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Abstract

In the following dissertation paper, I developed three economic environments in the presence of conflict, in order to analyze different contexts where conflict may change the expected outcome.

The chapters are organized as follows: Chapter 1 is an introduction in which an overview of the dissertation paper is provided. In chapter 2 and chapter 3, two economic models, both with conflict contexts, are developed, in order to study the drug fight in transit countries. Finally, in chapter 4, an economic model of private property in an appropriation environment is analyzed.

As it has been mentioned, the first model, in chapter 2, describes drug fight in transit countries, allowing an endogenous coalition formation. In this section, an external observer gives incentives to the State to fight against drug trafficking. It is shown, as a result, that an increase on State incentives reduces drug trafficking and violence in transit countries, assuming that only the grand coalition and the singleton structure can be formed. However, a simultaneous increase on State incentives and drug market parameters may increase drug trafficking and violence.

The second model, in chapter 3, shares context with the first one, but it displays a more general setting, where traffickers may form any kind of coalition structure may be formed by traffickers. An important result in this chapter is that when traffickers do not change their behavior, if State incentives increase, then drug trafficking and violence decrease. In contrast, when traffickers change their behavior, if State incentives increase then drug trafficking and violence.

The contribution of both chapters is the generalized model of coalition formation with conflict presence, which is not normally used. Therefore, the model is, by itself, an interesting result in the coalition formation and conflict literature.

In chapter 4, a model in which agents can create private property rights on a resource by making appropriative activities is explained. Here, it is shown that the value of any resource has a non-monotonic effect on the emergence of private property. When the resource is sufficiently valuable, agents have an incentive to leave a sharing agreement and private property can emerge. If the value of the resource increases, beyond a given threshold, deviations from the sharing agreement lead to a very costly confrontation. In order to avoid this, agents stay in the agreement and private property is no longer sustainable. Additionally, it is demonstrated that population size has an important effect on the size of the parameter set, in which private property is sustainable.

Resumen.

En esta tesis se desarrollan tres ambientes económicos, con presencia de conflicto entre agentes, para analizar diferentes contextos donde el conflicto puede cambiar el resultado esperado. Los capítulos de la tesis están organizado de la siguiente forma: Capítulo 1 es una introducción de la tesis. En el capítulo 2 y capítulo 3 se muestran dos modelos económicos en un contexto de conflicto, para estudiar la lucha contra las drogas en los países de tránsito. Finalmente, en el capítulo 4 se analiza un modelo económico de apropiación de propiedad privada.

Como se mencionó antes, el primer modelo del capítulo 2 describe la lucha contra las drogas en países de tránsito, en el cual se permite la formación endógena de coaliciones. En el modelo, un observador externo otorga incentivos al Estado para que combata el tráfico. En los resultados se muestra que un aumento en los incentivos del Estado reduce el tráfico y la violencia, suponiendo que sólo se puede formar la gran coalición o la coalición individual. Sin embargo, un incremento simultáneo en los incentivos y los parámetros del mercado de droga elevan el tráfico y la violencia.

En el capítulo 3 se expone un segundo modelo que comparte el contexto del primero, pero considera un escenario donde los traficantes pueden formar cualquier estructura de coalición. Uno de los resultados es que, si los traficantes no cambian su comportamiento, un aumento en los incentivos del Estado, reduce el tráfico y la violencia. En contraste, si los traficantes cambian su comportamiento, el incremento en los incentivos eleva el tráfico y la violencia.

La contribución de ambos capítulos es un modelo generalizado de formación de coaliciones en presencia de conflicto, lo cual no es utilizado comúnmente. Por esta razón, el modelo, por sí mismo, es un resultado dentro de la literatura de formación de coaliciones y la de conflicto.

En el capítulo 4, se presenta un modelo en el que los agentes pueden crear derechos de propiedad mediante actividades de apropiación. Aquí, se muestra que el valor del recurso tiene un efecto no monótono en la aparición de la propiedad privada; cuando el recurso es valioso, los agentes dejan el acuerdo de reparto comunal y surge la propiedad privada. Si el valor del recurso aumenta, por encima de un valor crítico, las desviaciones del acuerdo de reparto pueden provocar una confrontación muy costosa; para evitar esto, los agentes permanecen en el acuerdo y la propiedad privada no es sostenible. Adicionalmente, se demuestra que el tamaño de la población, tiene un efecto en el tamaño del conjunto de parámetros, en los cuales la propiedad privada es sostenible.

Contents

Chapter 1

Introduction

In this dissertation paper I develop three microeconomic models where agents are in a conflict framework. In these models I allow that agents can form endogenous coalitions. In other words, agents can either associate or be alone. As more coalitions are formed the intensity of the conflict is bigger, therefore it is more costly to form small coalitions rather than big coalitions. Therefore agents rationally decide whether to fight with each other, in order to avoid splitting the total payoff with more people, or make agreements to obtain the highest payoff possible.

In many contexts, conflicts affects economic variables, but also economics variables can provoke conflicts, so we can say that there is a relationship between conflicts and economics. For example, when agents or countries fights for a valuable economic resource, also there is a direct relationship between countries defense expenditure and conflict.

A first example, in Mexico's history is the land reform approved in 1992. Farmers could obtain the property rights, but there is some cases where they agree to keep the common land. This conflict may not have harmful consequences, however most of the farmers had to spend resources and time in lawyers and courts to obtain that rights. A second example is the recent drug fight that is affecting several economic decisions and the intensity of the conflict has been very high in the last years.

In the three games that I present in this (dissertation) paper, I model the conflict with Contest Success Function. This functions are used in most of the conflict context. The next two chapters are dedicated to develop theoretical model of drug fight in Mexico.

In chapter 2, I develop a model for drug fighting with an endogenous coalition formation framework. In the model, there is an external observer who gives incentives to the State to fight drug traffic. Traffickers only can behave either competitively or cooperatively. In this model I consider the drug policy variables change and drugs market variables change. I show that only in the case in which traffickers do not change their behavior, upon a change on State incentives then drug traffic and violence decrease in the transit country. However, a simultaneous increase on State incentives and drug market parameters generally induce a change in traffickers' behavior. Therefore, drug traffic and violence may increase.

In chapter 3, I also develop a model for drug fighting in transit countries which allows endogenous coalition formation. A difference with chapter one, in this model I use a sequential coalition formation game which allows that traffickers can form any kind of coalition structure. Another difference in this chapter is that I only consider the drug policy variables change. I show that when traffickers do not change their behavior, then an increase on State incentives reduces drug traffic and violence. However, when traffickers change their behavior, an increase on State incentives generally increase drug traffic and violence.

Finally in chapter 4, agents can create private property rights on a resource by making appropriative activities. We show that the value of the resource has a non monotonic effect on the emergence of private property. When the resource is sufficiently valuable, agents have an incentive to leave a sharing agreement and private property can appear. If the value of the resource increases beyond a given threshold, deviations from the sharing agreement lead to a very costly confrontation so in order to avoid that, agents stick to the agreement. In that case, private property is not sustainable. Another important result is that population size has an important effect on the size of the parameter set in which private property is sustainable.

Chapter 2

Conflict and Narcotrafic: A Simple Economic Enviroment for Drug Traffic

2.1 Introduction

Mexico's former President Felipe Calderon president initiated what he called a ”Drug War” in 2006. That policy was aimed at reducing drug trafficking in Mexico. As a result, the National Security Expenditure on Defense increased from 2009 to 2011 in Mexico, but with that increase a number of closely related phenomena were observed. Firstly, the strategic financial and military aid from the United States (US) considerably increased in 2008 under the Merida Initiative. It increased from 50 million dollars in 2007 to 378 million dollars in 2008. Secondly, cocaine price went up nearly to 50 % from 2006 to 2008. Thirdly, the drug cartels in Mexico were fragmented between 2008 and 2011, from 8 to 12, respectively. In those years of drug war, a growth in the number of killings was patent. From 2007 to 2008, the number of homicides went up from nine thousand to fourteen thousand. Furthermore, the number of homicides continued increasing until 2011, when 26 thousand of homicides were accounted, according to the Statistics National Institute in México (INEGI).

It's been stated that the state expenditure under a regime of prohibition may be related to levels of violence, for instance J. Miron (1999) finds that US spending associated with a severe compliance of the law against alcohol and drugs, it is positively related to homicide rates. Regarding the Mexican context, some authors have found that the policy of the ”War on Drugs” is clearly related to violence, Rios (2010) and Dell (2011). Other factors affecting the level of violence in Mexico are the anti-drug policies which are made in the drug-producing countries, such as Colombia, Mejia at al. (2011). According to these articles, the highest point related to violence is a result of anti-drug policies.

The underlying thesis of the quoted authors is based on the assumption that drug dealers behave competitively. Accordingly, traffickers need to monopolize crime in the territories where they operate. However, there is evidence that the drug cartels are associated with other criminal organizations, (Rios (2012)). In addition, Guerrero (2011) mentioned that major Drug Cartels in Mexico were fragmented and began intense conflicts with their former allies. For example, the Sinaloa cartel was fragmented into six cartels between 2008 and 2010, while the Gulf Cartel split in two in 2010. After the division of the Sinaloa Cartel, there was a dramatic increase in violence, Osorno

(2009).

