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Luis Armando Stack Sánchez

Advisor: Julen Berasaluce

WHY SHOULD I LET YOU STAY? A COMMON AGENCY PROBLEM WITH VERTICAL INTEGRATION

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1 Introduction

On September 2018 Walmart, the giant of the retail market, acquired Cornershop for 225 mdd, a startup company that buys groceries for you and takes them to your home, workplace or any place. Basically, Cornershop is a delivery application, his principal market is the grocery market. Cornershop works as follows: you go into the app, choose a store and then select all the things you need. There are several stores in the application. Also, Cornershop prices are, in most cases, the same prices that you find in the store. Cornershop profits comes from his distance fee and publicity.

However, Walmart already had an e-commerce application which was more popular than Cornershop at that time (considering app downloads in playstore and applestore in early 2019), so it becomes interesting to wonder why did Walmart buy Cornershop. Even more interesting than this question is the fact that Walmart allowed Cornershop to continue distributing other companies' products. With a vertical integration like this one, it is possible to imagine that Walmart would prefer to block his competitors from using Cornershop to reach the downstream market. So, why would Walmart let them stay?

There are different reasons why Walmart would allow Cornershop to distribute others principals products. First, allowing another supermarkets to distribute through Cornershop, Walmart can acquire their information demand. If this is the case, having information demand of the competitors gives Walmart a substantial advantage in the market. A second reason for allowing other principals' to distribute their product is a change in the Walmart business model toward Amazon's business model with low fixed cost and big scale economies. Amazon's business strategy is to distribute other principals' products even when Amazon has their own similar products. Amazons benefits are given by the retail fee and the membership from premium users.

The purpose of this work is to answer the question: which are the incentives for the Principal to let rival's product be distributed after vertical integration with the retailer. This work is organized as follow: second section is a literature review about common agency models, common agent as a collusion mechanism, exclusive dealing and an adverse selection model with vertical integration. Next section, I reproduced Martimort (1996) results regarding perfect information in order to obtain intuition about common agency models. Fourth section describes a common agency model with vertical integration of a principal with the agent. In the next section I describe the Cornershop case using the previous developed model of section four. Sixth section is about the previous model of vertical integration with imperfect information. Finally, I present the conclusions to this work.

2 Related Literature

In my work I will use a common agency model to describe the Cornershop case. Bernheim and Whinston (1986) common agency model arises as an alternative to the traditional bilateral agent model given that there are situations where the Principals choose incentive schemes noncooperatively. In their paper Bernheim and Whinston differentiated between intrinsic and delegated common agency. On the one hand, delegated common agency are those who the Principals given them the right to make certain decisions. On the other hand, intrinsic common agency are those who naturally have the capacity to make some decisions. Bernheim Whinston described equilibrium existence for each common agency type. These equilibria were efficiently reach, that is, equilibria were obtain at the lowest possible cost.

Regarding the common agency and collusion relationship, Bernheim and Whinston (1985) explore this in a context where firms choose their agents, product level, prices and compensation schemes non cooperatively. Common agency provides a collusion mechanism through which firms can collude. Bernheim and Whinston (1985) considers similar but differentiated goods as it is assumed in my paper. Nevertheless, unlike my work in Bernheim and Whinston (1985) I use a competition parameter that represents direct competition of the products in the downstream market. Another difference with my work is that the authors consider an stochastic demand. Finally, Bernheim Whinston (1985) does not explore the differences between common agency and vertical integration which is one of the purposes of my work.

Focusing on the incentives to use a common agent, Gal-Or (1991) argues that in an oligopoly common agency might not be the best option especially when agents have private information about their costs. In Gal-Or (1991), manufactures compete in the incentive provision to the common agent. Gal-Or analyzes an exclusive dealing and common agency structure regarding their benefits to the manufacturer. Also, Gal-Or (1991) uses differentiated goods, howsoever, she does not uses a competition parameter between the goods. Additionally, another difference between my work and this article is the private information: Gal-Or allows this private information to be different among agents when they are in an exclusive dealing structure. Agents have the same cost distribution, but the realization of the cost parameter can be different for each agent. Also, in Gal-Or (1991) the greater the uncertainty concerning the characteristics of the retailers and the more significant the differentiation among brands, the more likely it is that firms will establish exclusive channels of distribution for their products. In my model, a competition parameter increase between goods increases the incentives of the vertical integrated principal to distribute the competitors product. Finally, Gal-Or (1991) uses a stochastic demand which I do not consider in my model.

