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A DOWNSIDE RISK ASSET PRICING MODEL FOR THE MEXICAN STOCK MARKET

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Resumen

Este estudio plantea examinar el poder predictivo de la Pérdida Esperada (ES) sobre los rendimientos de portafolios en el mercado bursátil mexicano. Se crearon portafolios tipo Fama-French que podrían compararse con estudios previos que examinan el caso de México y se encuentra que el Modelo de Tres Factores propuesto por Fama y French es válido en el mercado nacional, durante el período 2000-2021. Tomando el modelo de Tres Factores como punto de referencia, se estiman regresiones de Fama-MacBeth para ese modelo y luego se incluye la beta implícita en ES, una medida de riesgo a la baja, para comparar sus desempeños fuera de la muestra. Además, se analiza el resultado de acomodar los portafolios por sus betas y, controlando por sus primas de tamaño y valor, se estudian los rendimientos futuros. Los resultados de estos dos ejercicios fuera de la muestra muestran que las betas implícitas en CAPM y ES son fuertes predictores de rendimientos, y la prima de tamaño es una variable de control destacada.

Palabras clave: Activos financieros, Modelo Fama-French, Pérdida Esperada, CAPM, México

Abstract

This study set out to examine the predictive power of Expected Shortfall over the returns of portfolios in the Mexican stock market. I created Fama-French portfolios that could be compared to previous studies examining the case of Mexico and found that the Three-Factor Model put forward by Fama and French is valid in the national market during the 2000-2021 period. Taking the Three-Factor Model as a benchmark, I estimated Fama-MacBeth regressions for that model and then included the ES-implied beta – a downside risk measure – in order to compare their out-of-sample performances. Additionally, I sorted portfolios by their betas and controlled for their size and value premia and studied their future returns. The results of these two out-of-sample exercises show that CAPM and ES-implied betas are strong predictors of returns, and the size premium is a salient control variable.

Keywords: Financial assets, Fama-French model, Expected Shortfall, CAPM, Mexico

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1. Introduction

In this study, I present a contribution to the literature on asset pricing in emerging markets by developing a model that incorporates a downside risk measure and the well-established controls of the Fama-French Three-Factor Model: size and value premia. The size premium refers to the observation that, on average and over the long-run, the returns on stocks of small market capitalization companies are higher than those for big market capitalization companies, whereas the value premium refers to the observation that, on average, the returns on stocks of high book-to-market companies are higher than those of low book-to-market ratio. This work develops by departing from recent developments, most notably (Liu 2019), and empirically examine the model's ability to predict returns for indexes in Mexican financial markets. In "Portfolio Selection," Harry Markowitz stated: "in trying to make variance small it is not enough to invest in many securities. It is necessary to avoid investing in securities with high covariances among themselves" (Markowitz 1952). Because covariance is higher in a bearish market, one should pay special attention to the differences between the upside and downside behavior of assets' returns. In a monograph for the Cowles Foundation published in 1959, Markowitz presented the concept of semivariance (the variance of returns below a certain threshold) and concludes that semivariance leads to better portfolios than variance. However, back in the day, computational capacity was a sufficient constraint to perform those estimations and financial economics had to conform to mean-variance analysis. Besides, it was an attractive simplification, as the concept of variance is easier to interpret. In this study, I work with the concept of Expected Shortfall, a tail-based measure of risk that has been widely adopted in financial markets, even imposed by regulators, as in the case of the banking system worldwide.

An intuitive justification for including a measure of downside risk into an asset pricing model is that the goal of diversification in portfolio management is to unload the downside risk, with little regard to the upside. The problem with standard deviation (and variance) is that it weights equally the upside and downside components of risk. However, there is more to this argument. As (Artzner et al. 1999) define it, a coherent risk measure has properties that help rank risks in a way that clearly identifies a strategy's superiority vis-à-vis its alternatives (leading to binary "yes or no" decisions) and estimates the stakes accurately. As they prove it, standard deviation is not a coherent risk measure. Neither does it make sense from an investor's standpoint. Hence, there is enough justification for resorting to a different kind of risk measure. (Ang, Chen, and Xing 2006) find that a downside risk beta is complementary to previously incorporated exogenous variables: liquidity, size and value, and macroeconomic variables. On the other hand, there has been little empirical examinations of the validity of multifactor asset pricing models in the Mexican markets. Working on an Arbitrage Pricing Theory model that includes Fama-French risk factors, (Trevino 2012) finds that market returns and exchange rates do explain risk premia in Mexico; and (Saucedo and González 2021) find evidence supporting a Fama-French model (i.e., size and value premia) while also examining the effect of macroeconomic variables and finding they affect portfolios composed of small firms the most.

Furthermore, there have been few studies exploring tail-based risk measures in Mexico. My contribution to this literature is the incorporation of recent developments in asset pricing models that account for tail risk. As Mexican financial markets come of age, one can expect that models that have been proven to work in developed markets will be more accurate descriptions of Mexico. Thus, this study contributes to this literature by providing newer evidence of the state of financial markets in the country. Therefore, I develop a Three Factor Model as in (Fama and French 1992) (3FM) and incorporate the ES-implied beta, which captures a portfolio's sensitivity to downside risk, to explain the part of return dispersion that is left out of 3FM. There may be several reasons for resorting to a 3FM, but the main motivation in this study is that previous work for the Mexican economy has focused on that family of models, thus allowing for comparisons (see literature review). I follow (Liu 2019) for the estimation of an ES-implied beta and its use in the model. The model is estimated using Fama-MacBeth regression procedures.

The remainder of the article is structured as follows. Section 2 provides a brief review of the recent literature on asset pricing for Mexico and Emerging Markets, downside risk asset pricing and the incorporation of expected shortfall (ES) into this literature. Section 3 introduces Value-at-Risk (VaR) and ES and derives the ES-implied correlation. Section 4 contains an explanation of how the variables of interest were formed and explains the Fama-MacBeth procedure as well as the inferences for key estimators. Next, in section 5 I detail the creation of the portfolios and market indexes used in this study, as well as some useful descriptions of the data. In section 6 I describe the results of a Three Factor Model over the 2000-2021 period, complementing the descriptions in section 5. Finally, sections 7 and 8 present the results of the Fama-MacBeth regressions. Section

7 presents the classic Fama-MacBeth procedure and finds evidence supporting CAPM, then section 8 incorporates the ES-implied beta, the downside risk measure of interest in this study, and finds evidence supporting its validity and predictive power in the sample. A discussion of results comes in section 9.

2. Literature Review

The aim of this study is to develop an asset pricing model for portfolios composed of stocks traded in the Mexican Stock Exchange that prices a tail-based risk measure. After the wave of market liberalizations of the 1980's, data on EM became available. (Claessens, Dasgupta, and Glen 1995) analyzed data compiled by the International Financial Corporation (IFC) on the stock markets of twenty emerging economies (including many Latin American markets). They use several tools for financial time series analysis, but do not develop an asset pricing model; however, they find evidence of high predictability in stock markets, due to low standards of market conduct. In contrast, (Harvey 1995) develops a CAPM using the same dataset. As a result of this availability of data, these two studies were among the firsts to analyze the risk-return trade-off in EM. According to these works, there are three key aspects in which EM differ from developed ones as of 1996: first, there is no association between EM's domestic stock returns and its degree of integration with the US market; second, predictability is significantly higher in EM; third, most of the variance in returns that is explained in the model can be explained by local information.

One of the first studies to incorporate downside risks for asset pricing in EM was Estrada (2001). He tackles the problem from the perspective of risk measurement and proposes the D-CAPM (downside risk approach), based on the semideviation (the standard deviation of returns below the mean) of stock returns with respect to the mean, studying the cross-section of stock returns. He considers three measures of risk: standard deviation of returns (total risk), semideviation with respect to the mean (downside risk), and the classical beta (systematic risk). Theoretically, in an integrated world market, the market beta would perfectly measure the cost of equity, whilst total risk is the tool for a fully segmented market. He argues that, back in 2000, EM would be better characterized as somewhere in between. Estrada (2001) argues downside risk is the right device for measuring the cost of equity under such circumstances. Another argument for downside risk is noteworthy: it is measured as the standard deviation for the subsample of observations when both domestic and world markets' returns are negative. (Estrada 2000) predicts the cost of equity for twenty-eight markets, finding returns of 10.6% when risk is measured by beta; 21.5% when measured by total risk (standard deviation of returns); and 19.5% when measured by downside risk. The results under CAPM are in line with the literature against its use in EM, for its results seem to underestimate the opportunity cost of investments. Both measures estimate cost of equity at around twice the estimates under systematic risk. However, (Estrada 2001) finds ambiguous evidence when analyzing cost of equity across industries for the three risk measures previously discussed. He argues that systematic risk serves well in explaining the cross-section of stock returns, while estimates under total risk do not.