In support of this view, Grossman and Mejia (2008) developed a model to study drug policies in producing countries. In their model, the state wants to reduce significantly the supply of drugs to consumer countries. Drug producers and the state are facing various conflicts to maximize their profits. The Grossman - Mejia model establishes a framework for evaluating policies against drugs and the effects of violence without overlooking the fact that drug traffickers also behave competitively.

In this chapter, I develop a cooperative model similar to Grossman–Mejia's model to study the conflict between the state and traffickers. This model is a three-stage sequential game. In the first stage, the outsider select the proportion of resources and the level of punishment to give incentives to the State to fight against drug traffickers. In the second stage, traffickers decide whether to behave cooperatively and form a cartel or to behave competitively and become individual traffickers. In the third stage, individual traffickers or the cartel confront openly against the state in a conflict over a drug trafficking network. We must also consider that there is an outside observer, which provides incentives to the state in order to combat trafficking.

In this chapter, I establish that State incentives, drug price, and size of the network, have an endogenous effect in the behavior of traffickers. Besides the features mentioned, one have to ponder that drug prohibition policies reduce traffic and violence when drug prices and the trafficking network remain constant. However, when drug prices and the trafficking network are altered, the results of the drug prohibition policy may have uncertain effects.

The chapter is organized as follows: In the following section, the economic environment for the traffickers game is set up. In Sections 3 and 4, I obtain, by the model proposed, respectively the optimal decision for the third stage, and the choice of a coalition structure. Finally, conclusions are set in Section 5.

2.2 Economic Environment

I model the traffickers' problem as a sequential three stages cooperative game. This model consider a conflict between traffickers and the State, in which traffickers obtain a benefit from trafficking drugs and the State wants to reduce the drugs that are trafficked.

In this game, there is an external observer, O, which gives incentives to the State to fight trafficking. I model the external observer as a nature player and because of that he does not receive payoffs. There is a set $N = \{1, 2, \ldots, n\}$ of *identical traffickers*, they are identical in all relevant aspects, which want to maximize their profits trafficking drugs to a consumer countries. There is also a State, S, which wants to reduce trafficking in his country.

Grossman and Mejía (2008) define an external observer. In this game, I model the external observer as a nature player. The nature player chooses the fraction of resources $1 - \Omega$, from an uniform distribution $U[0, 1]$, which the State receives as a help in its drug fight. It also selects the level of punishment h, from a $U[0, \infty]$, that the State receives if it does not fight traffickers.

Each trafficker chooses a number $\hat{a}_i \in [1, n]$ that represents the number of players of the coalition that he wishes to be. He also chooses the number of resources $x_i \in [0, \infty)$, valued in dollars, that he wants to spend in the conflict with the State. The State only chooses its level of investment $Z \in (0, \infty)$, valued in dollars, in the conflict with drug traffickers.

In the first stage, the nature selects the fraction of resources that it will give to the state and the punishment level, which is known by all players. In the second stage, the set N of traffickers form coalitions between them, these coalitions will be called cartels. I model the coalition formation with the Γ game from Hart and Kurz, (1983), which allows to the players to behave cooperatively. The game consists in different players who make lists of which other players they want to make a coalition with. If all players have the same list, then a coalition is formed. If they do not have the same list, then they become traffickers (singleton coalition)[1]. In this game, traffickers simultaneously choose a number $\hat{a}_i$ to form a coalition with that many players, after this a coalition

[1]For example consider that the set of players is $N = \{1, 2, 3\}$, player1 and player 2 list is $L_i = \{1, 2\}$, and player 3 list is $L_3 = \{1, 2, 3\}$. In this case, a coalition is not formed, and the coalition structure is $\Pi = \{1, 2, 3\}$. However, if player 3 list is $L_3 = \{3\}$, then players 1 and 2 form a coalition and player 3 becomes a singleton. The coalition structure, in this case, is $\Pi = \{12, 3\}$.

structure $\pi = \{a_1, a_2, \ldots, a_m\}$ is formed, this will be explained, with more detail, below. The number of cartels that traffickers form is m which is less or equal than n. This game is very restrictive because it gives veto power to any agent on the coalitions. This assumption will be relaxed in the next chapter.

In the third stage, the m cartels and the State engage in a conflict for the divisible network $W > 0$. In order to obtain a fraction of the network, each trafficker invests x_i resources. The resources that coalition j invest in the conflict is $X_j = \sum_{i \in a_j} x_i$. The State invests Z resources. I model the conflict with a standard Contest Success Function (CSF)[2],

$$p_j = \frac{\Phi_j X_j}{Z + \sum_{j=1}^{m} \Phi_j X_j}, \tag{2.1}$$

where $\Phi_j \equiv \Phi(a_j)$ is a function that represents the effectiveness of cartel j in the conflict, this function depends on the group size a_j. Once, cartel or traffickers acquire a fraction of the network, enables them to sell drugs in the consumer country and obtain C potential profits from traffic. Simultaneously, the State wants to destroy the network, in order to reduce the traffic and the supply of drugs.

Traffickers Payoffs

I assume that every cartel divides equally the profits with a *constant sharing rule*, $\rho = \frac{1}{a_j}$.*I assume that every player in the coalition makes an effort strictly positive.* If a trafficker that belongs to coalition a_j deviates then he receives an infinite punishment[3]. Therefore, free riding is not possible in this model. Traffickers payoffs are the share of the coalition potential profits from the drug market, less the conflict cost. Payoffs for each drug trafficker i who belongs to cartel a_j is

[2] Usually, a CSF considers $p(y_i, \{\mathbf{y_{-i}}\}) = \frac{\mathbf{1}}{\mathbf{n}}$ if $\{\mathbf{y}\}$ is zero. However, not invest strategy is weakly dominated, then I only consider the non-zero part. See Skaperdas (1996) for an axiomatization.

[3] This assumption may seem reliable in this context because if a trafficker does not pay the conflict cost from the coalition, then other coalition members try to kill him or people related to him (Family or collaborators), this is the punishment. Some stories related with drugs traffickers life mention that in 1992, Ismael Zambada didn't pay tariffs to Tijuana cartel, and then Arellano Felix brothers, Benjamin and Ramon, attempted to his life; Blancoornelas (2002). In 2004, Vicente Carrillo Fuentes didn't pay his security share with the Federacion cartel, and then a year later Sinaloa cartel members kill his brother; Hernandez, (2010).

$$R_{i \in a_j} = \rho C p_j W - x_i, \tag{2.2}$$

Additionally I assume that $CW > 1$.

State Payoffs

The state does not want to be punished by the nature; therefore he wants to reduce drug traffic. Without the state intervention, drugs that could be trafficked are λW, where λ represents the potential drug that may be trafficked in the network W. When the State intervenes trafficking is

$$D = \lambda \sum_{j=1}^{m} p_j W, \tag{2.3}$$

where $D \leq \lambda W$.

The State associates a probability to be punished given by $\frac{D}{\lambda W}$, then the expected punishment cost for the State is $\frac{hD}{\lambda W}$. The State expected payoff is the resources that he receives from the observer $(1 - \Omega)Z$, less the expected punishment and the resources spend in the conflict,

$$S = (1 - \Omega)Z - \frac{hD}{\lambda W} - Z.$$

Using equation 2.3 in the equation above, the State payoff is

$$S = -h \sum_{j}^{m} p_j - \Omega Z. \tag{2.4}$$

Then a maximization on State payoff implies a minimization in State cost of drug fight.

2.2.1 First Stage

I assume that the outside observer is a nature player, which selects the level of punishment h and the fraction of resources $1 - \Omega$, as described above.

Grossman and Mejia (2008) assume the existence of an *outside observer* which wants to minimize the drugs that are trafficked to the consumer country. The observer uses a stick and carrots strategy, which gives incentives to the State to fight against traffickers. If the state does not combat, then the observer labels it as a ”Narco-State”, which represents a cost to the state h; this is the stick. This observer provides aid[4] valued in dollars, to help the fight against drugs in the transit country; this is the carrot.

2.2.2 Second stage

In the second stage, the n drug traffickers form coalitions with the Γ game, which is explained above. A coalition structure is a partition from the set N.

The *identical traffickers* assumption implies that the main difference between coalitions is the number of players that belong to the same coalition. Then a coalition structure for this game can be written as $\pi = \{a_1, a_2, \ldots, a_m\}$, where a_j is the group size[5].

Once stated these settings, I modify the Γ game, in the following way. The players propose a number $\hat{a_i}$ of members who they want to be associated with. If each player can be assigned with a number of members that he's chosen, then a coalition structure $\pi = \{\hat{a_1}, \hat{a_2}, \ldots, \hat{a_m}\}$ is formed. If at least one player cannot be assigned, then a traffickers (singleton) structure $\pi = \{1, 1, \ldots, 1\}$ is formed.

[4]Money; training to the State police and army forces; intelligence reports; etc.