The decision of vertically integrate in an Adverse Selection model was explore by Gal-Or (1999). In her article, Gal-Or suggests that vertical separation of the sales force is more likely that vertical integration when goods are very substitutes. Alike my model, Gal-Or (1999) uses differentiated products, nevertheless, she does not explore the common agency structure. Furthermore, another difference with my model is the stochastic demand. In Gal-Or (1999) there are two effects that influence Principal decision regarding vertical integration: quantity produced and output schedule reaction function slope. While the former 'expected quantity' effect implies greater incentives for vertical integration the latter 'steepness' effect has the reverse implication of moderating the firm's incentives to vertically integrate. Gal-Or demonstrate that the degree of correlation between demand schedules determines which of the two opposing effects is dominant.

Finally, my work is closer to David Martimort (1996). Martimorts work question: What are the costs and benefits of exclusive dealing and why do manufacturers choose to organize their retailing markets in this way instead of taking a common retailer? Martimort (1996) first develops a theoretical model that studies competition between hierarchies under the assumption of secret wholesale contracts. Second, it analyzes a game of choice of retailing channels between rival manufacturers. Depending on the extent of the adverse selection problem and on the complementarily or substitutability of their brands, manufacturers prefer to use either a common or an exclusive retailer. The model that Martimort uses is an Adverse Selection model where retailers observe their cost information of distributing in the downstream market. Manufacturers do not have access to the cost information.Some of the main conclusions of Martimort (1996) are that equilibria characterization is sensitive to product differentiation, as well as three effects that influence the structure choice of the agent: competition as a retail discipline, lack of downstream coordination under exclusive dealing and the profit loss induced by non-cooperative behavior between the manufacturers under common agency. As in Martimort (1996) I consider how changes in the competition parameter affects the equilibrium of the model. Nevertheless, in my model, as mentioned before, a competition parameter increase between goods increases the incentives of the vertical integrated principal to distribute the competitors product, this is, it move us away from exclusive dealing. This result is not found in Martimort (1996) and is an implication of the vertical integration.

3 The contracting game with perfect information, reproducing Martimort (1996)

Consider a game with two principals and two retailers. Principals produce a good at cost $c(q_i)$, and use the retailers to distribute their product in the downstream market. In my case of study the supermarkets are the principals and the retailers are the platforms such as Cornershop or the supermarkets platform of e-commerce. Also, the principals cost $c(q_i)$ can be interpreted as the storage cost of the products. When distributing the product in the downstream market, agents obtain private information about their efficiency in supplying the final good. This private information can be model as a cost θ_i, which is distributed uniformly at $[\underline{\theta}, \overline{\theta}]$. This information is perfectly correlated between the retailers. So, to get their product to the downstream market, principals have to offer a contract to the retailers. This contract can be, without loss of generality, a nonlinear wholesale payment schedule $x_i(\hat{\theta}_i)$, which is a monetary transfer that the retailer must pay as a function of his choice of quantities of the intermediate good. The retailers have a profit function of the downstream market $v(q_i(\hat{\theta}_i), q_j(\hat{\theta}_j), \theta)$. Thus, the principals' i utility function is $-c(q_i) + x_i(\hat{\theta}_i)$. The retailer i utility function is $-x_i(\hat{\theta}_i) + v(q_i(\hat{\theta}_i), q_j(\hat{\theta}_j), \theta)$.

To guarantee some regularity to the problem is necessary to make some assumptions:

1 $v(q_1, q_2, \theta)$ is concave in q_1, and three times continuously differentiable, $c(q_i)$ is convex in q_i, and twice continuously differentiable.

2 v_{12} has a constant sign. If $v_{12} > 0$, one has $v_{11} + 2v_{12}$.

3 $v_\theta < 0$

4 $v_{1\theta} < 0$

5 v_θ is concave in q_1.