(Ang, Chen, and Xing 2006) test the predictive power of downside beta (β_D) using 12-month samples with daily data, calculated as the regression coefficient when market returns explain portfolios' returns, when market returns were below the mean. They propose that, in order for β_D to have predictive power, it must be persistent (exhibit autocorrelation), more than it is explained by firm characteristics. Also, they base their analyses on the relative downside beta, $\beta_D - \beta_{CAPM}$, to ensure they are not capturing the effects of the traditional beta. They find that portfolios made of stocks with high contemporaneous $\beta_D - \beta_{CAPM}$ achieve higher returns. They also conform portfolios according to the size and value premia. Estimating Fama-MacBeth regressions, they find that the premium on $\beta_D - \beta_{CAPM}$ holds even when portfolios are conformed following this procedure. (Lettau, Maggiori, and Weber 2014) explores the relative downside beta in foreign exchange markets. Within the Fama-MacBeth procedure, they calculate both CAPM and relative downside betas and then estimate λ in $\bar{r}_i = \lambda \bar{r}_m + \lambda^-(\beta_D - \beta_{CAPM}) + \alpha_i$. They test $H_0\colon \lambda^- = 0$ and reject the hypothesis, thus proving the Downside Risk CAPM for the USD.

An article that serves as a point of departure for this study is (Liu 2019), who further develops the concept of downside risk measures and builds a beta implied by the Expected Shortfall of an asset or portfolio. She estimates Expected Shortfall (ES) from the empirical distribution of assets, then the correlation coefficient for the market and the asset to derive an ES-implied beta (more on that in the following section). Constructing portfolios à la Fama-MacBeth, the author examines the predictive power of different betas: CAPM, the usual downside (as in (Ang, Chen, and Xing 2006; Lettau, Maggiori, and Weber 2014)), and the ES-implied. With data for the US stock markets, she finds that sorting portfolios by their ES-implied beta achieves a higher spread between high and low beta portfolios. The usual downside beta also performs better than the CAPM beta, and the three are accepted as significant sources of risk. This article contrasts with prior literature in tail-based risk measures because it incorporates Expected Shortfall, a measure that has become an industry standard even before its imposition to the banking system by the Basel Committee on Banking Supervision in 2013. The next section explains the concepts of Value at Risk and ES.

For the estimation of ES, there is a number of methods, from Gaussian to some non-parametric approaches. Because financial data exhibits skewness, it can be said that the Gaussian distribution assumption is not be the best description of reality. (De Roon and Karehnke 2017) develops a smooth half-normal distribution that accounts for the presence of skewness in returns data, for both upside and downside halves of the empirical distribution of returns. They find large differences in VaR and ES calculated under the assumption of a normal distribution, compared to their half-normal distribution, and differences become larger with the absolute value of skewness. Therefore, in this study the distribution of returns will not be assumed, and a kernel density is estimated from which ES is obtained. (Nadarajah, Zhang, and Chan 2013) propose several methods for estimation of ES, including the kernel method that will be used in this study (see Section 3).

Finally, for the Mexican economy the validity of 3FM variables has been tested, although the literature is not as vast. (Trevino 2012) prices macroeconomic factors, forming portfolios following the Fama-French procedure, a refinement of Fama-MacBeth's. She finds that the market risk is priced in portfolio returns and that result is robust to changes in estimation method and samples. Indeed, it is the most important risk factor even for portfolios composed of non-IPC stocks (her proxy for the market). 3FM's size premium was not significant, although the estimates had consistently positive signs. (Saucedo and González 2021) also create the portfolios in the Fama-French. Their focus is on testing macroeconomic factors as well, contrasting the 3FM with an augmented version that includes those sources of risk. Their results confirm (Trevino 2012): CAPM comes out positive and significant, and this is robust to changes in the model. For both baseline and extended models, the size and value premia were positive and significant, even during the crisis of 2008-2009. However, they identify a reduction in the magnitude of the coefficients, which they attribute to the fact that the Mexican market has matured through time: between 1997 and 2007, the IPC grew thirteen times; on the other hand, between 2010 and 2018, it grew slightly less than 50%. Other explanations for this slow growth are tight monetary policy and international politics (e.g., the US presidential election of 2016).

3. Expected Shortfall

The focus of this study is to analyze the risk-return trade-off in portfolio selection with an improved measure of risk, the ES-implied beta, put forward in (Liu 2019). The way it enters the model is as another predictor, besides the traditional CAPM beta and the explanatory variables proposed in the Fama-French model (the size and value premia explained below). (Liu 2019) has shown that the ES-implied beta contributes significantly to the predictive power of an asset pricing model for the G7 economies, and I test it for Mexico in this study. Although it can be used to analyze upside risks, upside correlation has been found to be significantly smaller than downside correlation (Ferreira Miguel and Gama 2010). In the rest of this section I present an explanation of ES and then I present the ES-implied beta and how to incorporate it to an asset pricing model.

3.1 Value-at-Risk and Expected Shortfall

VaR is the quantile function of the probability distribution from which profits are drawn. That quantile is the value of the bigger loss that can occur at a given probability level. ES complements VaR: in order to calculate ES, VaR must be calculated first. ES is the expected value of the assumed distribution, conditional on returns being equal or less than VaR. Thus, assume that returns, x, follow some distribution, F(x), that belongs to the location-scale family, and let $\alpha \in [0,1]$ be the probability associated to VaR as a quantile. Then, ES is the integral over all quantiles of the probability distribution function between $[0, \alpha]$. In this study, I consider continuous distributions only and perform empirical estimations with financial data that exhibits such behavior.

$$\begin{cases} VaR_\alpha \coloneqq inf\{x: F(x) \geq \alpha\} \\ ES_t(\alpha) \coloneqq E\big(x \big| x < VaR_\alpha(x)\big) = \dfrac{1}{\alpha}\displaystyle\int_0^\alpha VaR_t(v)dv \end{cases} \tag{1}$$

I estimate ES using a non-parametric approach. (De Roon and Karehnke 2017) show how sensitive VaR is to the assumption of a probability distribution and find that for a skewness of -0.6, $VaR_{5\%}$ would be 9.6% larger than estimated under the assumption that returns follow the normal distribution (i.e., assuming a skewness of zero). Moreover, when comparing $ES_{5\%}$, the difference is 14.6%. This raises another question on CAPM, since the model approximates risk as the dispersion of returns. Variance and standard deviation are simple measures, more intuitive than further concepts, and go well with the assumption of normally distributed returns which, at its turn, is also intuitive and simple. However, both fail to satisfy the four properties of a coherent risk measure. According to (Artzner et al. 1999), these properties are defined as follows:

Definition 3.1

Let Π be the set of all possible risks (portfolios) and ρ a measure of risk be a function mapping from Π to $\mathbb{R}$. Let X, Y ∊ Π be two portfolios, such that ρ(X), ρ(Y) ∊ $\mathbb{R}$. Then ρ is a coherent risk measure if it satisfies the following four properties:

1. *Monotonicity*: For X ≤ Y it follows that ρ(X) ≥ ρ(Y). Monotonicity implies that exposition to a portfolio (Y) that is at least as good as another portfolio (X) entails less risks.

2. *Subadditivity*: ρ(X) + ρ(Y) ≥ ρ(X + Y). Subadditivity implies that a portfolio adds at most as much risks as the sum of the assets or portfolios that compose it.

3. *Positive homogeneity*: Let a > 0. Then ρ(aX) = aρ(X). Positive homogeneity means that when exposure to a risk is modified in size (e.g., doubling or a=2), then the risk is equally modified.

4. *Translation invariance*: Let Λ be a risk-free asset with a sure payoff λ. Then the risk measure of a portfolio (Λ + X) is ρ(X + Λ) = ρ(X) – λ. Translation invariance means that adding a risk-free asset (such as cash or money market accounts) reduces risk by as much as its sure payoff.