[5]In this case, two different coalition structure, Π and Π', may have the same coalition structure π. For example, in the three players case, two different coalition structures are $\Pi = \{12, 3\}$ and $\Pi' = \{13, 2\}$, however they have the same coalition structure $\pi = \{2, 1\}$.

2.2.3 Third Stage

In the third stage, drug cartels and the State fight for a fraction of the drug trafficking network W, given the coalition structure π. The network represents territories, corrupt agents, ports where drugs arrive to the transit countries, laboratories, routes to the consumer country, etc. Traffic of drugs is proportional to the fraction network that each cartel control.

They compete accordingly with the CSF, equation 2.1. Cartel j controls a network of size $p_j W$ this fraction allows it to traffic drugs. The State wants to destroy the network, in order to reduce the probability to be punished.

I assume that there is a *superadditivity technology*[6] in traffickers' coalitions. In other words, I assume $\Phi(a_j)$ in equation 2.1 is a *continuous non-decreasing* function of the number of cartel members, a_j. I also assume, as well as Grossman and Mejía, (2008); that *traffickers are more effective* in the conflict than the State because they can use guerrilla techniques. This assumption implies that $\Phi(1) > 1$. Finally, I assume that conflict does not destroy the network[7]. However, an increment in conflict resources is positively related with violence rates, i.e. the number of homicides.

In this game, given the punishment level h and the resources from the external observer $1 - \Omega$, the traffickers strategy is a vector of members $\{\hat{\mathbf{a}}_\mathbf{i}\}$ of the possible coalition, and the resources x_i that he is willing to invest given the coalition structure π. The State strategy is the resources that he is willing to spend Z given π.

Before to solve this problem, I define *intensity of the conflict* as the effective resources spent in the conflict,

$$I \equiv Z + \sum_{j=1}^{m} \Phi_j X_j. \tag{2.5}$$

[6]See Skaperdas (1998), and Garfinkel and Skaperdas (2006) for a review on superadditivity coalitions games.

[7]It is possible to relax this assumption assuming that a more intensive conflict destroys a higher fraction of the network than less intensive conflicts. However, the assumption seems to fit as cartels replace their territories in less than three years in average; Rios, (2012).

This definition simplify the notation below, it only says that as the resources, considering the effectiveness function, increase, then the conflict between the State and cartels is more intense.

Following Miron (1999), the government expenditure in law enforcement against alcohol and drugs is directly related to the rates in homicides. Due to this fact, it is possible to define violence as the total amount of resources that the State and coalitions spend[8],

$$V = Z + \sum_{j}^{m} X_j. \tag{2.6}$$

As a consequence, any increment in the total amount of resources in the conflict increases the amount of violence. One must also remember that the number of traffickers' coalitions has an important effect on violence levels. Intuitively, one may think that when players cooperate they spend less money and in this case violence levels are lower.

2.3 Optimal Traffickers and State Expenditure

From the previous section, the observer (nature player) selects an $\{h, 1-\Omega\}$, from a uniform distribution. Traffickers choose the number of players which they want to be associated with, and the number of resources that he invests given the coalition π, i.e. $\{\hat{a}_i, x_{i \in a_j}\}$. Finally, the State decides the number of resources that he invests in the conflict, Z.

An equilibrium *in pure strategies*, in this game, consists in a level of punishment, and fraction of resources, $\{h, 1-\Omega\}$. A vector of $\hat{\mathbf{a}}_\mathbf{i}*$ which form a coalition π and a vector of resources spend in the conflict $\mathbf{x}_{\mathbf{i}\in\mathbf{a}_\mathbf{j}}, Z$. Then only symmetrical structures are formed. In other terms, at equilibrium the coalition structure is $\pi = \{a, a, \ldots, a\}$, where a is a divisor of n.

To see this consider the three player example, $N = \{1, 2, 3\}$. Let the coalition structures and individual payoffs to be: $\pi_{gc} = \{3\}$ and R_{gc} for the grand cartel; $\pi_s = \{1, 1, 1\}$ and R_s for the singleton structure; and $\pi = \{1, 2\}$, R_1 the payoff for the singleton, and R_2 the payoff for the

[8]This definition may seem restrictive and arbitrary, therefore I use two alternative definition of violence in appendix 2.

two-member coalition.

Suppose that $R_1 > R_2 > R_{gc} > R_s$. In this case all players want to be the singleton in coalition structure $\pi = \{1, 2\}$, then all of them has incentives to propose $\hat{a}_i = 1$ but it yields to the singleton structure where all are worse. Because they know that and there is no communication between them, they proposes $\hat{a}_i = 3$ which is the equilibrium. Suppose it is not and without loss of generality player 1 deviates and chooses $\hat{a_1} = 1$ as the other two players are choosing $\hat{a}_i = 3$, with $i = 1, 2$, then the coalition structure can not be formed and the protocol forms the singleton coalition structure with $R_s < R_{gc}$ therefore no one deviates. The argument is the same for $R_1 > R_{gc} > R_2 > R_s$, also to $R_2 > R_1 > R_{gc} > R_s$ and to $R_2 > R_{gc} > R_1 > R_s$. The argument is similar of the previous one, but in these cases players choose the singleton structure for $R_1 > R_2 > R_s > R_{gc}$, also to $R_1 > R_s > R_2 > R_{gc}$, this one to $R_2 > R_1 > R_s > R_{gc}$, and to $R_2 > R_s > R_1 > R_{gc}$. If the grand cartel or the singleton strategies are dominant then the result is trivial. Similar argument applies when the number of players is bigger and it is considered asymmetrical coalition structures.

In this chapter, it's only consider the grand cartel (Below I call this structure as the cartel) $\pi_{gc} = \{n\}$ and the traffickers structure $\pi_s = \{1, 1, \ldots, 1\}$. Therefore, if traffickers behave cooperatively, they form a cartel. If they behave competitively, a traffickers structure is formed.

I use backward induction to solve the problem. In the third stage traffickers maximize their individual payoffs, 2.2, given the coalition structure π. I divide the solution between the traffickers coalition structure π_s and the cartel coalition structure π_{gc}. At the end of this section, I summarize responses changes as parameters changes when coalition structure remains constant.

2.3.1 Traffickers Structure Solution

Here, I describe the solution for the traffickers coalition structure π_s. *Traffickers identical* assumption implies that they will spend the same resources and obtain the same network fraction in the traffickers structure. Then a expenditure vector $\{\mathbf{x_i}, Z\} = \{\mathbf{x_s}, Z_s\}$ and a fraction network vector $\{\mathbf{p_i}\} = \{\mathbf{p_s}\}$ is a solution for this problem in the third stage.

Traffickers solve the following program

$$\begin{aligned} R_i &= CWp_i - x_i, \\ s.t. \quad x_s &\leq CW. \end{aligned} \tag{2.7}$$

The Lagrangian of this system is

$$\mathcal{L} = CWp_i - x_i + \lambda(CW - x_i) \tag{2.8}$$

The Kuhn Tucker conditions are $\frac{\partial L}{\partial x_s} \leq 0$; $x_s \frac{\partial L}{\partial x_s} \leq 0$; $\frac{\partial L}{\partial \lambda} \geq 0$; and $\lambda \frac{\partial L}{\partial \lambda} \geq 0$. If $x_i = 0$ then $\lambda = 0$ because it is the unique value which satisfies the last condition. By the identical traffickers assumption every other trafficker should find that not invest is optimal then $\sum_i \Phi_s x_s = 0$ and should be true that $Z \geq CW\Phi_s$ by the two first condition.

Suppose that $Z \leq CW\Phi_s$ then the sufficient FOC are

$$\frac{\partial R_s}{\partial x_s} = \frac{CW\Phi_s}{I_s}(1 - p_s) - 1 = 0, \tag{2.9}$$

Where $I_s = Z + \sum_i \Phi_s x_s$ is the *intensity of the conflict*. In the equation above, I use that $\frac{Z + \sum_{-i} \Phi_s X_s}{I_s} = 1 - p_s$.

The State wants to minimize it's expected cost on drug fight, given the structure π_s, the FOC for the State is

$$\frac{\partial S}{\partial Z} = h \sum_i \frac{p_s}{I_s} - \Omega = 0. \tag{2.10}$$

Equations 2.9 and 2.10 solves the traffickers structure problem. Each trafficker obtains a fraction network

$$p_s = \frac{CW\Phi_s}{\frac{h}{\Omega}n + CW\Phi_s}, \tag{2.11}$$

the total network fraction under traffickers control is

$$\sum_i p_s = n\frac{CW\Phi_s}{\frac{h}{\Omega}n + CW\Phi_s}. \tag{2.12}$$

This problem is well defined as long as $\sum_i p_s \leq 1$. If this condition is not satisfied, then the State does not participate in the conflict, and traffickers control all the network. The following proposition characterize this possibility.