6 $v_{1\theta\theta} \leq 0$ and $v_{12\theta} \leq 0$

Also, the timing of the game is as follows:

1 In the first period, the retailers learn the value of their information.

2 After that, the principals simultaneously and non-cooperatively offer the secret contracts $(x_i(q_i))$ for $i \in 1, 2$ to their retailer.

3 Third, the retailers accept or refuse the proposal they respectively received and get their outside option which I will normalize to zero.

4 Fourth, the retailers make their reports to their respective principal, this is they report the value of their cost in the downstream market.

5 Finally, the transfer $(x_i(\hat{\theta}_i))$ is made by the agent to the respective principal, and outputs $q_i(\hat{\theta}_i)$, are implemented in the downstream market.

3.1 Equilibrium under exclusive dealing and perfect information

First instance, I assume perfect information so that every principal knows the true value of the retailers cost θ_i in the downstream market. The timing of the game is the one described in the previous section. The equilibrium is characterizes as follow: If the Agent accept (Y) Principals contract, then he will receive a payment of $-x_i(\hat{\theta}_i) + v(q_i(\hat{\theta}_i), q_j(\hat{\theta}_j), \theta)$ and the Principal will get $-c(q_i)+x_i(\hat{\theta}_i)$. If the Agent declines Principals contract, then both will get a payment equal to zero. Hence, the Agent will accept the Principals contract if $-x_i(\theta_i) + v(\cdot) \geq 0$. This is the individual rationality constraint of the Agent. So, the Principals contract has a range $x_i(\theta_i) \in [c(q_i), v(\cdot)]$. I the contract is less than $c(q_i)$ the principal will have a negative benefit, also if the contract is bigger than $v(\cdot)$, then the Agent will not accept it because he will have negative profits. In fact, the Principals will always choose the bigger contract possible, because this maximizes his profit.

Moreover, the Principal knows the Agents retail cost so he can verify the information that the Agent reports. As a result, we can use backward induction to find that the Principal will offer a contract so that $x_i(\theta_i) = v(\cdot)$. Therefore, the Agent is indifferent between accepting or refusing the Principals contract. In other words, the Principal extracts all the informational rent from the Agent. The Principal chooses the quantities solving:

$$Max_{q_i} \quad \Pi_i = -c(q_i) + x_i(\theta_i)$$
$$\text{s.t. -x}_i(\theta_i) + v(q_i(\theta_i), q_j(\theta_j)) \geq 0$$

The first order conditions:

$$-c'(q^F(\theta)) + v_i(q^F(\theta), q^F(\theta)) = 0, \qquad i \in \{1, 2\}$$

In order to obtain more intuition about the result, let's assume a specific function $v^i(q_i(\theta_i), q_j(\theta_j))$ for each principal that satisfies the initial assumptions over the retailers income function. Also assume a cost function $c(q_i) = cq_i$:

$$v^1(q_1(\theta_1), q_2(\theta_j), \theta) = (a_1 - \theta_1)q_1 - \tfrac{b_1}{2}q_1^2 - rq_1q_2$$

$$v^2(q_1(\theta_1), q_2(\theta_j), \theta) = (a_2 - \theta_2)q_2 - \tfrac{b_2}{2}q_1^2 - rq_1q_2$$

In these functions $(a_i - \theta_i)$ can be interpreted as the market size of the product, the b_i is a parameter of quality differentiation between the products and r is the competition cost between substitute products.

First order conditions of the exclusive dealing problem for each principal are described by:

$$-c + (a_1 - \theta_1) - b_1q_1 - rq_2 = 0 \tag{1}$$

$$-c + (a_2 - \theta_2) - b_2q_2 - rq_1 = 0 \tag{2}$$

I will assume some symmetries so that the goods have the same market size and same cost. Goods are substitutes but with a degree of differentiation: $a_1 = a_2 = a, \theta_1 = \theta_2 = \theta, c_1 = c_2 = c$. Solving for the equilibrium quantities under exclusive dealing:

$$q_1^E = \frac{(b_2 - r)(a - \theta - c)}{b_1 b_2 - r^2} \tag{3}$$

$$q_2^E = \frac{(b_1 - r)(a - \theta - c)}{b_1 b_2 - r^2} \tag{4}$$

The benefits under exclusive dealing with perfect information for the Principal 1 are given by:

$$\Pi_1^E = -cq_1^E + (a - \theta)q_1^E - \frac{b_1}{2}(q_1^E)^2 - rq_1^E q_2^E \tag{5}$$

