for a side payment, a proportion of the winner's profits, as it gets nothing in equilibrium.

Finally, in chapter 3 we extend the latter model to incorporate the business-cycle. In this section we want to analyze how the incentives to share information changes with the business-cycle. The economic literature focus mainly on how the banking system affects the macroeconomic scenario, although otherwise economic research is scarce. In order to model the business-cycle we use a beta distribution for the borrowers' type. Changes in parameters  $\alpha$  and  $\beta$  skew the Beta distribution. Thus, the share of low-risk borrowers increases during an expansion of the economy, an vice-versa. The equilibrium in this Bayesian game is very similar to the one in chapter 2 given that the uniform distribution is an specific case of the Beta distribution. Hence, most of the outcome is the same. Nevertheless, as the Beta distribution changes emulating the business-cycle, it is not the case that prices decreases with an improvement of the business-cycle, neither profits. Furthermore, the winner player can change, and so incentives to share information do.

# Contents

1 Introduction		oduction	5
2	Cha	pter 1 Information Sharing when Only One Player is Fully-Informed	17
	2.1	Introduction	17
	2.2	Literature Review	18
	2.3	Model: An asymmetric information game with only one fully-informed player	22
		2.3.1 A representation of the Bank & Platform game in extensive form	23
	2.4	Equilibrium	25
		2.4.1 Elimination of weakly dominated strategies	26
		2.4.2 New Reduced Game	28
		2.4.3 Equilibrium Analysis	36
	2.5	Conclusion	37
2	Cha		20
3	Cna	pter 2 Information Snaring when All Players Have Partial Information	38
	3.1		38
	3.2	Literature Review	40
		3.2.1 Information asymmetry equilibrium	42
		3.2.2 Switching costs in banking	42
		3.2.3 Winner's curse in banking	43
	3.3	Model: An asymmetric information game with different subsets of information	44
		3.3.1 Notation	46
		3.3.2 Initial remarks	48
	Unn	umbered Section	51
	3.4	Equilibrium	51
	Unn	umbered Section	54
		3.4.1 Example	54
	Unn	umbered Section	54
		3.4.2 Equilibrium Analysis	55
	Unn	umbered Section	59

	3.5	The value of information	59
	Unn	umbered Section	63
	3.6	Conclusion	63
4	Cha	pter 3 Sharing Information with Partial-Informed Players and the Business Cycle	65
	4.1	Introduction	65
	4.2	Literature Review	66
		4.2.1 Related Literature	66
	4.3	Model: An asymmetric information game with a changing business-cycle	68
		4.3.1 Notation	69
		4.3.2 Initial remarks	71
		4.3.3 How current expected cost and marginal expected cost chances as the busi-	
		ness cycles does	76
	Unn	umbered Section	78
	4.4	Equilibrium	79
		4.4.1 Profit maximization	79
	Unn	umbered Section	80
		4.4.2 Equilibrium Analysis	81
	Unn	umbered Section	88
	4.5	The value of information	88
Unnumbered Section			
	4.6	Conclusion	90
_			
5	Арр	endix	98
	5.1	Data Panel Estimation of the Nominal Interest Rate in Consumption Loans	98
	5.2	Proof of the integral in the cost function (26)	101
	5.3	Proof of Lemma 4.4	102
	5.4	Proof of Lemma 4.5	108

## **1** Introduction

Information is essential to compete, but it is increasingly relevant in contemporaneous markets. The most evident and current examples are digital markets; although information is fundamental to any other market. Big tech firms invest in knowledge and technology, in order to get more data and to turn it into information. This trend goes beyond social media and web sites, the internet of things is gathering data from users too, as well as payments systems. One example is the case of Walmart and Amazon. From Ezrachi and Stucke (2016): "The sentiment is that Walmart's distributional efficiencies from its brick-and-mortar store model do not translate to the data-driven analytics and dynamic pricing of the online world". About the latter, the authors state that Amazon's pricing algorithms get personal and market data constantly to price millions of products, while to do this by humans would take a lot of time. This tendency is widely and deeply used by digital providers, such that regulators now ask for personal data protection. The European Union Charter of Fundamental Rights considers the right to the protection of personal data.<sup>1</sup> Meanwhile, in the United States of America, there is no a federal law for privacy data protection; however, thirteen states had regulated on this issue.<sup>2</sup> In Mexico, there is a specific law for personal data protection since 2010.<sup>3</sup>

The financial markets are some of these markets where information is critical. This is true for traditional financial services suppliers and digital ones, like fintech firms. Information has been relevant to compete in these markets; but now, it is almost imperative as digital players are threatening traditional banks. Regulation like Open Banking and other measures by regulators confirm that information is relevant for firms to compete effectively in financial markets as well as for welfare to both sides of the markets. Open Banking is mainly the term for financial services users to allow access to their data to other financial suppliers. Regulators in the European Union and the United States have already a legal framework about it. In Mexico, the Open Banking regulation is part of the Fintech Law.<sup>4</sup>

On the other hand, the Mexican Antitrust Agency has concluded that lack of access to information

<sup>&</sup>lt;sup>1</sup>The European Parliament (2000)

<sup>&</sup>lt;sup>2</sup>Pittman (2023)

<sup>&</sup>lt;sup>3</sup>Cámara de Diputados (2010)

<sup>&</sup>lt;sup>4</sup>Cámara de Diputados (2018)

is a competition concern in financial markets, mainly in credit services. In a study on competition conditions in the financial services published in 2014, the Mexican Antitrust Agency identifies credit bureaus as one of the transversal elements that influences competition in the financial markets.<sup>5</sup> Furthermore, this antitrust office performed a research on the clearing house for credit and debit cards payment market, where it pointed out that information is an advantage for some players.<sup>6</sup>

Digital markets are increasingly used as the common space for firms and consumers to interact. These markets collect loads of data that could become high-value information to compete and to extract consumer surplus. Hence, data is becoming a key competitive advantage. This could lead to a "winner takes it all" condition, or at least, to anticompetitive practices, a concern for regulators, competitors and consumers. Regulators ask competitors to share data to improve market conditions, as it is the case for credit reports in financial markets. Likewise, would firms have an incentive to trade willingly data in order to increase their profits while they still compete? If it is the case, we ask under which conditions are competitors willing to exchange data?

We can see information is an advantage. Firms invest to get more information to be more competitive, as well as regulators consider information relevant for the efficiency of the markets. Hence, regulation looks to pave the information way for all players. But, in the presence of information asymmetry, better-informed players may refuse to share information as they know it is a competitive advantage. Nevertheless, is there any chance to share information voluntarily? This question is the main theme in this thesis. From here and on, we refer to information sharing when players exchange its own relevant information about current customers to competitors. Then we ask if a financial player has doubtless informational advantage, does it have incentive to exchange willingly its information to a competitor? This is the main question in Chapter 1. Then, we look to answer what about if it is no clear which player is better informed? In Chapter 2, we analyze information sharing when neither player has full information about the market and it is not straightforward which player has an informational advantage. Finally, if there is a chance for information sharing, does incentives change with business cycle? This is the last question we tackle in Chapter 3.

<sup>&</sup>lt;sup>5</sup>Comisión Federal de Competencia Económica (2014)

<sup>&</sup>lt;sup>6</sup>Comisión Federal de Competencia Económica (2023)

Our literature review starts at the discussion on intermediation, specifically the financial intermediation and how information is key to perform this role. Economic literature shows the conditions where there is opportunity for intermediation either the intermediary acts as a dealer or as platform operator.<sup>7</sup> Then, we follow with information literature given it is key for intermediaries and, of course, financial intermediaries, specially in credit markets. After this, we review the literature about information sharing among agents, as before, we focus on exchange of information by financial intermediaries. Finally, we survey the economic literature on how these three themes are changing with the irruption of digital markets, which are changing the financial services.

#### **Literature Review**

#### **Intermediation Literature**

We begin with the literature about intermediation in general as it shows the relevance of information. We survey some models for intermediation following Belleflamme and Peitz (2015). Then we focus on financial intermediation based on Freixas and Rochet (2008).

In a wide spectrum of intermediation analysis, Spulber (1996) enumerates the economic roles of the intermediaries. First, the author points out how much intermediation adds to the US economy. The importance of intermediation for the US economy is based on the GDP generated by sectors which are considered as intermediary markets, like retail and wholesale trade as well as finance and insurance. For the remaining of sectors, the author assumes that the labor force which is not working on the main activity is employed in intermediation activities. Secondly, there is a contribution of intermediaries in the price setting and clearing markets process. An intermediary with market power on both sides of the market sets bid and ask prices given its best estimate of the supply and demand (information about the difference between both sides of the market) and makes profits from this margin. An additional role is when intermediaries provide liquidity and immediacy by holding inventories and cash, intermediaries avoid the problem of buyer and seller trading between them at the same time. Even though the paper does not mention it, the latter characteristics are more relevant when the goods are rivals in the sense that the consumption of the good by one agent prevents the consumption by another one.

<sup>&</sup>lt;sup>7</sup>For a comprehensive review of the different roles of an intermediary see Belleflamme and Peitz (2015)

Furthermore, intermediaries coordinate buyers and sellers. About matching and searching; intermediation increases the chances of a successful match between members of each side of the market. In addition, the intermediary reduces the search cost (an information issue). In a market with different types of consumers and suppliers and random matching, the risk of lack of trading increases. It might happens when a low-valuation buyer meets a high-cost supplier. By setting a bid-ask spread, the intermediary makes each side of the market to self-select and only consumers and suppliers with values above and below the bid and ask prices respectively will be part of the market. Intermediation reduces search costs again with a bid-ask spread when buyers have different willingness-to-pay and sellers have different costs; each side of the market is heterogeneous in their types.

Finally, the author shows intermediation improves guaranteeing and monitoring performance. In particular, intermediation by depository institutions is performed when setting rates for loans and deposits, screening risks and monitoring repayments. When there is information asymmetry between the sides of the market, trade could be absent. Intermediation could provide the information to the market. Hence, an intermediary could capture gains from trade that would not exist otherwise. As the intermediary invests in monitoring, it gets information and reports it to each side of the market to make trade happens. Risk diversification and the high cost of monitoring borrowers for lenders makes financial intermediation more efficient as it is cheaper for the intermediary than for individual agents. This is particularly important in loan markets.

In most of these cases, there is opportunity for intermediation to improve thanks to information issues. For example, because different types of agents on each side of the market cannot identify preferred counterparts to match, or because of the higher costs of searching as well as the information asymmetry.

According to Berger, Molyneus, and Wilson (2019), banks have many roles. Here we mention some of them in non particular order. First, they act as intermediaries in the payments systems, between those who want to pay and the merchants which accept the payments. Secondly, depositors use banks as refuge for their funds or to save and earn some interest. Third, banks are mainly a source of credit, acting as an intermediary between depositors and borrowers. From all these roles, we focus on the banks' loan granting side of this latter intermediation function. We argue

that information issues are key to understand credit markets, this is the reason why we focus on it.

When we deal with financial intermediation, information topics become even more relevant. Economic literature comprehends many information edges in financial intermediation. D. W. Diamond (1984) shows that financial intermediation surges as this kind of intermediaries have a 'net cost advantage' relative to direct lenders. This is because diversification permits intermediaries to minimize the cost of monitoring the needed information to solve the agent-principal incentive problem between lenders and borrowers. Diversification of the projects funded is key to this outcome when agents are risk neutral. Besides, the author shows that the optimal contract is a debt one as it has the lowest delegation costs. A consequence of this model is that the intermediary's assets will be illiquid as the intermediary and the borrowers are the only ones which could know the performance of the projects.

Following D. W. Diamond (1984), Hellwig (2000) analyzes risk-aversion instead of risk neutrality in financial intermediation. One outcome is that in the presence of risk-aversion, optimal intermediation must shift risk from risk averse entrepreneurs and pass them to the risk-neutral intermediary or to final investors. The author looks to answer two questions. What is the impact of risk aversion on the viability of financial intermediation? And, what is the impact of risk aversion on the allocation of risk in a financial system based on intermediation? The paper answers the first query with its main conclusion: Financial intermediation is still valid in the presence of risk-averse intermediation. For the second question, the conclusion is that the intermediaty must take almost all risks by itself or take them to final investors.

Hölmstrom and Tirole (1997) argue that all forms of capital tightening, given capital constraints, have stronger effects on poorly capitalized firms, while large firms have alternatives like renegotiating or going to capital markets. The main purpose of the paper is to model some stylized facts and shed light on the effects of different kinds of capital constraints. Capital constraints make firms with low net worth go to financial intermediaries, who can ask for less collateral but increase monitoring. In this sense, monitoring becomes a substitute for collateral. Although intermediaries cannot monitor all firms. Authors analyze the effects of capital constraints on investment, interest rates, and other financing sources. Capital constraints and collateral are issues that make different this paper from others. All this research imply an asymmetry on information that generates an opportunity for financial intermediation to solve agency problems in different setups by monitoring borrowers performance.

#### **Information Literature**

Kamenica (2017) is a very brief review of the literature on economics of information. The author starts explaining two kinds of models about Information Acquisition. In search models, individuals incur costs to acquire information for private use, examples are Stigler (1961) and McCall (1970). He also mentioned P. A. Diamond (1971) to point out a critique about how distributions of options, a basic assumption of the first papers, arise. Following Sims (2003), Kamenica explains that rational inattention models endogenize agents' information. But more important, these models state that the cost of acquiring information is a function of Shannon entropy, which is the average information level of a random variable. Another way to model cost function in rational inattention framework is through sequential sampling models as Hébert and Woodford (2017) as well as Morris and Strack (2019) do. Instead of modeling individual information acquisition, Sanford J. Grossman and Joseph E. Stiglitz (1980) conclude that the value of information is lower if a large part of agents are informed. This is similar to our conclusions in chapter 2 and 3, although their research is focused on how price reflect information. In addition, herd behavior could be explained by information externalities as in Banerjee (1992) or Bikhchandani, Hirshleifer, and Welch (1992). In the edge of Asymmetric Information, indeed Harsany and others work on game theory applied to this issue, which is the basic setting of analysis for most of strategic models applied in this issue. This is precisely the framework we use in the three chapters. On going forward, authors like Akerlof (1970) and Spence (1973) develop this theme in the market for lemons and job signaling; while Joseph E Stiglitz (1975), Rothschild and J. Stiglitz (1976), and Joseph E. Stiglitz and Weiss (1981) applied in screening issues. The latter is particularly relevant for the loan markets. So far, these research has not deal with information sharing.

A crucial question is if agents have incentives to share their private information. Crawford, Sobel, and Sobeli (1982) model an informed agent sending a message costlessly, a cheap talk model, where the receiver then takes an action that affects welfare for both players. In equilibrium, the more informative signalling is the more similar preferences of both players are. Sanford J Grossman (1981) and P. R. Milgrom (1981) develop verifiable message models where the agent chooses how much information to share. In addition, technological issues are the concern for information transmission as in Cover and J. A. Thomas (2006) and Blume, Board, and Kawamura (2007). These types of communication forge a particularly relevant structure for the discussion about information sharing among competitors. Finally, the paper points out information design as one of recent developments in economic literature, which looks for the optimal information environment. Information design is where Bayes correlated equilibria and Bayesian persuasion converges, like in Bergemann and Morris (2013) in the first case and Kamenica and Gentzkow (2011) for the second one.

#### **Information Sharing Literature**

Here we discuss information sharing. Most of the economic literature on information sharing is about signaling. In these models there is usually a player than sends a signal and a receiver of it, who then takes an action that affects both parts. There is also a source of economic literature on information sharing where agents partake in negotiations to cooperate for a common benefit. We start this section with the first case to end up with the sharing for cooperation. To our knowledge, this literature focuses on an agent sending a signal and cooperation, while our models do not consider a signal but data sharing.

In Pareto's way, sharing information could help for better agreements. Nevertheless when the agreement involves opponents it is not clear how much information to reveal as it is also strategic. Crawford, Sobel, and Sobeli (1982) looks for an answer to this question as well as how the similarity of agents' interests influence this decision. In their model, a better-informed sender sends a noisy signal to a receiver, whose decision affects welfare to both agents. In equilibrium, the sender includes enough information to make the receiver to respond to it. After characterizing the set of Bayesian equilibria, the authors focus in the equilibrium which is Pareto-superior. In this equilibrium, they show that the more similar the agents' preferences are, the more informative the equilibrium signal is. Hence, they conclude that direct communication between agents is more likely, the more their goals are alongside; which is not case when players compete against each other.

D. W. Diamond (1985), in opposite sense to previous economic literature, shows that voluntary release of information by firms makes all traders better off. This is explained because it is costly for traders to get private information. Thus, if we assume that it is cheaper for firms to produce information, an information release policy exists such that all traders increased expected utility. Although the agents share information to the shareholders not to competitors, we consider it because it is one of the first papers to take information sharing in financial markets. Besides, the author uses a general equilibrium model, while we use an asymmetric information game.

Another reason to share information is to persuade other agents to take a different action from the one she would decide without a signal. In this context, Kamenica and Gentzkow (2011) try to answer if a sender can persuade a receiver to change her mind and what is the optimal way to do it. They show that the sender benefits as two conditions are fulfilled. First, the default option of the receiver is not the preferred action by the sender; and second, the receiver action does not change around the prior. For example, with a prior for a judge that the probability of someone to be guilty is 30%, after receiving the signal, she verdicts 60%. In addition, the sender benefits depending on the concavity or convexity of her payoff as a function of the receiver's beliefs. Like in Crawford, Sobel, and Sobeli (1982), the sender's gain depends on the alignment of the preferences of both agents.

#### **Digital Intermediation Literature**

Nowadays information is more relevant as new digital players can gather more data and more efficiently to transform it into information, changing the financial markets. Stulz (2019) points out some advantages for fintech, like those related to information, which come mainly from technology. For example, big data can be implemented with relative low levels of investment. Another edge is that digital technology has built-in economies of scale, making the marginal cost very low. This is not restricted to fintech or big tech firms, but their information born digital. A report to the President of the United State on big data and privacy states that born digital information is specifically made for computer processing; while information born analog is bounded to the minimum for immediate purposes.<sup>8</sup> Banks have more information of the latter kind, while digital players of

<sup>&</sup>lt;sup>8</sup>Executive Office of the President (2014)

the first one, which drives to decreasing marginal costs faster for the latter. Doerr et al. (2023) analyze big techs in financial markets, whose potential is based on non-traditional data and machine learning. The collected data by these agents and their new credit scoring procedures allows them to supply the unattended share of the market and improve financial inclusion. There are increasing returns to scale for the big techs as the average cost decreases with the number of users; at the same time, users value more the big techs' services the larger the network is. There are more advantages and disadvantages for digital players in credit markets; however, information is becoming more relevant in financial markets.

#### **Probability of Default Literature**

A main concept in each of the three chapters of this thesis is the probability of default. Henceforth, we devote a specific literature review on this theme from the beginning. We can see references to this type of risk in academic research as well as public policies.

The probability of defaulting a credit is not only a contingent cost. Regulated lenders have to establish reserves any time they grant a credit, although just a fraction of the amount.<sup>9</sup> Besides, this is usually a risk management policy in financial institutions. This materializes the cost, even if the credit is payed back, as provisions of the the loan cannot bring earnings meanwhile. Of course, if the credit is not payed back, the cost increases *ex post*. Regardless if the lender reserve credits using a constant fraction of the amount granted or an increasing one, a loan to a consumer with a higher probability of default is more expensive than to one with a lower probability of default as the credit supplier has to provision a larger part of that debt.

Following Freixas and Rochet (2008), banks deal with microeconomic as well as macroeconomic risks. In the first case, we can find default, liquidity, interest rate and market risks. These authors explain that default risk materializes if a debt is not paid back by the borrower. When banks try to diversify the default risk, the probability of default is the main concern. One early reference is Merton (1974) that shows that the value of corporate debt depends on the probability of default, along with return on riskless debt and other characteristics like maturity and coupon. In this risk structure theory of the interest rate, the risk is a function of the changes of the probability of default.

<sup>&</sup>lt;sup>9</sup>In Mexico, the whatchdog for the financial system considers the probability of default to compute the reserves for each loan. See Comisión Nacional Bancaria y de Valores (2023)

Hence, the interest rate depends on the risk, which depends on the probability of default.

The use of the probability of default in the economic and finance literature is so extensive that research focused on other issues take it as given. Particularly, in this thesis, cited references like Broecker (1990), Raghuram G. Rajan (1992), Petersen and Raghuram G Rajan (1995), Jiang and Li (2022), Thadden (2004) as well as Frankel and Jin (2015) consider this concept in their framework when they established that there is a probability that a borrower does not pay back the loan.

On the other hand, international regulation has been considering the probability of default as an input in the public policy. Based on the Bank of International Settlements, the Basel Committee "is the primary global standard setter for the prudential regulation of banks."<sup>10</sup> In Basel II, the determination of the internal rating-based (IRB) approach is a function of the probability of default, while the advanced IRB uses estimation of loss according to default. This framework issued in 2004 considers that interest rates may cover part of unexpected losses, but not all of them; that is why minimum capital levels are required.<sup>11</sup> Banks can estimate expected losses according to probability of default, exposure at default and loss given default. Afterwards, Basel III toughens the IRB approach, but it essentially still depends on the probability of default.<sup>12</sup>

In Mexico, regulation fulfills the aforementioned international framework.<sup>13</sup> Besides, the central bank, Banco de Mexico, considers the probability of default in the analysis of the financial system and acknowledges that default risk is a financial cost. In the Financial Stability Report of June 2020, Banco de Mexico states that financial costs faced by firms comprises different risk primes, for example the probability of default. Moreover, the central bank takes into account the probability of default in its analysis about stress tests or as an explanation in changes of non-performance balances in different issues of the Financial Stability Report.<sup>14</sup>

The probability of default is only one of the different types of risk in the credit markets. The former impacts on the required capital level for any regulated credit institution as we already showed above. Furthermore, as a risk, the probability of default reduces profits, that is, it is a cost. In the

<sup>&</sup>lt;sup>10</sup>https://www.bis.org/bcbs/

<sup>&</sup>lt;sup>11</sup>Basel Committee on Banking Supervision (2004)

<sup>&</sup>lt;sup>12</sup>Basel Committee on Banking Supervision (2017)

<sup>&</sup>lt;sup>13</sup>See Comisión Nacional Bancaria y de Valores (2023)

<sup>&</sup>lt;sup>14</sup>Banco de México (2020)

following chart, we show how the financial margin diminishes when the risks are considered in the Mexican banking system according to figures from Comision Nacional Bancaria y de Valores (CNBV), the Mexican banking watchdog. For example, the gross financial margin reached 33 billion US dollars in 2022, but the net financial margin was only 27 billion US dollars once we discount credit risk. Banco de Mexico explains that the increase in the financial margin in March 2023 was enough to compensate the greater provisions for credit risk, which is function of the probability of default.<sup>15</sup>



Figure 1: Risk-Adjusted Financial Margins in Mexico

Last of all, in a different exercise, we estimate the relation between the probability of default and the nominal interest rates in the consumption loans offered by banks in Mexico. Using monthly data for each bank in the Mexican market since April 2011 to April 2023 from CNBV, we estimate a simple panel model where the interest rates on consumption loans granted by banks depends on the probability of default and we use the reference rate as a control. Only motor vehicle, personal, durable goods and payroll loans are considered in this exercise. <sup>16</sup>

In each case, the probability of default has a positive coefficient as expected. A striking point is

<sup>&</sup>lt;sup>15</sup>See https://www.banxico.org.mx/publicaciones-y-prensa/resumenes-visuales/ reporte-de-estabilidad-financiera-html/reporte-estabilidad-financi00002.html

<sup>&</sup>lt;sup>16</sup>see Annex Z for data and estimation details

Туре	Model	constant	reference rate	probability of default	Prob > chi2
	fixed effects	19.54	0.0103	0.0942	0.0000
		(0.00)	(0.55)	(0.00)	
All types of	random effects	19.25	0.0103	0.0942	0.0000
loans		(0.00)	(0.55)	(0.00)	
	maximum-likelihood	19.23	0.0103	0.0942	0.0000
		(0.00)	(0.55)	(0.00)	
	fixed effects	13.21	0.2240	0.1337	0.0000
		(0.00)	(0.10)	(0.00)	
durable	random effects	17.21	0.2117	0.1322	0.0000
goods	random encous	(0.00)	(0.12)	(0.00)	0.0000
	maximum-likelihood	17.21	0.2110	0.1321	0.0000
	maximum-likelinood	(0.00)	(0.12)	(0.00)	0.0000
	fixed effects	13.02	0.0062	0.0514	0.0000
		(0.00)	(0.36)	(0.00)	0.0000
motor	random effects	13.38	0.0065	0.0514	0.0000
vehicle		(0.00)	(0.34)	(0.00)	
	maximum-likelihood	13.38	0.0064	0.0514	0.0000
		(0.00)	(0.34)	(0.00)	0.0000
	fixed effects	26.53	-0.1844	0.1575	0.0000
payroll		(0.00)	(0.00)	(0.00)	0.0000
	random effects	26.74	-0.1840	0.1575	0.0000
		(0.00)	(0.00)	(0.00)	0.0000
	maximum-likelihood	24.75	-0.1840	0.1575	0.0000
		(0.00)	(0.00)	(0.00)	
personal	fixed effects	22.58	-0.1458	0.2108	0.0000
		(0.00)	(0.00)	(0.00)	
	random offects	22.89	-0.1458	0.2108	0.0000
	random enects	(0.00)	(0.00)	(0.00)	
	maximum-likelihood	22.89	-0.1458	0.2107	0.0000
		(0.00)	(0.00)	(0.00)	

Figure 2: Panel regression consumption loans interest rates

that the probability of default has a greater effect on interest rate as the loan seems riskier. In motor vehicle and durable goods we can see the lower effect of the probability of default, these types of loans usually considers the acquired good as a collateral. Meanwhile, payroll and personal loan show greater coefficients of the probability of default. In all cases, it is clear that this risk affects interest rates, that is, it is a cost that affects prices. These outcomes harmonize with the use of collateral as screening device which helps to screen risk types better, lowering interest rate; , like in Bester (1985).