Proposition 2.3.1. *A necessary and sufficient condition for the State to fight against traffickers is that*

$$\frac{h}{\Omega} \geq \frac{n-1}{n}CW\Phi_s \tag{2.13}$$

Proof : From 2.12, it is **necessary and sufficient** that $nh \geq CW\Omega\Phi_s(n-1)$. Notice that $\Omega \leq 1$, then a sufficient condition is that $h \geq \frac{n-1}{n}CW\Phi_s$. ■

Proposition 2.3.1 tells that, if State incentives are high enough, then the State fights against traffickers. Else the State does not participate and accept the punishment. Below, I assume that this condition is satisfied, always.

Using, 2.11 and 2.10, in 2.5 the conflict intensity is

$$I_s = n\frac{\frac{h}{\Omega}CW\Phi_s}{\frac{h}{\Omega}n + CW\Phi_s}. \tag{2.14}$$

Using the two expressions above and 2.1 is easy to find the resources that traffickers and the State spend in the conflict,

$$x_s = \frac{p_s I_s}{\Phi_s} = n\frac{\frac{h}{\Omega}C^2W^2\Phi_s}{\left(\frac{h}{\Omega}n + CW\Phi_s\right)^2}, \tag{2.15}$$

and

$$Z_s = (1 - np_s)I_s = n\frac{\frac{h}{\Omega}CW\Phi_s[n\frac{h}{\Omega} - CW\Phi_s(n-1)]}{\left(\frac{h}{\Omega}n + CW\Phi_s\right)^2} \tag{2.16}$$

Both expressions must be non-negative. State expenditure is positive, by the condition in proposition 2.3.1. Traffickers' profits come from 2.15 and 2.11 in 2.2

$$R_s = CWp_s^2 = CW\left(\frac{\Phi_s CW}{\frac{h}{\Omega}n + CW\Phi_s}\right)^2. \tag{2.17}$$

Drugs traffic is easily obtained from 2.12 in 2.3,

$$D_s = \lambda W n p_s = \lambda W n \frac{CW\Phi_s}{\frac{h}{\Omega}n + CW\Phi_s}. \tag{2.18}$$

Finally, using both expenditures, 2.15 and 2.16, it is easy to obtain the violence expression 2.6

$$V_s = I_s - np_sI_s + \frac{np_sI_s}{\Phi_s} = I_s\frac{\frac{h}{\Omega}n + CW - (n-1)(\Phi_s - 1)CW}{\frac{h}{\Omega}n + CW\Phi_s} \tag{2.19}$$

These equations are the solution in the third stage for the traffickers structure problem.

2.3.2 Cartel Coalition Structure Solution

In the cartel structure the expenditure vector $\{\mathbf{x_i}, X_{gc}, Z\} = \{\frac{\mathbf{X_{gc}}}{\mathbf{n}}, X_{gc}, Z\}$ and the fraction p_{gc} is a solution for traffickers game given π_{gc}. In this problem traffickers FOC are

$$\frac{\partial R_i}{\partial x_i} = \frac{CW\Phi_{gc}}{I_{gc}}(1 - p_{gc}) - 1 = 0, \tag{2.20}$$

Where $I_{gc} = Z + \Phi_{gc}X_{gc}$ is the *intensity of the conflict*. As well as in the traffickers section the FOC are sufficient if the State expenditure is lower than the potential profits considering the effectiveness of the grand cartel. $Z < CW\Phi_{gc}$. Notice that for the *no free riding* assumption if the cartel invest in the conflict then every member spend a positive amount on it.

The State minimize costs, given the coalition structure π_{gc}, the FOC for the State is

$$\frac{\partial S}{\partial Z_{gc}} = h\frac{p_{gc}}{I_{gc}} - \Omega = 0. \tag{2.21}$$

Equations 2.20 and 2.21 solves this problem; the cartel obtains a fraction network

$$p_{gc} = \frac{CW\Phi_{gc}}{\frac{h}{\Omega}n + CW\Phi_{gc}}, \tag{2.22}$$

in this case p_{gc} represents the total traffickers network, also. This problem is always well defined because $p_{gc} \leq 1$.

Using, 2.22 in 2.21, the conflict intensity is

$$I_{gc} = \frac{\frac{h}{\Omega}CW\Phi_{gc}}{\frac{h}{\Omega}n + CW\Phi_{gc}}. \tag{2.23}$$

Using the two expression above and 2.1 is easy to find the resources that traffickers and the State spend in the conflict,

$$X_{gc} = \frac{p_{gc}I_{gc}}{\Phi_{gc}} = \frac{\frac{h}{\Omega}C^2W^2\Phi_{gc}}{\left(\frac{h}{\Omega}n + CW\Phi_{gc}\right)^2}, \tag{2.24}$$

$$Z_{gc} = (1 - p_{gc})I_{gc} = n\frac{\left(\frac{h}{\Omega}\right)^2 CW\Phi_{gc}}{\left(\frac{h}{\Omega}n + CW\Phi_{gc}\right)^2} \tag{2.25}$$

Both expressions are positive. The State always fights against the cartel. Individual traffickers' profits come from equations 2.15 and 2.11 in 2.2

$$R_{gc} = \frac{CWp_{gc}}{n} - \frac{p_{gc}I_{gc}}{n\Phi_{gc}} = \frac{CWp_{gc}}{n}\left(\frac{\frac{h}{\Omega}(n-1) + CW\Phi_{gc}}{\frac{h}{\Omega}n + CW\Phi_{gc}}\right) \tag{2.26}$$

Drugs traffic is easily obtained from 2.22 in 2.3,

$$D_{gc} = \lambda W p_{gc} = \lambda W \frac{CW\Phi_{gc}}{\frac{h}{\Omega}n + CW\Phi_{gc}}. \tag{2.27}$$

Finally, from both expenditures, 2.24 and 2.25, violence expression 2.6 is obtained

$$V_{gc} = (1-p_{gc})I_{gc} + \frac{p_{gc}I_{gc}}{\Phi_{gc}} = I_{gc}\frac{\frac{h}{\Omega}n + CW}{\frac{h}{\Omega}n + CW\Phi_{gc}} \tag{2.28}$$

2.3.3 Incentives Changes

The Solution for both coalition structures, traffickers and cartel respectively, are in parameters. In this section, parameters change are analyzed, when traffickers do not change their behavior. In other words, I assume that the second stage of the game does not exist, in this section, and I analyze the effects on the results. Conditions under which the coalition structure changes are provided in proposition 2.4.1, in the next section.

The analysis of this section is important because most of the studies which analyze the impact on drugs policy in drug traffic (e.g. Grossman and Mejia (2008); Rios (2013); Dell (2015)), assume that the coalition structure does not change. Most of them obtain that if there is an increase in international resources or punishment then drugs traffic decreases (which is the result on proposition 2.3.2), or if the potential profits from drugs sells decrease then it reduces traffic activities (the result on proposition 2.3.3). I obtain a similar results to them when traffickers are not allowed to change the coalition structure. But in the next section, I show that these results are not so obvious when they are allow to change between coalitions.

Proposition 2.3.2. *If coalition structure does not change, whenever State punishment (international resources) increases (increase), then State expenditure increases; traffickers in-*

dividual profits and drugs traffic decrease. As a collateral effect violence increases.

Proof : See Appendix 1

Proposition 2.3.2 summarizes the results when the coalition structure remains constant, and punishment or international resources increase[9]. In other words, when the State incentives increase.

The results are expected by the policy. As the observer gives more incentives, the State spends more resources in the conflict, then traffickers network fraction decreases, which reduces drugs traffic and traffickers' profits. Finally, an undesirable effect is that violence rates increase.

Proposition 2.3.3. *If coalition structure does not change, whenever network size (potential profits) increases (increase), then traffickers individual profits and drug traffic increase. However, State expenditure and violence may decrease.*

Proof: See Appendix 1

Remark A necessary but not sufficient condition for a decreasing violence in the traffickers structure is $\Phi_s > 2$; and in the cartel structure is $\Phi_{gc} > (n-1)\Phi_s + 2$.

Proposition 2.3.3 summarizes the results when the coalition structure remains constant and potential profits or the network size increase. i.e. traffickers incentives increase.

These results are as expected, also. With more incentives coalitions spend more resources in the conflict, then traffickers network fraction increases, which increases drugs traffic and traffickers profits. However, in this case, the State expenditure may decrease, and then violence change is uncertain.

2.4 Coalition Structure Choice

In the second stage, traffickers compare between traffickers and cartel structures and choose the one with higher profits.

[9] An increment on international resources implies that Ω decreases

Proposition 2.4.1. *Traffickers prefer the cartel to the traffickers structure if*

$$\frac{h}{\Omega\Phi_{gc}}\frac{n-1}{n} \geq CW\left[\left(\frac{\frac{h}{\Omega}\frac{n}{\Phi_{gc}}+CW}{\frac{h}{\Omega}\frac{n}{\Phi_{s}}+CW}\right)^2-\frac{1}{n}\right] \tag{2.29}$$

Proof: Traffickers prefer the cartel structure if $R_{i\in gc} > R_s$. From 2.17 and 2.26 this implies that

$$\frac{1}{n}\left(\frac{(h\frac{n-1}{\Phi_{gc}}+CW\Omega)CW\Omega}{(h\frac{n}{\Phi_{gc}}+CW\Omega)^2}\right) \geq \left[\frac{CW\Omega}{h\frac{n}{\Phi_s}+CW\Omega}\right]^2$$

.