As (Ortmann 2016) states, variance fails the four properties and standard deviation fails monotonicity and translation invariance. Eventually, finance and economics moved beyond mean-variance analysis and adopted Value-at-Risk (VaR) as their predilected risk measure. VaR is a quantile of the probability distribution of returns, specified such that losses can be greater than or equal to VaR with probability α.

Nonetheless, as (Artzner et al. 1999) demonstrate, VaR itself fails the subadditivity axiom. There are other problems with VaR. It indicates clearly where the problem begins and that cutoff can be written both in percentage points (returns) or levels (dollar loss or worth), thus making for an intuitive tool. What VaR belies is the true size of the problem: how bad is bad? That is why expected shortfall (ES) became the accepted risk measure, including a shift from VaR to ES in the 2013 Basel Committee session. ES is the expected value of returns beyond the VaR quantile. (Artzner et al. 1999) and (Acerbi and Tasche 2002) show ES is a coherent risk measure, meaning it fulfills all four properties. Given the theoretical appeal of ES as a measure of risk, it is natural to think of it as a candidate for inclusion into CAPM.

3.2 The ES-implied beta

Following (Liu 2019), I depart from the traditional CAPM beta, $\beta_{CAPM} = \frac{cov(r_i, r_m)}{var(r_m)} = \rho_{i,m}\frac{\sigma_i}{\sigma_m}$, and calculate another beta for returns below VaR. (Liu 2019) calls it the ES-implied beta and defines it as $\beta_{ES,\alpha} = \frac{cov(r_i, r_m)}{var(r_m)} = \rho_{ES,\alpha}\frac{\sigma_i}{\sigma_m}$ for all $r_m < VaR_\alpha$, where $\rho_{ES,\alpha}$ is the ES-implied correlation, that is calculated as follows for a two-asset portfolio:

$$\rho_{ES,\alpha} = \frac{\left(ES_{p,\alpha} - \mu_p\right)^2 - w_1^2\left(ES_{1,\alpha} - \mu_1\right)^2 - w_2^2\left(ES_{2,\alpha} - \mu_2\right)^2}{2w_1 w_2\left(ES_{1,\alpha} - \mu_1\right)\left(ES_{2,\alpha} - \mu_2\right)} \tag{2}$$

Although this correlation could be generalized to n assets, the two-asset portfolio is enough since the interest is in analyzing the joint behavior of a portfolio and the market. As shown in (Liu 2019), it is derived in the following manner.

The correlation coefficient of a two-asset portfolio, with weights w_1, w_2, is modeled as:

$$\rho_p = \frac{\sigma_p^2 - (w_1^2\sigma_1^2 + w_2^2\sigma_2^2)}{2w_1 w_2 \sigma_1 \sigma_2} \tag{3}$$

Let x_i (the random vector of returns for asset i) follow some distribution in the location-scale family, characterized as:

$$x_i = \mu_i + \sigma_i Z_i \tag{4}$$

where Z_i has mean zero and standard deviation one. Thus,

$$ES_{i,\alpha} = \mu_i + \sigma_i ES(Z_i)_\alpha \tag{5}$$

The ES-implied correlation is obtained from (5) and (3). Since portfolios are a collection of securities, its ES is determined by their correlation. In fact, for any α, when the correlation among them increases, the ES of the portfolio might be greater than the weighted sum of the individual assets. Finally, if it was the case that returns conform to a multivariate normal distribution, then this correlation would be the same as Pearson's. Thus, with equation (2), the ES-implied beta is calculated as:

$$\beta_{ES,\alpha} = \rho_{ES,\alpha} \frac{\sigma_i}{\sigma_m} \; \forall \, r_m < VaR_\alpha \tag{6}$$

Finally, as (Liu 2019) highlights, one should account for the information contained in both CAPM and ES-implied betas. The suggestion is to work with the relative ES-implied beta, defined as the following subtraction:

$$\beta_{ES}^{rel} = \beta_{ES} - \beta_{CAPM} \tag{6'}$$

3.3 Kernel Method for Estimating ES

There are some well-established techniques for estimating Expected Shortfall. As mentioned in the literature review, I compute Kernel Density Estimations (KDE) to approximate the probability distribution function (PDF) of returns, following (Nadarajah, Zhang, and Chan 2013). The kernel method is a non-parametric estimation and, as such, it helps discard many strong assumptions about data and is easy to calculate. Let $x_{(1)}, \dots, x_{(T)}$ be the order statistics of the vector of returns on the i-th asset for the whole sample period, h be a bandwidth, denote by $K(\cdot)$ a symmetric kernel function, $K_h(u) = \frac{1}{h} K\left(\frac{u}{h}\right)$ and define $A(x) = \int_{-\infty}^{x} K(u) du; A_h(h) = A\left(\frac{u}{h}\right)$. Thus, as in (Nadarajah, Zhang, and Chan 2013) the ES estimator is given by:

$$\begin{cases} \widehat{\mathrm{ES}}_\alpha(\mathrm{x}) = \dfrac{1}{\mathrm{T}\alpha} \sum_{\mathrm{i}=1}^{\mathrm{T}} \mathrm{x_i} \mathrm{A_h}(\hat{\mathrm{q}}(\alpha) - \mathrm{x_i}) \\ \hat{\mathrm{q}}(\alpha) = \sum_{\mathrm{i}=1}^{\mathrm{T}} \left[\int_{\mathrm{i}-1/\mathrm{T}}^{1/\mathrm{T}} \mathrm{K_h}(\mathrm{t} - \alpha) \mathrm{dt} \right] \mathrm{x_{(i)}} \end{cases} \tag{7}$$

In this study, I work with Epanechnikov's kernel function, $K(u) = \frac{3}{4}(1 - u^2)$. The ES-implied correlation in (5) requires that we calculate the ES for every individual portfolio, the market portfolio, and the equal-weight portfolio composed of the market and the i-th portfolio.

4. Empirical Strategy

In this article, I estimate Fama-MacBeth regressions to compare the performance of two risk-pricing strategies: the traditional Fama-French Three Factor Model and one that incorporates the ES-implied beta. The benchmark model is a Fama-French Three Factor Model (3FM). Thus, a test of CAPM will be performed indirectly. In this section, I present the calculation of the ES-implied beta following the explanation of the ES-implied correlation provided in the Expected Shortfall section, then I provide an explanation of how I form the size and value premia variables. Then, I present the Fama-MacBeth regression procedure and test statistics for these regressions.

4.1 ES-implied betas

In this procedure, the first step is to calculate the ES-implied correlation (2) of a portfolio and the market, to the left of a given quantile ($VaR_{50\%}$). Following (Liu 2019), I use the 50% quantile and a 12-month window for the estimation procedure. As explained above, because the ES-implied correlation requires the formation of a portfolio, I calculate a portfolio's $\rho_{ES,\alpha}$ as the average of all the resulting correlations of combining that portfolio one-on-one with the market portfolio. Thus $\beta_{ES_\alpha} = \rho_{ES,M,i}\frac{\sigma_i}{\sigma_M}$.

4.2 Size and value premia

For the 3FM variables, I follow (Fama and French 1992) in the creation of the size and value premia variables. I split stocks according to size (small and big, by market capitalization) and value (high, medium, and low, by book-to-market ratio), and create six portfolios (rebalanced yearly): small/high (SH); big/high (BH); small/medium (SM); big/medium (BM); small/low (SL); big/low (BL). Next, the size premium is calculated as the average returns of 'small' portfolios minus the average returns of the 'big' ones (a 3FM excludes the 'medium' portfolios). Correspondingly, the value premium is calculated as the difference in average returns between the portfolios with high book-to-market ratio (thus, low market value) and those with a low ratio. Finally, market excess returns (RX) are simply the difference between the stock market (Mkt_t) return and the CETES28 rate (the risk-free asset).

$$\begin{aligned} \mathrm{SMB_t} &= \frac{(\mathrm{SL_t} + \mathrm{SM_t} + \mathrm{SH_t}) - (\mathrm{BL_t} + \mathrm{BM_t} + \mathrm{BH_t})}{3} \\ \mathrm{HML_t} &= \frac{(\mathrm{SH_t} + \mathrm{BH_t}) - (\mathrm{SL_t} + \mathrm{BL_t})}{2} \\ \mathrm{RX_t} &= \frac{\mathrm{Mkt_t} - \mathrm{Mkt_{t-1}}}{\mathrm{Mkt_{t-1}}} - \mathrm{CETES28_t} \end{aligned} \tag{8}$$