# 2 Chapter 1 Information Sharing when Only One Player is Fully-Informed

#### 2.1 Introduction

Antitrust policy prevents cooperation among competitors, in particular collusion. A collusion agreement requires a way to verify the fulfilment of the coordination conditions. The mechanisms to verify an agreement imply information sharing, for example exchanging information about prices, market shares and so on. This is why antitrust agencies do not usually approve information sharing among competitors. Nevertheless, there is evidence of cooperation instructed by competition agencies. One case is the case of United States vs Terminal Railroad Association of St. Louis, which concluded that every player in the train market must have access to the terminal in order to effectively compete.<sup>17</sup> This decision is considered the beginning of the essential input doctrine.

The decision about access to Terminal Railroad in St. Louis was later revisited as it was estimated that the outcome would be sharing the market power instead of increasing competition.<sup>18</sup> Furthermore, another problem is to set the conditions to access to a common essential input. Despite this case, it is now commonly accepted that access to essential facilities increases the ability to compete and foreclosure of them surges as an entry barrier. Nowadays guaranteeing access to essential facilities became a cornerstone of the antitrust policy. But, is there a chance for sharing information voluntarily among competitors without any instruction by the regulator?

A case where information sharing was particularly relevant is the acquisition of Whatsapp by Facebook, since one of the main asset involved in the transaction was probably information. This was considered a data-driven business in the Report of the Digital Competition Expert Panel.<sup>19</sup> Gautier and Lamesch (2021) pointed out that advertising is the main source of income for Facebook and for Google such that specific information about users is used for a better addressed advertising.

<sup>&</sup>lt;sup>17</sup>United States v. Terminal Railroad Association of St. Louis (1912)

<sup>&</sup>lt;sup>18</sup>Reiffen and Kleit (1989)

<sup>&</sup>lt;sup>19</sup>See Furman et al. (2019)

Sharing information implies that at least one player does not have full information. In order to analyze incentives to share, we use models of asymmetric information in different circumstances. We start with a two-players model in a credit market. In this game, one player is fully-informed about customers' risk, and another one who is partially informed. In this model of informational asymmetry between two lenders, we show that there are not incentives to share information, when one player has full information. We use the probability of default as a measure of credit risk, which defines the type of a customer in credit markets. In a one-stage simultaneous game, the informed player's profit is positive, while the expected profit for the other player is zero. If they exchange information, competition becomes à la Bertrand and both get no profit. Hence, there is no incentives for the informed player to share its information about the types of customers. Furthermore, if competitors exchange information and compete à la Bertrand, there is a loss in the supply side of this market; although credit rationing disappear. Therefore, the only incentive to share information is to collude for increasing profits.

This work looks for a chance to exchange relevant information among players in the credit markets when there is asymmetric information about the cost of granting a loan.

#### 2.2 Literature Review

An agent with more information about the market or consumers behavior has a competitive advantage. Information technology, which is focused to get information, can be used to take advantage and to apply different degrees of price discrimination, it also has network effects.<sup>20</sup> More knowledge about consumers helps lenders to screen the borrowers' types, which is useful when demand is heterogeneous. For example, information could help to know the willingness to pay or how likely it is that the consumer would demand a good or a service. In the loan market, information is commonly used to know the probability of default. Most of the time credit scores are built with ex-post data. Furthermore, regulation asks for ways to measure probability of default in financial markets.

The closest works to our model are Ferrerira and Pinto (2008) and Broecker (1990). In the second case, banks compete for firms which have a project with different probability of success, or

<sup>&</sup>lt;sup>20</sup>Varian and Farrel (2007)

probabilities of default. Each bank has its own imperfect and independent credit-worthiness test to screen the type of the firms. In a simultaneous one-stage game, banks announce a specific interest rate for each type of firm according to its own test; then firms take the best rate. If all banks set the same rate, there is an incentive for a bank to lower its rate. Firms takes the cheaper credit, hence subsequent banks will attend only rejected firms by the first bank, which are riskier on average. We present a taxonomy of this work and ours given the number of similarities.

The difference about the borrowers is an abstraction *per se*, that is between modelling the success of firms or just taking the probability that the borrower pays back the loan. When we consider consumption loans we can abstract from the borrower behavior, there is only a probability that she paybacks the loan or not. In addition, the relevance of quantity of data is quite higher in consumption loans than in the case of firms as the former is a larger market measured by the number of clients.

While in Broecker (1990) both players have incomplete information about the risk types; in our model on payer has full information while the opponent does not. This is a key assumption for information sharing analysis as there must be at least one player with more information;<sup>21</sup> while in Broecker's model there is no informational advantage. One consequence of this is that there is a possibility for pure strategy equilibrium depending on the value of the parameters that define the proportion of low-risk consumers, and the marginal costs; although the mixed-strategy equilibrium is more probable. This is one difference of our model in contrast to Broecker's.

<sup>&</sup>lt;sup>21</sup>Notice we refer to more information, not necessarily better information

Table 1. Model Taxonomy			
Concept	Broecker (1990)	Ours	
Supply side	Two banks	Two banks	
Demand side	Continuum of firms	Two consumers	
Information asymmetry	Both player do not know	One player knows the type	
	the type of borrowers	of customers and the other	
		one does not	
Borrower types	Probability of success	Probability of default	
	(1 - probability of default)		
Distribution of types	Common knowledge	Common knowledge	
Rates (prices)	Two rates, one for each	Informed player two rates,	
	type	uninformed player one	
Game	Simultaneous one-stage	Simultaneous one-stage	
Equilibrium	Only mixed strategies	Pure and mixed strategies	
Self-selection devices	None	None	
Screen sharing	No	No	
Winner's curse	Possible by assumption	No	
Generalization	Outcome for <i>n</i> players	No	
Information sharing analy-	No	Yes	
sis			
Welfare analysis	No	Yes	
Pooling equilibria	No way	Depending on the parame-	
		ters	

Ferrerira and Pinto (2008) model the price strategy of two competing firms with differentiated products, whose technology could have a high or low cost according to a known distribution. They show that this Bayesian game has only one equilibrium, in which prices increases with their costs and expected profits with the variance of their respective costs. Besides the Bayesian game, this work is like ours in that they model the price strategy of two competing firms. Nevertheless, in

their model no firm can be sure about the cost, that is, there is no information advantage; on the contrary, in our setting there is one player fully informed. Furthermore, they conclude there is only one equilibrium; while we have at least three equilibria depending on the proportion of low and high risk (low and high costs), and the difference between marginal cost for each player. In addition, in our model loans are perfect substitutes, such that players have incentives to offer a lower opponent's rate, which leads to the uninformed player to expected zero profits and the high-risk borrower to get a rate equal to marginal cost. Finally, this model does not deal with information differences, and does not involve information sharing analysis.

We already have devoted a specific literature review for the probability of default that is used along this thesis. Here, we cite some reference that are focused on bank competition with different information asymmetries but that they use the function form of probability of default that we employ in this setting. Once more, Broecker (1990) analyzes the potential to repay credit in a competitive credit market, where banks compete in prices, interest rates. In the one-stage game, there are two types of firms, each with different probability of success for their projects:  $p_i$  for i = a, b; and  $0 \le p_i \le p_j \le 1$ .

The more information a bank has about its own customers, the deeper the relationship between the lender and the borrowers. As a consequence, competition focuses on new firms driving capital to lower quality firms. In this model, Sharpe (1990) sets a continuum of risk-neutral entrepreneurs whose distribution of returns is binary: a firm that invest I units of capital earns g(I)I with probability p and 0 otherwise. Raghuram G. Rajan (1992) began by acknowledging that benefits from bank financing are well understood, while costs are not. Informed banks exert bargain power, so firms diversify their financial sources to less informed options. The latter is also a strategy to limit the bank's bargaining power. In this model, we can see the same kind of function: a good state (G) with probability q and a bad one with complementary probability. In a similar way, Petersen and Raghuram G Rajan (1995) show the importance of lending relationships in the competition of credit markets. In a risk-neutral world there are good and bad firms. They choose a safe or a risky project in the first stage. The good firm can re invest in a safe project if it chose a safe one in the first stage. On the other hand, if it chose a risky project, this could pay out  $R_1$  with probability p or it may fail with complementary probability.

In order to point out an error by Sharpe (1990), Thadden (2004) showed that in Sharpe's model the game has only one mixed strategy equilibrium. The outcome is a partial informational lockin. To proof this equilibrium, Thadden (2004)used the same framework as Sharpe (1990), where the default function is alike. As in previous cited research, Frankel and Jin (2015) stress the informational advantage to compete. However, securitization permits remote banks to compete, which is consistent with empirical evidence. In these model we can find an alike default function to where a project returns  $\rho > 1$  if it succeeds and zero otherwise, alike it pays  $\rho$  with probability p and nothing with 1 - p. Beyond the probability of default, all this research highlight the informational advantage of a bank with a longer relationship with its clients. On an empirical work, Jiang and Li (2022) finds that the better knowledge a bank has of an industry the easier to supply other firms in the same sector; that is, there is an informational advantage.

Our model is a contribution because we show that there is more than one equilibrium in Bayesian games when there is one full-informed player about the types of borrowers. More important, we show that there are no incentives to share information, as the informed player will lose its informational advantage. As a result, the low-risk borrower pays a rate higher than marginal cost, while the high-risk pays marginal cost. This does not necessarily mean that the former pays a higher rates. Furthermore, if player exchange information, competition becomes à la Bertrand and there is a welfare loss for the supply side; although borrowers get cheaper loans. Finally, there is a possibility of pooling equilibrium, which has been discarded in previous research like Broecker (1990).

# 2.3 Model: An asymmetric information game with only one fully-informed player

We propose a model that aims to identify if there is a chance for information sharing in the presence of asymmetric information. In this game there are two loan providers, one supplies the market through a digital platform and the other one is a traditional bank. The digital lender, the most informed player, can systemically get more data from its customers' behavior thanks to their algorithms. The bank could also get a lot of data from its customers and its algorithms, but not as large as the digital lender. We suppose the more stock of data, the better the algorithms to get more and better information about their behavior for a better screening (the big data argument). These lenders compete for the two different kinds of consumers, who have different probability to default a loan for buying a consumption bundle. The higher the probability of default the higher the cost for the credit provider because they have to back loans with provisions as risk management and financial regulation require. Hence we have a low-risk consumer (*L*) and a high-risk consumer (*H*). This encourages loan providers to get more data to transform it into information for a better screening when they do not have full information about the types.

The consumers ask for a credit to the lender which offers the lowest interest rate. If rates are the same the demand splits in two equal shares for each lender. Both loan providers know there are two different types of consumers. Nevertheless, only the digital platform supplier can screen each consumer type because it has more data about the consumers' behavior. The other one only knows that there are two types.

Given the information asymmetry, players face the following Bayesian Game. There are two players,  $j = \{A, B\}$ , let's say a financial digital platform and a brick-and-mortar bank. Two types of consumers who look for a loan,  $i = \{L, H\}$ , one with a low-probability of default, L, and the other with a high-probability of default, H. Players offer loans at rates  $r_i$  but face different costs:  $c_L$  for the low-risk and  $c_H$  for the high-risk consumers. Both players know  $0 < c_L < c_H < 1$ , but only player A can screen the consumer type. Consumers' demand function:  $L_i(r) = 1 - r_i$ , where  $r_i$  is the nominal interest rate  $\forall i \in \{L, H\}$ . Nature states are 1:  $Prob(c_i = c_L) = \theta$  and 2:  $Prob(c_i = c_H) = 1 - \theta$ , which is common knowledge for both players.

#### 2.3.1 A representation of the Bank & Platform game in extensive form

Nature chooses among states L (low-risk) and H (high-risk). Player A chooses its strategy for low-risk and high-risk consumers. At the same time, player B opts for its optimal interest rate.

We present a graphical representations for these profit functions with parameters  $\theta = \frac{1}{2}$ ,  $c_L = \frac{1}{3}$  and  $c_H = \frac{2}{3}$  (Figure 4 and Figure 5).

Let's define profit functions for each player:



Figure 3: Extensive form of the Bank & Platform game

• Player *A*'s profit function:

$$\pi_i^A(r_i^A, r^B) = s_i^A(1 - r_i^A)(r_i^A - c_i) \forall i \in \{L, H\}$$
(1)

where

$$s_i^A(r_i^A, r^B) = \begin{cases} 1 & r_i^A < r^B \\ \frac{1}{2} & r_i^A = r^B \quad \forall i \in \{L, H\} \\ 0 & r_i^A > r^B \end{cases}$$

- Player A's profit function for each state with parameters  $\theta = \frac{1}{2}$ ,  $c_L = \frac{1}{3}$  and  $c_H = \frac{2}{3}$
- Player *B*'s profit function:

$$\pi^{B}(r^{B}, r_{i}^{A}) = (1 - r^{B}) \sum_{i \in \{L, H\}} \theta_{i} s_{i}^{B}(r^{B} - c_{i}) \forall i \in \{L, H\}$$
(2)



Figure 4: Example of player *A*'s profit functions

where

$$s_{i}^{B}(r^{B}, r_{i}^{A}) = \begin{cases} 1 & r^{B} < r_{i}^{A} \\ \frac{1}{2} & r^{B} = r_{i}^{A} \ \forall i \in \{L, H\} \\ 0 & r^{B} > r_{i}^{A} \end{cases}$$

• Player *B*'s profit function with parameters  $\theta = \frac{1}{2}$ ,  $c_L = \frac{1}{3}$  and  $c_H = \frac{2}{3}$ 

### 2.4 Equilibrium

A Bayesian Nash Equilibrium (BNE) of Bank & Platform game consists of a triad  $(r_L^{*A}, r_H^{*A}, r^{*B})$  such that:

- For any  $r^B \in [0,1]$  :  $\pi^A(r_L^{A*}, r_H^{A*}, r^B) \ge \pi^A(r_L^A, r_H^A, r^B)$ ; and
- For any pair  $r_L^A \in [0,1]$  and  $r_H^A \in [0:1]$ :  $\pi^B(r^{B*}, r_L^A, r_H^A) \ge \pi^B(r^B, r_L^A, r_H^A)$ .



Figure 5: Example of player B's profit function

#### 2.4.1 Elimination of weakly dominated strategies

Let's begin by eliminating weakly dominated strategies, where discarded strategies lead to zero profit. In a one-stage game, a rate lower than marginal cost brings losses to the lender.<sup>22</sup> This is straightforward for player A, taking (1) and estimating marginal revenue. Take the derivative of  $\pi_i^A$  respect to  $L_i$ , the amount of credit demanded

$$\frac{\partial \pi_i^A}{\partial L_i} = s_i^A (r_i^A - c_i) \equiv M R_i^A \forall i \in \{L, H\}$$

and

$$MR_{i}^{A}(r_{i}^{A}, c_{i}) = \begin{cases} \leq 0 & 0 < r_{i}^{A} \leq c_{i} \\ \geq 0 & c_{i} < r_{i}^{A} < 1 \end{cases} \quad \forall i \in \{L, H\}.$$

Hence, we can say that any rate such that  $r_i^A \leq c_i$  is weakly dominated by  $r_i^A > c_i \ \forall i \in \{L, H\}$ . Likewise, the maximum profit will be given by the monopoly rate:

<sup>&</sup>lt;sup>22</sup>In a deterrence game this could be no true, see for example P. Milgrom and Roberts (1982)

$$r_i^{A*} = \frac{1+c_i}{2} \equiv \hat{r}_i^A \forall i \in \{L, H\}$$
(3)

Maximizing  $\pi_i^A$ 

$$\frac{\partial \pi_i^A}{\partial r_i^A} = (1 - r_i^A) - (r_i^A - c_i) = 1 - 2r_i^A + c_i$$

and the second-order condition, to prove is a maximum, is

$$\frac{\partial^2 \pi_i^A}{\partial r^{A^2}} = -2 < 0$$

Therefore, player A's monopoly rate, namely  $\hat{r}_i^L$ , dominates any higher rate. Now, the strategy space for player A is reduced to any rate equal or higher than the marginal cost and lower or equal to the monopoly rate for each state; that is  $c_i \leq r_i^A \leq \hat{r}_i^A$ .

On the other hand, player *B* never plays an interest rate lower than the marginal cost in state *L* as this strategy brings losses knowing that player *A* never sets a rate lower than this marginal cost; that is,  $r^b \ge c_L$ . We could extend this argument to state *H*; nevertheless, player *B* could play a rate lower than marginal cost in state *H* if that rate make it to gain a share of the market in state *L*. In other words, player *B* could make losses in state *H* if gains from state *L* compensate that deficit. This is due to the information asymmetry, making player *B* decide its strategy in terms of expected profit. Hence, let's define the expected marginal cost as  $c_w = \theta c_l + (1 - \theta)c_H$ . Taking (2) and getting marginal revenue:

$$\frac{\partial \pi^B}{\partial L_i} = \sum_{i \in \{L,H\}} \theta_i s_i^B (r^B - c_i) \equiv M R^B \forall i \in \{L,H\}$$

and

$$MR^{B}(r^{B}, c_{i}) = \begin{cases} \leq 0 & 0 < r^{B} \leq c_{w} \\ \geq 0 & c_{w} < r^{B} < 1 \end{cases} \quad \forall i \in \{L, H\}$$

Thus, any rate lower than weighted marginal cost is weakly dominated by higher rates. Besides, any rate higher than player *B*'s weighted monopoly rate,  $\hat{r}^B$ , is also weakly dominated by the latter. Maximizing  $\pi^B$ :

$$\frac{\partial \pi^B}{\partial r^B} = \theta[(1 - r^B) - (r^B - c_L)] + (1 - \theta)[(1 - r^B) - (r^B - c_H)]$$

using  $c_w$ , we have that:

$$r^{B*} = \theta \frac{1 + c_L}{2} + (1 - \theta) \frac{1 + c_H}{2} = \frac{1 + c_w}{2} \equiv \hat{r}^B$$
(4)

and the second-order condition, to prove is a maximum, is

$$\frac{\partial^2 \pi^B}{\partial r^{B^2}} = -2 < 0$$

The strategy space for player *B* is also reduced to any rate equal or higher than the weighted marginal cost and lower or equal to the weighted monopoly rate for both states; that is  $c_w \leq r^B \leq \hat{r}^B$ .

Iterating once more, player A knows that player B will not set a rate lower its weighted marginal cost,  $c_w$ . Indeed, the game with narrowed strategies spaces for each player is:

- Player A in state L:  $r_L^A \in [c_w \varepsilon, \hat{r}_L^A]$ , and for state H:  $r_H^A \in [c_H, \hat{r}_H^A]$ ;
- Player B:  $r^B \in [c_w, \hat{r}^B]$ ; where  $c_w = \theta c_L + (1 \theta)c_H$ .

#### 2.4.2 New Reduced Game

Once we have eliminated of weakly dominated strategies, we simplify this game. The Bank & Platform game is mostly the same as before, the only change is the strategy space. Next we show the new game in extensive form.

In state L player A's strategies go from  $c_w$  to  $\hat{r}_L^A$ . In the state H, the range goes from  $c_H$  to  $\hat{r}_H^A$ . For player B, the reduced space of strategies goes from  $c_w$  to  $\hat{r}^B$ .



Figure 6: Extensive form of the reduced Bank & Platform game

In this narrowed version of the Bank & Platform game, we can see that the triad of strategies where each player set rates equal to monopoly rates,  $s(\hat{r}_L^A, \hat{r}_H^A, \hat{r}^B)$ , results in positive profit for player Aand positive profit for player B given that  $\hat{r}_L^A < \hat{r}^B < \hat{r}_H^A$  and  $\hat{r}^B > c_H$ . On the other hand, if each lender plays strategies equal to marginal costs,  $s(c_L, c_H, c_w)$ , then player A will make no profits at all, while player B will earn negative profits given that  $c_L < c_w < c_H$ .<sup>23</sup> Nevertheless, none of these strategies can be a BNE because both players have incentives to deviate.

Let  $r_L^{A*}$  and  $r_H^{A*}$  be respectively player *A*'s optimal rates in state *L* and state *H*. For player *B*, let  $r^{B*}$  be the optimal rate for both states. Now we can identify three different equilibria.

#### Pure-strategy equilibrium

**Proposition 1.** In the presence of asymmetric information between two competing lenders, where only player A is fully informed about each customer's risk;  $s^*(r_L^{A*}, r_H^{A*}, r^{B*}) = (\hat{r}_L^A, c_H, c_H)$  is a BNE in pure strategies if the player A's monopoly rate in state L,  $\hat{r}_L^A$ , is lower or equal to player B's weighted marginal cost,  $c_w$ , that is, when  $\hat{r}_L^A < c_w$ .

*Proof.* For small values of  $\theta$  and a large difference between  $c_L$  and  $c_H$  it could be that  $\hat{r}_L^A < c_w$ . To see this, note that  $\lim_{\theta \to 0} \theta c_L + (1 - \theta) c_H = c_H$  and there exist values of  $c_L$  and  $c_H$  that satisfy  $c_H - c_L > 1 - c_H$  what makes  $\hat{r}_L^A$  the most profitable strategy for player A in state L as player B will not set a price lower than  $c_w$ . Therefore  $r_L^A = \hat{r}_L^A$  is a dominant strategy for player A in state

<sup>&</sup>lt;sup>23</sup>The latter is the case in perfect competition with no informational asymmetry.