Simplifying this expression, it is obtained the expression in the proposition. ■

Notice that, if the term in brackets, in equation 2.29, on the right hand side is negative, then cooperation is a dominant strategy in the second stage.

The left hand side on the condition is increasing with respect to State incentives[10].The right hand side is decreasing with State incentives and increasing with traffickers incentives.

Remark: *If the State incentives increase, then traffickers have more incentives to behave cooperatively, i.e. collusion incentives increase. If traffickers incentives increase, then traffickers have more incentives to behave competitively.*

This result is very intuitive. When State incentives increase; traffickers prefer, behave cooperatively to fight a stronger enemy. If traffickers incentives increase, then it is more difficult to keep a cooperative agreement because deviations from it are more profitable.

Figure 2.1 shows the change on the dependent variables as State incentives change, as well as the coalition structure choice. In this figures crosses lines (x) represent Cartel solution, and pluses lines (+) represent traffickers solution. The continuous lines are the equilibrium outcome at different levels of the State incentives of the game. Figure 2.1.a shows individual traffickers' profits. Figure 2.1.b illustrate the increasing State expenditure. Figure 2.1.c and 2.1.d shows drugs that are

[10]It is increasing with respect to h and decreasing with respect to Ω

trafficked and violence levels respectively.

Figure 2.1 shows the intuition in proposition 2.4.1. State expenditure increases as the State incentives do. Profits in the traffickers structure decrease faster than cartel structure profits, then traffickers prefer to collude than fight each other. Drugs traffic and violence equilibrium paths are discontinuous as coalition structure changes. Therefore, as State incentives increase there can be a big reduction on trafficking and violence.

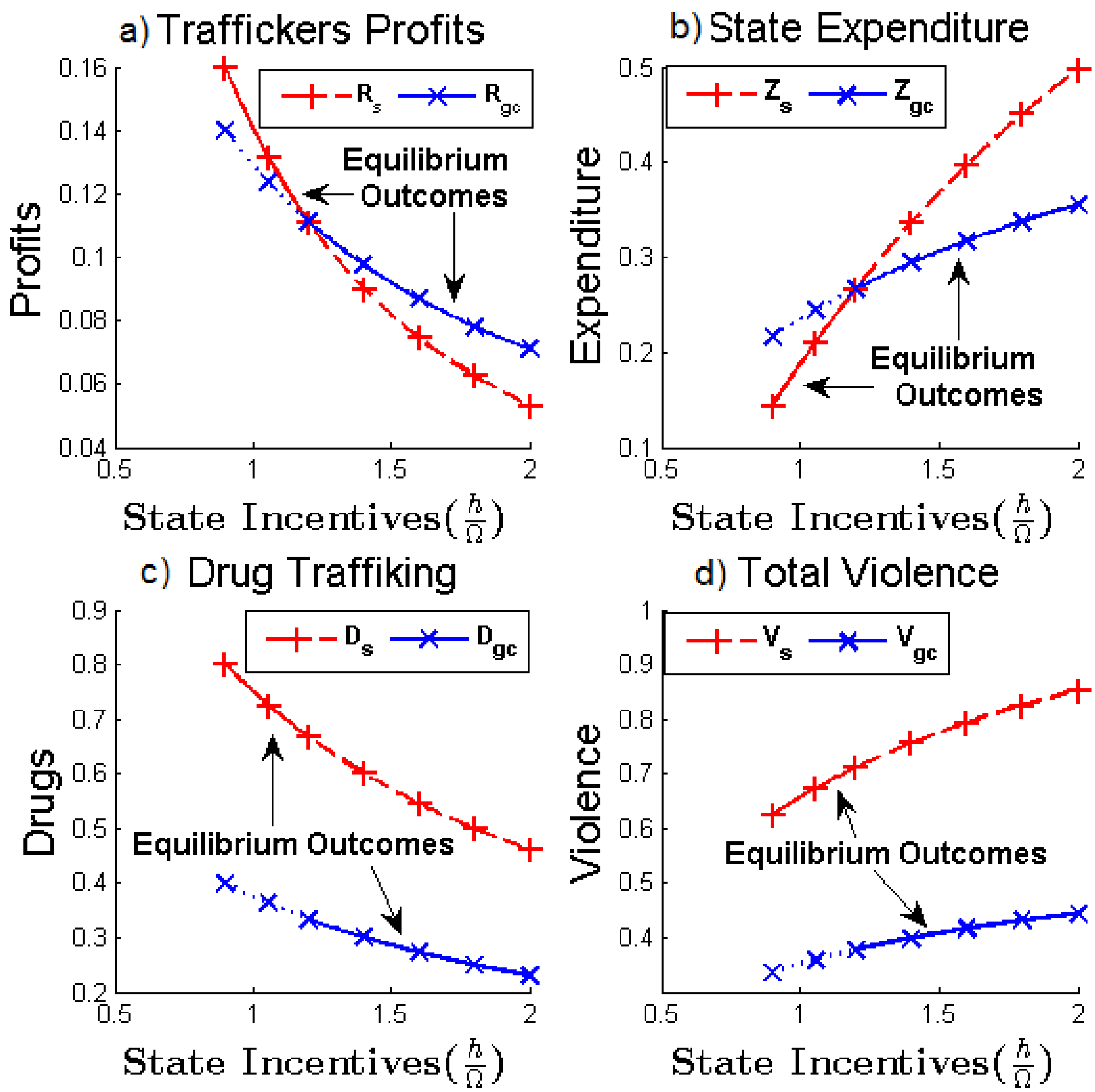

Figure 2.1: State incentives changes and coalition structure choice. a) Individual Traffickers Profits; b) State Expenditure; c) Drugs Trafficked; d) Violence. For this example: $\lambda = 1$; $Q = 1$; $W = 1$; $n = 2$; and $\Phi_s = \Phi_{gc} = 1.2$

Proposition 2.4.2. *Suppose that $\Phi_s = \Phi_{gc} = \Phi < n + 1$. In this case, drug traffic and violence are higher in the traffickers structure than in the cartel structure.*

Proof If $\Phi_{gc} = \Phi_s = \Phi$, then $p_{gc} = p_s$, which proofs the drugs traffic part of the proposition because $p_{gc} < \sum_i p_s$. In the second part of the proposition I want to show that $V_s - V_{gc} > 0$, If $\Phi_{gc} = \Phi_s = \Phi$ then $I_s = nI_{gc}$, then from 2.19 and 2.28 it is sufficient to show that

$$n(\frac{h}{\Omega}n + CW - CW(\Phi - 1)(n-1)) - (\frac{h}{\Omega}n + CW) > 0. \tag{2.30}$$

Notice that this equation is increasing with respect to $\frac{h}{\Omega}n$. By equation 2.29 I know that $\frac{h}{\Omega}n \geq (n-1)CW\Phi$, then the expression 2.30 is bigger than

$$(n+1)CW - CW\Phi \tag{2.31}$$

Which is bigger than zero, because $\Phi < n+1$. Hence, violence in the traffickers structure is higher than violence in the cartel. ■

These results are expected. When traffickers behave competitively, trafficking activities and violence rates are bigger than when they behave cooperatively. Thus, an increment in State incentives, when everything else remains constant, increases cooperative incentives between traffickers, which reduces drugs traffic and violence rates. However, when state incentives and traffickers incentives increase simultaneously, may lead to undesired results, as traffickers may change from the cartel to the singleton (here and below I use singleton coalition structure as same as traffickers coalition structure). The following proposition shows how the coalition structure change affect the outcomes.

Proposition 2.4.3. *Suppose that $\Phi_s = \Phi_{gc} = \Phi < n+1$ and $h_f \geq h_0$; $\Omega_f \geq \Omega_0$; $C_f \geq C_0$; and $W_f \geq W_0$, where the subindex f is the final state and the subindex 0 is the initial state.*

If State and traffickers incentives increase simultaneously, i.e $\frac{h_f}{\Omega_f} > \frac{h_0}{\Omega_0}$ and $C_f W_f > C_0 W_0$, such that there is a coalition structure change from cartel to singleton structure, then drugs traffic and violence rates increase.

proof: Assume $\Phi_{gc} = \Phi_s$. Let $\frac{h_f}{\Omega_f} > \frac{h_0}{\Omega_0}$ and $C_f W_f > C_0 W_0$, then $D_{s_f} > D_{gc_0}$ and $V_{s_f} > V_{gc_0}$

Claim: $D_{s_f} > D_{gc_0}$.

It is sufficient to show that

$$nC_0W_0(C_fW_f^2\Phi_s - \frac{h_f}{\Omega_f}W_0) + C_fW_f(\frac{h_0}{\Omega_0}W_fn^2 - C_0W_0^2\Phi_s) > 0. \tag{2.32}$$

Initially traffickers choose the cartel structure, from 2.29 $\frac{h_0}{\Omega_0} > C_0W_0\Phi_s$. At the final period, they choose singleton structure, then $\frac{h_f}{\Omega_f} < C_fW_f\Phi_s$, from equation 2.29. These conditions satisfy the inequality[11].