4.3 Fama-MacBeth Regression

The baseline model in this study is a traditional 3FM, established as a benchmark for comparing the models that incorporate tail-based risks. For all models, Fama-MacBeth regressions require a two-stage procedure. In the first stage, the variables above (β_{ES} and 3FM premia) are estimated. This stage is valuable since it allows for a check of the validity of results by comparing to previous studies. The second stage is a regression where portfolio returns are regressed against the variables calculated in the first stage for a given period, then taking the mean of the resulting time series of coefficients. That average is the estimate of the risk premium on any risk factor and the interest is on the significance of the models and its estimated coefficients.

$$\overline{x_t} = \left(\overline{x_{1,t}}, \overline{x_{2,t}}, \dots \overline{x_{n,t}}\right)' = \begin{bmatrix} \lambda_{1,B_1} & \cdots & \lambda_{1,B_m} \\ \vdots & \ddots & \vdots \\ \lambda_{n,B_1} & \cdots & \lambda_{n,B_m} \end{bmatrix} \begin{pmatrix} B_{1,t} \\ \vdots \\ B_{\mathrm{m},t} \end{pmatrix} + \begin{pmatrix} \nu_{1,t} \\ \vdots \\ \nu_{m,t} \end{pmatrix} \tag{9}$$

The left-hand side of (9) is a vector of average excess returns in the subperiod t for the n assets and the first matrix in the right-hand side is an $n \times m$ matrix of risk premia coefficients to be estimated, the second matrix is a $m \times 1$ risk of risk factors (the average of the time series variables estimated above). The vector of errors ν_t captures the effects not represented in the model and is assumed to be independent of the risk factors (i.e., $E(B_t{}'\nu_t) = 0$). Thus, in stage two the cross-section estimation of the price of each of the time series risk factors estimated in stage one. Once all cross-section estimators are obtained, inferences can be made to test their significance.

4.4 Testing the variables in the model

The t-statistic for our estimates can then be expressed as the ratio of the estimated coefficient and its standard deviation, as in (10). More importantly, a time series test is required for downside betas. All standard errors in the regressions reported in this study are corrected for heteroskedasticity and correlation using the Newey-West procedure. Thus, in Fama-MacBeth regressions, the tests of the coefficients are based on:

$$\begin{cases} \widehat{\lambda_h} = \frac{1}{T}\sum_{t=1}^{T} \lambda_{h,t} \\ \hat{\sigma}\left(\widehat{\lambda_h}\right) = \sqrt{\sum_{t=1}^{T}\left(\widehat{\lambda_{h,t}} - \widehat{\lambda_h}\right)^2} \\ t\left(\widehat{\lambda_h}\right) = \frac{\widehat{\lambda_h}}{\sigma\left(\widehat{\lambda_h}\right)} \end{cases} \tag{10}$$

The interest here is to find whether the risk loadings estimated in the first stage have predictive power over future returns. I achieve this by estimating stage-one regressions for a 12-month window, and then using those coefficients as the data that feeds stage-two regressions. The second stage is then a test of predictive power for 3FM factors and the ES-implied beta.

Furthermore, I check the outcomes of investing on linear combinations of portfolios (portfolios of portfolios), formed according to their betas over the past 12 months, sorting them by tertiles, and then comparing the average aggregated returns among tertiles. (Liu 2019) finds that average returns increase monotonically with their betas: the greater the size of the beta, the higher the return. She also finds that the ES-implied beta achieves a higher spread between highest and lowest quintiles than does the traditional CAPM beta. This is an interesting test since an asset pricing model can serve the purpose of determining whether returns can be predicted (as opposed to efficient markets' hypothesis). In this study, I question whether CAPM and ES-implied betas have predictive power, and if so, do portfolios sorted by the ES-implied beta outperform those sorted by the CAPM beta?

5. Data

5.1 Data Processing

In this study, I work with data from Economatica. Cetes28 data was consulted from Banco de México Economic Information System's website. The selection of data follows the procedure described below, highly influenced by (Saucedo and González 2021):

1. Economatica provides a list of 185 stocks. Portfolios are compiled with data from January 2000 and are rebalanced every year. The first elimination is for companies with no report of market capitalization as of December 31, 1999. The second round is for companies that have

a streak of less than 4 consecutive months towards the end of the year, or less than six consecutive months of market cap reported during the year.

2. Based on (Fama and French 1992), I also exclude financial firms. They argue that the levels of leverage that are normal in the finance industry, are unreasonable for most other corporations.

3. For every year, stocks are chosen based, first, on having a closing price at least 150 days (out of approximately 252 operations days) reported; second, I use Economatica month-end's market capitalization to check the conditions mentioned in 1.

4. Sorting the companies by total assets, stocks are split into as Big (highest half) or Small. If there is and odd number of stocks, the median goes into the category to which it is closer.

5. Next, stocks are sorted by their book-to-market (BTM) ratio. Book value is calculated as Total Assets minus Total Liabilities. Market value is the stock's market cap. Then, BTM = (Book Value)/(Market Value). I sort into three categories: High, Medium, and Low, where High is the highest 33%. If a category is to be larger or smaller than the other two, the Medium one takes the extra stock or the cut.

6. Now, all stocks are split in six portfolios, marked by the intersection of their BTM (value) and Assets (size): BH, BM, BL, SH, SM and SL. With these portfolios I create the size and value premia, as described in Section 4.

7. CAPM betas are created as the time series slopes, resulting from the regression: $r_{i,t} - r_{f,t} = \beta_m[r_{m,t} - r_{f,t}] + \delta_1 HML + \delta_2 SMB + \epsilon_{i,t}$. Where t comprises the past 12 months, m is the market portfolio and i is the i-th portfolio. Together, β_m, δ_1 and δ_3 will be our 3FM variables that will pass into Fama-MacBeth regressions as "data."

8. For the same window, the ES-implied correlation, as well as the standard deviations for the i-th portfolio and market returns are calculated as in Section 4. I estimate them using the Empirical Cumulative Distribution Function (kernel method) rather than the Gaussian approach, to avoid the problems that come with the assumption of normality. The portfolio is composed equally of the market and the i-th portfolio.

9. I analyze the six portfolios created above to circumvent the complexities and excessive noise that come from working with individual stocks. Finally, EWMkt refers to an equally-weighted market index composed all available stocks in the sample. As can be appreciated in the graph in the time series plot below, it exhibits the volatility clustering generated by IPC, but it includes more than the thirty-five stocks included in IPC.

Table 1 shows descriptive statistics for the returns of the market index and the portfolios that are used in this study. Remarkably, the SM portfolio (small by assets, medium by book-to-market) is the one with the higher volatility and wider range; on the other hand, the BL portfolio is the more stable of them. N refers to the number of daily observations available for every portfolio.