*L* of the narrowed version of the Bank-Platform game. Once player *B* knows that it could not set a competitive price in state *L*, it competes against player *A* only for high-risk consumers, in state *H*. This drives to compete à la Bertrand, with rates going to marginal cost. Thus the equilibrium for this game under condition  $\hat{r}_L^A < c_w$  is  $s^*(\hat{r}_L^A, c_H, c_H)$ . To see this, remember  $r_L^A = \hat{r}_L^A$  is a dominant strategy for player *A* in state *L*, so it does not have any incentives to deviate. In state *H*, if any player sets its price above  $c_H$  it will get no market share and so zero profit. Furthermore, if any of them sets a price below  $c_H$  it can get the whole market but with loses. Consequently, there is no incentive to deviate from  $s^*(\hat{r}_L^A, c_H, c_H)$ .

In this equilibrium,  $s^*(\hat{r}_L^A, c_H, c_H)$ , player A's profit is strictly positive:

$$\pi^{A} = s_{L}^{A} (1 - r_{L}^{A}) (r_{L}^{A} - c_{L}) + s_{H}^{A} (1 - r_{H}^{A}) (r_{H}^{A} - c_{H})$$
  
$$\pi^{A} = 1 (1 - \hat{r}_{L}^{A}) (\hat{r}_{L}^{A} - c_{L}) + \frac{1}{2} (1 - c_{H}) (c_{H} - c_{H})$$
  
$$\pi^{A} = (1 - \hat{r}_{L}^{A}) (\hat{r}_{L}^{A} - c_{L}) > 0$$

On the other hand, player *B*'s expected profit is zero,  $\pi^B = 0$ .

$$\pi^{B} = (1 - r^{B})[\theta_{L}s_{L}^{B}(r^{B} - c_{L}) + \theta_{H}s_{H}^{B}(r^{B} - c_{H})]$$
  
$$\pi^{B} = (1 - c_{H})[\theta_{L}0(c_{H} - c_{L}) + \theta_{H}\frac{1}{2}(c_{H} - c_{H})]$$
  
$$\pi^{B} = (1 - c_{H}) * 0 = 0$$

This case implies that there is a very small share of low-risk consumers; which decreases the information advantage.

#### Mixed-strategy equilibria

The most likely case is when  $c_w < \hat{r}_L^A$ , such that  $r_L^A = \hat{r}_L^A$  is not longer a dominant strategy for player A in state L, as we saw before. In this case there is no pure-strategy equilibrium given that player B can play anything between  $c_w$  and  $\hat{r}_L^A$  and get market L. Thus, player B will set

 $c_w \leq r^B \leq \hat{r}^B$  as we discussed above. Now, the best-response for player A is to set  $r_L^A = r^B - \varepsilon$ and, at the same time, the best-response for player B to this strategy is to set  $r^B = r_L^A - 2\varepsilon$ , and so on until player B sets  $r^B = c_w$  and player A sets  $r^A_L = c_w - \varepsilon$ . Nevertheless, the best-response for player B to  $r_L^A = c_w - \varepsilon$  is to set  $r^B = c_H \le r_H^A$ , to avoid losses.<sup>24</sup> To see this remember that for player B playing  $c_w$  is worthwhile only if it can get the low-risk market gains to compensate the losses in state H. However, if  $r^B \leq \hat{r}_L^A$ , the best-response for player A is to set  $r_L^A = r^B - \varepsilon$  onto  $r^B = c_w$  and  $r_L^A = c_w - \varepsilon$ , starting over.<sup>25</sup>

This is why, whenever  $c_w < \hat{r}_L^A$ , equilibrium must be in mixed strategies. Let  $F_i^A(r)$  be the mixed strategy that A plays in state i; and let  $F^B(r)$  be the mixed strategy that lender B plays for both states. Hence,  $F_i^A(r)$  is the cumulative probability that player A sets a rate below or equal to r in state *i*. Likewise,  $F^B(r)$  brings the cumulative probability that player *B* sets a rate below or equal to r for both states.

Now we face two scenarios. The first one is when  $c_w < \hat{r}_L^A \le c_H$  and the second one when  $c_H < c_H$  $\hat{r}_L^A$ . In both cases we use Proposition 142.2 (Characterization of mixed strategy Nash equilibrium) from Osborne (2004), which is used for mixed strategies with a continuum of actions.

**Proposition 2.** In the presence of asymmetric information between two competing lenders, where only player A is fully informed about each customer's risk; and its monopoly rate in state L,  $\hat{r}_L^A$ , is greater or equal to player B's weighted marginal cost,  $c_w$ , but lower than marginal cost in state H, that is,  $c_w < \hat{r}_L^A \leq c_H$ , the BNE is given by:

$$\sigma^*(r_L^{A*}, r_H^{A*}, r^{B*}) = (F_L^{A*}, c_H, F^{B*})$$
(5)

<sup>&</sup>lt;sup>24</sup>Up to now,  $c_H \le r_H^A \le \hat{r}_L^A$ <sup>25</sup>If  $r^B > \hat{r}_L^A$ , the best-response for player A is  $r_L^A = \hat{r}_L^A$ , but again, the best-response for player B is  $r^B = r_L^A - \varepsilon$ and so on.
where

$$F_L^{A*}(r) = 1 - \frac{1 - \theta}{\theta} \frac{c_H - r}{r - c_L}, \forall r \in [c_w, c_H]$$

$$F^{B*}(r) = \begin{cases} 1 - \frac{(1 - c_w)(c_w - c_L)}{(1 - r)(r - c_L)} \frac{(1 - c_w)(c_w - c_L)}{(1 - r)(r - c_L)} & \hat{r}_L^A < r \end{cases}$$

*Proof.* First, we define the expected profit functions for each player:

$$\begin{aligned} \hat{\pi}_{i}^{A}(r_{i}^{A}, r^{B}) &= (1 - r_{i}^{A})(r_{i}^{A} - c_{i})[s_{i}^{A}(r_{i}^{A}, r^{B} \leq r_{i}^{A})F^{B}(r) + s_{i}^{A}(r_{i}^{A}, r_{i}^{A} \leq r^{B})(1 - F^{B}(r))], \forall i \in \{L, H\} \end{aligned}$$

and

$$\begin{aligned} \hat{\pi}^{B}(r^{B}, r_{L}^{A}, r_{H}^{A}) &= (1 - r^{B}) \\ & \left[\theta(r^{B} - c_{L})[s_{L}^{B}(r^{B}, r_{L}^{A} \le r^{B})F_{L}^{A}(r^{B}) + s_{L}^{B}(r^{B}, r_{L}^{A} \le r^{B})(1 - F_{L}^{A}(r^{B}))] \\ & + (1 - \theta)(r^{B} - c_{H})[s_{H}^{B}(r^{B}, r_{H}^{A} \le r^{B})F_{H}^{A}(r^{B}) + s_{H}^{B}(r^{B}, r_{H}^{A} \le r^{B})(1 - F_{H}^{A}(r^{B}))] \right] \end{aligned}$$

Given that  $s_i^j(r_i^j, r^{-j} \leq r_i^j)F^{-j}(r_i^j) = 0$  and  $s_i^j(r_i^j, r^j < r_i^{-j}) = 1, \forall i \in \{L, H\}$  and  $\forall j \in \{A, B\}$ ;<sup>26</sup> profit functions simplify to:

$$\hat{\pi}_i^A(r_L^A, r_H^A, r^B) = (1 - r_i^A)(r_i^A - c_i)(1 - F^B(r)), \forall i \in \{L, H\}$$
(6)

$$\hat{\pi}^{B}(r^{B}, r_{L}^{A}, r_{H}^{A}) = (1 - r^{B})[\theta(r^{B} - c_{L})(1 - F_{L}^{A}(r^{B})) + (1 - \theta)(r^{B} - c_{H})(1 - F_{H}^{A}(r^{B}))]$$
(7)

Then, let's look for each player's best response. The expected profit for player A in state L of playing  $r_L^A = c_w$  is:

$$\hat{\pi}_L^A = (1 - c_w)(c_w - c_L)(1 - F^B(c_w)) = (1 - c_w)(r_i^A - c_w)$$

<sup>&</sup>lt;sup>26</sup>Note that if the player j's rate is higher than player -j's rate, then  $s_i^j = 0$  and  $F^j(r) = 0$  for a specific point.

 $F^B(c_w) = 0$  because the cumulative probability that player *B* set a rate between 0 and  $c_w$  is zero. Yet, the expected profit for player *A* in state *L* of playing  $r_L^A$  is:

$$\hat{\pi}_L^A(r_L^A) = (1 - r_L^A)(r_L^A - c_L)(1 - F^B(r)) > 0$$
(8)

Then, player *B*'s mixed strategy that makes indifferent player *A* among its set of feasible rates.

$$\hat{\pi}_L^A(c_w) = \hat{\pi}_L^A(r_L^A)$$

$$(1 - c_w)(c_w - c_L) = (1 - r_L^A)(r_L^A - c_L)(1 - F^B(r_L^A))$$

$$F^B(r) = 1 - \frac{(1 - c_w)(c_w - C_L)}{(1 - r)(r - c_L)}$$

So, we have that

$$F^{B*}(r) = \begin{cases} 1 - \frac{(1-c_w)(c_w - C_L)}{(1-r)(r-c_L)} & c_w \le r \le \hat{r}_L^A \\ \frac{(1-c_w)(c_w - C_L)}{(1-r)(r-c_L)} & \hat{r}_L^A < r \end{cases}$$
(9)

For player *B*, the expected profit of playing  $r^B = c_w$  and  $r^B = c_H$  is:

$$\hat{\pi}^B(c_w) = (1 - c_w)[\theta(c_w - c_L)(1 - F_L^A(c_w)) + (1 - \theta)(c_w - c_H)(1 - F_H^A(c_w))]$$

 $F_{H}^{A}(c_{w}) = 0$  because player A will not set  $r_{H}^{A} < c_{H}$ . Hence

$$\hat{\pi}^B(c_w) = (1 - c_w) [\theta(c_w - c_L)(1 - F_L^A(c_w)) + (1 - \theta)(c_w - c_H)] \ge 0$$

and

$$\hat{\pi}^B(c_H) = (1 - c_H)[\theta(c_H - c_L)(1 - F_L^A(c_H)) + (1 - \theta)(c_H - c_H)] = 0$$
(10)

 $F_L^A(c_H) = 1$  because player A will not set  $r_L^A > c_H$  as we argued above at the beginning of the mixed strategy equilibrium section.

We look for a player *A*'s mixed strategy which makes indifferent player *B* among its available choices.

$$\hat{\pi}^{B}(c_{H}) = \hat{\pi}^{B}(c_{w})$$

$$0 = (1 - c_{w})[\theta(c_{w} - c_{L})(1 - F_{L}^{A}(c_{w})) + (1 - \theta)(c_{w} - c_{H})]$$

$$F_{L}^{A*}(r) = 1 - \frac{1 - \theta}{\theta} \frac{c_{H} - r}{r - c_{L}}$$

Let's check if there is incentive to deviate from this strategies. In state *L*, if player *A* deviates to any  $r_L^A < c_w$ , it would get  $\pi_L^A(c_H < r_L^A) = 0 < \hat{\pi}_L^A(c_w)$ . In state *H*, player *A* never plays  $r_L^A < c_H$ because  $\pi_H^A(r_H^A < c_H) < 0$  which it could avoid by setting  $r_H^A = c_H$ . Neither  $c_H < r_L^A$  is profitable as  $\pi_H^A(c_H < r_H^A) = 0 = \pi_H^A(c_H)$ .

For player *B* there is also no profitable deviation. We already showed that no player would set a rate lower than marginal cost, that is, any  $r^B < c_w$  leads to  $\pi^B(r^B < c_w) < 0 < \hat{\pi}^B(c_w) = 0$ . On the other hand,  $\pi^B(c_H < r^B) = 0 = \hat{\pi}^B(c_H)$ . Then, in equilibrium  $s^*(\hat{r}_L^A, c_H, F^B(r))$ .

In this equilibrium,  $\sigma^*(r_L^{A*}, r_H^{A*}, r^{B*}) = (F_L^{A*}, c_H, F^{B*})$ , profit is strictly positive for player A, while player B's profit is zero, the same that it is in the previous equilibrium.

**Proposition 3.** In the presence of asymmetric information between two competing lenders, where only player A is fully informed about each customer's risk; and its monopoly rate in state L,  $\hat{r}_L^A$ , is greater than marginal cost in state H, that is,  $c_H < \hat{r}_L^A$ , the BNE is given by:

$$\sigma^*(r_L^{A*}, r_H^{A*}, r^{B*}) = (F_L^{A*}, F_H^{A*}, F^{B*})$$
(11)

where

$$F_L^{A*}(r) = 1 - \frac{1 - \theta}{\theta} \frac{c_H - r}{r - c_L}, \forall r \in [c_w, c_H]$$

$$F_H^{*A}(r) = \begin{cases} 1 - \frac{(1 - c_h)(c_h - c_L)}{(1 - r)(r - c_L)} & c_h \le r \le \hat{r}_L^A \\ \frac{(1 - c_h)(c_h - c_L)}{(1 - r)(r - c_L)} & \hat{r}_L^A < r \end{cases}$$

$$F^{B*}(r) = \begin{cases} 1 - \frac{(1 - c_w)(c_w - c_L)}{(1 - r)(r - c_L)} & c_w \le r \le \hat{r}_L^A \\ \frac{(1 - c_w)(c_w - c_L)}{(1 - r)(r - c_L)} & \hat{r}_L^A < r \end{cases}$$

*Proof.* Expected profit functions are the same for both players as in Proposition 2. Take equation(6):

$$\hat{\pi}_i^A(r_i^A, r^B) = (1 - r_i^A)(r_i^A - c_i)(1 - F^B(r_i^A)) > 0$$
(12)

and equation (7):

$$\hat{\pi}^{B}(r^{B}, r_{L}^{A}, r_{H}^{A}) = (1 - r^{B})[\theta(r^{B} - c_{L})(1 - F_{L}^{A}(r^{B})) + (1 - \theta)(r^{B} - c_{H})(1 - F_{H}^{A}(r^{B}))] > 0$$
(13)

Applying the same arguments in Proposition 2, we have

$$F^{B*}(r) = \begin{cases} 1 - \frac{(1 - c_w)(c_w - C_L)}{(1 - r)(r - c_L)} & c_w \le r \le \hat{r}_L^A \\ \frac{(1 - c_w)(c_w - C_L)}{(1 - r)(r - c_L)} & \hat{r}_L^A < r \end{cases}$$
(14)

and

$$F_i^{A*}(r) = 1 - \frac{1-\theta}{\theta} \frac{\hat{r}_i^A - r}{r - c_i} \forall i \in \{L, H\}$$

$$(15)$$

As before, player A's expected profit is positive. However, this time player B's expected profit is also positive as player A does not play  $c_H$  in state H for sure.

There may be many other BNE. However, we will not discuss all of them. This is not a proof for all equilibria, nor of uniqueness. Furthermore, we focus on equilibria that are relevant for the discussion about information sharing. Up to now, we can conclude that the information advantage results in a positive expected benefit for player *A*; although not necessarily for player *B*. How large is the expected profit from having more information depends on how different the two states are, or how worthwhile is to distinguish both types of consumers; that is, the proportion of high and low risk.

#### 2.4.3 Equilibrium Analysis

In none of the equilibria there is a chance for information sharing. If player A shares its information, it loses its informational advantage and both players compete à la Bertrand as credit is a homogeneous good and there will be not asymmetric information. In this case, both players earn zero profit, which is a loss for the fully-informed player at least. In the other way, player B does not have any useful information for player A. Despite this, if there is information sharing, borrowers are better off by paying rates equal to marginal costs.

In addition, in only one of these equilibria, when  $\hat{r}_L^A \leq c_w$ , the fully-informed player sets a monopoly rate. Despite the informational advantage, the presence of the opponent's prevents monopoly rates in the three cases for the high-risk borrower and in two of them equilibria for the low-risk too.

A striking conclusion is that the low-risk borrower always pays a supramarginal rate; while the high-risk borrower can get a rate equal to marginal cost. In two of these equilibria, whenever  $\hat{r}_L^A \leq c_H$ , the high-risk borrower pays a competitive rate; while the low-risk consumer does not. Furthermore, it could be that the lower-risk consumer pays the high-risk rate. Thus, there could be pooling equilibrium in addition to separating equilibria. This is the case when player A's monopoly rate in state L is greater than player B' weighted marginal cost, that is  $c_w < \hat{r}_L^A$ , although this is an extreme case. This outcome is similar to the pooling and separating equilibria in Bester (1985) and to the conclusion in Joseph E. Stiglitz and Weiss (1981) that the low-risk type pays a higher rate; although those authors do not analyze information sharing.

The worst case is when the proportion of low-risk types is very small such that player A's monopoly rate is less or equal to player B's weighted marginal cost, that is,  $c_w \ge \hat{r}_L^A$ , as this borrower pays precisely the highest rate, the monopoly one.

Finally, this informational asymmetry gives positive profit to the fully-informed player in any case; while the uninformed player expected zero profits in most of the cases. Thus, when it is clear one player has informational advantage, there is no chance for information sharing, keeping this advantage is profitable.

# 2.5 Conclusion

We conclude that there is no chance for voluntary information sharing in the presence of information asymmetry when one of two lenders is fully informed about the types of borrower. The intuition is simple, uninformed player does not have any worthy information for the fully informed lender. Nevertheless, the competition of the uninformed player inhibits monopolistic behavior of the lender with informational advantage, at least for the high-risk borrower.

On the other hand, low-risk borrower always pays a greater interest rate than marginal cost rate. Asymmetric information makes this type of consumer to cross-subsidy high-risk borrowers. Despite information sharing makes at least one lender worst-off; it makes borrowers better-off, at least one of them. Hence, information sharing will benefit borrowers, but not the lenders.

# **3** Chapter 2 Information Sharing when All Players Have Partial Information

# 3.1 Introduction

In the previous chapter we conclude that there are no incentives to share information between two competitors when one of them is fully-informed and the other one has only partial information about the types of the consumers asking for a loan. In this chapter we propose a model where none of the players has full information about the types of consumers. Thus, *ex-ante* it is not clear if asymmetric information is an advantage for one of the firms. In this model, competitors have already a share of the consumption loan market, but there is still a part of the demand that is not attended. Consumers ask for consumption loans, but each of them has a different probability of paying the credit back. We assume that both loan suppliers know the specific probability of default for every single of its current customers, although they do not know this probability for new clients. In this sense, each player has a share of information about consumers' probability of default or types. This rises an informational asymmetry among credit providers as they know only a portion of the market and a different one. In this model, we look for incentives to share information and, if it is the case, how worthy that information is.

The probability of default is a cost for any loan supplier as it is necessary to set provisions for a fraction of the credit amount. This is mandatory for regulated credit institutions.<sup>27</sup> At the same time, it is a common risk management practice in credit institutions. Granting a loan to a consumer with a higher probability of default is more expensive than to a safer borrower, given the lender has to reserve a larger portion of that debt. Hence, if players are currently supplying different types of consumers, that is consumers with different probabilities of default, they have different costs. Furthermore, they will face different costs of granting loans to new customers. The informational asymmetry is given by this cost asymmetry.

<sup>&</sup>lt;sup>27</sup>In Mexico, the whatchdog for the financial system considers the probability of default to compute the reserves for each loan. See Comisión Nacional Bancaria y de Valores (2023)

The main goal of this section is to identify if competitors could be better off by sharing information and how much they value it in a market characterized by asymmetric information and heterogeneous demand. We deal with heterogeneous demand because each consumer has a different quality, that is, a different probability of default such that granting a loan has a different cost for each borrower. On the other hand, the lack of informational symmetry among competitors, given by which share of the market each player knows, biases the cost estimation of granting loans to new customers as loaners cannot be sure of the type of those customers Hence, sharing information seems to be a good idea if loan granters can minimize or even eliminate cost bias estimation. Nevertheless, each competitor could be afraid of losing its competitiveness to fight for the rest of the market by sharing information. So, it is not clear that the trade-off will be non negative for at least one player, that is, if there is a Pareto improvement.

Our model is a contribution to economic literature by considering a variable marginal cost function in order to know the possibility of information sharing and how worthy it is. Previous work on information sharing based on cost asymmetry, Vives (1984); Vives (1990), Gal-Or (1985); Gal-Or (1986) and Shapiro (1986), considers constant marginal cost function, a difference in one parameter of the demand function or different signals about the quality of the borrower. In credit markets, the probability of defaulting a loan is not constant among customers; making the marginal cost variable. We show that when players cannot screen unattended demand in a credit market, they will bias expected marginal cost, and previous outcomes no longer remain. Then, asymmetric information biases the expected marginal cost, such that in equilibrium one player sets a lower interest rate and gets the rest of the market. In this case, the winner has no incentives to share its information because it could lose the market, but the looser does as it can get a side-payment.

Furthermore, in the presence of switching costs and lock-in, the winner overestimates the value of the residual demand, what results possibly in winner's curse. This is a new source of winner's curse, previous literature on this issue, like **Shaffer1983**, shows that winner's curse emerges as rejected poor quality borrowers are granted by subsequent banks. Besides, we set that the possibility of winner's curse in this framework depends on the relation of the Lerner index and the competitor's market share. Cornelli et al. (2023) show that there is a positive relation between the fintech and big tech credit volumes with the Lerner index, which our model can explain theoreti-

cally. Despite winner's curse, the winner still has incentives to get the competitors information as it can avoid it, or at least, decrease the negative effect.

## 3.2 Literature Review

Research about informational asymmetry, given by costs or by a specific characteristic of the demand, has been developed long time ago. For example, Vives (1984); Vives (1990) and Gal-Or (1985) set information asymmetry based on different signaling of a parameter of the demand function. They conclude that in equilibrium there is information sharing in Bertrand competition, while in Cournot framework there is not when goods are substitutes and the contrary when goods are complements. Although, Gal-Or (1985) focuses only on Cournot competition and Vives (1990) is concerned with the rules established by an association to share information.

#### **Information sharing**

Likewise, Gal-Or (1986) and Shapiro (1986) deal with information sharing in the presence of private cost asymmetry among competitors. In a Cournot competition framework, both authors conclude that competitors can share information through an association. This increases efficiency in the supply side by supplying more lower cost firms; while consumer surplus is reduced because the variability of the output decreases.

Indeed, the models in this referenced literature are quite abstract because the only difference is if competition is in prices or quantities, without specifying any kind of industry. Nevertheless, they focus on the differences between Bertrand and Cournot equilibria within a monopolistic competition market. In addition, the authors consider constant marginal cost, which is not the case for the credit markets if we consider the probability of defaulting a loan as we already explained.

#### Asymmetric information equilibrium

Our model is an asymmetric information game, given by asymmetric costs. The equilibrium for this model is based on the solution in Belleflamme and Peitz (2015). At the same time, Hansen (1988), Spulber (1995) and Lofaro (2002), are the sources for that solution. Our model is different from the latter in that they do not justify the cost asymmetry, they just take it as given.

Besides, in these models costs cannot change and the implications of this constant cost assumption remain. Furthermore, and more important, they assume constant marginal cost, which is difficult to justify in the credit market as each customer has a different risk, thus a different cost. Constant marginal cost could be the case if players face the same cost of funding, for example the reference rate, regardless of others way of funding like public sources. But in credit markets, marginal cost also depends on the probability of default, which is not constant among customers. In this sense, the lender's marginal cost changes given it grants loans to more customers. Hence, the marginal cost function is not constant, even more, it is not monotonic. This is because the marginal cost can increase if the marginal customer is riskier or the marginal cost decreases if the marginal customer is safer. In addition, in the equilibria from cited research, there is no way that expected profit would be negative as each player sets a price above marginal cost, which is certain; while if costs are uncertain it is not straightforward for the price to be higher than the cost realization.

Considering that a variable cost function has different outcomes as players already have different marginal costs; they will have different expected costs for the rest of the market. In other words, expected cost changes depending on the kind and amount of information a player has. The expected marginal cost could be higher or lower depending on the information they acquired. This could drive players to overestimate or underestimate the cost of attending the residual demand.