Claim: $V_{s_f} > V_{gc_0}$.

If $\Phi_{gc} = \Phi_s$ then violence in the cartel structure increases with traffickers incentives, by proposition 2.3.3. From propositions 2.3.2 and 2.3.3, violence in the cartel structure increases, i.e. $\Delta V_{gc} = V_{gc_f} - V_{gc_0} > 0$. Finally, total violence after the fragmentation of the cartel increases, i.e. $\Delta V = V_{s_f} - V_{gc_0} > V_{gc_f} - V_{gc_0} > 0$, by proposition 2.4.2. ■

Proposition 2.4.3 shows that if traffickers and state incentives change simultaneously, may change traffickers behavior. Therefore, drugs[12] and violence rates are higher, as traffickers behave competitively instead of cooperatively. Next figure 2.2 shows this result.

Figure 2.2 shows drugs and violence change as potential profits and state incentives increase. In this case, potential profits increase 50%. Crosses (x) and pluses (+) lines represent cartel and singleton solutions, respectively, before the increment on potential profits, at Q_0. Asterisks (*) and circles (o) lines represent cartel and singleton solutions, respectively, after potential profits increase. D_0 and V_0 are the drug traffic and violence, respectively, before the change. D_1 and V_1 represent the first hypothetical change on drugs and violence, respectively, when the coalition structure does not change. D_2 and V_2 represent the second hypothetical change on drug and violence when the coalition structure changes.

Figure 2.2 shows two situations. In the first situation, coalition structure does not change. In this case, trafficking activities may decrease if the State incentives change is big enough. However, violence increases, because the State and the cartel expenditure increase with both incentives. In the second situation, the coalition structure changes. In this case, drugs and violence increase. This

[11] If the network size decreases, then the result is uncertain, then the nondecreasing assumption is necessary in this case

[12] Drugs result is weaker than violence result with respect to $\Phi_{gc} = \Phi_s$ assumption. If the technology is increasing, $\Phi_{gc} > \Phi_s$, the result may reverse.

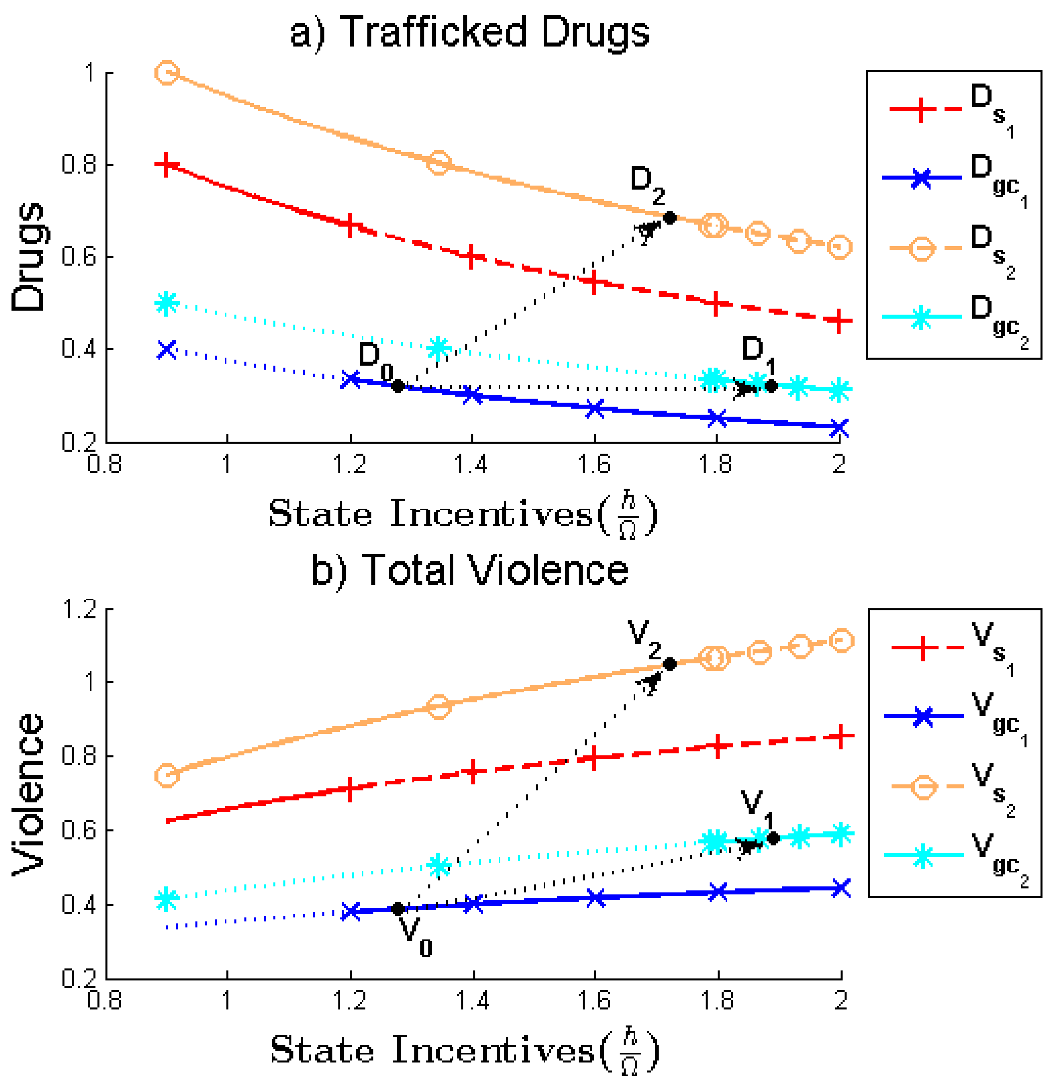

Figure 2.2: Drugs and Violence change as state incentives and potential profits increases. a) Trafficked Drugs and b) Total Violence. For this example: $Q_0 = 1$; $Q_f = 1.5$; $\lambda = 1$; $W_0 = W_f = 1$; $n = 2$; and $\Phi_s = \Phi_{gc} = 1.2$.

increment on drug traffic activities is unexpected by prohibition policies.

In this section, I show that an increase in State incentives, with everything else constant, increases cooperation incentives between traffickers. Therefore, violence rates and traffic activities decrease. However, if state incentives and traffickers incentives increase simultaneously, such that traffickers fragment, then traffic activities and violence rates increase.

2.5 Conclusion

In this chapter, I develop a sequential cooperative game with three stages where players endogenously choose, in order to study drug trafficking in transit countries. The model takes into consideration that there is a conflict between the State and traffickers for a drug traffic network. However, in the second stage traffickers may decide whether, to behave cooperatively and form a cartel or to behave competitively and become single traffickers.

It is shown, that players rationally cooperate when they expect that the State will invest a lot of resources in the conflict. However, if the prize (potential profits from the network) is high enough they behave competitively. Therefore traffickers behavior may change when there is a simultaneously change in State and traffickers incentives.

As it is shown, in this chapter as well as other works, when there is an increase in state incentives, with any other variables constant, this scenario increases the collusion incentives between traffickers. Therefore, drug trafficking and violence decreases. However, if the State and drug traffickers incentives increase simultaneously; in such a way that traffickers change from a cooperative to competitive behavior, then drug trafficking and violence increase. This conclusion may fit well in the Mexican drug fight results from 2006 to 2012.

In this (model), it is only assumed that drug traffickers can form two conceivable coalition structures: grand cartel when traffickers agree to have full cooperation with each other; and single traffickers when they decide to behave absolutely competitive against each other. I also assume that drug prices are exogenous. In future work, this model could consider applied endogenous prices and as a consequence the drug trafficker's behavior would be determined by the policy variables of the model.

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2.7 Apendix 1

In this section, I provide the proofs in section 3. First notice that the fraction network that each coalition structure obtain may be rewritten as

$$p_i = \frac{B\Phi_i}{An + B\Phi_i}, \tag{2.33}$$

where $A = \frac{h}{\Omega}$ are the State incentives, $B = CW$ are the traffickers incentives, and $i = \{s, gc\}$[13]. The intensity on the conflict under coalition structure i is

$$I_i = m_i \frac{AB\Phi_i}{An + B\Phi_i}, \tag{2.34}$$

where m_i is the number of coalitions in the structure π. In traffickers structure $m_s = n$ and in cartel structure $m_{gc} = 1$. The network fraction is decreasing with respect to A,

$$\frac{\partial p_i}{\partial A} = -n \frac{B\Phi_i}{(An + B\Phi_i)^2} < 0, \tag{2.35}$$

and increasing with respect to B,

$$\frac{\partial p_i}{\partial B} = \frac{An\Phi_i}{(An + B\Phi_i)^2} > 0. \tag{2.36}$$

Analogously, intensity in the conflict is increasing with respect to A,

$$\frac{\partial I_i}{\partial A} = m_i \left(\frac{B\Phi_i}{An + B\Phi_i} \right)^2 > 0, \tag{2.37}$$

and with respect to B,

$$\frac{\partial I_i}{\partial B} = m_i \frac{A^2 n\Phi_i}{(An + B\Phi_i)^2} > 0, \tag{2.38}$$

Now the State expenditure is $Z_i = (1 - m_i p_i) I_i$, then

[13] I do a little abuse of notation, i index was used for players. In this appendix, I use i index to denote different coalition structures.

$$\frac{\partial Z_i}{\partial A} = I_i \frac{\partial(1 - m_i p_i)}{\partial A} + (1 - m_i p_i) \frac{\partial I_i}{\partial A} > 0, \tag{2.39}$$

Using equations 2.35 and 2.37 the right hand side of the above expression is positive. Therefore, State expenditure is increasing with respect to A. Now the State expenditure change with respect to B is

$$\frac{\partial Z_i}{\partial B} = I_i \frac{\partial(1 - m_i p_i)}{\partial B} + (1 - m_i p_i) \frac{\partial I_i}{\partial B}. \tag{2.40}$$

Using that $I_i = m_i A p_i$ the above expression is

$$\frac{\partial Z_i}{\partial B} = \frac{m_i A}{An + B\Phi_i} [An - B\Phi_i(2m_i - 1)] \frac{\partial p_i}{\partial B},$$

The term in brackets can be negative. Therefore, State expenditure may decrease when B changes if A is not big.