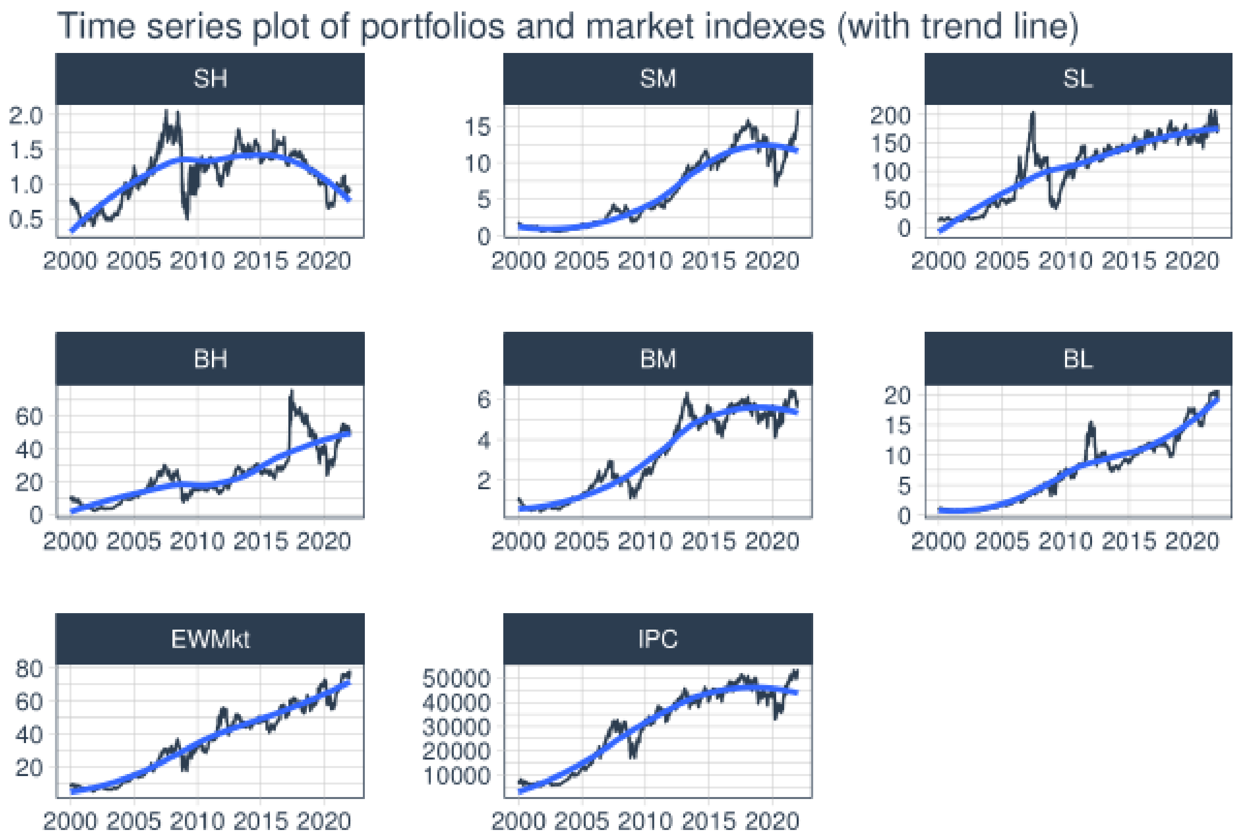
Table 1: Descriptive statistics of the market index and portfolios

Portfolio	N	Mean	St. Dev.	Min	Max
EWMkt	5,494	0.001	0.012	−0.116	0.092
HML	5,494	−0.00001	0.025	−0.176	0.248
SMB	5,494	0.0003	0.018	−0.141	0.175
SH	5,494	0.001	0.044	−0.338	0.521
SM	5,494	0.001	0.020	−0.308	0.404
SL	5,494	0.001	0.018	−0.309	0.283
BH	5,494	0.001	0.020	−0.207	0.138
BM	5,494	0.0005	0.014	−0.273	0.158
BL	5,494	0.001	0.015	−0.107	0.136

5.2 Time Series Analysis of Portfolios and Market Indexes

In this study, I use the indexes described above to analyze the pricing of risks in the Mexican Stock Exchange. For developed markets there are numerous studies and well-established data sources that provide all the risk factors and portfolio returns. The most salient is Kenneth French's website. However, in the case of Mexico there is little information available, and it is important that the data is carefully created, and estimations are validated, to ensure a minimum of errors. Hence, I now present a time series analysis of the data used.

The level of the indexes is irrelevant. In practice, index creators adjust the value of an index when the underlying portfolio is rebalanced, to avoid abrupt steps and bumps due to the inclusion or exclusion of assets, while keeping the behavior of the index intact. All portfolios presented here are equal-weighted. There are two explanations for the difference in the trends of EWMkt and IPC. The first is the equal-weights creation of EWMkt (IPC is weighted by many factors, including company size and liquidity). The second is that EWMkt includes more information than IPC since the latter focuses on the thirty-five most important stocks in the market. All portfolios were impacted negatively (albeit to different degrees) by the 2008 financial crisis and news about Covid-19 in early 2020.



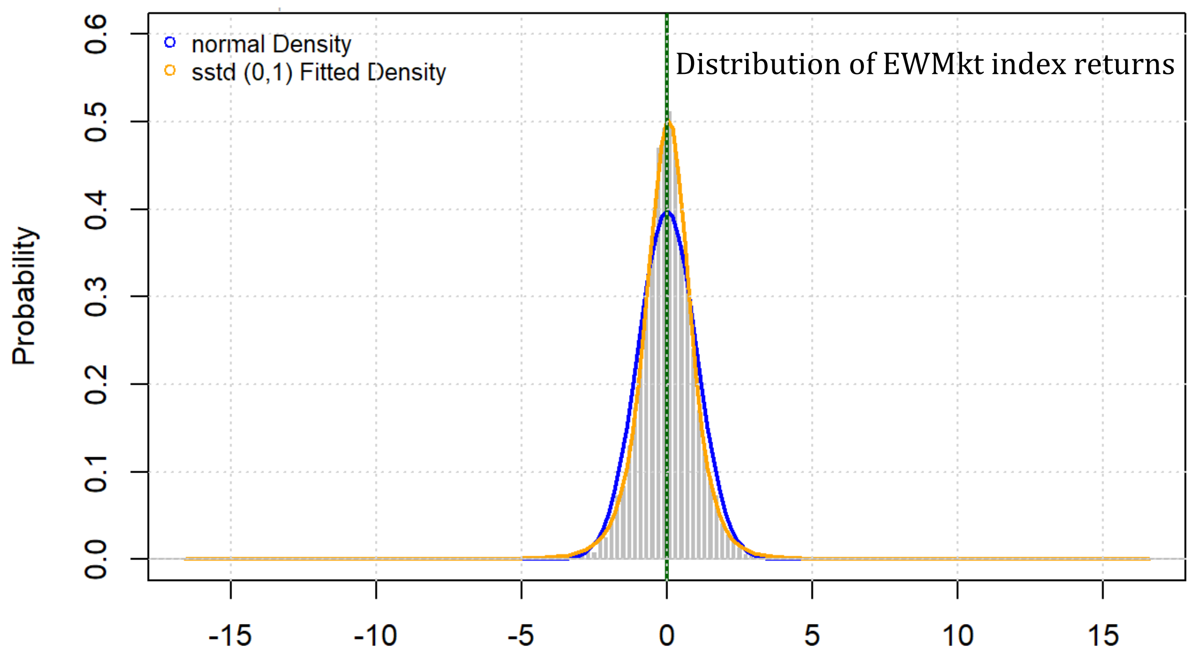
On the other hand, Table 2 shows augmented Dickey-Fuller statistics for the returns on these indexes. Together, the chart and the unit-root tests prove that these portfolios behave as most financial time series do: the series is continuous with little abrupt changes (most of which can be explained by public information, like the news), and the natural transformation (returns) makes them stationary.

Table 2:

	SH	SM	SL	BH	BM	BL	EWMkt	IPC
Statistic	-14.809	-15.461	-16.217	-15.785	-17.480	-16.541	-16.062	-17.358
DF	17	17	17	17	17	17	17	17
p.value	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Stationary	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Augmented Dickey-Fuller Test: Returns of Size-Value portfolios and market indexes

Finally, in the next chart I provide further justification for thinking about alternatives to the assumption that returns follow the normal distribution. In most cases, the assumption works well, except for three instances: the mean, which happens more often than predicted by the normal distribution; the downside, which occurs with little frequency; and the upside. In the latter two cases, the theoretical normal density fails again by assuming symmetry. Instead, this chart models returns as following a skew-Student (SSTD) distribution.



6. Three-Factor Model over the Full Sample

Table 6 (see in Appendix: Three-Factor Models) presents the 3FM estimation for the small and big portfolios, calculated over the full 2000-2021 period. This analysis is important for making comparisons, particularly with (Saucedo and González 2021), a recent study that works with the same portfolios. Unlike them, (Trevino 2012) estimates an APT that includes β_{CAPM} and δ_{size}, but chooses portfolios differently, complicating comparisons. I used Newey-West Heteroskedasticity and Autocorrelation Consistent (HAC) standard errors because of the large number of observations. Even though financial data is not particularly problematic in terms of autocorrelation, over the long run such problems may arise. Besides, heteroskedasticity is a well-known issue when developing these models. Shapiro-Wilk tests for the six regressions confirmed normality of residuals.