Secondly, in our model there is a cost estimation process, this is different from other models which take the cost as given The discussion of cost estimation process is similar to that in Hansen (1988) about the price formation. In a model with asymmetric costs, the author points out that auction theory better explains the price-formation process than classical oligopoly models. The equilibrium quantities and prices in the latter models do not have tendency to change and they do not explain how the equilibrium prices are reached, agents just get there. On the other hand, auction models show an equilibrium process, the path that agents follow to reach the equilibrium. Similarly, in the proposed model, agents look for information in order to have a less unbiased cost estimation. Although it is not a price-formation method, this cost-estimation process could change the profit and even the equilibrium as initial parameters change.

#### 3.2.1 Information asymmetry equilibrium

The most similar research to our work is He, Huang, and Zhou (2023), that follows Broecker (1990). The authors assess the open-banking policy for information sharing among financial competitors. There are similarities in assumptions like cost asymmetry, Bayesian-Nash equilibrium, the loan market and focus on information sharing. Moreover, some outcomes are very alike; mainly the presence of winner's curse and that information sharing could damage consumers. Nevertheless, once more, a substantial difference is the assumption about the expected marginal cost that is variable and non monotonic in our model. Besides, another difference is the process of costformation like the price-formation in Hansen (1988), which we already discussed in the last paragraph. This is important because the information advantage is not an assumption like in He, Huang, and Zhou (2023); but it is an outcome of the cost-formation process. In addition, in our model winner's curse is the outcome of an overestimation given by the presence of switching costs which are very common in the banking industry. Besides, we establish a condition for the presence of the winner's curse related to the Lerner index resulting from equilibrium. They conclude that open banking benefits at least the less risky borrowers, although this policy could over-empower fintech such that they will have monopoly power. In contrast, we conclude that consumers are worse-off despite which loan supplier improves by sharing information. Last of all, we focus on the incentives to share information independently among competitors and not because they are forced by regulation.

#### 3.2.2 Switching costs in banking

Lock-in is a crucial assumption in our model, which we consider is common in the banking industry because of different sources of switching costs. Indeed, switching costs have been a widespread issue in the industrial organization, antitrust, banking theory as well as policy literature. Farrell and Klemperer (2007) is a very extensive work on switching costs. For the banking industry there is also a lot of literature about it, both theoretical and empirical, as well as in the deposit and credit sides of financial markets. Vives (2001) cover different cases of switching costs, for example, the author mentions this kind of costs as a source of friction in retail banking; in addition, electronic and SME (small and medium enterprises) banking are subject to switching costs.

Zephirin (1994), Sharpe (1997) and Stenbacka and Takalo (2019) cover switching costs in bank deposits. In the first case, switching costs are endogenous as there is a trade-off between interest rates and service quality; in this setting, if banks are forward looking they set higher interest rates. Sharpe (1997) supports empirically the theoretical results in Klemperer (1987a) that switching costs have an effect on the level of competition and also in concentration. On the other hand, Stenbacka and Takalo (2019) show that the stability of the financial system is affected by switching costs depending on if competition among banks is for the relationships with customer or competition is after the relationship is given. Besides, Shy (2002) proposes a general method to estimate switching costs. Indeed the author gives an empirical example in the banking industry, the switching cost is between 0 and 11% of the average balance of the accounts.

On the credit side, Stango (2002) estimates switching cost in credit cards, the author finds that this kind of cost explains more than one quarter of the variation in credit card rates; in addition, the results point out a positive relationship between the probability of default and switching costs. This author refers to Calem and Mester (1995) to mention some sources of switching costs. First, those borrowers with high credit balances require a larger decrease in interest rate to switch. A second source is that lenders take usually into account debt-to-income ratios to approve loans, then borrowers with high level of this ratio find it difficult to switch. The third source is that debt-laden borrowers have liquidity constraints. Two more are annual fees and switching checks. Besides, Kim, Kliger, and Vale (2003) estimate switching cost in the loan market, which depends on the scale of the banks, where the bank-borrower relationship plays a key role. This work presents relationship baking as an additional source of switching cost. In Bouckaert and Degryse (2004) reducing switching costs decreases competition in a two-period game as banks compete less vigorously in the first-period because they know they will have access to borrowers in the second stage. Finally, the model in Vesala (2007) predicts that switching costs have a non-monotonously effect on loans because of the adverse selection problem given by borrowers of different quality.

#### 3.2.3 Winner's curse in banking

One result in our model is that winner's curse could arise given that expected profits could be negative. To our knowledge, the first reference to winner's curse is in Capen et al. (1971) in an

analysis of firms competing for oil contracts. Most of the references and analysis about winner's curse could be found in auction theory in the presence of common values. We can find a simple explanation in Krishna (2009), where the author highlights that winner's curse is the outcome of miscalculation by the bidders and not the equilibrium. Another reference is Thaler (1988), where winner's curse is considered as an anomaly given that rational players should not face this phenomenon; nevertheless it happens. In banking framework, there is also literature showing that winner's curse could surge mainly because of adverse selection. Shaffer (1998) points out that as rejected loan applicants look for a credit in other banks, the latter will face worse borrowers. The main conclusions are that the larger the number of banks, the larger the expected loan loss rate; despite this, expected aggregate bank profit increases; and *de novo* banks will suffer a higher winner's curse. In a critique to Sharpe (1990), Thadden (2004) shows that winner's curse must be taken into account in the analysis of asymmetric information in bank lending markets.

# **3.3 Model:** An asymmetric information game with different subsets of information

In this model consumers demand loans to finance their consumption. The demand function for loans is L(r) = 1 - r, where L is the loan amount demanded and r is the interest rate or price. Each consumer has a different probability of defaulting the loan. This probability of default goes from 0 to 1. That is, a consumer with probability of default equal to 0 will pay back the loan for sure. On the other hand, if the consumer borrowed has a probability of default of 1, she will not repay the credit at all.

The probability of default defines the type of the consumer. There is a continuum of types which goes from 0 to 1. Let *t* denotes the type of the consumer, that is her probability of default. Hence, *t* is a random variable with probability density function f(t) and domain [0, 1] in the whole market.

<sup>c</sup> There are two incumbents firms in the market, a digital platform A, and a brick-and-mortar bank B. These undertakings compete for consumers by offering homogeneous loans for an interest rate r. Both lenders have been in the market for a long time and each player j has a market share,  $s_j$ , but there is still a part of it that has not been attended. To simplify, we set the fixed cost of providing the credit at zero. We can think that competitors have covered their fixed costs with the earnings



Figure 7: Example of the density function of types

from previous credits. The variable costs of financing consumption usually depend on the funding interest rate and the premium risk. The first one is the cost of money for financial inter r this risk. Let the funding interest rate to be constant at zero. By now, we are not interested in a competitive advantage based on the cost of funding, so we can assume it is the same for both players; while we assume the premium risk as the consumer's probability of default, that could be anything between 0 and 1 according to f(t).

We consider a mature market where each player has a different set of borrower types in their portfolio. This rises an informational asymmetry as each player knows a different portion of the market. The density function of types for the whole market, f(t), is common knowledge, although they do not know which part of the market the competitor has. They can screen perfectly only

the type *t* of customers they are currently attending; nevertheless they cannot screen the type of any other consumer. Thus, they also know that they do not know the rest of the market. Given this asymmetry, which becomes a cost asymmetry, it is not initially clear which firm has better information, nor which one has an informational advantage to compete. Given that each firms has different a share of the market, they also have different subsets of information about the distribution of consumers' types, and as marginal cost is given by the probability of default, we have a price competition with uncertain costs.

#### 3.3.1 Notation

We can define the basics of the model. In the rest of the chapter we use the following notation. From the demand side, the demand function for consumption loans is L(r) = 1 - r, where L is the amount of the loan demanded and r is the nominal interest rate. The probability of defaulting the credit defines the type, t, of the consumers. T is the set of all types, where T = [0 : 1] and  $t \in T$ . Thus, t is a random variable with density function f(t). For simplicity, we assume that  $t \sim U(0, 1)$ .

From the supply side, there are two players denoted by j, a financial digital platform A and a brickand-mortar bank B, that is  $j \in \{A, B\}$ . Given we have a continuum of types, the probability of a specific type is 0, this is why we have to deal with ranges of types, let us call them tranches. Let  $\omega_j \in \Omega_j$  denote a tranche of the market which player j is already attending. Let  $\Omega_j = \{1, 2, ...\}$ , where  $|\Omega_j|$  is the cardinality of  $\Omega_j$ , be the set of tranches for player j's market share. The number of tranches does not have to be the same for both players. Each player has already a share,  $s_j$ , of the market, which is the sum of all segments or tranches of the market in its portfolio. The player j's market share,  $s_j$ , satisfies the following conditions:

- 1. Let  $\bar{t}_{\omega_j}$  be the supremum and  $\underline{t}_{\omega_j}$  be the infimum in tranche  $\omega_j$  of player *j*'s market share, so  $s_j = \sum_{\omega_j} (\bar{t}_{\omega_j} - \underline{t}_{\omega_j})$  (see Figure 2);
- 2. Tranches are not empty, that is,  $\underline{t}_{\omega_j} < \overline{t}_{\omega_j} \ \forall \omega_j \in \Omega_j$  such that  $s_j > 0$ ;
- 3. There is no intersection among tranches,  $\bar{t}_{\omega_j} < \underline{t}_{\omega_j+1} \forall \omega_j \in \Omega_j$ ;
- 4. Thus, the market share for each player,  $s_j$ , is closed but not necessarily convex, unless a

player's market share is formed by only one tranche;

- 5.  $s_j \subset T \ \forall j \in \{A, B\}$ , this is why we will refer to market shares as subsets of information;
- Furthermore, s<sub>j</sub> ∪ s<sub>-j</sub> = S ⊂ T. S can be interpreted as the level of credit penetration and indicates that there is a part of the market that is not attended;

The cost of funding a consumer is her probability of default, t; such that c = t, but we have to deal with tranches for the continuum of types conditions. Thus, the expected cost for player j of granting a loan to a current customer in tranche  $\omega_j$  is  $c_{\omega_j} = E_{\omega_j}(t)$ :

$$c_{\omega_j} = E_{\omega_j}(t) = \int_{\underline{t}_{\omega_j}}^{\overline{t}_{\omega_j}} tf(t), dt$$

Player *j*'s total expected cost for its market share is:

$$c_j = \sum_{\omega_j} \left( \int_{\underline{t}_{\omega_j}}^{\overline{t}_{\omega_j}} tf(t), dt \right)$$

For firm j, the expected marginal cost of the rest of the market,  $\bar{c}_j = E(t|s_j)$ , is:

$$\bar{c}_j = \int_0^1 tf(t), dt - \sum_{\omega_j} \left( \int_{\underline{t}_{\omega_j}}^{\overline{t}_{\omega_j}} tf(t), dt \right)$$

or simply  $\bar{c}_j = \frac{1}{2} - c_j$  when  $t \sim U(0, 1)$ .

Firm j's profit from granting credit at  $r_j$  is:

$$\pi_j = (1 - r_j)(r_j - c_j)$$

The profit function for each player j of the rest of the market is:  $\pi'_j = (1 - r_j)\theta_j(r_j - \bar{c}_j)$ , where

$$\theta_{j} = \begin{cases} 1 & r_{j} < r_{-j} \\ \frac{1}{2} & r_{j} = r_{-j} & \forall j \in \{A, B\} \\ 0 & r_{j} > r_{-j} \end{cases}$$

Hence, the expected profit for player j is  $\bar{\pi}_j = (1 - r_j)(r_j - c_j) \Pr(r_j < r_{-j})$ 



Figure 8: A density function of types for different subsets of information

With this setup, we can point out some initial remarks for this market.

#### 3.3.2 Initial remarks

We mentioned that one of the main differences of this model in comparison with previous literature is that marginal cost is not constant. We already set fixed cost at zero and the probability of default as the only variable cost, which defines the type of the borrower at the same time. Suppose a consumer asks for a loan for the first-time to player j. Her type could be anything from 0 to 1. For instance, suppose her type is 0.5. Then a second consumer asks for a loan to player j too. The latter consumer has a type anything from 0 to 1, but 0.5. Thus, the marginal cost of the latter could be higher or lower than the first consumer's cost; but not the same. That is, the marginal cost of attending an additional customer is not constant; furthermore, it is not monotonic as it could be higher or lower than the cost of the inframarginal customer. There are two exceptions. First, when the inframarginal customer has type 0, then it is clear that marginal cost is higher; and when the inframarginal borrower's type is 1 any other customer has a lower cost. Nevertheless, each single type has a probability of 0 as we are dealing with a continuum of types. This is another difference between our model and He, Huang and Zhou (2023) as they consider only two types of borrowers, low and high type related to the probability of default.

Then, each player estimates an expected type for a consumer in a range or tranche  $\omega$  as:

$$c_{\omega} = E_{\omega}(t) = \int_{\underline{t}_{\omega}}^{\overline{t}_{\omega}} tf(t), dt$$

as we already stated. When  $t \sim U(0, 1)$  this becomes:

$$c_{\omega} = \frac{1}{2}(\bar{t}_{\omega}^2 - \underline{t}_{\omega}^2)$$

And the expected cost for an additional tranche  $\omega'$  is:

$$c_{\omega'} = \frac{1}{2}(\bar{t}_{\omega'}^2 - \underline{t}_{\omega'}^2)$$

namely expected marginal cost, which is higher o lower than  $c_{\omega}$  depending on the parameters  $\bar{t}$  and  $\underline{t}$  of the tranche  $\omega'$ . Hence, the expected marginal cost will be higher or lower, but not the same as the inframarginal cost; moreover, it is not monotonic as we already stated.

Now, given player j's cost function and  $f(t) \sim U(0, 1)$ , we have:

$$c_j = \sum_{\omega_j} \left( \int_{\underline{t}_{\omega_j}}^{\overline{t}_{\omega_j}} tf(t), dt \right)$$
$$c_j = \sum_{\omega_j} \left( \int_{\underline{t}_{\omega_j}}^{\overline{t}_{\omega_j}} t, dt \right)$$
$$c_j = \frac{1}{2} \sum_{\omega_j} \left( \overline{t}_{\omega_j}^2 - \underline{t}_{\omega_j}^2 \right)$$

such that we can state.

**Proposition 4.** Given that  $t \sim U(0, 1)$  and that tranches are not empty, that is,  $\underline{t}_{\omega_j} < \overline{t}_{\omega_j} \forall \omega_j \in \Omega_j$ such that  $s_j > 0$ , furthermore,  $\overline{t}_{\omega_j} < \underline{t}_{\omega+1_j} \forall \omega_j \in \Omega_j$ ; the player with the higher current expected



Figure 9: Current Expected Cost

cost will estimate a lower expected marginal cost for the rest of the market. That is, if  $c_j > c_{-j}$ , then  $\bar{c}_j < \bar{c}_{-j}$ .

*Proof.* If  $c_j > c_{-j}$ , then

$$\sum_{\omega_j} (\int_{\underline{t}_{\omega_j}}^{\overline{t}_{\omega_j}} tf(t), dt) > \sum_{\omega_{-j}} (\int_{\underline{t}_{\omega_{-j}}}^{\overline{t}_{\omega_{-j}}} tf(t), dt)$$

and  $\bar{c}_j = \frac{1}{2} - c_j$ , then  $\bar{c}_j < \bar{c}_{-j}$ , or

$$\int_{0}^{1} tf(t), dt - \sum_{\omega_{j}} (\int_{\underline{t}_{\omega_{j}}}^{\overline{t}_{\omega_{j}}} tf(t), dt) > \int_{0}^{1} tf(t), dt - \sum_{\omega_{-j}} (\int_{\underline{t}_{\omega_{-j}}}^{\overline{t}_{\omega_{-j}}} tf(t), dt)$$

rearranging

$$-\sum_{\omega_j} \left(\int_{\underline{t}_{\omega_j}}^{\overline{t}_{\omega_j}} tf(t), dt\right) > -\sum_{\omega_{-j}} \left(\int_{\underline{t}_{\omega_{-j}}}^{\overline{t}_{\omega_{-j}}} tf(t), dt\right)$$

or

$$\sum_{\omega_j} \left( \int_{\underline{t}_{\omega_j}}^{\overline{t}_{\omega_j}} tf(t), dt \right) < \sum_{\omega_{-j}} \left( \int_{\underline{t}_{\omega_{-j}}}^{\overline{t}_{\omega_{-j}}} tf(t), dt \right)$$

From  $\bar{c}_j = \frac{1}{2} - c_j$ , we also have

$$\frac{\partial \bar{c}_j}{\partial c_j} = -1 < 0$$



Figure 10: Marginal Expected Cost

# 3.4 Equilibrium

We look for a Bayes Nash Equilibrium in this cost asymmetric game. The equilibrium strategy must be a price function, increasing on the marginal expected cost. We take the equilibrium for a game with asymmetric costs from Belleflamme and Peitz (2015). Their proof is based on Hansen (1988), Spulber (1995) and Lofaro (2002). They look for a price function of marginal costs, to solve for the symmetric Bayesian Nash equilibrium. Their price function is strictly increasing on marginal cost in the interval [0, 1]. Thus, each player solves:

$$\max_{r_j} \quad (1 - r_j)(r_j - \bar{c}_j) \Pr(r_j < r_{-j})$$

Given  $t \sim U(0, 1)$ , then  $\Pr(r_j < r_{-j}) = 1 - r_j$ , the problem to solve becomes:

$$\max_{r_j} (1 - r_j)(r_j - \bar{c}_j)(1 - r_j)$$

or

$$\max_{r_j} (r_j - \bar{c}_j)(1 - r_j)^2$$

The first-order condition:

$$(1 - r_j)^2 - 2(r_j - \bar{c}_j)(1 - r_j) = 0$$

Then

$$r_j^* = \frac{1 + 2\bar{c}_j}{3}$$

or

$$r_j^* = \frac{2 - \sum_{\omega_j} (\bar{t}_{\omega_j}^2 - \underline{t}_{\omega_j}^2)}{3}$$

in terms of player j's tranches of information.

This strategy satisfies the more general solution in Belleflamme and Peitz (2015) for this asymmetric cost game, assuming the uniform distribution case:



Figure 11: Equilibrium Interest Rate

$$r_j^* = \frac{1 + nc_j}{n+1}$$

Besides, this price function is strictly increasing in  $\bar{c}_j$  in the range [0, 1]. The same conclusions apply:

- Prices are above expected marginal cost.
- The lower the marginal cost, the higher the price-cost margin,  $m_j$ .

$$m_j = \frac{1+nc_j}{n+1} - c_j = \frac{1-c_j}{n+1}$$
$$\frac{\partial m_j}{\partial c_j} = -\frac{c_j}{n+1} < 0$$

Nevertheless, we are interested in identifying which of these players has an advantage given its market share or how worthy is the information currently owned by each player.

In equilibrium, the player with the lowest expected marginal cost,  $c_j$ , offers the lowest price and will win the rest of the market. Marginal cost is the expected cost or type, t, from the density

function of type, f(t), once each player takes out the types of consumers which are currently in its portfolio.

The intuition is straightway: The player that knows the more expensive types of customers, expects the rest of the market to have a lower probability of default, then a lower cost, and vice versa.

#### 3.4.1 Example

For example, take Figure 8. Player *j* has currently in its portfolio types from  $\underline{t}_{1_j} = 0.10$  to  $\overline{t}_{1_j} = 0.25$  and from  $\underline{t}_{2_j} = 0.55$  to  $\overline{t}_{2_j} = 0.65$ , while player -*j* has types from  $\underline{t}_{1_{-j}} = 0.35$  to  $\overline{t}_{1_{-j}} = 0.45$  and from  $\underline{t}_{1_{-j}} = 0.75$  to  $\overline{t}_{1_{-j}} = 0.90$ . In this case we have:

$$c_j = \int_{0.10}^{0.25} t \, dt + \int_{0.55}^{0.65} t \, dt = \frac{1}{2} ((0.25^2 - 0.10^2) + (0.65^2 - 0.55^2)) = 0.1725$$

and

$$c_{-j} = \int_{0.35}^{0.45} t \, dt + \int_{0.75}^{0.90} t \, dt = \frac{1}{2} ((0.45^2 - 0.35^2) + (0.90^2 - 0.75^2)) = 0.3275$$

such that  $c_j < c_{-j}$ . Then

$$\bar{c}_j = \frac{1}{2} - 0.1725 = 0.3275$$

and

$$\bar{c}_{-j} = \frac{1}{2} - 0.3275 = 0.1725$$

such that  $\bar{c}_{-j} < \bar{c}_j$ .

Despite that player *j* has a safer share of the market,  $s_j$ , or information about better quality of types, its marginal cost is higher than player *-j*'s. Thus, lender *j* will set a higher price losing the rest of the market. In general, the player with the greater mass mean of types in its current portfolio has the lower expected marginal cost and it takes the rest of the market.

#### 3.4.2 Equilibrium Analysis

Given  $\bar{c}_j < \bar{c}_{-j}$ , the price range for each player is also different and it is wider for player j. In addition, the infimum in the rate range for player j is lower than for player -j's. Hence, player j could always underprice player -j.

**Proposition 5.** Given that  $t \sim U(0, 1)$  and that tranches are not empty, that is,  $\underline{t}_{\omega_j} < \overline{t}_{\omega_j} \forall \omega_j \in \Omega_j$ such that  $s_j > 0$ , furthermore,  $\overline{t}_{\omega_j} < \underline{t}_{\omega+1_j} \forall \omega_j \in \Omega_j$ , and none of the players is fully-informed; the player with the lowest expected cost has a wider range of prices above expected marginal cost. This is, if:

$$\bar{c}_j < \bar{c}_{-j}$$

or

$$\int_{0}^{1} tf(t), dt - \sum_{\omega_{j}} (\int_{\underline{t}_{\omega_{j}}}^{\overline{t}_{\omega_{j}}} tf(t), dt) < \int_{0}^{1} tf(t), dt - \sum_{\omega_{-j}} (\int_{\underline{t}_{\omega_{-j}}}^{\overline{t}_{\omega_{-j}}} tf(t), dt)$$

and let

$$\bar{r}_j \in [\bar{c}_j, r_j^M] \forall j \in \{A, B\}$$

where  $r_j^M = \frac{1+\bar{c}_j}{2}$  is the monopoly rate. Then, the range or distance, d, from marginal expected cost,  $\bar{c}_j$  and the player's j monopoly rate,  $r_j^M$ , is greater than for player's -j

$$d[\bar{c}_j, r_j^M] > d[\bar{c}_{-j}, \bar{r}_{-j}^M] \forall j \in \{A, B\}$$

*Proof.* Let  $\bar{r}_j$  be the rate that player j sets for a new customer from the rest of the market. Given that the profit function is  $\pi_j = (1 - r_j)(r_j - c_j)$ , the monopoly rate for player j is  $r_j^M = \frac{1 + \bar{c}_j}{2}$ . From Proposition 4, we already know  $\bar{c}_j < \bar{c}_{-j}$ , then:

$$d[\bar{c}_j, r_j^M] = r_j^M - \bar{c}_j = \frac{1 - \bar{c}_j}{2}$$

and

$$\frac{1-\bar{c}_j}{2} > \frac{1-\bar{c}_{-j}}{2}$$

This Proposition shows that the player with the highest current expected cost, in equilibrium could always underprice the competitor for the rest of the market. Despite the winner player can always underprice the other player, it only knows the equilibrium rate,  $\bar{r}_j^*$ , is below the competitor's monopoly rate, as long as it does not know the rates that the competitor offer to its current customers.

**Proposition 6.** The equilibrium rate,  $\bar{r}_j^*$  is always less than the player -j's monopoly rate,  $r_{-j}^M$ .