3.2 The common agency game with perfect information

Let's consider a situation in which each principal offer a common contract to the retailer, so that the retailer can distribute both products in the downstream market. By distributing both products, the Agent has a benefit in the downstream market described by $v^c(q_1, q_2, \theta)$, so his total benefits are described as $-x_1(q_1) - x_2(q_2) + v^c(q_1, q_2, \theta)$. The order of the game is as follow:

1 In the first period, the Agent learns the value of his information.

2 The Principals offer simultaneously a common contract.

3 The common retailer accepts or refuses simultaneously both contract offers.If the Agent refuses, he gets his outside option which I normalize to zero. The Principals also get a benefit equal to zero.

4 The Agent reports his retail cost to the Principals.

5 Finally, the transfer $(x_i(\hat{\theta}_i))$

Using the same logic than in the previous section and by backwards induction it is possible to deduce that the Principals will offer a contract such that $x_i(q_i) = v^c(q_i, q_j, \theta)$. Also, Principals problem to solve for quantities is:

$$Max_{q_i} \quad \Pi_i = -c(q_i) + x_i(\theta_i) \qquad i \in \{1, 2\}$$
$$\text{s.t. -x}_j(\theta_j) - x_i(\theta_i) + v^c(q_1, q_2, \theta) \geq 0$$

First order conditions:

$$-c'(q_i(\theta)) + v_i^c(q_(\theta), q_j(\theta), \theta) \qquad i \in \{1, 2\} \tag{6}$$

Now, I will assume a specific function that satisfies the assumptions made before. The common benefit function for the retailer under the common agency game:

$$v^c(q_1, q_2, \theta) = (a_1 - \theta_1)q_1 - \frac{b_1}{2}q_1^2 + (a_2 - \theta_2)q_2 - \frac{b_2}{2}q_2^2 - rq_1q_2$$

With $\quad a - \theta_i - c > 0, \quad b_1 > r, \quad b_2 > r, \quad b^2 > r^2, \quad c > 0$

This specifications emphasize the fact that, there is no cost advantage of the horizontal integration of the retailers activities through a common agency structure. Also, there is no contracting opportunities for the principals because no information other than what is available under exclusive dealing is obtained.

The maximizing problem of the Principal i:

$$Max_{q_i} \quad \Pi_i = -cq_i + x_i(\theta_i)$$

$$s.t. - x_j(\theta_j) - x_i(\theta_i) + (a_i - \theta_i)q_i - \frac{b_i}{2}q_i^2 + (a_j - \theta_j)q_j - \frac{b_j}{2}q_j^2 - rq_iq_j \geq 0$$

Solving for each Principal and imposing symmetries $a_1 = a_2 = a, \quad \theta_1 = \theta_2 = \theta, \quad c_1 = c_2 = c,$ the equilibrium quantities are given by:

$$q_1^C = \frac{(b_2 - r)(a - \theta - c)}{b_1 b_2 - r^2} \tag{7}$$

$$q_2^C = \frac{(b_1 - r)(a - \theta - c)}{b_1 b_2 - r^2} \tag{8}$$

Note that exclusive dealing and common agency equilibria under perfect information are the same, so principals profits are also the same.

4 The common agency game with vertical integration

Now assume that the Principal 1 and the common retailer are vertically integrated. Then the timing of the game is as follows:

1 Agent 1 and the Principal 1 get vertically integrated.

2 The Agent learns the value of his information.

3 Principal 2 offers the Agent a contract to distribute his product.

4 The Agent accepts or refuse or refuses the Principals contract. If the Agent declines the Principals offer, he gets his outside option which is the Principal 1 benefits under exclusive dealing and perfect information. If the Agent refuses Principal 2 offer, then Principal 2 will get a payment equal to zero.