Individual profits are rewritten as $R_i = \frac{Bp_i}{a_i}\left(1 - \frac{m_i}{n}(1 - p_i)\right)$ remember that $a_i = 1$ in traffickers structure, and $a_i = n$ in cartel structure. Individual profits change with respect to A are,

$$\frac{\partial R_i}{\partial A} = \frac{B}{a_i}\left[\left(1 - \frac{m_i}{n}(1 - p_i)\right)\frac{\partial p_i}{\partial A} + \frac{m_i}{n} p_i \frac{\partial p_i}{\partial A}\right], \tag{2.41}$$

by equation 2.35 this is negative. Individual profits are increasing with respect to B,

$$\frac{\partial R_i}{\partial B} = \frac{1}{a_i}\left[p_i\left(1 - \frac{m_i}{n}(1 - p_i)\right) + B\left(1 - \frac{m_i}{n}(1 - p_i)\right)\frac{\partial p_i}{\partial B} + \frac{m_i}{n} B p_i \frac{\partial p_i}{\partial B}\right]. \tag{2.42}$$

Drugs traffic is $D_i = \lambda W m_i p_i$,

$$\frac{\partial D_i}{\partial A} = \lambda W m_i \frac{\partial p_i}{\partial A} < 0, \tag{2.43}$$

from equation 2.35 it is easy to see that trafficking decreases, with respect to A.

Finally violence is $V_i = (1 - m_i p_i) I_i + m_i \frac{I_i p_i}{\Phi_i}$. First notice that $I_i = m_i A p_i$, then $p_i I_i = m_i A p_i^2$. The change of $p_i I_i$ with respect to A is

$$\frac{\partial p_i I_i}{\partial A} = m_i p_i^2 + 2 m_i A p_i \frac{\partial p_i}{\partial A} = m_i p_i^2 \frac{B\Phi_i - An}{An + B\Phi_i}. \tag{2.44}$$

The violence change with respect to A is

$$\frac{\partial V_i}{\partial A} = \frac{\partial I_i}{\partial A} - \frac{m_i}{\Phi_i}(\Phi_i - 1)\frac{\partial p_i I_i}{\partial A}, \tag{2.45}$$

from equations 2.37 and 2.44,

$$\frac{\partial V_i}{\partial A} = m_i p_i^2 \left[1 - \frac{m_i(\Phi_i - 1)}{\Phi_i}\left(\frac{B\Phi_i - An}{An + B\Phi_i}\right)\right],$$

if $i = s$, then the second term in brackets is positive by proposition 2.3.1. if $i = gc$, then the term in brackets becomes, $\frac{An(2\Phi_{gc} - 1) + B\Phi_{gc}}{\Phi_{gc}(An + B\Phi_{gc})}$ which is positive. **Therefore, violence is increasing with** A.

the change on violence with respect to B is

$$\frac{\partial V_i}{\partial B} = \frac{\partial I_i}{\partial B} - \frac{m_i}{\Phi_i}(\Phi_i - 1)\frac{\partial p_i I_i}{\partial B}, \tag{2.46}$$

the change of $p_i I_i$ with respect to B is

$$\frac{\partial p_i I_i}{\partial B} = 2 m_i A p_i \frac{\partial p_i}{\partial B}, \tag{2.47}$$

using this expression and 2.38 in 2.46,

$$\frac{\partial V_i}{\partial B} = \frac{m_i A}{An + B\Phi_i}\left[An - B(2m_i(\Phi_i - 1) - \Phi_i)\right]\frac{\partial p_i}{\partial B}$$

Notice that the term in brackets may be negative. From proposition 2.3.1, $An \geq (n-1)B\Phi_s$ *assume that the condition is satisfied with equality*. A necessary but not sufficient condition for decreasing violence in the traffickers structure is $\Phi_s > 2$. Analogously, a necessary but not sufficient condition for decreasing violence in the cartel structure is $\Phi_{gc} > (n-1)\Phi_s + 2$.

Proposition 2.3.2 mention that, If State punishment (international resources) increases, i.e. $\Delta h = h_f - h_0 > 0$ ($\Delta\Omega < 0$), where f is the final state and 0 is the initial state, whenever $\pi_f = \pi_0$, then Z_i increases; R_i and D_i decrease. V_i increases, also.

Proof to Proposition 2.3.2:

Case 1: Punishment increment. It is sufficient to check the infinitesimal change with respect to h. Notice that

$$\frac{\partial\{\mathbf{y_i}\}}{\partial h} = \frac{\partial\{\mathbf{y_i}\}}{\partial A}\frac{\partial A}{\partial h},$$

where $\{\mathbf{y_i}\}$ is the interest vector variable, i.e. $\{\mathbf{y_i}\} = \{\mathbf{Z_i}, \mathbf{R_i}, \mathbf{D_i}, \mathbf{V_i}\}$. As $\frac{\partial A}{\partial h} = \frac{1}{\Omega} > 0$, then equations 2.39, 2.45, 2.41 and 2.43 have the same sign. Therefore, State expenditure and violence are increasing; and individual profits and drugs traffic are decreasing, with respect to punishment.

Case 2: International resources increment. When international resources increase, Ω decreases. In this case, the infinitesimal change, with respect to Ω, has the opposite sign to punishment because

$$\frac{\partial\{\mathbf{y_i}\}}{\partial \Omega} = \frac{\partial\{\mathbf{y_i}\}}{\partial A}\frac{\partial A}{\partial \Omega},$$

and $\frac{\partial A}{\partial \Omega} = -\frac{h}{\Omega^2} < 0$. From equations 2.39 and 2.45, if Ω decrease, then State expenditure and violence increase. Individual profits and drugs traffic decrease, by equations 2.41 and 2.43, as Ω

decreases. ■

Notice that traffickers expenditure decreases as State incentives increase. However, violence rates increase, this implies that total traffickers expenditure decreases at a lower rate than State expenditure.

Proposition 2.3.3 mention that, If network size (drug price) increases, i.e. $\Delta W > 0$ ($\Delta C > 0$, whenever $\pi_f = \pi_0$, then R_i and D_i increase, but Z_i, and V_i change is uncertain.

Proof to Proposition 2.3.3:

Case 1: Network size increment. In this case $\frac{\partial B}{\partial W} = C > 0$, from equation 2.42, individual profits are increasing, with respect to W. However, State expenditure and violence rates change is uncertain because equations 2.40 and 2.46 may be positive or negative. Drugs traffic change with respect to W is

$$\frac{\partial D_i}{\partial W} = \lambda m_i p_i + \lambda W m_i \frac{\partial p_i}{\partial B}\frac{\partial B}{\partial W}, \tag{2.48}$$

which is positive. Then drugs traffic is increasing with W.

Case 2: drug price increment. This case is analogous to the previous.

Both cases show that individual profits and drug trafficking are increasing with W (C). However, State expenditure and violence rates change are uncertain. ■

Remark: If $C_f > C_0$ and $W_f < W_0$, such that $C_f W_f > C_0 W_0$, then the change on drugs is uncertain, because

$$\Delta D_i = \frac{\lambda m_i}{(An + B_f\Phi_i)(An + B_0\Phi_i)}\left[An(C_f W_f^2 - C_0 W_0^2) - \Phi_i C_f W_f C_0 W_0 (W_0 - W_f)\right]. \tag{2.49}$$

The term in brackets is negative when $An(B_f W_f - B_0 W_0) < \Phi_i B_f B_0 (W_0 - W_f)$, i.e. when

$B_f W_f < B_0 W_0$ or $An < \Phi_i \frac{B_f B_0 (W_0 - W_f)}{B_f W_f - B_0 W_0}$ with $B_f W_f - B_0 W_0 > 0$. This implies that even when traffickers incentives increase, reductions on the network size may decrease drug traffic. However, individual profits increase.