Because of how portfolios were constructed (see section 5.1), it is expected that columns 1 through 3 in Table 6 have a positive δ_{size}, and a negative sing for columns 4 through 6. The sign of δ_{value} should be positive for columns 1 and 4, negative for columns 3 and 6, but there is no expectation for the signs in columns 2 and 5. β_{CAPM} is expected to be positive and significant for all portfolios, although their magnitude cannot be expected, except for BL, a portfolio composed of big companies with a low book-to-market value. For that portfolio, we would expect a beta that is consistently close to one. Finally, the intercept is an interesting feature of any Multifactor Model. The intuitive interpretation of a statistically significant intercept (the investment's α) is that the chosen sources of risk are not capturing the full variation in returns. Thus, a positive value means there were returns in excess of what the model (3FM in this study) predicted.

An inspection of Table 6 confirms the hypotheses stated above. These results go in line with (Saucedo and González 2021), who estimates this table for 1997-2018. The six models are significant, and all model coefficients are found significant. Small companies' portfolios have a positive and significant δ_{size}, while it is negative for big companies' portfolios. δ_{value} is positive and significant for SH, SM, BH and BM, but negative and significant for SL and BL. β_{CAPM} is close to one in most cases (BL's is the greater) and its magnitude is similar in the cases of mirror portfolios: BL and SH (two exact opposite portfolios) and SL and BH. Interestingly, the portfolios composed of low BTM (SL, BL) had positive and significant, albeit small, α, suggesting there is room for improvement in this model. The 3FM is thus an adequate model to price the factors driving the risk-return trade-off in the Mexican stock market, at least during the 2000-2021 period. However, this is an in-sample analysis. In what follows I present a Fama-MacBeth analysis that performs an out-of-sample, predictive analysis to test whether 3FM and the ES-implied beta are strong predictors of future returns.

7. Dynamic Fama-MacBeth Analysis

As explained in section 4.3, the Fama-MacBeth procedure is an out-of-sample analysis. In the Appendix: Three-Factor Models, I present the results of regressing the times series of returns on the 3FM variables over the past 5 years (Table 7 through Table 11); in the second stage, these coefficients become the data for a cross-section analysis every month, over the next 4 years. The average of the resulting coefficients is known as the price, in terms of returns, of that particular risk factor. In this section, I estimate Fama-MacBeth regressions as follows: first-stage coefficients estimated for subperiods 2000-2004, 2004-2008, 2008-2012 and 2012-2016; risk premia estimated monthly for the subperiods are 2005-2008, 2009-2012, 2013-2016 and 2017-2020. Thus, coefficients for 2000-2004 feed the monthly estimations of subperiod 2005-2008, and so forth, constituting an out-of-sample analysis. This is the classic way to Fama-MacBeth regressions.

7.1 First-Stage

Subsequent tables in the Appendix: Three-Factor Models show the estimation of 3FM for the subperiods, that allow to study the stability of the full-sample results analyzed above. All betas are computed with daily returns in the past year. Tables 7 through 11 in the appendix show the results of these exercises. All portfolios had a positive, significant and relatively stable market beta through subperiods. Mirror portfolios (SH and BL; SL and BH) still have a β_{CAPM} that closely follow each other. Whenever the sign of δ_{value} or δ_{size} does not comply with the expected sign, the coefficient is non-significant. All models are significant for every subperiod. The significance of size and value premia is highly volatile across periods, especially for the medium value portfolios (SM, BM). This raises the question of whether δ_{size}, δ_{value} will come up as significant predictors of returns in the second stage.

The value premium, calculated as high BTM minus low BTM, has the smaller effect on SM and BM, which demonstrates consistency. The sign of the corresponding coefficients is positive, implying a closeness to the high BTM category. These effects are only significant in 2012-2016 and 2016-2020. The size premium is more persistent, particularly in the small companies' portfolios where it is always significant. Although there is not a consistency in magnitudes, one could conjecture that this risk factor will come up as significant in second stage regression.

7.2 Second-Stage

Table 3 shows the results of regressing monthly returns on the coefficients shown in the 3FM tables. For subperiod 2005-2008, the data are those coefficients estimated in the first stage for the 2000-2004 subperiod and so on, with the coefficients of each estimation subperiod feeding the following second-stage subperiod as data (hence it is an out-of-sample analysis). In Table 3, column 1 shows the results for this exercise, and only β_{CAPM} comes close to being statistically significant. The t-statistics were calculated using the Newey-West procedure for Heteroskedasticity and Autocorrelation Consistent standard errors. Returns are expressed in percentage so the coefficients can be interpreted in those terms. These results are not directly comparable to (Liu 2019), for they do not report Fama-MacBeth for 3FM variables alone, and because their estimation window is over the past 12-months. However, it is interesting that λ_{Size} is negative (opposite sign) and λ_{CAPM} is significant. In most studies, this is the only estimate to come out as significant. This result suggests that in predicting future returns, only market sensitivity and the size factor have been able to consistently distinguish the signal from the noise in the Mexican stock exchange over the past two decades. The negative sign of λ_{size} means that stocks of big companies have paid the best returns (i.e., an investor expects a negative return from exposure to small companies' stocks). On the other hand, the magnitude of λ_{CAPM} means that, on average, gaining exposure to the overall market is a strategy with a significant positive return (5%). Importantly, the market proxy used here is a value-weighted market that is more comprehensive than IPC.

Now, the estimation window can be changed to consider the twelve past months (immediate to the month in question). Column 2 of Table 3 shows the results of this exercise. Notice that standard errors are larger consistent with the fact that every coefficient is estimated monthly, contrary to the 5-year window of column one. Interestingly, all coefficients are stable in value, which reinforces the results of column one. In summary, this study finds the 3FM as a proper description of Mexican financial markets (see the previous subsection), and the market beta (i.e., β_{CAPM}) is a strong predictor of returns. This is, after all, an out-of-sample analysis.

Table 3: Fama-MacBeth Second Stage

	Dependent variable:	
	Return	
	5Y-window	12M-window
	(1)	(2)
λ_{CAPM}	0.028	0.065**
	(0.026)	(0.027)
λ_{value}	0.064**	0.025
	(0.027)	(0.033)
λ_{size}	0.026	0.026
	(0.034)	(0.033)
R^2	0.310	0.367
Note:	*p<0.1; **p<0.05; ***p<0.01	

8. ES-implied beta

β_{ES} was not calculated inside the Fama-French model regression, hence its absence in the Appendix: Three-Factor Models. β_{ES} was obtained as in equation (6), from the $ES_{50\%}$-implied correlation between the market and the portfolios, and β_{ES}^{rel} resulted from subtracting CAPM from the ES-implied beta. The justification for adding this measure is twofold. First, it is a well-known fact that correlation among stocks increases during a bearish market, thus β_{ES}^{rel} computes the sensitivity of stocks and portfolios to the market, when "bad scenarios" (VaR_{α}) occur. Second, β_{CAPM} measures risk equally to the right and the left of returns distributions. One need only look at the Min and Max columns of table 1 to confirm the relevance of this coefficient: when returns fall, they go further away from the mean than when they rise. To calculate β_{ES}, I subtracted the risk-free rate to portfolio and market returns, to ensure comparability with β_{CAPM}. Except for SM, there is an inverse relationship between R^2 of the 3FM model by portfolio and its associated β_{ES}^{rel}, further suggesting the gains in predictive power that come with adding this predictor into second-stage regressions.

8.1 Fama-MacBeth Second Stage Revisited

Now, if β_{CAPM} proved a strong predictor of out-of-sample returns in the context of a 3FM, a natural question is whether β_{ES}^{rel} can enhance its predictions. The risk premia for the size and value loadings were non-significant in the rolling-window exercise, leaving room for some

Table 4: Fama-MacBeth Second Stage

	Dependent variable:				
	Return				
	(1)	(2)	(3)	(4)	(5)
λ_{CAPM}	0.080***	0.076***	0.078***	0.077***	0.059**
	(0.029)	(0.026)	(0.025)	(0.029)	(0.026)
λ_{value}	0.014		0.019		−0.024
	(0.033)		(0.035)		(0.033)
λ_{size}		0.003	0.024		−0.083***
		(0.032)	(0.033)		(0.032)
λ_{relES}	0.133***	−0.038		0.014	0.068**
	(0.032)	(0.032)		(0.034)	(0.030)
Observations	1,512	1,512	1,512	1,512	1,512
R^2	0.404	0.381	0.396	0.341	0.448
Note:			*p<0.1; **p<0.05; ***p<0.01		

improvements to the model. I now present the results of estimating second-stage regressions for the 2001-2021 period. Model 3 of Table 4 is the same as in Model 2 of Table 3, but the results are different because Table 3 reports results for 2005-2021. It reinforces the notion that 3FM confirms CAPM but has room for improvement in the Mexican stock market. From left to right, Table 4 provides compelling evidence supporting the complementary nature of CAPM and ES-implied betas. Model 5 is the one with the highest (mean) R^2 among them. Consistent with the intermittency of the deltas in 3FM by subperiods, the price on size and value loadings are not consistently significant, and only $\lambda_{size}$is significant at one point. Although the 3FM gains substantially from the addition of the β_{ES}^{rel}, any model from 1 to 4 already explains around 40% of future returns, while adding the Fama-French loadings barely improves the model in 5 percentage points.