5 The Agent reports his retail cost.

6 The Agent make the transfer.

Furthermore, Principal 2 knows that the outside option of the Agent is the Principal 1's benefit under exclusive dealing and perfect information. Thus, Principal 2 contract will give the Agent at least this benefit. Otherwise, the Agent would not accept Principals 2 contract. Such logic is shown in the participation constraint.

$$Max_{q1,q_2} \qquad \Pi_2 = -c(q_2) + x_2(q_2)$$

$$s.t. - x_2(q_2) - x_1(q_1) + v^c(q_1, q_2, \theta) \geq 0$$

$$s.t. \qquad - c(q_1) + x_1(q_1) \geq \alpha$$

α is Principals 1 benefit under exclusive dealing

Replacing the restrictions in the problem and taking first order conditions:

$$-c'(q_1(\theta)) + v_1^c(q_1(\theta), q_2(\theta), \theta) = 0 \tag{9}$$

$$-c'(q_2(\theta)) + v_2^c(q_1(\theta), q_2(\theta), \theta) = 0 \tag{10}$$

These conditions characterize a Perfect Subgame Equilibrium in this game.

Moreover, in order to obtain a better intuition about the vertical integration solution I will use a specific function: the common agency function I used before.

$$v^c(q_1, q_2, \theta) = (a_1 - \theta_1)q_1 - \frac{b_1}{2}q_1^2 + (a_2 - \theta_2)q_2 - \frac{b_2}{2}q_2^2 - rq_1q_2$$

With $\quad a - \theta_i - c > 0, \quad b_1 > r, \quad b_2 > r, \quad b^2 > r^2, \quad c > 0$

The equilibrium quantities are given by:

$$q_1 = \frac{(b_2 - r)(a - \theta - c)}{b_1b_2 - r^2} \tag{11}$$

$$q_2 = \frac{(b_1 - r)(a - \theta - c)}{b_1b_2 - r^2} \tag{12}$$

Which are the same quantities as in exclusive dealing and common agency with perfect information. This is because with perfect information principal 2 is able to extract all the informational rent from the agent when distributing his product. Nevertheless, the outside option changes, so Principal 2 has to give Agent 1 at least the same benefit that he has when vertically integrates and makes an exclusive deal with Principal 1. Note that the mathematically implication is that $c(q_1)$ appears in the maximization problem. Also, an interesting implication of the vertical integration between Principal 1 and the agent is that under vertical integration Principal 1 knows the retail cost of Principal 2.

5 The Cornershop case

In this section I propose a model to analyze the Cornershop acquisition by Walmart. Cornershop is an online application where people can buy their groceries from distinct supermarkets including Walmart and other competitors. Once the order is send, Cornershop employees go to the supermarket you choose and buy your groceries for you. Cornershop employees do not require an authorization from the supermarket. This can be interpreted as if the supermarkets can not refuse to distribute their product, so they do not obtain a transfer from the Agent. In the model this is $x_2(\hat{\theta}) = 0$. Also, the game is no longer a sequential game given that Principal 2 does not make any choice. Thus, the solution of the game is a Nash equilibrium.

The retailers payment by accepting the common contract of the principal is $-x_1(q_1) - x_2(q_2) + v^c(q_1, q_2, \theta)$. I assume perfect information, so Principal 2 knows the retailer cost of distributing his product in the downstream market. Note that vertical integration implies that the fact that double marginalization is avoided, this is $x_1(_1)$, is not in the Agents benefit function.

Given that the Principal 2 does not make any choice, Agent 1 is the one that decides the quantities distributed of each product in the market. Also, Principal 2's costs do not show in the maximization problem. As a result, Agent 1's maximization problem is given by:

$$Max_{q_1,q_2} \qquad \Pi_1 = -c(q_1) - x_2(q_2) + v^c(q_1, q_2, \theta)$$

As we mention before, Principal 2 cannot decide if he wants to have his product distributed by the retailer. This assumption implies that $x_2(\hat{\theta}) = 0$. This means that no informational rent is transferred to the principal 2. Note that Π_1 is a concave function where $v^c_{11} v^c_{22} > (v^c_{12})^2$. First order conditions of this program with respect to q_1 and q_2:

$$-c'(q_1) + v^c_1(q^v_1(\theta), q^v_2(\theta), \theta) = 0 \tag{13}$$

$$v^c_2(q^v_1(\theta), q^v_2(\theta), \theta) = 0 \tag{14}$$

These first order conditions characterize the equilibrium quantities $q_1(\hat{\theta})$ and $q_2(\hat{\theta})$. The Nash equilibrium of this game:

$$EN_1^v = \left\{x_2(\hat{\theta}) = 0, \left(q_1(\hat{\theta}), q_2(\hat{\theta})\right)\right\}$$