*Proof.* Take  $r_{-j}^M = \frac{1+2c_{-j}}{2}$  and  $\bar{r}_j^* = \frac{1+2\bar{c}_j}{3}$  then

$$\frac{1+2c_{-j}}{2} \stackrel{\geq}{\geq} \frac{1+2\bar{c}_j}{3} \tag{16}$$

$$\frac{1+2c_{-j}}{2} \gtrsim \frac{1+2(\frac{1}{2}-c_j)}{3} \tag{17}$$

$$\frac{1+2c_{-j}}{2} \stackrel{\geq}{\stackrel{\geq}{_{\sim}}} \frac{2}{3}(1-c_j) \tag{18}$$

$$3(1+c_{-j}) \stackrel{>}{<} 1-c_j \tag{19}$$

$$2 + c_{-j} \gtrsim -c_j \tag{20}$$

given that  $0 < c_j < \frac{1}{2} \ \forall j \in A, B$ 

$$2 + c_{-j} > -c_j$$

As we already stated, in presence of switching costs, like strong banking relationship or credit constraints; even though player -j is charging the monopoly rate over its residual demand, namely  $r_{-j}^{M}$ ; there is no way to get the opponent's share of the market. Another reason could be simply that consumers who already have a credit do not shop around to change its balance to a different institution for a lower interest rate. Because of the lack of mobility, the winner player overestimates the size of the residual demand.

What about profits? Once the player *j* with the lowest expected marginal cost sets its price and gets the rest of the market, the expected benefit,  $\hat{\pi}_j$ , will be:

$$\hat{\pi}_j = (1 - r_j^*) (r_j [\int_0^1 dt - \sum_{\omega - j} (\int_{\underline{t}_{\omega - j}}^{\overline{t}_{\omega - j}} dt) - \sum_{\omega j} (\int_{\underline{t}_{\omega_j}}^{\overline{t}_{\omega_j}} dt)] - \bar{c}_j \sum_{\omega j} (\int_{\underline{t}_{\omega_j}}^{\overline{t}_{\omega_j}} dt))$$

or simply

$$\hat{\pi}_j = (1 - r_j^*)(r_j[1 - s_{-j} - s_j] - \bar{c}_j(1 - s_j)).$$

Notice that player -j's market share has not been included in the profit function as player j cannot attend those consumers because they are already supplied by player -j. This could be the case in the presence of switching costs or because those consumers are already credit constrained.

**Proposition 7.** In equilibrium, the expected profit is not necessarily positive. There is a chance for winner's curse. That is,

$$\hat{\pi}_j \gtrless 0$$

*Proof.* Let  $s_u$  denote the share of the market that is not attended yet. That is:

$$s_u = 1 - s_j - s_{-j}$$

Take  $\hat{\pi}_j$ 

$$\hat{\pi}_j = (1 - r_j^*) \Big[ r_j (1 - s_{-j} - s_j) - \bar{c}_j (1 - s_j) \Big]$$
$$= (1 - r_j^*) (r_j^* s_u - \bar{c}_j) (1 - s_j)$$

Demand,  $(1 - r_j^*)$ , is strictly positive

$$1 - r_j^* = 1 - \frac{1 + 2\bar{c}_j}{3} = \frac{2 + 2\bar{c}_j}{3} > 0$$

as  $0 < \bar{c}_j < \frac{1}{2}$ . The second part:

$$r_j^* s_u - \bar{c}_j (1 - s_j) \geq 0$$

$$r_j^* s_u \geq \bar{c}_j (1 - s_j)$$

$$\frac{s_u}{1 - s_j} \geq \frac{\bar{c}_j}{r_j^*}$$

$$\frac{s_u}{1 - s_j} \geq \frac{3\bar{c}_j}{1 + 2\bar{c}_j}$$

Once more, given that  $0 < \bar{c}_j < \frac{1}{2}$ ,

$$0 < \frac{3\bar{c}_j}{1+2\bar{c}_j} < \frac{3}{4}$$

Hence, it is possible that  $\frac{s_u}{1-s_j} < \frac{3\bar{c}_j}{1+2\bar{c}_j}$ , making  $\hat{\pi}_j < 0$  also a possibility.

The possibility of winner's curse has been considered in Broecker (1990) and Shaffer (1998). Shaffer (1998) developed a model based precisely on Broecker (1990), in order to verify empirically the winner's curse in the banking industry in the United States from 1986 to 1995. The author concluded that new banks experience winner's curse with a higher probability, which is consistent with the model where a rejected borrower, that has a higher probability of being a bad type, looks for a loan with a subsequent bank.

**Corollary 3.0.1.** There is winner's curse whenever the total share of the market, the ratio of player -j' share,  $s_{-j}$ , to the rest of the market from player j's perspective,  $1-s_j$  is greater than the Lerner index. Furthermore, there is no chance of winner curse if  $\frac{s_u}{1-s_j} \ge \frac{3}{4}$ , or there is huge part of the market unattended.

*Proof.* The second part of this statement is evident from the proof of the previous proposition. Also from that proposition, there is a possibility of winner's curse whenever  $\frac{s_u}{1-s_j} < \frac{\bar{c}_j}{\bar{r}_j^*}$ . Given that  $s_u = 1 - s_j - s_{-j}$ , we have:

$$\frac{1 - s_j - s_{-j}}{1 - s_j} < \frac{\bar{c}_j}{\bar{r}_j^*}$$

Arranging

$$1-\frac{\bar{c}_j}{\bar{r}_j^*} < \frac{s_{-j}}{1-s_j}$$

 $\frac{\bar{r}_{j}^{*} - \bar{c}_{j}}{\bar{r}_{i}^{*}} < \frac{s_{-j}}{1 - s_{j}}$ 

or

Depending on the market shares already attended,  $s_j$  and  $s_{-j}$ ; in equilibrium, one player does not increase its market share and the other one expects positive or even negative profits. In this scenario, incentives to share information must exist if both increases their profits. This is an important difference from a Pareto improvement, because the latter requires only one of the agents to get better off without the other one no gets worse off; while to boost information sharing each agent must gain something.

If both players share full information, they will estimate the same cost for the unattended demand; thus, we will get the case of Bertrand competition for the rest of the market. Hence, we analyze information sharing only from one player to the other.

# 3.5 The value of information

We have seen that the player with the lowest expected marginal cost, the lowest expected type given its information subset, will win the rest of the unattended demand. This player sets its rate thinking in getting the whole of the rest of the market, although it only gets a fraction of it because switching costs do not let competitor's borrowers to switch. This decreases the expected profit, which could be even negative. In any case, this player overestimated the size of the market, when it does not have information about the opponent's market share.

The possibility of getting negative profits is *per se* an incentive to get more information about the types of the customers who are not in the portfolio of the winner player. On the other hand, the player that losses the rest of the market could have incentives to get also more information in order

to get at least part of the unattended demand.

**Proposition 8.** In equilibrium, the player that looses the market gets no additional profits, then this lender has incentives to share its information for a side-payment in the presence of switching costs. On the other hand, the player that gained the market has incentives to get the opponent's information to increase its profits or avoid winner's curse.

*Proof.* Suppose player j estimated the lower expected marginal cost for the rest of the market. Then, player -j's profit is zero. If this player shares its information to player j, the latter will estimate a different expected marginal cost,  $\vec{c}_j$ :

$$\vec{c}_j' = \sum_{\omega_j} \left( \int_{\underline{t}_{\omega_j}}^{\overline{t}_{\omega_j}} t, dt \right)$$
$$\vec{c}_j' = \int_0^1 t f(t), dt - \sum_{\omega_j} \left( \int_{\underline{t}_{\omega_j}}^{\overline{t}_{\omega_j}} t f(t), dt \right) - \sum_{\omega_{-j}} \left( \int_{\underline{t}_{\omega_{-j}}}^{\overline{t}_{\omega_{-j}}} t f(t), dt \right)$$

Or simply  $\bar{c}'_j = \frac{1}{2} - c_j - c_{-j}$  given  $f(t) \sim U(0, 1)$ 

Then, the new equilibrium rate is:

$$r_j^{*'} = \frac{1 + 2\bar{c}_j'}{3}$$

the expected profit with this information is

$$\hat{\pi}'_j = (1 - r_j^{*'})(r_j^{*'} - \bar{c}'_j)s_u$$

which is always positive. The first-term is strictly positive:

$$(1 - r'_{j}) \stackrel{\geq}{\geq} 0$$

$$1 \stackrel{\geq}{\geq} r'_{j}^{*}$$

$$1 \stackrel{\geq}{\geq} \frac{1 + 2\vec{c}'_{j}}{3}$$

$$3 \stackrel{\geq}{\geq} 1 + 2\vec{c}'_{j}$$

$$2 \stackrel{\geq}{\geq} 2\vec{c}'_{j}$$

$$1 \stackrel{\geq}{\geq} \vec{c}'_{j}$$

as  $0 < \vec{c}_j' < \frac{1}{2}$ , then

 $1 > \bar{c}'_j$ 

and

 $1 - r_{j}^{\prime *} > 0$ 

The second-term is also strictly positive

$$r_j^{\prime *} - \vec{c}_j^{\prime} \gtrless 0$$
$$\frac{1 + 2\vec{c}_j^{\prime}}{3} - \vec{c}_j^{\prime} \gtrless 0$$
$$\frac{1 - \vec{c}_j^{\prime}}{3} \gtrless 0$$

as  $0 < \overline{c}'_j < \frac{1}{2}$ , then

$$1 - \bar{c}'_j > 0$$

the third-term is also strictly positive as we suppose there is a part of the market not attended yet. The new expected profit is greater than the one without sharing information:

# 

$$(1 - r_j^*)(r_j^* s_u - \bar{c}_j(1 - s_j)) \stackrel{\geq}{\leq} (1 - r_j^{*'})(r_j^{*'} - \bar{c}_j')s_u$$
$$(1 - r_j^*)(r_j^* - \bar{c}_j)s_u - \bar{c}_j s_{-j} \stackrel{\geq}{\geq} (1 - r_j^{*'})(r_j^{*'} - \bar{c}_j')s_u$$
$$(1 - r_j^*)(r_j^* - \bar{c}_j)s_u \stackrel{\geq}{\geq} (1 - r_j^{*'})(r_j^{*'} - \bar{c}_j')s_u + \bar{c}_j s_{-j}$$

Dividing by  $(1-r_j^{*'})(r_j^{*'}-\bar{c}_j')s_u$ 

$$\frac{(1-r_j^*)(r_j^*-\bar{c}_j)}{(1-r_j^{*'})(r_j^{*'}-\bar{c}_j')} \gtrsim 1 + \frac{\bar{c}_j s_{-j}}{(1-r_j^{*'})(r_j^{*'}-\bar{c}_j')s_u}$$

given that  $\bar{c}_j' < \bar{c}_j$ , then  $r_j^{*'} < r_j^*$ , so we have

$$\frac{1 - r_j^*}{r_j^{*'}} < 1$$

and

$$\frac{r_j^* - \bar{c}_j}{r_j^{*'} - \bar{c}_j'} < 1$$

then

$$\frac{(1-r_j^*)(r_j^*-\bar{c}_j)}{(1-r_j^{*'})(r_j^{*'}-\bar{c}_j')} < 1$$

which implies

 $\hat{\pi}_j < \hat{\pi}'_j$ 

or

$$\hat{\pi}_j' - \hat{\pi}_j > 0$$

Then, for any  $0 < \alpha < 1$ , a side-payment  $\alpha(\hat{\pi}'_j - \hat{\pi}_j)$  from player j to player -j; makes both players better-off than in case with no information sharing.

In equilibrium, the loser player only earns its current profit, it gets nothing additional; then, this player can share its information for a payment. If this lender shares its information, it cannot lose its current customers because of the switching cost. On the other side, the winner lender can eliminate the probability of winner's curse or increase expected profit with that information. Note that the presence of switching cost is necessary to share information without collusion.

On the other hand, player j does not have any incentive to share its information subset,  $s_j$ .<sup>28</sup> If it does it, the expected marginal cost of the competitor will be lower. As a consequence, player j will lose the rest of the market.

We propose as a measure of the value of the information the difference between the expected profit that the winner player gets if it has the rest of known information about types of consumers and the current expected profit.

$$V_j(s_{-j}) = \hat{\pi}'_j - \hat{\pi}_j \tag{21}$$

The winner player has incentives to invest a share of this difference in the opponent's firm to have better information, and the loser lender has incentives to accept that investment. This outcome can explain mergers like Whatsapp by Facebook<sup>29</sup> or joint-ventures like Banorte with Rappi <sup>30</sup>.

### **3.6** Conclusion

We have shown that there is space for information sharing when lenders have asymmetric information about the quality of borrowers and none of them is fully-informed in the presence of switching

<sup>&</sup>lt;sup>28</sup>Except for collusion

<sup>&</sup>lt;sup>29</sup>Furman et al. (2019) and Gautier and Lamesch (2021)

<sup>&</sup>lt;sup>30</sup>See https://www.banorte.com/wps/portal/gfb/Home/noticias-banorte/2021/ rappicard

costs. Nevertheless information sharing is given only in one direction. The winner player will not share its information to the opponent because it will earn nothing and it loses profit as competition becomes à la Bertrand for the unattended demand. However, the loser player has incentives to share information for a side-payment, which is better than zero profits that it will earn without the side payment. In this case, both players improve their profits. The payment is a fraction of the value of information, which we define as the difference between the expected profits once information have been shared and expected profit without it for the winner player.

In our model, more information and even information about less risky borrowers is not necessarily an informational advantage. Hence, neither having more information, nor knowing safer borrowers is a guarantee to get the unattended consumer in the presence of asymmetric information. The player with the current highest expected cost will estimate the lowest marginal expected cost for the rest of potential demand for loans offering a lower interest rate. Nevertheless, it is possible that winner's curse surges as this player overestimates the remaining demand.

We derive a rule to know if winner's curse could surge in this setting. If the margin that the winner player gets from the remaining demand is less than the ratio of the rival's current share to the unattended demand there will be winner's curse. That is, if the winner player gets a Lerner index is lower than the rival's share in the residual demand. Furthermore, if the unattended demanded is large enough there is no way that winner's curse appears.

Finally, the information is worthier the larger the unattended market share. This outcome highlights the relevance of public policy favoring information sharing to compete fairly, although any rule on this must take into consideration the costs of getting that information. If players have to share its information for free, incentives for investment to get more information decrease. Besides, public information on aggregate banking data can help to avoid winner's curse, for example, market shares and data about the quality of borrowers.

# 4 Chapter 3 Sharing Information with Partial-Informed Players and the Business Cycle

## 4.1 Introduction

Most of the literature deals with the macroeconomic effect of the banking industry. The question to be answered normally is how the banking industry affects real economic activity at macroeconomic level or how the business cycle changes because of the banks' behavior. There is a narrow literature concerned with the opposite way, on how macroeconomic conditions influence the banking industry. In this field, the most common source is the monetary policy effect on interest rates, changing funding conditions for banks and incentives for saving, borrowing and investing to household and firms. Nevertheless, to our knowledge, there is no previous work to tackle how the business cycles affects the lenders' incentives for sharing information on the borrowers' quality. In this chapter we focus on how the business cycle affects the probability of defaulting a loan by the borrowers, such that this distribution also changes along the business cycle. This, at the time, modifies lenders' knowledge about borrowers they previously had. Thus, incentives to share information among lenders may also change along the business cycle.

We propose a game of asymmetric information about the types of customers for analyzing how information sharing changes with the business cycle. Two lenders compete for new borrowers, These players have a share of the market already, although they do not supply entirely the market. Given that each of them has a share of the market, they also know the type of its own customers; thus, they can only estimate the type of the unattended demand. The cost of granting a loan is equal to the risk of defaulting the credit. Thus, we have a game of asymmetric information, given by asymmetric costs.

In equilibrium, the lender with the higher current expected cost computes a lower expect marginal cost for the rest of the market, which allows it to win the unattended consumers. Nevertheless, the equilibrium interest rate does not necessarily decrease with an improvement in the business cycle. We propose as an improvement in the business cycle when the high-risk borrowers are less

than before, or an increase in the low-risk portion of the market . As a consequence, profits could increase or decrease. Furthermore, the winner player could also change with the business cycle. Indeed information advantage could become a disadvantage.

This outcome contradicts the idea that credit markets are necessarily procyclical. Our model explains why the positive correlation between the business cycle and different measures of loan activity is not always true. Nuño and C. Thomas (2013) document a positive correlation between leverage ratio and the GDP in the United States, although very low (0.12 to 0.36). Moreover, the model explains why interest rates may not decrease when the business cycle improves. That is, for a constant funding cost, the interest rates could increase (decrease), despite high-risk borrowers decrease (increase) as the business cycle improves (deteriorates).

As a consequence, the uncertainty about profits increases, and it is not clear if sharing information generates greater profits. Furthermore, in the presence of switching cost, the probability of winner's curse could increase with the business cycle.

# 4.2 Literature Review

#### 4.2.1 Related Literature

Cetorelli and Blank (2019) survey empirical literature which deals with the real effects of banking activity. They showed that the initial discussion about a causal or associative relationship between banking behavior and economic activity has vanished thanks to development on econometric techniques.<sup>31</sup> Despite discussion about causality, the works cited by these authors imply a close relation between banking activity and real activity. Specifically, competition is one of the most explored themes to explain causality at the micro level; although there are opposite outcomes in the literature.<sup>32</sup> The authors also deal with how bank behavior creates economic crisis at macroeconomic level.<sup>33</sup>

<sup>&</sup>lt;sup>31</sup>They cite King and Levine (1993), Levine and Zervos (1998), Beck, Levine, and Loayza (2000), Levine, Loayza, and Beck (2000), Zingales (2003), Kroszner and Strahan (1999) and Jayaratne and Strahan (1996), for example

<sup>&</sup>lt;sup>32</sup>Here, some references are Petersen and Raghuram G Rajan (1995), Cetorelli and Peretto (2012), Cetorelli and Gambera (2001) and Raghuram G Rajan and Zingales (1998)

<sup>&</sup>lt;sup>33</sup>B. Bernanke and Gertler (1989), Kiyotaki and Moore (1997), B. S. Bernanke, Gertler, and Gilchrist (1999), Cúrdia and Woodford (2016) are some examples cited by the authors

However, in Cetorelli and Blank (2019) we can find two references about macroeconomic conditions changing risks, but not focused on information sharing among banking players. The first one is Gertler and Kiyotaki (2010), they show that a real shock deteriorates net worth because a moral hazard constraint becomes a leverage constraint, which restricts a recovery from the initial shock. This is probably the most common issue on how macroeconomics influences banking behavior. In addition, Stein (2012) shows that ex ante regulation to prevent instability makes banks unstable because they issue more short-term debt than it is optimal.

For the Finnish economy, Jakubík (2006) analyzed the relationship between macroeconomic indicators and the default rate. Using a latent factor model, the author proved there is a relationship between macroeconomic data and probability of default at least for the manufacturing, construction and trade sectors.

In the discussion about causality between financial performance and growth, Raghuram G Rajan and Zingales (1998) develop a model to identify if industries which need large amount of external funding grow faster in countries with more developed financial markets. They conclude that financial development has a substantial influence on economic growth. Besides, financial imperfections have an impact on investment and growth. Lastly, a well-developed financial market is a source of comparative advantage in industries which depends more on external financing.

Claessens et al. (2005) develop a competition model for the banking system in order to know if industries dependent on external funding grow more in markets with a higher level of competition. Given that theory is somewhat ambiguous about the effect of competition in banking markets, they develop a competition indicator and prove for 16 countries the relationship between the level of competition and the performance of highly-dependent industries for external financing. They find the more competition in banking markets, the faster highly-dependent firms grow. On the other hand, the concentration level in the banking system is not helpful to forecast industrial growth.
# 4.3 Model: An asymmetric information game with a changing businesscycle

On the demand side of a credit market, consumers ask for funds in order to finance a consumption basket. The demand for credit is given by L(r) = 1 - r, where L is the amount of the loan demanded at interest rate r. The demand is heterogeneous as each consumer has a different probability of defaulting the credit. A consumer pays back the loan for sure when her probability of default is zero; while a borrower with a probability of default equal to one does not pay back the credit at all. Thus, the borrowers' type is given by her probability of default. There is a continuum of types from 0 to 1. Let t denotes the type of the consumer, t is a random variable with probability density function f(t) and domain [0, 1].

On the supply side, there are two incumbent lenders who compete for consumers by offering homogeneous loans for an interest rate r. They supply unlimited funds for this rate. For simplicity, we assume that the fixed cost of supplying credit is zero. The variable costs of granting a loan usually splits into the funding costs and the premium risk. The funding costs are commonly referenced to the monetary policy rate; and the premium risk depends on the probability of default for each customer as loan suppliers must provision each loan to cover this risk.<sup>34</sup> Let the funding interest rate to be constant at zero. By now, we are not interested in a competitive advantage based on the cost of funding, nor the effects of the monetary policy. Thus, we can assume it is the same for both players. On the other hand, we also assume that the premium risk is the consumer's probability of default, that could be anything between 0 and 1 according to f(t). Let c = t be the cost of granting a loan with a probability of default t.

These two lenders are already supplying some borrowers, but there is still an unattended share of the market. In addition, borrowers are supplied by only one lender as each consumer fully borrows from the cheapest lender. Despite the fact that both banks know the distribution of types, they can only identify its own share of the market. They can screen perfectly only the type t of customers they are currently attending; nevertheless they cannot specifically screen the type of any other consumer, they can only have an expectation. Thus, they have asymmetric information about

<sup>&</sup>lt;sup>34</sup>In Mexico, the supervisor for the financial system considers the probability of default to compute the reserves for each loan. See Comisión Nacional Bancaria y de Valores (2023)

the main characteristic of the potential demand. Given that the cost of a loan is the probability of default, this asymmetry becomes a cost asymmetry. Then, we have an asymmetric information game.

#### 4.3.1 Notation

We can define the basics of the model. In the rest of this chapter we use the following notation. From the demand side, the demand function for consumption loans is L(r) = 1 - r, where L is the amount of loan demanded and r is the nominal interest rate or price. The probability of defaulting the credit defines the type, t, of the consumers. T is the set of all types, where T = [0 : 1] and  $t \in T$ . Thus, t is a random variable with density function f(t).

From the supply side, there are two players denoted by j, a financial digital platform A and a brickand-mortar bank B, that is  $j \in \{A, B\}$ . Given that we have a continuum of types, the probability of a specific type is 0, this is why we have to deal with ranges of types, let us call them tranches. Let  $\omega_j \in \Omega_j$  denotes a tranche of the market that player j is supplying. Let  $\Omega_j = \{1, 2, ...\}$ , where  $|\Omega_j|$ , the cardinality of  $\Omega_j$ , is the total number of tranches for player j's market share, the number of tranches does not have to be the same for both players. Each player has already a share,  $s_j$ , of the market, which is the sum of all segments or tranches of the market in its portfolio. The player j's market share,  $s_j$ , satisfies the following conditions:

- 1. Let  $\bar{t}_{\omega_j}$  be the supremum and  $\underline{t}_{\omega_j}$  be the infimum of tranche  $\omega_j$  of player *j*'s market share, so  $s_j = \sum_{\omega_j} (\bar{t}_{\omega_j} - \underline{t}_{\omega_j})$  (Figure 13);
- 2. Tranches are not empty, that is,  $\underline{t}_{\omega_i} < \overline{t}_{\omega_j} \forall \omega_j \in \Omega_j$  such that  $s_j > 0$ ;
- 3. There is no intersection among tranches,  $\bar{t}_{\omega_j} < \underline{t}_{\omega_j+1} \forall \omega_j \in \Omega_j$ ;
- 4. Thus, the share of the market for each player,  $s_j$ , is closed but not necessarily convex, unless a player market share is formed by only one tranche;
- 5.  $s_j \subset T \ \forall j \in \{A, B\}$ , this is why we refer to market shares as subsets of information;
- 6. Furthermore,  $s_j \cup s_{-j} = S \subset T$ , S can be interpreted as the level of credit penetration and shows there is a part of the market that is not attended;

The cost of lending to a consumer is her probability of default, t; such that c = t, but we have to deal with tranches for the continuum of types conditions. Thus, the expected cost for player j of granting a loan to a current customer in tranche  $\omega_j$  is  $c_{\omega_j} = E_{\omega_j}(t)$ :

$$c_{\omega_j} = E_{\omega_j}(t) = \int_{\underline{t}_{\omega_j}}^{\overline{t}_{\omega_j}} tf(t), dt.$$

Player *j*'s total expected cost for its market share is:

$$c_j = \sum_{\omega_j} \left[ \int_{\underline{t}_{\omega_j}}^{\underline{t}_{\omega_j}} tf(t), dt \right].$$

For firm j, the expected marginal cost for the rest of the market,  $\bar{c}_j = E(t|s_j)$ , is:

$$\bar{c}_j = \int_0^1 tf(t), dt - \sum_{\omega_j} \left[ \int_{\underline{t}_{\omega_j}}^{\overline{t}_{\omega_j}} tf(t), dt \right]$$

or simply  $\bar{c}_j = E(t) - c_j$ .