8.2 Do ES-implied betas outperform market betas?

Tables 5 (below) and 11 (see in Appendix: Equal-Weighted Portfolios by Risk Loading) report the returns of holding the equal-weighted portfolio consisting of the intersection of factor loadings from 3FM and ES-implied beta. Because such loadings were calculated with data for the twelve past months, these returns are the prediction of returns for the month ahead. Table 11 is a matrix that contains four different sorting. The upper-left quadrant corresponds to the high-to-low β_{ES}^{rel} controlling for the high-to-low δ_{size}. The upper-right quadrant controls for the δ_{value}. The lower quadrants report the returns of portfolios sorted by their β_{CAPM}, controlling for δ_{size} and δ_{value}, from left to right respectively.

It is important to notice there is a monotonic increase from left to right in the left quadrant, for both betas. In words, this means that the portfolios with the highest Size effect are predicted by this model to report the highest returns, at least for the next month. On the other hand, this monotonicity is not observed in the right quadrants and that is no coincidence. In the Fama-MacBeth regressions, δ_{size} resulted a significant predictor of returns. Moreover, both β_{ES}^{rel} and β_{CAPM} exhibit a distinctive pattern that can be observed in the four quadrants: the medium beta portfolios have the lowest returns. This is also true of δ_{value}, although portfolios with a high δ_{value} earned more than those with a low delta. These results suggest that a strategy is to opt for portfolios with the smallest β (CAPM or ES-implied) and the greatest $delta$ (size or value).

As a matter of fact, Table 5 (below) shows the same analysis when sorting portfolios by their β_{ES}^{rel} and controlling for β_{CAPM}. Of all rows and columns, the third row and third column have the greatest returns. Their intersection offered the best reward during the 2000-2021 period. There is an important constraint to this strategy. I conducted the same analysis for longer holding periods (3- and 6-month), and there was no clear pattern. The clear limitation, as in most asset pricing studies comes from the fact that portfolio rebalancing is costly. The best hope for this strategy that rebalances every month is that such activity may not happen as frequently and there are two reasons why it may be the case. First, portfolios' betas and deltas are highly persistent since they were estimated using a rolling-window procedure. Second, these portfolios are created yearly (as explained in the Data section). Hence, a 1-month holding may be a good strategy in the end.

Table 5: Average returns on a combination of portfolios, sorted by their relative ES-implied beta (rows) and CAPM beta (columns). All returns are expressed in percentage points. β_{ES}^{-} refers to β_{ES}^{rel}.

	β_{CAPM}^{high}	β_{CAPM}^{med}	β_{CAPM}^{low}
$\beta_{ES}^{-,high}$	1.8502	1.7157	1.9961
$\beta_{ES}^{-,med}$	1.4634	1.3289	1.6093
$\beta_{ES}^{-,low}$	1.8779	1.7435	2.0239

9. Discussion

This study set out to examine the predictive power of Expected Shortfall over the returns of portfolios on the Mexican Stock Exchange. A stylized fact of stock price time series is that their correlation is higher during a bearish market, which is an observation as old as formal portfolio theory. However, back in the day, the incorporation of downside risk measures into asset pricing was computationally exhaustive, leading practitioners and even academics to conform to traditional mean-variance portfolio optimization models, like CAPM. With the development of risk management came new measures of financial risks that allow for better portfolio decisions, but these measures, such as Expected Shortfall, are more complex than mean-variance analysis.

Thus, the aim was to find out whether ES was a good instrument in pricing stocks in Mexico. There were two good reasons to suspect so: first, because ES is mandated to the banking system by the Basel Committee on Banking Supervisions as their tool for making risk provisions; second, because downside risk measures make intuitive sense, from an investors' standpoint. Hence, the conjecture was that investors in the Mexican market were accounting for such risks. In such a case, the ES-implied beta that was derived in this study should have predictive power over future returns.

Therefore, I created Fama-French portfolios that could be compared to previous studies examining the Mexican market. The literature studying asset pricing in Mexico is not vast and there are so few resources available, that I chose and created portfolios so as to make them closely comparable. For transparency, a brief description of the data was also added. It would be helpful if researchers in financial economics elaborated more on their procedures for creating portfolios. I also provided evidence supporting the need for relaxing the assumption of a normal distribution of returns was provided.

I found results consistent with a recent examination that is comparable, and that is reassuring of the next results. I found that the Three-Factor Model proposed in (Fama and French 1992) is valid in the case of Mexico during the 2000-2021 subperiod. However, because the ES-implied beta is not estimated within the model, I estimated the 3FMs Fama-MacBeth regressions and compared them when I included the ES-implied beta. I found there is greater predictive power (judging by their goodness-of-fit statistic) when this downside risk measure is incorporated. The ES-implied beta was calculated from the left part of the returns' empirical distribution function. Moreover, because CAPM beta contains some of that information (since CAPM accounts for the full distribution of returns, but is estimated under the assumption of normality), I subtracted it from the ES-implied beta, thus working with the relative beta instead. This way, the new variable entering Fama-MacBeth second stage regressions is bringing new information only.

The results of this out-of-sample exercise show that the best model in predicting future returns is to account for CAPM and ES-implied betas together with the size and value premia deltas. Consistent with the results of the first stage calculated in their classic subperiods, where the deltas (size and value premia coefficients) were not consistently significant across periods and portfolios, their coefficients were also not significant in the second stage.

Moreover, I performed a second exercise to test these results. If the CAPM and ES-implied betas together with the size and value deltas have predictive power over future returns, then the returns of portfolios (linear combinations of the Fama-French portfolios) sorted by their betas and deltas should exhibit some monotonicity. In this case, the results show that the only sorting that achieved monotonicity is when controlling for the size delta. This is consistent with the results of the Fama-MacBeth regressions, where only the size delta resulted significant (see Table 4, model 5).

Therefore, the ES-implied beta is confirmed as a variable with the ability to predict future returns in the Mexican Stock Exchange. However, further research is necessary to elucidate its role in the risk-return tradeoff. I propose two questions deserving greater detail: Does this predictive power stem from the risk management strategies implemented by banks? How does the ES-implied beta interact with the effects of macroeconomic variables in this relationship? It can also be a tool for studying contagions, both from domestic and imported shocks. Expected Shortfall is, after all, a measure describing how bad can bad days get.

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Appendix: Three-Factor Models

Table 6 reports 3FM estimations for the six portfolios in the sample. They were estimated with monthly data for 2000-2021. Standard errors are Heteroskedasticity and Autocorrelation Consistent (HAC), following the Newey-West procedure. Then, Tables 7 through 11 report the same model for the subperiods 2000-2004, 2004-2008, 2008-2012, 2012-2016 and 2016-2020.