For the vertical integration problem I consider the same common benefit function for the retailer as in the common agency problem and the same cost:

$$v^c(q_1, q_2, \theta) = (a_1 - \theta_1)q_1 - \frac{b_1}{2}q_1^2 + (a_2 - \theta_2)q_2 - \frac{b_2}{2}q_2^2 - rq_1q_2, \quad cq_1$$

Note that in this case, Agent 1 chooses both quantities in the downstream market. Also, Agent 1 has a particular program where q_1 generates a cost $c(q_1)$. This cost can be interpreted as a storage cost. Nevertheless, q_2 does not have a storage cost. I will explore the implications of this latter. The program for Principal 1:

$$Max_{q_1q_2} \quad \Pi_1 = -cq_1 + (a_1 - \theta_1)q_1 - \frac{b_1}{2}q_1^2 + (a_2 - \theta_2)q_2 - \frac{b_2}{2}q_2^2 - rq_1q_2 \tag{15}$$

The firt order conditions of the problem:

$$-c + (a - \theta_1) - b_1q_1 - rq_2 = 0 \tag{16}$$

$$(a_2 - \theta_2) - b_2q_2 - rq_1 = 0 \tag{17}$$

Solution of this program is given by:

$$q_1^v = \frac{(a_1 - \theta_1 - c)b_2 - (a_2 - \theta_2)r}{b_1 b_2 - r^2} \tag{18}$$

$$q_2^v = \frac{(a_2 - \theta_2)b_1 - (a_1 - \theta_1 - c)r}{b_1 b_2 - r^2} \tag{19}$$

Assuming symmetry in the parameters a_i, θ_i, the equilibrium quantities are:

$$q_1^v = \frac{(b_2 - r)(a - \theta - c) - rc}{b_1 b_2 - r^2} \tag{20}$$

$$q_2^v = \frac{(b_1 - r)(a - \theta - c) + b_1 c}{b_1 b_2 - r^2} \tag{21}$$

Note that the quantities in this equilibrium are different from both last equilibria. In particular, the quantity of q_2 in the downstream market is bigger under vertical integration that under exclusive dealing and the standard common agency program. Also, note that the quantity of q_1 in the market is smaller under vertical integration. The quantity q_1 is affected by the term rc which is the competition cost.

Thus, it is necessary to compare profits between exclusive dealing and the vertical integration problem. It is possible to show that conditions exist for which the difference between $\Pi_1^v - \Pi_1^E$ is positive.

Proposition 1: $r < b_2 < \dfrac{r(2a - 2\theta - c)}{2(a - \theta - c)}$ Is a sufficient condition for the vertical integration to be profitable.

Proof: Let be $x = (b_2 - r), z = (b_1 b_2 - r^2), d = (a - \theta - c), y = (b_1 - r)$

$$\Pi_1^V - \Pi_1^E = \frac{-rc}{z}[a - \theta + \frac{rcb_1}{2z}] + \frac{rc}{z}[\frac{ryd + rcb_1 + cz}{z}] + \frac{yd + b_1 c}{z}[a - \theta - \frac{b_2}{2}(\frac{yd + b_1 c}{z})]$$

Analyzing the first and second terms we get a positive difference if

$$r < b_2 < \frac{r(2a - 2\theta - c)}{2(a - \theta - c)}$$

This been satisfied, the condition for the third term to be always positive is also satisfied. Then the difference of benefits is always positive.

A positive difference means that the Agent prefers to distribute Principal 2's product rather than make exclusive dealing with Principal 1. The next proposition will clarify the intuition behind this result.

Prpoposition 2: If vertical integration is profitable, a more aggressive competition between the products move us away from exclusive dealing contracts. In other words, q_2 increases when the competition parameter increases, this is $\dfrac{\partial q_2^v}{\partial r} > 0$.