The profit function for firm j is:

$$\pi_j = (1 - r_j)(r_j - c_j).$$

The profit function for each player j for the rest of the market is:  $\pi'_j = (1 - r_j)\theta_j(r_j - \bar{c}_j)$ , where

$$\theta_{j} = \begin{cases} 1 & r_{j} < r_{-j} \\ \frac{1}{2} & r_{j} = r_{-j} \quad \forall j \in \{A, B\} \\ 0 & r_{j} > r_{-j} \end{cases}$$

Hence, the expected profit for player j is  $\bar{\pi}_j = (1 - r_j)(r_j - c_j) \Pr(r_j < r_{-j})$ .

With this setup, we can point out some initial remarks for this market.

#### 4.3.2 Initial remarks

There is a inverse relation between the current expected cost and the expected marginal cost for the residual demand in this market. The player with the highest current cost will estimate a lower expected marginal cost for the unattended portion of the market. Suppose a lender is currently supplying the ten percent of the market with the highest probability of default. Then, this player expects the rest of the market to be safer than its own portfolio, with a lower probability of default. On the contrary, a lender serving the lowest-risk ten percent of the market, only expects that the risk increases with the unattended consumers. We can generalize this insight with the following lemma.

**Lemma 4.1.** Given that  $t \sim f(t)$  and that tranches are not empty, that is,  $\underline{t}_{\omega_j} < \overline{t}_{\omega_j} \forall \omega_j \in \Omega_j$  such that  $s_j > 0$ , furthermore,  $\overline{t}_{\omega_j} < \underline{t}_{\omega+1_j} \forall \omega_j \in \Omega_j$ ; the player with the higher current expected cost will estimate a lower expected marginal cost for the rest of the market. That is, if  $c_j > c_{-j}$ , then  $\overline{c}_j < \overline{c}_{-j}$ .

*Proof.* Given  $c_{-j} < c_j$ ; we have

 $E(t) + c_{-j} < E(t) + c_j$ 

rearranging

$$E(t) - c_j < E(t) - c_{-j}$$

Taking

$$\bar{c}_j = E(t) - c_j$$

such that

 $\bar{c}_j < \bar{c}_{-j}$ 

Or simply

$$\frac{\partial \bar{c}_j}{\partial c_j} = -1$$

This lemma tell us that the expected marginal cost could be higher or lower than the current expected cost. Suppose the lender has the lowest-risk ten percent of the market, then its marginal expected cost will be higher as there are only worse types left. On the other hand, if the player has the ten percent highest-risk borrowers, this lender could only expect better types, making expected marginal cost to decrease. Furthermore, when a lender is currently attending a range in the middle of the distribution, it might be that the additional borrower is riskier or safer. Thus, the expected marginal cost is not constant. In addition, it is not monotonic in borrowers, nor in information tranches as we stated before. The expected marginal cost mainly depends on the condition of the marginal consumer's type in relation to the lender's current portfolio. This is, if the marginal consumer's probability of default is greater or lower than the mean of the lender's current customers. If the marginal borrowers is riskier or safer than the mean of the current customers of a given lender.

**Lemma 4.2.** Given  $t \sim f(t)$  and  $t \in [0, 1]$ , the expected marginal cost could be lower or higher than the current expected cost,  $c_j$ . That is,

$$\bar{c}_j \stackrel{>}{\leq} c_j \tag{22}$$

*Proof.* Take expected marginal cost,  $\bar{c}_j = E(t) - c_j$ 

$$E(t) - c_j \gtrless c_j$$
$$E(t) \gtrless 2c_j$$
$$c_j \gtrless \frac{E(t)}{2}$$

Indeed,  $c_j$  could be higher, equal or lower than half E(t) given that  $c_j \in [0, 1]$ 

Notice that this lemma does not apply if the lender's portfolio has only one tranche at either tail of the distribution.

We want to analyze incentives for information sharing when the distribution of types changes as the business cycle changes too. For this reason, we use a Beta density function for the distribution of types. The Beta density function is very common for random variables that take values continuously between zero and one like the probability of default does. One advantage of using this density function is that we can modify its parameters to simulate changes in the business cycle. This density function has two parameters,  $\alpha$  and  $\beta$ , when this parameters changes, the form of this density functions also varies. Hence, we can assume that if the business cycle is in expansion, then low-risk (low probability of default) borrowers increases; on the contrary, high-risk (high probability of default) consumers increases as the business cycle is contracting. In the first case, the parameter  $\beta > \alpha$ ; and when  $\alpha > \beta$  the distribution of types skews to bad risk borrowers (see Figure 1). A special case of the Beta density function is when its parameter are both equal to 1, then we have a uniform density function. To model the change in the business cycle we can modify one of the parameters or both, to simplify we will deal only with variations of parameter  $\beta$ .



Figure 12: Beta distribution of types depending on stages of the business cycle

When employment or wages are increasing, we suppose that the probability of default tends to decrease. Those consumers who get a job or get higher earning diminishes their probability of defaulting a credit. Some high-risk borrowers becomes low-risk as the probability of default decreases, this makes the share of good-risk borrowers greater than before. In general, the expected cost for the full distribution of types varies as the business cycle changes.

**Lemma 4.3.** When  $t \sim B(\alpha, \beta)$ , the expected cost for the whole distribution of types, E(t), decreases as the business cycles improves; that is, as low-risk borrowers increases. On the other hand, as the high-risk borrowers increases because of a contraction of the business cycle; the expected cost of the full distribution of types, E(t), increases.

*Proof.* Given that  $t \sim B(\alpha, \beta)$ , then

$$E(t) = \frac{\alpha}{\alpha + \beta} \tag{23}$$

An improve (deterioration) of the business cycle could be modeled as an increases (decrease) of the parameter  $\beta$  or a decrease (increase) of the parameter  $\alpha$  of the Beta distribution. Hence, we have the following derivatives:

$$\frac{\partial E(t)}{\partial \beta} = -\frac{\alpha}{(\alpha + \beta)^2} < 0 \tag{24}$$

and

$$\frac{\partial E(t)}{\partial \alpha} = \frac{\beta}{(\alpha + \beta)^2} > 0 \tag{25}$$

Thus, when the business cycles improves, that is when the portion of high-risk borrowers decreases (Figure 12c), the expected cost decreases for the whole distribution of types. In a similar way, when low-risk borrowers decrease, the expected cost increases (Figure 12a).



Figure 13: Beta distribution of types depending on parameters  $\alpha$  and  $\beta$ 

First thing to analyze is if the expected cost for each lender given its market share changes with the business cycle. For example, lender j has a different expected cost when the business cycle is bad (Figure 13a) than when it is good (Figure 13c) according to its share,  $s_j$ .

To simplify, let's take  $\alpha = 1$  and evaluate changes in  $\beta$  only for integers greater than 1. Then we have  $t \sim B(1,\beta)$ , where  $\beta \in \{2,3,...\}$ . First, the current expected cost for the player's j,  $c_j = E(t|s_j)$ , depends on the parameters  $\beta$ ,  $\bar{t}_{\omega_j}$  and  $\underline{t}_{\omega_j}$ . Let  $c_j = E(t|s_j)$ , then we have:

$$c_j = \sum_{\omega_j} \left[ \int_{\underline{t}_{\omega_j}}^{\overline{t}_{\omega_j}} t \frac{(1-t)^{\beta-1}}{B(1,\beta)}, dt \right]$$
(26)

as  $\beta \in \{1,2,3,\ldots\}$ 

$$c_j = \frac{\Gamma(1+\beta)}{\Gamma(1) + \Gamma(\beta)} \sum_{\omega_j} \left[ \int_{t_{\omega_j}}^{\bar{t}_{\omega_j}} t(1-t)^{\beta-1}, dt \right]$$

$$c_j = \beta \sum_{\omega_j} \left[ \frac{-(1-t)^{\beta}(t\beta+1)}{\beta(\beta+1)} \Big|_{\underline{t}_{\omega_j}}^{\overline{t}_{\omega_j}} \right]$$

$$c_j = \sum_{\omega_j} \left[ \frac{-(1-t)^\beta (t\beta+1)}{(\beta+1)} \Big|_{\underline{t}_{\omega_j}}^{\overline{t}_{\omega_j}} \right]$$

$$c_j = \sum_{\omega_j} \left[ \frac{(1 - \underline{t}_{\omega_j})^\beta (1 + \beta \underline{t}_{\omega_j}) - (1 - \overline{t}_{\omega_j})^\beta (1 + \beta \overline{t}_{\omega_j})}{1 + \beta} \right]$$

Proof of the integral in equation (26) in the appendix. See Figure 14 for a graphical representation of  $c_j$ .

Note that when  $\beta = 1$ ,  $c_j$  follows the uniform distribution studied in Chapter 2.

Now, given that  $\bar{c}_j = E(t) - c_j$  and taking (23), the marginal expected cost,  $\bar{c}_j$  becomes:

$$\bar{c}_j = E(t) - c_j$$
$$\bar{c}_j = \frac{1}{1+\beta} - c_j$$
$$\bar{c}_j = \frac{1}{1+\beta} - \sum_{\omega_j} \left[ \frac{(1-\underline{t}_{\omega_j})^\beta (1+\beta \underline{t}_{\omega_j})}{(1+\beta)} - \frac{(1-\overline{t}_{\omega_j})^\beta (1+\beta \overline{t}_{\omega_j})}{(1+\beta)} \right]$$

or

$$\bar{c}_j = \frac{1 + \sum_{\omega_j} (1 - \bar{t}_{\omega_j})^\beta (1 + \beta \bar{t}_{\omega_j}) - \sum_{\omega_j} (1 - \underline{t}_{\omega_j})^\beta (1 + \beta \underline{t}_{\omega_j})}{1 + \beta}$$
(27)

For  $\beta \in \{1, 2, ...\}$ , this expected marginal cost function is bounded above by the mean of the whole distribution of types, E(t), and bounded below by zero. The expected marginal cost is equal to the mean in particular for entrants, that is, when the borrower has not information about the types at all. On the other hand, when a player has full information about the types, the expected marginal cost is zero. In Figure 15 you can find a graphical representation of  $\bar{c}_j$ . Again, note that when  $\beta = 1$ , we have the expected marginal cost studied in Chapter 2. We present the graph for the expected marginal cost in Figure 15 for different values of  $\beta$ . Those charts show that the minimum value of the marginal expected cost is zero and the maximum cost is equal to the mean E(t), despite the value of  $\beta$ .

# 4.3.3 How current expected cost and marginal expected cost chances as the business cycles does

How the current expected cost changes as the business cycles does?

**Lemma 4.4.** Given  $t \sim B(1, \beta)$ , the current expected cost for the lender j given its market share

 $s_j$ ,  $c_j = E(t|s_j)$ , could decrease or increase as the business cycle improves. That is,

$$\frac{\partial c_j}{\partial \beta} = \frac{\partial}{\partial \beta} \sum_{\omega_j} \left( \frac{(1 - \underline{t}_{\omega_j})^\beta (1 + \beta \underline{t}_{\omega_j}) - (1 - \overline{t}_{\omega_j})^\beta (1 + \beta \overline{t}_{\omega_j})}{1 + \beta} \right) \gtrless 0$$

or

$$\frac{\partial c_j}{\partial \beta} = \sum_{\omega_j} \left( \frac{(1 - \underline{t}_{\omega_j})^\beta \left[ (\underline{t}_{\omega_j} - 1) + (1 + \beta)(1 + \beta \underline{t}_{\omega_j}) \ln (1 - \underline{t}_{\omega_j}) \right]}{(1 + \beta)^2} \right)$$

$$-\sum_{\omega_{j}} \left( \frac{(1-\bar{t}_{\omega_{j}})^{\beta} \left[ (\bar{t}_{\omega_{j}}-1) + (1+\beta)(1+\beta\bar{t}_{\omega_{j}}) \ln (1-\bar{t}_{\omega_{j}}) \right]}{(1+\beta)^{2}} \right) \gtrless 0$$

Proof in the appendix. In Figure 16 we show the derivative of the current expected cost with respect to  $\beta$ . There you can see the function can take values positive or negative for different values of  $\beta$ . Note that given that tranches are not empty,  $\underline{t}_{\omega_j} < \overline{t}_{\omega_j}$ , this derivative cannot be zero. Figure 16 shows graphical representations of this derivative for  $\beta \in \{2, 3, 4, 5\}$ .

The space to be positive is very narrow, nevertheless it is not empty. This space decreases with greater values of  $\beta$ . The better the business cycle, the lower the probability that the current expected cost increases; or the lower the proportion of high-risk borrowers, the higher the probability that the current expected cost decreases.

This lemma shows that any player's current expected cost could increase or decrease with the business cycle. This may change incentives to share information. Furthermore, it could be that the player that in a given stage of the business cycle have a lower expected cost; once  $\beta$  changes, then the opponent's expected cost is lower. However, we still have that  $\bar{c}_j < \bar{c}_{-j}$  or  $\bar{c}_j > \bar{c}_{-j}$ . Note that the probability of  $\bar{c}_j = \bar{c}_{-j}$  is zero.

Consequently, the same happens for the marginal expected cost,  $\bar{c}_j$ , which can increase or decrease with the business cycle, given it depends on  $c_j$ . Figure 17 shows the derivative of  $\bar{c}_j$  for  $\beta \in \{2, 3, 4, 5\}$ .

**Lemma 4.5.** Given  $t \sim B(1, \beta)$ , the expected marginal cost for the lender j given its market share  $s_j$ ,  $\bar{c}_j$ , could decrease or increase as the business cycle improves. That is,

$$\frac{\partial \bar{c}_j}{\partial \beta} = \frac{\partial}{\partial \beta} \frac{1}{1+\beta} - \frac{\partial}{\partial \beta} c_j \ge 0$$
(28)

Proof in the appendix (5.4). In Figure 17 we show the derivative of the marginal expected cost respect to  $\beta$ . There you can see the function can take values positive or negative for different values of  $\beta$ .

We associate higher levels of employment, wages, investment or even better welfare conditions with the expansion of business cycle. More employment and higher wages may diminish the probability of defaulting loans. As a result, the ratio of higher-risk to the total borrowers becomes lower. Henceforth, the lending cost must decrease as the risk does. In short, intuition tells us that the borrowing cost decreases when the business cycles comes along. Nevertheless, lemmas 4.4 and 4.5 do not necessarily follow this intuition. Indeed, the expected cost and the expected marginal cost decrease with the improvement of the business cycle. However, these lemmas also show that it may be the case that expected cost or expected marginal cost increases, despite macroeconomic conditions are better. The outcome depends on the tranches in the players' portfolios or the unattended market share.

When the business cycle improves, high risk borrowers decreases, both, known and unknown risky consumers. At the same time, known and unknown low-risk borrowers increases. This is true for each player. The fewer the known borrowers, the greater the unknown ones. If known types borrowers decrease (increase) proportionally more than unknown ones increase (decrease), the current expected cost is lower (higher); hence, the marginal expected cost is higher (lower). In Figure 18 we present three cases of how the distribution of types changes with the business cycle and the information tranches.

Now we can estimate the equilibrium of this game of asymmetric information.

## 4.4 Equilibrium

#### 4.4.1 **Profit maximization**

We take the equilibrium for a game with asymmetric costs following Belleflamme and Peitz (2015). Their proof is based on Hansen (1988), Spulber (1995) and Lofaro (2002). They look for a price function of marginal costs, to solve for the symmetric Bayesian Nash equilibrium. The price function must be strictly increasing on the interval [0, 1]. In this case, each player solves:

$$\max_{r_j} (1 - s_j)(1 - r_j)(r_j - \bar{c}_j) \Pr(r_j < r_{-j})$$

or

$$\max_{r} \quad (1 - s_j)(1 - r)(r - \bar{c}_j)(1 - F(r))$$

The cumulative distribution function is:

$$F(r) = \frac{1}{B(\alpha,\beta)} \int_0^{r_j} r^{\alpha-1} (1-r)^{\beta-1}, dr$$
(29)

When  $\alpha = 1$ , then

$$F(r) = \frac{1}{B(1,\beta)} \int_0^{r_j} (1-r)^{\beta-1} dr$$
  
=  $\frac{1}{B(1,\beta)} \left[ -\frac{1}{\beta} (1-r)^{\beta} \right] \Big|_0^{r_j}$   
=  $\frac{1}{B(1,\beta)} \frac{1}{\beta} \left[ -(1-r_j)^{\beta} + (1)^{\beta} \right]$   
=  $\frac{1}{B(1,\beta)} \frac{1}{\beta} \left[ 1 - (1-r_j)^{\beta} \right]$ 

as  $\beta \in \{2, 3, ...\}$ , then

$$B(1,\beta) = \frac{\Gamma(1)\Gamma(\beta)}{\Gamma(1+\beta)} = \frac{0!(\beta-1)!}{\beta!} = \frac{1}{\beta}$$
(30)

such that (30) becomes:

$$1 - (1 - r_j)^{\beta}$$
 (31)

Now, our problem to solve is

$$\max_{r_j} (1 - s_j)(1 - r_j)(r_j - \bar{c}_j)(1 - r_j)^{\beta}$$

or

$$\max_{r_j} \quad (1 - s_j)(r_j - \bar{c}_j)(1 - r_j)^{\beta + 1}$$

The first-order condition:

$$(1-s_j)\Big[(1-r_j)^{\beta+1} - (\beta+1)(r_j - \bar{c}_j)(1-r_j)^{\beta}\Big] = 0$$

solving for  $\boldsymbol{r}$ 

$$r_j^* = \frac{1 + (1+\beta)\bar{c}_j}{2+\beta}$$
(32)

or

$$r_j^* = \frac{2 + \sum_{\omega_j} \left[ (1 - \bar{t}_{\omega_j})^\beta (1 + \beta \bar{t}_{\omega_j}) - (1 - \underline{t}_{\omega_j})^\beta (1 + \beta \underline{t}_{\omega_j}) \right]}{2 + \beta}$$
(33)

This rate increases as the expected marginal cost,  $\bar{c}_j$ , also increases. In Figure 19 we plot equilibrium interest rate for  $\beta \in \{2, 3, 4, 5\}$  and one tranche of information.

#### 4.4.2 Equilibrium Analysis

First, note that the equilibrium interest rate is always less than the monopoly rate.

**Proposition 9.** The equilibrium interest rate  $r_j^*$  is always less than the monopoly rate  $r_j^M$ , that is

$$r_j^* < r_j^M$$

Proof.

$$r_j^* = \frac{1 + (1+\beta)\bar{c}_j}{2+\beta} \gtrless r_j^M = \frac{1+2\bar{c}_j}{2}$$
$$2 + 2(1+\beta)\bar{c}_j \gtrless 2(1+2\bar{c}_j) + \beta(1+2\bar{c}_j)$$
$$2\bar{c}_j \gtrless 4\bar{c}_j + \beta$$
$$0 < 2\bar{c}_j + \beta$$

Note that this proposition is true even if  $\bar{c}_j = 0$ , this is the case when the player has full information about the types. Hence, in equilibrium, there is no chance for an interest rate to be equal to the monopoly rate.

This proposition shows that monopoly interest rate is not possible at equilibrium. Independently of which player has the informational advantage, that lender cannot set rates equal to the monopoly price. The presence of the competitor makes monopoly rates unavailable.

In order to know how incentives to share information change along the business cycle, we have to start to know how the equilibrium interest rate changes with the business cycle too. Let's take expression (34) to see how  $r_j^*$  depends on  $\beta$  and information tranches:

$$r_{j}^{*} = \frac{2 + \sum_{\omega_{j}} \left[ (1 - \bar{t}_{\omega_{j}})^{\beta} (1 + \beta \bar{t}_{\omega_{j}}) - (1 - \underline{t}_{\omega_{j}})^{\beta} (1 + \beta \underline{t}_{\omega_{j}}) \right]}{2 + \beta}$$
(34)

**Proposition 10.** If the distribution of the types of borrowers is a Beta Distribution,  $t \sim B(1, \beta)$ ,

 $\beta \in \{2, 3, ..\}$ , and none of the players is fully-informed; the equilibrium interest rate can increase or decrease, when the business cycle improves, that is, when high-risk borrowers decrease:

$$\frac{\partial r_j^*}{\partial \beta} = \frac{\partial}{\partial \beta} \frac{1 + (1+\beta)\bar{c}_j}{2+\beta} \frac{\partial}{\partial \beta} \bar{c}_j \stackrel{\geq}{\geq} 0 \tag{35}$$

or

$$\begin{aligned} \frac{\partial r_j^*}{\partial \beta} &= \frac{(1 - \bar{t}_{\omega_j})^\beta (1 + \beta \bar{t}_{\omega_j}) \log(1 - \bar{t}_{\omega_j}) - (1 - \underline{t}_{\omega_j})^\beta (1 + \beta \underline{t}_{\omega_j}) \log(1 - \underline{t}_{\omega_j})}{2 + \beta} \\ &+ \frac{(1 - \bar{t}_{\omega_j})^\beta \bar{t}_{\omega_j} - (1 - \underline{t}_{\omega_j})^\beta \underline{t}_{\omega_j}}{2 + \beta} - \frac{2 + (1 - \bar{t}_{\omega_j})^\beta (1 + \beta \bar{t}_{\omega_j}) - (1 - \underline{t}_{\omega_j})^\beta (1 + \beta \underline{t}_{\omega_j})}{(2 + \beta)^2} \gtrless 0 \end{aligned}$$

Proof.

$$\begin{split} \frac{\partial r_j^*}{\partial \beta} &= \frac{\partial}{\partial \beta} \frac{1 + (1+\beta)\bar{c}_j}{2+\beta} \frac{\partial}{\partial \beta} \bar{c}_j \\ &= \frac{-[1+(1+\beta)\bar{c}_j][\frac{\partial}{\partial \beta}(2+\beta)] + (2+\beta)(\frac{\partial}{\partial \beta}[1+(1+\beta)\bar{c}_j])}{(2+\beta)^2} \frac{\partial}{\partial \beta} \bar{c}_j \\ &= \frac{-[1+(1+\beta)\bar{c}_j] + (2+\beta)\bar{c}_j}{(2+\beta)^2} \frac{\partial}{\partial \beta} \bar{c}_j \\ &= \frac{\bar{c}_j - 1}{(2+\beta)^2} \frac{\partial}{\partial \beta} \bar{c}_j \end{split}$$

The first term is negative given that  $0 \le \bar{c}_j < 1$  as intuition make us expect. However,  $\bar{c}_j$  depends on  $\beta$ . From lemma 4.5 we know that the second term could be either positive or negative. Then,

$$\frac{\partial r_j^*}{\partial \beta} \gtrless 0$$

Beyond causality discussion as in Cetorelli and Blank (2019), one expects that interest rate decreases with an improvement in the business cycle. This outcome shows it is highly probable, but a greater interest rate is also possible. It is straightforwardly true whenever there is full information about the types of borrowers or about the risks. Nevertheless, this equilibrium interest rate emerges from an asymmetric information game, where none of the players has full information. Furthermore, both lenders together do not have all the information. When the business cycle improves and reduces the unknown high risk borrowers and increases the proportion of known types, then the marginal expected cost reduces and so does the equilibrium interest rate. However, if the the marginal expected cost increases, as lemma 4.5 shows, the equilibrium interest rate. In the first case, the probability of an unknown high risk borrowers decreases, while known types increases. In the second, unknown high risk borrowers decreases, but also known types; such that the own data are less informative, and lenders increases rates for this uncertainty.