Table 6: Traditional Three-Factor Model (2000-2021)

	Dependent variable:					
	SH	SM	SL	BH	BM	BL
	(1)	(2)	(3)	(4)	(5)	(6)
β_{CAPM}	0.954***	0.841***	0.799***	0.887***	0.664***	1.043***
	(0.080)	(0.051)	(0.054)	(0.057)	(0.040)	(0.029)
δ_{value}	0.782***	0.104**	−0.242***	0.728***	0.163***	−0.248***
	(0.087)	(0.043)	(0.053)	(0.050)	(0.035)	(0.026)
δ_{size}	0.922***	0.621***	0.763***	−0.367***	−0.120***	−0.207***
	(0.132)	(0.066)	(0.066)	(0.061)	(0.043)	(0.031)
α	0.0003	0.00002	0.0003***	0.0002	0.0001	0.0002**
	(0.0002)	(0.0001)	(0.0001)	(0.0002)	(0.0001)	(0.0001)
R^2	0.844	0.558	0.514	0.659	0.551	0.855
F Statistic (df = 3; 260)	469.911***	109.442***	91.507***	167.739***	106.208***	510.254***

Note: *p<0.1; **p<0.05; ***p<0.01

Table 7: Traditional Three-Factor Model (2000-2004)

	Dependent variable:					
	SH	SM	SL	BH	BM	BL
	(1)	(2)	(3)	(4)	(5)	(6)
β_{CAPM}	0.900***	0.928***	0.869***	0.963***	0.740***	0.994***
	(0.063)	(0.094)	(0.108)	(0.099)	(0.071)	(0.052)
δ_{value}	0.516***	0.234	−0.535***	0.575***	0.014	−0.374***
	(0.087)	(0.143)	(0.106)	(0.121)	(0.108)	(0.047)
δ_{size}	0.517***	0.742***	0.177	−0.751***	−0.401***	−0.412***
	(0.102)	(0.158)	(0.293)	(0.265)	(0.117)	(0.077)
α	0.0004	−0.0004	0.001*	0.001	0.0001	0.0002
	(0.0003)	(0.0003)	(0.0004)	(0.0004)	(0.0002)	(0.0001)
R^2	0.843	0.658	0.570	0.758	0.722	0.900
Residual Std. Error (df = 56)	0.002	0.003	0.003	0.003	0.002	0.001
F Statistic (df = 3; 56)	100.456***	35.910***	24.704***	58.370***	48.483***	167.369***

Note: *p<0.1; **p<0.05; ***p<0.01

Table 8: Traditional Three-Factor Model (2004-2008)

	Dependent variable:					
	SH	SM	SL	BH	BM	BL
	(1)	(2)	(3)	(4)	(5)	(6)
β_{CAPM}	0.727***	1.085***	1.027***	1.176***	0.787***	0.876***
	(0.081)	(0.093)	(0.196)	(0.150)	(0.054)	(0.053)
δ_{value}	0.765***	0.049	−0.567***	0.406***	0.102	−0.262***
	(0.068)	(0.110)	(0.152)	(0.129)	(0.101)	(0.039)
δ_{size}	0.795***	0.626***	1.178***	0.046	−0.110	−0.337***
	(0.078)	(0.123)	(0.143)	(0.092)	(0.138)	(0.039)
α	0.001***	−0.0002	−0.0004	−0.0004	−0.0001	0.001***
	(0.0002)	(0.0002)	(0.001)	(0.0004)	(0.0003)	(0.0002)
R^2	0.872	0.710	0.749	0.698	0.594	0.863
Residual Std. Error (df = 56)	0.002	0.003	0.003	0.003	0.002	0.001
F Statistic (df = 3; 56)	127.100***	45.693***	55.802***	43.218***	27.268***	117.610***
Note:						*p<0.1; **p<0.05; ***p<0.01

Table 9: Traditional Three-Factor Model (2008-2012)

	Dependent variable:					
	SH	SM	SL	BH	BM	BL
	(1)	(2)	(3)	(4)	(5)	(6)
β_{CAPM}	1.080***	0.695***	0.641***	0.737***	0.504***	1.176***
	(0.124)	(0.126)	(0.133)	(0.145)	(0.074)	(0.101)
δ_{value}	0.870***	−0.058	−0.222	0.692***	0.115	−0.217***
	(0.161)	(0.184)	(0.167)	(0.101)	(0.119)	(0.080)
δ_{size}	1.085***	0.572***	0.939***	−0.301**	0.052	−0.155
	(0.162)	(0.197)	(0.205)	(0.152)	(0.099)	(0.101)
α	0.001*	0.0001	−0.0002	−0.0005	0.001*	0.0005
	(0.0004)	(0.0003)	(0.001)	(0.001)	(0.0003)	(0.0005)
R^2	0.909	0.535	0.546	0.583	0.502	0.908
Residual Std. Error (df = 56)	0.003	0.003	0.004	0.003	0.002	0.002
F Statistic (df = 3; 56)	185.703***	21.483***	22.453***	26.137***	18.809***	183.755***
Note:						*p<0.1; **p<0.05; ***p<0.01

Table 10: Traditional Three-Factor Model (2012-2016)

	Dependent variable:					
	SH	SM	SL	BH	BM	BL
	(1)	(2)	(3)	(4)	(5)	(6)
β_{CAPM}	0.935***	0.515***	0.743***	0.771***	0.459**	0.963***
	(0.130)	(0.083)	(0.114)	(0.124)	(0.206)	(0.180)
δ_{value}	0.733***	0.354***	−0.146	0.911***	0.238**	−0.210***
	(0.063)	(0.091)	(0.089)	(0.084)	(0.103)	(0.072)
δ_{size}	0.684***	0.560***	0.910***	−0.222	−0.175	−0.449***
	(0.154)	(0.093)	(0.139)	(0.141)	(0.181)	(0.173)
α	−0.0001	0.001***	0.0004***	0.001***	0.0004	0.00002
	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)
R^2	0.738	0.438	0.523	0.607	0.268	0.778
Residual Std. Error (df = 56)	0.001	0.001	0.002	0.002	0.002	0.001
F Statistic (df = 3; 56)	52.699***	14.553***	20.503***	28.800***	6.836***	65.535***
Note:				*p<0.1; **p<0.05; ***p<0.01		

Table 11: Traditional Three-Factor Model (2016-2020)

	Dependent variable:					
	SH	SM	SL	BH	BM	BL
	(1)	(2)	(3)	(4)	(5)	(6)
β_{CAPM}	0.909***	0.975***	0.941***	0.967***	0.923***	0.935***
	(0.090)	(0.198)	(0.242)	(0.180)	(0.138)	(0.079)
δ_{value}	0.489***	0.363***	−0.150	1.007***	0.048	−0.353***
	(0.050)	(0.069)	(0.167)	(0.137)	(0.112)	(0.064)
δ_{size}	0.636***	1.175***	0.528***	−0.317**	−0.134	−0.209***
	(0.153)	(0.246)	(0.156)	(0.137)	(0.087)	(0.072)
α	−0.0001	0.0003	0.0003	0.0005**	0.00001	0.0001
	(0.0001)	(0.0003)	(0.0002)	(0.0002)	(0.0002)	(0.0002)
R^2	0.811	0.697	0.392	0.823	0.615	0.783
Residual Std. Error (df = 56)	0.001	0.002	0.002	0.002	0.002	0.001
F Statistic (df = 3; 56)	80.140***	42.882***	12.038***	86.786***	29.787***	67.359***
Note:				*p<0.1; **p<0.05; ***p<0.01		

Appendix: Equal-Weighted Portfolios by Risk Loading

Each month, risk loadings are calculated with data for the past year (prior to the current month), by estimating a 3FM (β_{CAPM}, δ_{size} and δ_{value}) and β_{ES} (as explained in the ES-implied beta section). The returns reported below correspond to the returns on the equal-weights linear combination of portfolios, sorted by their beta (β_{ES} for the first three rows and β_{CAPM} for the last three), and then controlling for their delta (δ_{size} for the first three columns and δ_{value} for the last three). Therefore, Table 12 reports a matrix with four quadrants, each containing nine cells with the mean of returns for the portfolio resulting from the intersection of both criteria (beta and delta).

Table 12: Average returns on an equal-weight combination of portfolios by tertile of their relative ES-implied beta (first three rows) or CAPM beta (last three rows) and their size and value delta (first and last three columns, respectively). All returns are expressed in percentage points.

	δ_{size}^{high}	δ_{size}^{med}	δ_{size}^{low}	δ_{value}^{high}	δ_{value}^{med}	δ_{value}^{low}
$\beta_{ES}^{-,high}$	2.0781	1.7452	1.7388	2.0094	1.6788	1.8737
$\beta_{ES}^{-,med}$	1.6913	1.3583	1.3520	1.6226	1.2920	1.4869
$\beta_{ES}^{-,low}$	2.1059	1.7729	1.7665	2.0372	1.7066	1.9015
β_{CAPM}^{high}	1.9546	1.6216	1.6153	1.8859	1.5553	1.7502
β_{CAPM}^{med}	1.8201	1.4872	1.4808	1.7515	1.4208	1.6157
β_{CAPM}^{low}	2.1005	1.7676	1.7612	2.0319	1.7013	1.8962

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