Proof: Analyzing q_2^v respect the competition parameter:

$$\frac{\partial q_2^v}{\partial r} = \frac{[2rb_1(a - \theta) - (a - \theta - c)(b_1 b_2 + r^2)]}{(b_1 b_2 - r^2)^2} \geq 0 \tag{22}$$

A sufficient condition for $\dfrac{\partial q_2^v}{\partial r} > 0$ is given by $r < b_2 < \dfrac{2b_1 r(a - \theta) - r^2(a - \theta - c)}{b_1(a - \theta - c)}$ Which is satisfied when the sufficient condition of proposition 1 is satisfied, then proposition 2 follows.

This result implies that the competition parameter (r) influences the quantity of q_2 in the market. An increase in the competition parameter (r) causes the Agent 1 to allow a bigger quantity of the Principals 2 product to be distributed by the retailer. In other words, more aggressive competition increases the incentive of Principal 1 to avoid a exclusive dealing contract with the retailer.

The intuition behind this result is the following: Principal 1 has a cost of producing q_1^v but does not has a cost when q_2^v is produced, because the cost for q_2^2 is paid by the Principal 2. Is possible to think this cost as an inventory cost. Also, the benefit function for distributing the product under symmetry and the sufficient conditions for the vertical integration to be profitable implies that the increase in q_2^v brings more benefits that the increase in q_1^v. So, when products are more substitutes (r increases) the Principal 1 prefers to distribute q_2^v, because he gets more benefits and less inventory cost. Let's think for a moment in Amazons' business model, which has low inventory costs and high revenue from retailing products from other producers. Therefore, it makes sense that the Principal 1 will let the q_2^v product be distributed in the downstream market. This result suggest that more aggressive competition between products moves us away from exclusive dealing contracts.

6 The common agency game with vertical integration and imperfect information

Regarding the retail information of the principals, it is possible to notes that the informational structure has changed: principal 1 is integrated with the agent so he knows his retail cost. This implies that there is no double marginalization of q_1 in the downstream market. Nevertheless, principal 2 still does not know his retail price. This information is known by the agent and in consequence by the principal 1. This context might be because the principal 1 is more efficient that other retailers after the vertical integration. This problem can be modeled as if principal 2 offers a truth telling contract to principal 1.

First, I look for a Nash equilibrium in direct revelation mechanism:

$$\theta \in argmax_{\theta_2} - x_2(\hat{\theta_2}) + v^c(q_1(\theta), q_2(\hat{\theta_2}), \theta)$$

$$\Rightarrow -\dot{x_2}(\theta) + \dot{q_2}(\theta)v_2^c(q_1(\theta), q_2(\theta), \theta) = 0$$

Second, I described the utility function of the principal 1 after vertical integration with the agent and its first order condition with respect to θ:

$$U_1(\theta) = -c(q_1(\theta)) - x_2(\theta) + v^c(q_1(\theta), q_2(\theta), \theta)$$

$$\dot{U_1}(\theta) = -c_1'(q_1(\theta))\dot{q_1}(\theta) - \dot{x_2}(\theta) + v_1^c(\cdot)\dot{q_1}(\theta) + v_2^c(\cdot)\dot{q_2}(\theta) + v_\theta(\cdot)$$

Using the previous condition of the nash equilibrium in the truth telling mechanism it is possible to rearrange the expression to get:

$$\dot{U_1}(\theta) = -c_1'(q_1(\theta)) + v_1^c(\cdot)\dot{q_1}(\theta) + v_\theta(\cdot)$$

Usually, under standard conditions this expression would be affected by the so call competing effect, but in this case is also affected by principal 1 cost. Therefor, principals 2 problem can be expressed as:

$$max_{q_2,U_1} \qquad \int_{\underline{\theta}}^{\overline{\theta}} f(\theta)\left[-c_2(q_2(\theta)) + v^c(q_1(\theta), q_2(\theta), \theta) - U_1(\theta)\right] d\theta$$

$$s.t. \qquad \dot{U}_1(\theta) = -c_1'(q_1(\theta)) + v_1^c(\cdot)\dot{q}_1(\theta) + v_\theta(\cdot)$$

$$U_1(\theta) \geq 0 \qquad \forall\theta$$

After solving the Hamiltonian it is possible to get:

$$\dot{q}_1(\theta) = \frac{c_2'(q_2(\theta)) - v_2^c(\cdot) - \dfrac{F(\theta)}{f(\theta)} v_{2\theta}(\cdot)}{v_{12}^c(\cdot)} \frac{f(\theta)}{F(\theta)}$$