The expected profit now is

$$\hat{\pi}_j = (r_j - \bar{c}_j)(1 - r_j)^{\beta + 1}$$

which also changes with the business cycle.

**Proposition 11.** When the distribution of the types of borrowers is a Beta Distribution,  $t \sim B(1,\beta)$ ,  $\beta \in \{2,3,..\}$  and none of the players is fully-informed; at equilibrium, expected profits could increase or decrease, despite the business cycle is improving in the sense that high-risk borrowers decreases. That is,

$$\frac{\partial \hat{\pi}_j}{\partial \beta} = \frac{\partial}{\partial \beta} (r_j - \bar{c}_j) (1 - r_j)^{\beta + 1} \stackrel{\geq}{\stackrel{\geq}{\stackrel{\sim}{=}} 0$$
(36)

Proof.

$$\frac{\partial \hat{\pi}_j}{\partial \beta} = (1 - r_j^*)^{\beta + 1} (r_j^* - \bar{c}_j) \log(1 - \bar{c}_j) \left( -\frac{\partial r_j^*}{\partial \beta} \right) + (1 - r_j^*)^{\beta + 1} \left[ \frac{\partial r_j^*}{\partial \beta} - \frac{\partial \bar{c}_j}{\partial \beta} \right]$$

As  $0 < r_i^* < 1$  and  $0 < \bar{c}_j < r_j^*$ , then

$$(1 - r_j^*)^{\beta + 1} (r_j^* - \bar{c}_j) > 0$$

but

$$\log(1-\bar{c}_i) < 0$$

such that

$$(1 - r_j^*)^{\beta + 1} (r_j^* - \bar{c}_j) \log(1 - \bar{c}_j) < 0$$

from the Proposition 10 we know that

$$\frac{\partial r_j^*}{\partial \beta} \gtrless 0$$

such that the sign of the first term is ambiguous, depending on parameters  $\beta, \underline{t}_{\omega_j}$  and  $\overline{t}_{\omega_j}$ . In the same way, as  $0 < r_j^* < 1$ , then

$$(1 - r_j^*)^{\beta + 1} > 0$$

0

but we also have that

$$\frac{\partial r_j^*}{\partial \beta} \gtrless$$

and

$$\frac{\partial \bar{c}_j}{\partial \beta} \gtrless 0$$

such that

$$\frac{\partial \hat{\pi}_j}{\partial \beta} \gtrless 0$$

These propositions contradict the intuition that the credit market is pro cyclical. For example, in a working paper, Nuño and C. Thomas (2013) show the cyclical characteristics of the balance sheets of financial intermediaries in the United States. They point out four stylized facts, but let's highlight two of them. Based on data from financial intermediaries in the United states, assets and leverage are positively correlated to the business cycles. Besides, leverage and assets are procyclical. Despite correlation is positive between these variables and Gross Domestic Product, it is relative low, between 0.12 and 0.36. In our equilibrium, the economy could improve and interest rates go up or down independently of the funding cost, as well as profits could increase or decrease. In our model, the ambiguous relation between the business cycle and the interest rate depends on the quantity of information and which tranches of information the winner player has in its portfolio, as well as the information the winner player cannot get because it comes from borrowers locked-in.

Now, we adjust the expected profit in the presence of switching costs, which are frequent in the credit market as we already discussed in the literature review.<sup>35</sup> Now we have

$$\hat{\pi}_j = (1 - r_j^*)^{\beta + 1} \left[ r_j^* (1 - s_j - s_{-j}) - \bar{c}_j (1 - s_j) \right]$$

Player j must establish reserves for the share of the market it is not currently attending,  $1 - s_j$ . Nevertheless, it will earn only the share of the market unattended by both players,  $1 - s_j - s_{-j}$ , because opponent's borrowers are locked-in. We can also think those customers are credit constrained or they do need more credit at all.

**Proposition 12.** In equilibrium, there is a chance for winner's curse in the presence of switching costs. That is,

$$\hat{\pi}_j \stackrel{\geq}{\leq} 0$$

Proof. The demand part of expected profits is always positive

$$(1 - r_j^*)^{\beta+1} = \left[1 - \frac{1 + (1 + \beta \bar{c}_j)}{2 + \beta}\right]$$
$$= \frac{2 + \beta - 1 - (1 + \beta)\bar{c}_j}{2 + \beta}$$
$$= \frac{1 + \beta(1 - \bar{c}_j) - \bar{c}_j}{2 + \beta}$$
$$= \frac{(1 - \bar{c}_j)(1 + \beta)}{2 + \beta}$$

given that  $0 \leq \bar{c}_j \leq \frac{1}{2}$  and  $\beta \in \{2, 3, ...\}$ , then

$$(1 - r_j^*)^{\beta + 1} \ge 0$$

<sup>&</sup>lt;sup>35</sup>See Shy (2002), Stenbacka and Takalo (2019), Vesala (2007), Stango (2002), and Zephirin (1994)

Now, let  $s_u = 1 - s_j - s_{-j}$ , the second part of the expected profit becomes

$$\begin{aligned} r_j^* s_u - \bar{c}_j (1 - s_j) &\gtrless 0 \\ r_j^* s_u &\gtrless \bar{c}_j (1 - s_j) \\ \frac{s_u}{1 - s_j} &\gtrless \frac{\bar{c}_j}{r_j^*} \\ \frac{s_u}{1 - s_j} &\gtrless \frac{(2 + \beta)\bar{c}_j}{1 + (1 + \beta)\bar{c}_j} \end{aligned}$$

Once more, given that  $0 \le \overline{c}_j \le \frac{1}{2}$  and  $\beta \in \{2, 3, ...\}$ , then

$$0 \leq \frac{(2+\beta)\bar{c}_j}{1+(1+\beta)\bar{c}_j} \leq \frac{2+\beta}{3+\beta} < 1$$

Hence, there is chance for

$$\frac{s_u}{1-s_j} < \frac{(2+\beta)\bar{c}_j}{1+(1+\beta)\bar{c}_j}$$
(37)

making  $\hat{\pi}_j < 0$  a possibility

Note that when  $\beta = 1$ , we have the same upper limit as in the uniforms distribution case in Chapter 2.

Whenever prices are higher than marginal cost, we expected positive profits. In this case, the winner player set an interest rate higher than the marginal expected cost based on own information or market share. Hence, expected profits must be positive. This player offers only one interest rates to any borrower. The expectation is that the margin from low-risk borrowers compensates the loses from high-risk customers. However, because our assumption of switching cost, some low-risk borrower do not switch. This could be large enough to make expected profit negative.

**Corollary 4.5.1.** The room for winner's curse increases with the business cycle, that is, when high-risk borrowers decreases.

$$\frac{\partial}{\partial\beta} \frac{(2+\beta)\bar{c}_j}{1+(1+\beta)\bar{c}_j} > 0$$

Proof. Take inequality (20), which is the upper bound for the possibility of winner's curse, whose

derivative is positive respect to  $\beta$ 

$$\frac{\partial}{\partial\beta} \frac{(2+\beta)\bar{c}_j}{1+(1+\beta)\bar{c}_j} = \frac{1}{3+\beta} > 0$$

When the business cycle improves, the proportion of low-risk borrowers increases relative to highrisk ones. The winner player has a larger proportion of high-risk borrowers than the competitor, but as the business cycles increases low-risk borrowers the possibility of an overestimation of the market by the winner player also increases. This is because if the portion of less risky borrowers increases, those constrained borrowers or locked-in also increase, which makes larger the overestimation by the winner player.

**Corollary 4.5.2.** There is winner's curse whenever the ratio of the opponent's share and the market share unattended by the winner player is greater than the Lerner index in equilibrium. That is

$$\frac{r_j^* - \bar{c}_j}{r_j^*} < \frac{s_{-j}}{1 - s_j}$$

*Proof.* Take inequality (20), which can be expressed as:

$$\begin{aligned} \frac{s_u}{1-s_j} &< \frac{(2+\beta)\bar{c}_j}{1+(1+\beta)\bar{c}_j} \\ \frac{1-s_j-s_{-j}}{1-s_j} &< \frac{\bar{c}_j}{r_j^*} \\ 1-\frac{s_{-j}}{1-s_j} &< \frac{\bar{c}_j}{r_j^*} \\ 1-\frac{\bar{c}_j}{r_j^*} &< \frac{s_j}{1-s_j} \\ \frac{r_j^*-\bar{c}_j}{r_j^*} &< \frac{s_j}{1-s_j} \end{aligned}$$

Note that the derivative of the Lerner index respect to  $\bar{c}_j$  is negative. Hence, the larger the expected marginal cost, the greater the room for winner's curse. On the other hand, as we know from lemma

4.5, the derivative of the expected marginal cost,  $\bar{c}_j$ , with respect to the  $\beta$ , could be positive or negative. Thus, the derivative of the Lerner index respect to  $\beta$ , could be positive or negative too. As a consequence, the probability of winner's curse could increase or decrease with the business cycle. Intuitively, the margin given by the Lerner index must be high enough to compensate the uncertainty of the market share that could be not taken by the competitors because it is locked-in.

#### 4.5 The value of information

If both players exchange information, they will have the same information set; thus, they will have the same marginal expected cost for the unattended market. With the same cost, this competition is à la Bertrand, the outcome is well-known in this case. However, Is there any chance for information sharing in some way?

**Proposition 13.** The player that looses the market has incentives to share its information for a sidepayment as it gets nothing. On the other hand, the player that gained the market has incentives to get the opponent's information to increase its profits or avoid winner's curse.

*Proof.* Suppose player j estimated the lower expected marginal cost for the rest of the market. Then, player -j's expected profit is zero. If this player share its information to player j, the latter will estimate a different expected marginal cost,  $\vec{c}'_{j}$ .<sup>36</sup>

$$\bar{c}'_j = \frac{1}{1+\beta} - c_j - c_{-j}$$

Then, the new equilibrium rate is:

$$r_{j}^{*'} = \frac{1 + (1 + \beta)\bar{c}_{j}'}{2 + \beta}$$

the expected profit with this information is

$$\hat{\pi}'_j = (1 - r_j^{*'})(r_j^{*'} - \bar{c}'_j)s_u$$

<sup>36</sup>Given  $f(t) \sim B(1, \beta)$ 

which is always positive.

The player that loses the competition for the unattended share of the market earns nothing else than its current profit. When there is switching cost, this lender keeps its profit even if it shares its information to the opponent and could earn something else for that information. The winner player can diminish the overestimation of the market if it has more information, reducing the probability of winner's curse or increasing positive expected profit.

Information sharing in the opposite way is not profitable for the winner player as it does nor earn the profit from the unattended share of the market. This happens because the expected marginal cost of the competitor will be lower.

Our measure of information value is the difference between the expected profit with additional information about the borrowers' type and the expected profit without it in equilibrium.

$$V_j(s_{-j}) = \hat{\pi}'_j - \hat{\pi}_j$$

Note that this measure allows for negative additional information. Remember that Proposition 12 show that profit can be decrease with changes in information sets given by the dynamics of the business cycle.

This measure shows that the winner player has incentives to invest a fraction of this difference in the opponent's firm to have better information, and the loser lender has incentives to accept that investment. This can explain mergers like Whatsapp by Facebook or joint-ventures like Banorte with Rappi.

Nevertheless, the value of information changes with the business cycle. From Proposition 12, additional information is not necessarily better as profits could decrease. On the other hand, the value of the same information set could increase with the business cycle. In a static framework, like that studied in Chapter 2, it is clear that information sharing is better for both players; but now it is not necessarily good. This will depend on current information sets to exchange and the expectation on the business cycle.

#### 4.6 Conclusion

In a game of asymmetric information about the risk types of borrowers, two lenders compete for new customers. None player has full information, neither sharing their information sets they would not have all information available because some customers are unattended in the market by either player. This makes each lender estimate types based on its own data. When the business cycle changes, reducing the proportion of high-risk borrowers for example, lenders also change expectations about the types of unattended consumers. This expectation could be higher or lower, even if the business cycles improves. As a consequence, equilibrium interest rate and expected profit could be higher or lower. This outcomes explains why credit markets are not necessarily pro cyclical, contrasting with the vast majority of the literature. In the literature, for instance in Nuño and C. Thomas (2013), it is commonly accepted that financial markets are pro cyclical; despite correlation is positive among some performing financial variables and the GDP, that correlation is usually low or there are short periods where it is not positive at all. This model could help explain why this could happen.

Furthermore, this model tell us that both, interest rate and profits, depends on the information that lenders have about the risk types of customers. This could be the reason for the intense competition about getting data between traditional banks and fintech. Finally, there is still incentives to share information at least for one player to increase profits; but as the business cycle changes, so does its value. Despite this, there is always a chance to improve for both players by sharing information, although these incentives can be modest in certain phases of the business cycle.

Winner's curse is also present in this model as the winner lender over estimates the size of the unattended market. The winner player set its interest rate expecting to get some of the low-risk borrowers, but the switching costs prevent this, such that the lender does not get all the expected profit. An improvement of the business cycle does not necessarily decrease the chance of winner's curse. This depends on the whether the relative number of high risk consumers increases in the population. This also can explain why interest rates increases or profits do not necessarily are larger in good times, despite funding costs do not change. Once more, the outcome depends on the information owned by lenders.

At the same time, this model can explain why financial services penetration is bounded. Given the information asymmetry and that it is costly, the expected profit could not justify the expected cost of supplying the tails of the distribution.

Our model is a framework to explain investment by traditional banks on fintech and other digital players to get data or knowledge about an unknown part of the market. The business cycle could increase uncertainty, making information more worthy. Hence, the side payment proposal is equivalent to an acquisition, a merger, or a joint venture. We have already seen this in different markets. In Mexico, the Sociedades Financieras de Objeto Limitado (Sofoles) were focused on segments of the market unattended by the banks: although they were finally acquired by the banks. Now, we can also see some joint ventures between banks and digital players like Banorte and Rappi. Internationally, mergers like Facebook and Wahtsapp, is an example in this direction.





(b)  $\beta = 3$ 



Figure 14: Current expected cost function for different values of  $\beta$ 









Figure 15: Marginal expected cost function for different values of  $\beta$ 



(b)  $\beta = 3$ 



Figure 16: Derivative of expected cost function for different values of  $\beta$ 



(b)  $\beta = 3$ 



(c)  $\beta = 4$ 

(d)  $\beta = 5$ 

Figure 17: Derivative of marginal expected cost function for different values of  $\beta$ 



Figure 18: Beta distribution of types for different values of  $\beta$ 





(c)  $\beta = 4$ 



(d)  $\beta = 5$ 



Figure 19: Equilibrium interest rate for different values of  $\beta$ 







Figure 20: Derivative of the equilibrium interest rate for different values of  $\beta$ 

## 5 Appendix

# 5.1 Data Panel Estimation of the Nominal Interest Rate in Consumption Loans

We got interest rates for consumption loans and probability of default from April 2011to April 2023 from the CNBV (the Mexican financial market supervisor) and reference rates from Banco de Mexico (the Mexican central bank). The data set contains 68,398 registers from four different credit products and from 80 different credit institutions, banks, fintech and others. These data are public and available in the Portafolio de Informacion in the CNBV web page.<sup>37</sup> Each register is the average rate for each product by each institution for each month for each probability of default range. The probability of default ranges are 5 basic points long.

Nominal Interest Rate										
Year	Durable goods	Motor	Payroll	Personal	Total					
2011	56.3	13.9	30.4	39.2	29.8					
2012	58.5	13.5	28.4	38.4	28.4					
2013	32.2	13.2	26.7	36.4	26.5					
2014	36.0	12.9	26.7	36.2	27.2					
2015	36.2	12.9	26.8	36.3	27.5					
2016	26.5	12.9	26.2	35.6	26.5					
2017	65.8	13.4	26.1	33.2	27.1					
2018	57.1	13.3	26.8	31.3	25.1					
2019	31.9	13.5	25.6	30.2	24.2					
2020	13.8	13.5	25.8	30.1	23.1					
2021	17.1	13.8	26.9	29.7	23.2					
2022	0.0	14.1	26.1	18.6	13.9					
2023	0.0	14.1	23.7	21.0	11.9					
Total	33.5	13.4	26.7	33.6	25.6					

Figure 21: Mean of consumption loans interest rates by year

In Table 21 we present the average of nominal interest rates for each of the four loan products analyzed. In Table 22 we show the mean of probability of default for the same loan products in each year from 2011 to 2023.

We use the reference rate as a control. The consumption loan interest rates are usually based on an inter-bank rate, namely TIIE 28 or TIIE 91, plus some based points. Although, as you can see in Figure 23, the correlation is very high among these rates.

<sup>&</sup>lt;sup>37</sup>https://portafolioinfo.cnbv.gob.mx/Paginas/Inicio.aspx

Probability of Default									
Year	Durable goods	Motor	Payroll	Personal	Total				
2011	22.0	7.9	8.3	8.4	9.0				
2012	24.4	8.2	8.8	8.6	9.3				
2013	26.2	8.2	8.9	8.8	9.3				
2014	24.6	8.3	8.9	8.8	9.2				
2015	22.5	8.3	9.0	8.8	9.2				
2016	20.8	8.2	8.9	8.8	9.2				
2017	22.1	8.3	9.0	8.7	9.4				
2018	21.3	8.4	9.2	8.7	9.5				
2019	20.6	8.4	9.4	8.8	9.4				
2020	19.7	8.5	9.4	8.9	9.3				
2021	19.3	8.6	9.3	8.9	9.4				
2022	17.2	7.1	5.6	5.3	8.3				
2023	16.1	6.0	1.0	3.4	8.3				
Total	21.1	8.3	9.0	8.7	9.3				

Figure 22: Mean of consumption loans probability of default by year

Given we have data by credit institutions and by month we estimated a panel data. We estimate panel data with fixed effects, random effects and maximum likelihood. We did this for the whole data set, as well as for each loan product. In any case we have more than a thousand of observations, but for payroll and personal credits we have more than 20 thousand observations.



Figure 23: Mean of consumption loans probability of default year

## **5.2** Proof of the integral in the cost function (26)

*Proof.* Take equation (26) in Chapter 3 and that  $t \sim B(1, \beta)$ 

$$c_{j} = \sum_{\omega_{j}} \left( \int_{\underline{t}_{\omega_{j}}}^{\overline{t}_{\omega_{j}}} t \frac{(1-t)^{\beta-1}}{B(1,\beta)}, dt \right)$$
$$c_{j} = \frac{\Gamma(1+\beta)}{\Gamma(1) + \Gamma(\beta)} \sum_{\omega_{j}} \left( \int_{\underline{t}_{\omega_{j}}}^{\overline{t}_{\omega_{j}}} t(1-t)^{\beta-1}, dt \right)$$
$$c_{j} = \beta \sum_{\omega_{j}} \left( \int_{\underline{t}_{\omega_{j}}}^{\overline{t}_{\omega_{j}}} t(1-t)^{\beta-1}, dt \right)$$

Let x = 1 - t and dx = -dt

$$c_{j} = \beta \sum_{\omega_{j}} \left( \int_{\underline{t}_{\omega_{j}}}^{\overline{t}_{\omega_{j}}} (x-1) x^{\beta-1}, dx \right)$$

$$c_{j} = \beta \sum_{\omega_{j}} \left( \int_{\underline{t}_{\omega_{j}}}^{\overline{t}_{\omega_{j}}} x^{\beta} - x^{\beta-1}, dx \right)$$

$$c_{j} = \beta \sum_{\omega_{j}} \left( \int_{\underline{t}_{\omega_{j}}}^{\overline{t}_{\omega_{j}}} x^{\beta}, dx - \int_{\underline{t}_{\omega_{j}}}^{\overline{t}_{\omega_{j}}} x^{\beta-1}, dx \right)$$

$$c_{j} = \beta \sum_{\omega_{j}} \left( \frac{x^{\beta+1}}{\beta+1} - \int_{\underline{t}_{\omega_{j}}}^{\overline{t}_{\omega_{j}}} x^{\beta-1}, dx \right)$$

$$c_{j} = \beta \sum_{\omega_{j}} \left( \frac{x^{\beta+1}}{\beta+1} - \frac{x^{\beta}}{\beta} \Big|_{\underline{t}_{\omega_{j}}}^{\overline{t}_{\omega_{j}}} \right)$$

Substitute x = 1 - t

$$c_{j} = \beta \sum_{\omega_{j}} \left( \frac{-(1-t)^{\beta}(t\beta+1)}{\beta(\beta+1)} \Big|_{\underline{t}_{\omega_{j}}}^{\overline{t}_{\omega_{j}}} \right)$$

$$c_{j} = \beta \sum_{\omega_{j}} \left( \frac{(1-\underline{t}_{\omega_{j}})^{\beta}(1+\beta\underline{t}_{\omega_{j}})}{\beta(1+\beta)} - \frac{(1-\overline{t}_{\omega_{j}})^{\beta}(1+\beta\overline{t}_{\omega_{j}})}{\beta(1+\beta)} \right)$$

$$c_{j} = \sum_{\omega_{j}} \left[ \frac{(1-\underline{t}_{\omega_{j}})^{\beta}(1+\beta\underline{t}_{\omega_{j}}) - (1-\overline{t}_{\omega_{j}})^{\beta}(1+\beta\overline{t}_{\omega_{j}})}{1+\beta} \right]$$

## 5.3 Proof of Lemma 4.4

**Lemma 4.4** Given  $t \sim B(1, \beta)$ , the current expected cost for the lender j given its market share  $s_j$ ,  $E(t|s_j) = c_j$ , decreases as the business cycle improves. That is,

$$\frac{\partial c_j}{\partial \beta} \gtrless 0$$

or

$$\frac{\partial c_j}{\partial \beta} = \sum_{\omega_j} \left( \frac{(1 - \underline{t}_{\omega_j})^\beta \left[ (\underline{t}_{\omega_j} - 1) + (1 + \beta)(1 + \beta \underline{t}_{\omega_j}) \ln (1 - \underline{t}_{\omega_j}) \right]}{(1 + \beta)^2} - \frac{(1 - \overline{t}_{\omega_j})^\beta \left[ (\overline{t}_{\omega_j} - 1) + (1 + \beta)(1 + \beta \overline{t}_{\omega_j}) \ln (1 - \overline{t}_{\omega_j}) \right]}{(1 + \beta)^2} \right) \ge 0$$

Proof.