And the corresponding payment from principal 1 to principal 2 is given by:

$$x_2(\theta) = v^c(\cdot) - c_1(q_1) + \int_{\underline{\theta}}^{\overline{\theta}} -c_1'(q_1(\theta))\dot{q}_1(\theta) + v_1^c(\cdot)\dot{q}_1(\theta) + v_\theta(\cdot)\, du$$

It is possible to see that the contract that the Principal 2 offers ($x_2(\theta)$) subtracts part of Principals 1 costs of production. This did not happen before without vertical integration. This implies that there is some production internalization of the rival as in common agency.

Using the specific function for $v^c(\theta)$ and assuming a uniform function for theta I can express this results as:

$$\dot{q}_1(\theta) = \frac{c_2 - \left[(a-\theta) - b_2 q_2 - r q_1\right] + \left(\dfrac{1}{\theta - \underline{\theta}}\right)}{-r} \left[\theta - \underline{\theta}\right] \tag{23}$$

Note that q_2 is a variable that P2 can choose freely. For simplicity, I will assume symmetry so that $q_1 = q_2$ and solve for q_1. Remember that the decision of how much of q_2 is distributed is subject to P1 approval. Let $\dot{q}_1(\theta) = 0$ and solving for q_1:

$$q_1^I = \frac{(a - \theta - c_2)(\theta - \underline{\theta}) - 1}{(\theta - \underline{\theta})(b_2 - r)} \tag{24}$$

Now let's compare the q_1 of vertical integration with perfect information with the one resulting from imperfect information. As I assume $q_1 = q_2$ in perfect information I will do the same for imperfect information, this implies that $b_1 = b_2$, then:

$$q_i^I < q_i^v \quad i = 1, 2$$

The quantity produces under perfect information is bigger than the one with imperfect information.

7 Conclusions

In this work I have developed a common agency model with vertical integration. With perfect information, the model has the same results that exclusive dealing and common agency models. Nevertheless, one of the principal results of this work is that, in the Cornershop case, vertical integration is profitable for the Principal that integrates with the Agent. In this perspective, I also showed that an increase in the competition parameter between substitutes results in an increase of the rival's product through Cornershop. This is because rival's product distribution does not imply a production cost for the Principal that is integrated. In this context, the model suggests that Walmarts decision of distributing rivals products through Cornershop can be in line with this logic. That is Walmart is trying a business model more like the one that Amazon has. Moreover, common agency can remain as a collusion mechanism in the context that it is profitable for Principal 1 to distribute Principal 2's product in the downstream market.

Finally, when I add imperfect information to the model with vertical integration, Principals 2 contract changes. The contract has a smaller profit for Principal 2 because it internalizes part of Principal 1 cost of production. Moreover, quantities under imperfect information are lower that under perfect information. Nevertheless, it is important to mention that I have assume cooperative behaviour in the imperfect information and vertical integration model. Next steps is to include Principal 1's best response in the problem in order to deviate the implications of strategic behaviour in this model. Also, further work in this field should try to incorporate this analysis with informational design literature in order to answer the second hypothesis about Walmart decision to distribute the competition products through Coernershop.

8 Literature

Bernheim, B. Douglas, and Michael D. Whinston. "Common Agency." Econometrica 54 (July 1986): 923-42.

Bernheim, B. Douglas, and Michael D. Whinston. "Common Marketing Agency as a Device for Facilitating Collusion." The RAND Journal of Economics (1985) 16:269-81.

Gal-Or, Esther. "A Common Agency with Incomplete Information." The RAND Journal of Economics 22 (1991): 274-86.

Gal-Or, Esther. "Vertical integration or separation of the sales function as implied by competitive forces." International Journal of Industrial Organization 17 (1999): 641-62.

Martimort, David. "Exclusive Dealing, Common Agency, and Multiprincipals Incentive Theory." The RAND Journal of Economics 27 (Spring 1996): 1-31.

Bernheim, B. Douglas, and Michael D. Whinsgon. "EXCLUSIVE DEALINGG." NBER WORKING PAPER SERIES, July 1996.