$$\begin{split} \frac{\partial c_j}{\partial \beta} &= \frac{\partial}{\partial \beta} \frac{\sum_{\omega_j} \left[ (1 - \underline{t}_{\omega_j})^{\beta} (1 + \beta \underline{t}_{\omega_j}) - (1 - \bar{t}_{\omega_j})^{\beta} (1 + \beta \bar{t}_{\omega_j}) \right]}{1 + \beta} \\ &= \frac{\partial}{\partial \beta} \sum_{\omega_j} \left[ \frac{(1 - \underline{t}_{\omega_j})^{\beta} (1 + \beta \underline{t}_{\omega_j})}{1 + \beta} \right] - \frac{\partial}{\partial \beta} \sum_{\omega_j} \left[ \frac{(1 - \bar{t}_{\omega_j})^{\beta} (1 + \beta \bar{t}_{\omega_j})}{1 + \beta} \right] \\ &= \sum_{\omega_j} \left[ \frac{-(1 - \underline{t}_{\omega_j})^{\beta} (1 + \beta \underline{t}_{\omega_j}) \left[ \frac{\partial}{\partial \beta} (1 + \beta) \right] + (1 + \beta) \left[ \frac{\partial}{\partial \beta} \left( (1 - \underline{t}_{\omega_j})^{\beta} (1 + \beta \underline{t}_{\omega_j}) \right) \right]}{(1 + \beta)^2} \right] \\ &- \sum_{\omega_j} \left[ \frac{-(1 - \bar{t}_{\omega_j})^{\beta} (1 + \beta \bar{t}_{\omega_j}) \left[ \frac{\partial}{\partial \beta} (1 + \beta) \right] + (1 + \beta) \left[ \frac{\partial}{\partial \beta} \left( (1 - \bar{t}_{\omega_j})^{\beta} (1 + \beta \bar{t}_{\omega_j}) \right) \right]}{(1 + \beta)^2} \right] \\ &= \sum_{\omega_j} \left[ \frac{(1 + \beta) \left[ \frac{\partial}{\partial \beta} (1 - \underline{t}_{\omega_j})^{\beta} (1 + \beta \underline{t}_{\omega_j}) \right] - (0) (1 - \underline{t}_{\omega_j})^{\beta} (1 + \beta \bar{t}_{\omega_j}) - (1) (1 - \bar{t}_{\omega_j})^{\beta} (1 + \beta \bar{t}_{\omega_j})}{(1 + \beta)^2} \right] \\ &- \sum_{\omega_j} \left[ \frac{(1 + \beta) \left[ \frac{\partial}{\partial \beta} (1 - \bar{t}_{\omega_j})^{\beta} (1 + \beta \bar{t}_{\omega_j}) \right] - (0) (1 - \bar{t}_{\omega_j})^{\beta} (1 + \beta \bar{t}_{\omega_j}) - (1) (1 - \bar{t}_{\omega_j})^{\beta} (1 + \beta \bar{t}_{\omega_j})}{(1 + \beta)^2} \right] \\ &= \sum_{\omega_j} \left[ \frac{(1 + \beta) \left[ \frac{\partial}{\partial \beta} (1 - \underline{t}_{\omega_j})^{\beta} (1 + \beta \bar{t}_{\omega_j}) \right] - (1 - (1 - \bar{t}_{\omega_j})^{\beta} (1 + \beta \bar{t}_{\omega_j})}{(1 + \beta)^2} \right] \\ &- \sum_{\omega_j} \left[ \frac{(1 + \beta) \left[ \frac{\partial}{\partial \beta} (1 - \bar{t}_{\omega_j})^{\beta} (1 + \beta \bar{t}_{\omega_j}) \right] - (1 - \bar{t}_{\omega_j})^{\beta} (1 + \beta \bar{t}_{\omega_j})}{(1 + \beta)^2} \right] \\ &- \sum_{\omega_j} \left[ \frac{(1 + \beta) \left[ \frac{\partial}{\partial \beta} (1 - \bar{t}_{\omega_j})^{\beta} (1 + \beta \bar{t}_{\omega_j}) \right] - (1 - \bar{t}_{\omega_j})^{\beta} (1 + \beta \bar{t}_{\omega_j})}{(1 + \beta)^2} \right] \\ &- \sum_{\omega_j} \left[ \frac{(1 + \beta) \left[ \frac{\partial}{\partial \beta} (1 - \bar{t}_{\omega_j})^{\beta} (1 + \beta \bar{t}_{\omega_j}) \right] - (1 - \bar{t}_{\omega_j})^{\beta} (1 + \beta \bar{t}_{\omega_j})}{(1 + \beta)^2} \right] \\ &- \sum_{\omega_j} \left[ \frac{(1 + \beta) \left[ \frac{\partial}{\partial \beta} (1 - \bar{t}_{\omega_j})^{\beta} (1 + \beta \bar{t}_{\omega_j}) \right] - (1 - \bar{t}_{\omega_j})^{\beta} (1 + \beta \bar{t}_{\omega_j})}{(1 + \beta)^2} \right] \right] \\ &- \sum_{\omega_j} \left[ \frac{(1 + \beta) \left[ \frac{\partial}{\partial \beta} (1 - \bar{t}_{\omega_j})^{\beta} (1 + \beta \bar{t}_{\omega_j}) \right] - (1 - \bar{t}_{\omega_j})^{\beta} (1 + \beta \bar{t}_{\omega_j})}{(1 + \beta)^2} \right] \\ \\ &- \sum_{\omega_j} \left[ \frac{(1 + \beta) \left[ \frac{\partial}{\partial \beta} (1 - \bar{t}_{\omega_j})^{\beta} (1 + \beta \bar{t}_{\omega_j}) \right] - (1 - \bar{t}_{\omega_j})^{\beta} (1 + \beta \bar{t}_{\omega_j})} \right] \\ \\ &- \sum_{\omega_j} \left[ \frac{(1 + \beta) \left[ \frac{\partial}{\partial \beta} (1 - \bar{t}_{\omega_j})^{\beta} (1$$

it continues on the next page
$$\begin{split} &= \sum_{\omega_j} \left[ \frac{(1+\beta) \left[ (1+\beta t_{\omega_j}) \frac{\partial}{\partial \beta} (1-t_{\omega_j})^{\beta} + (1-t_{\omega_j})^{\beta} \frac{\partial}{\partial \beta} (1+\beta t_{\omega_j}) \right] - (1-t_{\omega_j})^{\beta} (1+\beta t_{\omega_j})}{(1+\beta)^2} \right] \\ &- \sum_{\omega_j} \left[ \frac{(1+\beta) \left[ (1+\beta t_{\omega_j}) \frac{\partial}{\partial \beta} (1-t_{\omega_j})^{\beta} \log (1-t_{\omega_j}) + (1-t_{\omega_j})^{\beta} \frac{\partial}{\partial \beta} (1+\beta t_{\omega_j}) \right] - (1-t_{\omega_j})^{\beta} (1+\beta t_{\omega_j})}{(1+\beta)^2} \right] \\ &= \sum_{\omega_j} \left[ \frac{(1+\beta) \left[ (1+\beta t_{\omega_j}) (1-t_{\omega_j})^{\beta} \log (1-t_{\omega_j}) + (1-t_{\omega_j})^{\beta} \frac{\partial}{\partial \beta} (1+\beta t_{\omega_j}) \right] - (1-t_{\omega_j})^{\beta} (1+\beta t_{\omega_j})}{(1+\beta)^2} \right] \\ &- \sum_{\omega_j} \left[ \frac{(1+\beta) \left[ (1+\beta t_{\omega_j}) (1-t_{\omega_j})^{\beta} \log (1-t_{\omega_j}) + (1-t_{\omega_j})^{\beta} (0+t_{\omega_j}) \right] - (1-t_{\omega_j})^{\beta} (1+\beta t_{\omega_j})}{(1+\beta)^2} \right] \\ &= \sum_{\omega_j} \left[ \frac{(1+\beta) \left[ (1+\beta t_{\omega_j}) (1-t_{\omega_j})^{\beta} \log (1-t_{\omega_j}) + (1-t_{\omega_j})^{\beta} (0+t_{\omega_j}) \right] - (1-t_{\omega_j})^{\beta} (1+\beta t_{\omega_j})}{(1+\beta)^2} \right] \\ &- \sum_{\omega_j} \left[ \frac{(1+\beta) \left[ (1+\beta t_{\omega_j}) (1-t_{\omega_j})^{\beta} \log (1-t_{\omega_j}) + (1-t_{\omega_j})^{\beta} (0+t_{\omega_j}) \right] - (1-t_{\omega_j})^{\beta} (1+\beta t_{\omega_j})}{(1+\beta)^2} \right] \\ &= \sum_{\omega_j} \left[ \frac{(1+\beta) \left[ (1+\beta t_{\omega_j}) (1-t_{\omega_j})^{\beta} \log (1-t_{\omega_j}) + (1-t_{\omega_j})^{\beta} t_{\omega_j} \right] - (1-t_{\omega_j})^{\beta} (1+\beta t_{\omega_j})}{(1+\beta)^2} \right] \\ &= \sum_{\omega_j} \left[ \frac{(1+\beta) \left[ (1+\beta t_{\omega_j}) (1-t_{\omega_j})^{\beta} \log (1-t_{\omega_j}) + (1-t_{\omega_j})^{\beta} t_{\omega_j} \right] - (1-t_{\omega_j})^{\beta} (1+\beta t_{\omega_j})}{(1+\beta)^2} \right] \\ &= \sum_{\omega_j} \left[ \frac{(1-t_{\omega_j})^{\beta} \left[ (1+\beta) (1+t_{\omega_j}) \log (1-t_{\omega_j}) + (1-t_{\omega_j})^{\beta} t_{\omega_j} \right] - (1-t_{\omega_j})^{\beta} (1+\beta t_{\omega_j})}{(1+\beta)^2} \right] \\ &= \sum_{\omega_j} \frac{(1-t_{\omega_j})^{\beta} \left[ (1+\beta) (1+\beta t_{\omega_j}) \log (1-t_{\omega_j}) + (t_{\omega_j}-1) \right]}{(1+\beta)^2}} \\ &= \sum_{\omega_j} \frac{(1-t_{\omega_j})^{\beta} \left[ (1+\beta) (1+\beta t_{\omega_j}) \log (1-t_{\omega_j}) + (t_{\omega_j}-1) \right]}{(1+\beta)^2} \\ &= \sum_{\omega_j} \frac{(1-t_{\omega_j})^{\beta} \left[ (1+\beta) (1+\beta t_{\omega_j}) \log (1-t_{\omega_j}) + (t_{\omega_j}-1) \right]}{(1+\beta)^2} \\ &= \sum_{\omega_j} \frac{(1-t_{\omega_j})^{\beta} \left[ (1+\beta) (1+\beta t_{\omega_j}) \log (1-t_{\omega_j}) + (t_{\omega_j}-1) \right]}{(1+\beta)^2} \\ &= \sum_{\omega_j} \frac{(1-t_{\omega_j})^{\beta} \left[ (1+\beta) (1+\beta t_{\omega_j}) \log (1-t_{\omega_j}) + (t_{\omega_j}-1) \right]}{(1+\beta)^2} \\ &= \sum_{\omega_j} \frac{(1-t_{\omega_j})^{\beta} \left[ (1+\beta) (1+\beta t_{\omega_j}) \log (1-t_{\omega_j}) + (t_{\omega_j}-1) \right]}{(1+\beta)^2} \\ &= \sum_{\omega_j} \frac{(1-t_{\omega_j})^{\beta} \left[ (1+\beta) (1+\beta t_{\omega_j}) \log (1-t_{\omega_j}) + (t_{\omega_j}-1) \right]}{(1+\beta)^2} \\ &= \sum_{\omega$$

Now, is  $\frac{\partial c_j}{\partial \beta}$  positive or negative? To answer let us simplify this derivative. Suppose there is only one information tranche. Then we have

$$\frac{\partial c_j}{\partial \beta} = \frac{(1 - \underline{t}_{\omega_j})^{\beta} \Big[ (1 + \beta)(1 + \beta \underline{t}_{\omega_j}) \log(1 - \underline{t}_{\omega_j}) + (\underline{t}_{\omega_j} - 1) \Big]}{(1 + \beta)^2} \\ - \frac{(1 - \overline{t}_{\omega_j})^{\beta} \Big[ (1 + \beta)(1 + \beta \overline{t}_{\omega_j}) \log(1 - \overline{t}_{\omega_j}) + (\overline{t}_{\omega_j} - 1) \Big]}{(1 + \beta)^2}$$

or simply

$$\frac{\partial c_j}{\partial \beta} = \frac{(1-\underline{t})^{\beta} \Big[ (1+\beta)(1+\beta\underline{t})\log(1-\underline{t}) + (\underline{t}-1) \Big]}{(1+\beta)^2} - \frac{(1-\overline{t})^{\beta} \Big[ (1+\beta)(1+\beta\overline{t})\log(1-\overline{t}) + (\overline{t}-1) \Big]}{(1+\beta)^2}$$

We want to show that for the given market share,  $s_j$ , or the given information set: the cost can increase or decrease with the business cycle. Then, suppose player j's market share goes from 0 to 0.5. That is,  $\underline{t} = 0$  and  $\overline{t} = 0.5$ . Given that  $\beta \in \{1, 2, 3, ...\}$ , we evaluate the cost function for values of beta from 1 to 10 with parameters  $\underline{t} = 0$  and  $\underline{t} < \overline{t}$ .

In Table (1) you can see that the cost increases with low values of  $\beta$ , then, depending on parameters <u>t</u> and *bart* it decreases as  $\beta$  still increases.

We present cost evaluation for the case the player has a market share with high-risk borrowers, that is with parameters  $\underline{t} = 0.25$  and and  $\underline{t} < \overline{t}$  in Table (2).

We can see that the cost increases for low values of  $\beta$  and then it decreases while  $\beta$  increases for the same values of parameters  $\underline{t}$  and  $\overline{t}$ .

In addition, we include tow sets of graphs (Figure 24a and Figure 24) for these examples.

$\overline{t}$	0.20	0.20	0.20	0.20	0.20	0.20
_						
$\bar{t}$	0.25	0.33	0.50	0.66	0.75	1.00
$\beta$	с	с	с	с	с	с
1	0.0113	0.0345	0.1050	0.1978	0.2613	0.4800
2	0.0174	0.0503	0.1320	0.2093	0.2466	0.2987
3	0.0202	0.0552	0.1267	0.1755	0.1921	0.2048
4	0.0209	0.0540	0.1100	0.1377	0.1443	0.1475
5	0.0202	0.0496	0.0910	0.1060	0.1085	0.1092
6	0.0188	0.0439	0.0735	0.0813	0.0822	0.0824
7	0.0170	0.0378	0.0585	0.0625	0.0629	0.0629
8	0.0151	0.0320	0.0463	0.0483	0.0485	0.0485
9	0.0132	0.0268	0.0365	0.0375	0.0376	0.0376
10	0.0114	0.0222	0.0288	0.0293	0.0293	0.0293
β	$\Delta$	$\Delta$	$\Delta$	$\Delta$	$\Delta$	$\Delta$
1	0.0113	0.0345	0.1050	0.1978	0.2613	0.4800
2	0.0062	0.0158	0.0270	0.0115	-0.0147	-0.1813
3	0.0028	0.0049	-0.0053	-0.0338	-0.0545	-0.0939
4	0.0007	-0.0012	-0.0167	-0.0378	-0.0478	-0.0573
5	-0.0007	-0.0044	-0.0190	-0.0318	-0.0359	-0.0382
6	-0.0014	-0.0057	-0.0175	-0.0247	-0.0263	-0.0268
7	-0.0018	-0.0060	-0.0149	-0.0187	-0.0193	-0.0195
8	-0.0019	-0.0058	-0.0122	-0.0142	-0.0144	-0.0144
9	-0.0019	-0.0053	-0.0098	-0.0108	-0.0109	-0.0109
10	-0.0018	-0.0046	-0.0078	-0.0083	-0.0083	-0.0083

Table 1: Cost & Business Cycle Estimation (example 1)



Figure 24: Cost function examples

t	0.25	0.25	0.25	0.25	0.25	0.25
$\bar{t}$	0.38	0.42	0.50	0.58	0.63	0.75
$\beta$	с	с	с	с	с	с
1	0.0391	0.0549	0.0938	0.1370	0.1641	0.2500
2	0.0534	0.0725	0.1146	0.1542	0.1758	0.2292
3	0.0549	0.0722	0.1064	0.1338	0.1467	0.1719
4	0.0503	0.0643	0.0891	0.1059	0.1127	0.1234
5	0.0433	0.0539	0.0708	0.0805	0.0839	0.0882
6	0.0359	0.0436	0.0546	0.0601	0.0617	0.0634
7	0.0290	0.0344	0.0415	0.0444	0.0452	0.0458
8	0.0230	0.0268	0.0312	0.0328	0.0331	0.0334
9	0.0180	0.0206	0.0233	0.0241	0.0243	0.0244
10	0.0140	0.0157	0.0174	0.0178	0.0179	0.0179
0						
$\beta$	<u>Δ</u>	<u>Δ</u>	<u>Δ</u>	<u>Δ</u>		<u>Δ</u>
1	0.0391	0.0549	0.0938	0.1370	0.1641	0.2500
2	0.0143	0.0176	0.0208	0.0173	0.0117	-0.0208
3	0.0015	-0.0003	-0.0081	-0.0204	-0.0291	-0.0573
4	-0.0046	-0.0080	-0.0174	-0.0279	-0.0339	-0.0484
5	-0.0070	-0.0104	-0.0183	-0.0254	-0.0288	-0.0352
6	-0.0074	-0.0103	-0.0161	-0.0204	-0.0222	-0.0248
7	-0.0069	-0.0091	-0.0131	-0.0156	-0.0165	-0.0175
8	-0.0060	-0.0077	-0.0103	-0.0117	-0.0121	-0.0125
9	-0.0050	-0.0062	-0.0079	-0.0086	-0.0088	-0.0090
10	-0.0040	-0.0049	-0.0059	-0.0063	-0.0064	-0.0065

Table 2: Cost & Business Cycle B Estimation (example 2)

## 5.4 Proof of Lemma 4.5

**Lemma 4.5** Given  $t \sim B(1, \beta)$ , the expected marginal cost for the lender j given its market share  $s_j$ ,  $\bar{c}_j$ , could decrease or increase as the business cycle improves. That is,

$$\frac{\partial \bar{c}_j}{\partial \beta} = \frac{\partial}{\partial \beta} \frac{1}{1+\beta} - \frac{\partial}{\partial \beta} c_j \ge 0$$
(38)

*Proof.* We only need to prove that there is a case where

$$\frac{\partial \bar{c}_j}{\partial \beta} < 0$$

and a case where

$$\frac{\partial \bar{c}_j}{\partial \beta} > 0$$

Then

$$\begin{split} \frac{\partial \bar{c}_{j}}{\partial \beta} &= \frac{\partial}{\partial \beta} \frac{1}{1+\beta} - \frac{\partial}{\partial \beta} c_{j} \\ &= -\frac{1}{(1-\beta)^{2}} \\ &- \sum_{\omega_{j}} \left( \frac{(1-\underline{t}_{\omega_{j}})^{\beta} \Big[ (\underline{t}_{\omega_{j}}-1) + (1+\beta)(1+\beta \underline{t}_{\omega_{j}}) \ln (1-\underline{t}_{\omega_{j}}) \Big]}{(1+\beta)^{2}} \right) \\ &- \frac{(1-\bar{t}_{\omega_{j}})^{\beta} \Big[ (\bar{t}_{\omega_{j}}-1) + (1+\beta)(1+\beta \overline{t}_{\omega_{j}}) \ln (1-\bar{t}_{\omega_{j}}) \Big]}{(1+\beta)^{2}} \Big) \\ &= -\frac{1}{(1-\beta)^{2}} \\ &- \sum_{\omega_{j}} \frac{(1-\underline{t}_{\omega_{j}})^{\beta} \Big[ (\underline{t}_{\omega_{j}}-1) + (1+\beta)(1+\beta \underline{t}_{\omega_{j}}) \ln (1-\underline{t}_{\omega_{j}}) \Big]}{(1+\beta)^{2}} \\ &+ \sum_{\omega_{j}} \frac{(1-\bar{t}_{\omega_{j}})^{\beta} \Big[ (\bar{t}_{\omega_{j}}-1) + (1+\beta)(1+\beta \overline{t}_{\omega_{j}}) \ln (1-\bar{t}_{\omega_{j}}) \Big]}{(1+\beta)^{2}} \\ &= \sum_{\omega_{j}} \frac{(1-\bar{t}_{\omega_{j}})^{\beta} \Big[ (\underline{t}_{\omega_{j}}-1) + (1+\beta)(1+\beta \underline{t}_{\omega_{j}}) \ln (1-\bar{t}_{\omega_{j}}) \Big]}{(1+\beta)^{2}} \\ &- \sum_{\omega_{j}} \frac{(1-\underline{t}_{\omega_{j}})^{\beta} \Big[ (\underline{t}_{\omega_{j}}-1) + (1+\beta)(1+\beta \underline{t}_{\omega_{j}}) \ln (1-\underline{t}_{\omega_{j}}) \Big]}{(1+\beta)^{2}} \\ &- \frac{1}{(1+\beta)^{2}} \end{split}$$

Suppose that the winner player has only one information tranche. Then we have

$$\frac{\partial \bar{c}_j}{\partial \beta} = \frac{(1-\bar{t})^\beta \Big[ (\bar{t}-1) + (1+\beta)(1+\beta\bar{t})\ln(1-\bar{t}) + (1+\beta)^2 \Big]}{(1+\beta)^2} - \frac{(1-\underline{t})^\beta \Big[ (\underline{t}-1) + (1+\beta)(1+\beta\underline{t})\ln(1-\underline{t}) + (1+\beta)^2 + (1+\beta)^2 \Big]}{(1+\beta)^2}$$

Now let's estimate this derivative for  $\underline{t} = 0.20$  and  $\overline{t} \in \{0.25, 0.33, 0.50, 0.66, 0.75\}$ . In the following table we present the values of this function for  $\beta \in \{1, 2, ..., 10\}$ .

$\overline{t}$	0.20	0.20	0.20	0.20	0.20	0.20
_						
$\bar{t}$	0.25	0.33	0.50	0.66	0.75	1.00
$\beta$	с	с	с	с	с	с
1	0.4888	0.4656	0.3950	0.3022	0.2388	0.0200
2	0.3159	0.2831	0.2013	0.1241	0.0867	0.0347
3	0.2298	0.1948	0.1233	0.0745	0.0579	0.0452
4	0.1791	0.1460	0.0900	0.0623	0.0557	0.0525
5	0.1464	0.1171	0.0757	0.0607	0.0582	0.0574
6	0.1240	0.0990	0.0694	0.0616	0.0607	0.0605
7	0.1080	0.0872	0.0665	0.0625	0.0621	0.0621
8	0.0960	0.0791	0.0648	0.0628	0.0627	0.0626
9	0.0868	0.0732	0.0635	0.0625	0.0624	0.0624
10	0.0795	0.0688	0.0622	0.0616	0.0616	0.0616
β	$\Delta$	$\Delta$	$\Delta$	$\Delta$	$\Delta$	$\Delta$
1	0.4888	0.4656	0.3950	0.3022	0.2388	0.0200
2	-0.1728	-0.1825	-0.1937	-0.1781	-0.1520	0.0147
3	-0.0861	-0.0882	-0.0780	-0.0496	-0.0289	0.0105
4	-0.0507	-0.0488	-0.0333	-0.0122	-0.0022	0.0073
5	-0.0327	-0.0290	-0.0144	-0.0016	0.0025	0.0049
6	-0.0224	-0.0181	-0.0063	0.0009	0.0024	0.0030
7	-0.0161	-0.0118	-0.0029	0.0009	0.0015	0.0016
8	-0.0120	-0.0081	-0.0017	0.0003	0.0005	0.0006
9	-0.0092	-0.0058	-0.0013	-0.0003	-0.0002	-0.0002
10	-0.0073	-0.0045	-0.0013	-0.0008	-0.0008	-0.0008

Table 3: Cost & Business Cycle Estimation (example 3)

In Table 3, we show that for  $\underline{t} = 0.20$  and  $\overline{t} = 0.66$  the value of the expected marginal cost decreases up to  $\beta = 5$ , then it increases.

In Figure (25) the expected marginal cost decreases in two of the three cases for low values of  $\beta$ . Then, when  $\beta > 4$ , the expected marginal cost increases.



Figure 25: Cost function from Table 3

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