Notice that when traffickers are highly effective then violence rates decrease, as traffickers incentives increase. In this case, total traffickers expenditure increases but not as much as State expenditure decreases because traffickers are so effective that a small increase in resources produces a big increment on fraction network.

2.8 Appendix 2

2.8.1 Alternative Definitions on Violence.

In the text, I consider that the State and traffickers expenditure increase violence levels. J. Miron (1999), find a positive relationship between homicides and enforcement expenditure in alcohol and drugs. By this reason, I assume that the total expenditure in the conflict is perfectly and symmetrically related with violence.

This is a strong assumption. In addition, this definition may be criticized because the main objective to the State expenditure is the enforcement on law offenders, which should reduce violence in the long run. Besides, competitive behavior increases traffickers expenditure and homicides rates, but it is endogenous to the model, and it is not clear that the State can avoid it[14].

In this section, I propose two alternative definitions on violence, which are extreme cases of the two critics mentioned. The first definition considers that the State expenditure does not affect violence levels, in other words only traffickers expenditure has an effect on violence[15]. The second definition considers that only the State expenditure affect violence, this is an extreme result of J.

[14] Data in homicides rates include general population homicides. There is no a direct link with traffickers and State conflict. However, if one can assume that these homicides rates are linked with the diversification of crime markets, then this definitions may be generalized.

[15] Lindo and Padilla-Romo (2015) results suggest that most of the violence is caused by traffickers competition, then an alternative definition is considered only the expenditure on traffickers conflict. In this case the result is obvious, only traffickers structure has an effect on violence.

Miron (1999) paper[16].

Accordingly to the first alternative definition on violence, I assume that violence is perfectly correlated with traffickers expenditure, but not with State expenditure,

$$\widetilde{V} = \sum_{j=1}^{m} X_j.$$

From the text, traffickers expenditure may be rewritten as

$$X_j = m_i \frac{AB^2\Phi_i}{(An + B\Phi_i)^2}, \tag{2.50}$$

where A, B, i, and m_i are as same as in appendix 1. Using 2.50 in violence definition[17],

$$\widetilde{V} = m_i^2 \frac{AB^2\Phi_i}{(An + B\Phi_i)^2}. \tag{2.51}$$

Proposition A2.1 *If the coalition structure remains constant, whenever the traffickers incentives increase, then violence increases.*

Proof: From equation 2.51 is easy to see that it is increasing with traffickers incentives, B. ■

Proposition A2.1 *If the coalition structure remains constant, whenever the State incentives increase, then violence decreases in the traffickers structure, but the effect is uncertain in the cartel structure.*

Proof: From equation 2.51 the change with respect to A, is

$$\frac{\partial \widetilde{V}}{\partial A} = m_i^2 \frac{B^2\Phi_i(B\Phi_i - A)}{(An + B\Phi_i)^3}.$$

[16] A better definition is to split the expenditure between traffickers conflict and the State-traffickers conflict and use the latter as the violence definition. However, it is not possible to split the traffickers expenditure in the traffickers structure.

[17] The violence in the traffickers and the cartel structure are, $\widetilde{V_s} = n^2 \frac{AB^2\Phi_i}{(An+B\Phi_i)^2}$ and $\widetilde{V_{gc}} = \frac{AB^2\Phi_i}{(An+B\Phi_i)^2}$, respectively

From proposition 2.3.1, this expression is negative in the traffickers structure. But in the cartel structure $B\Phi_{gc}$ may be greater than An, therefore it is uncertain. ■

Remark A2.1 If $\Phi_s = \Phi_{gc} = \Phi$, then violence is decreasing in both structures.

Proposition A2.3 *If superadditivity technology is constant and the parameters are non-decreasing, whenever the traffickers and State incentives increase simultaneously, such that coalition changes from cartel to singleton structure, then violence rates increase.*

Proof: Let $\Phi_s = \Phi_{gc} = \Phi$. Besides $A_f > A_0$ and $B_f > B_0$, where A_0 and A_f are the initial and final State incentives, respectively. Similarly, B_0 and B_f are the initial and final traffickers incentives. Such that, $A_0 \geq B_0\Phi$ and $A_f \leq B_f\Phi$, then initially traffickers choose the cartel structure, and in the final state they choose the traffickers structure accordingly to proposition 2.4.1. Then,

$$n^2\frac{A_f B_f^2\Phi}{(A_f n + B_f\Phi)^2} \geq n^2\frac{A_f B_f^2\Phi}{(B_f\Phi n + B_f\Phi)^2} > \frac{A_0 B_0^2\Phi}{(B_0\Phi n + B_0\Phi)^2} \geq \frac{A_0 B_0^2\Phi}{(A_0 n + B_0\Phi)^2}.$$

Notice that the first expression is the violence in the traffickers structure at the final state, and the last expression is the violence in the traffickers structure at the initial state. The first and third inequalities are satisfied by proposition 2.4.1. Finally, the second inequality is satisfied because $n^2 A_f > A_0$. ■

Qualitative results are similar to the text, with this definition of violence. The main difference is that if State incentives increase, then violence decreases, in general, when everything else is constant.

Accordingly to the second alternative definition on violence, I assume that violence is not correlated with traffickers expenditure, and it is perfectly correlated with State expenditure, that is

$$\widehat{V} = Z.$$

From proposition 2.3.2 in the text, if State incentives increase, then violence does, when the coalition structure remains constant.

Proposition A2.4 *If superadditivity technology is constant, and traffickers behave competitively, whenever the traffickers incentives increase, then violence decreases.*

Proof: Let $\Phi_s = \Phi_{gc} = \Phi$. From equation 2.40, the traffickers structure violence hat change is

$$\frac{\partial \widehat{V_s}}{\partial B} = \frac{An}{An + B\Phi}[An - B\Phi(2n-1)]\frac{\partial p_s}{\partial B}.$$

This expression is negative because traffickers behave competitively, then by proposition 2.4.1, $B\Phi > A$, therefore the term in brackets is negative. ■

Proposition A2.5 *If superadditivity technology is constant, and traffickers behave cooperatively, whenever the traffickers incentives increase, then violence does.*

Proof: This proof is analogous to the previous one, the difference here is that cartel structure violence hat change is

$$\frac{\partial \widehat{V_{gc}}}{\partial B} = \frac{A}{An + B\Phi}[An - B\Phi]\frac{\partial p_{gc}}{\partial B}.$$

By proposition 2.4.1, $A > B\Phi$, then the term in brackets is positive. ■

From the last two propositions, when the traffickers incentives increase there is a mixed result. If traffickers behave competitively, then violence decreases, but if they behave cooperatively, then violence increases. From the text, I know that the State considers the competition between traffickers. Therefore, if they compete among them, then the State reduces its expenditure. Conversely, If traffickers cooperate, then the State increment its expenditure.

Proposition A2.6 *If superadditivity technology is constant and the parameters are non-decreasing, whenever traffickers and State incentives increase simultaneously, such that there is a coalition change from cartel to singleton structure, then violence effect is uncertain.*

Proof: Let $\Phi_s = \Phi_{gc} = \Phi$. Besides $A_f > A_0$ and $B_f > B_0$, where A_0 and A_f are the initial and final State incentives, respectively. Similarly, B_0 and B_f are the initial and final traffickers incentives. Such that, $A_0 \geq B_0\Phi$ and $A_f \leq B_f\Phi$, i.e. initially traffickers choose the cartel structure, and in the final state they choose the traffickers structure accordingly to proposition 2.4.1.

In this case there is no an strict relationship between $\widehat{V_s}$ and $\widehat{V_{gc}}$. In order to see this, first assume that $A_f = A_0$ and $B_f = \gamma B_0$, with $1 < \gamma \leq \frac{A_0 n}{B_0\Phi(n-1)}$, such that $\widehat{V_{s_f}} \geq \widehat{V_{gc_0}}$, then the following inequality has to be satisfied,

$$\frac{\gamma A_0 B_0(A_0 n - \gamma B_0\Phi(n-1))}{(A_0 n + \gamma B_0\Phi)^2} \geq \frac{A_0^2 B_0}{(A_0 n + B_0\Phi)^2}$$

Let $\gamma = \frac{A_0 n}{B_0\Phi(n-1)}$ then the left hand side is $0 < A_0$ which contradicts the assumption. Second, assume that $\widehat{V_{s_f}} \leq \widehat{V_{gc_0}}$. In this case, the inequality, in the above expression, changes of direction. Let γ goes to one, then the inequality goes to $A_0 < B_0\Phi$ which is false by the initial conditions. Moreover, by proposition A2.4 the left hand side is continuously decreasing with γ. Therefore with $\gamma = 1$, it is an upper bound, the lower bound is achieved when $\gamma = \frac{A_0 n}{B_0\Phi(n-1)}$. Notice that the lower bound is 0 which is lower that $\widehat{V_{gc_0}}$.

Now assume $A_f > A_0$, in this case $\frac{A_f}{B_0} < \gamma \leq \frac{A_f n}{B_0\Phi(n-1)}$. By proposition 2.3.2 the left hand side from the inequality is increasing, then the upper bound is higher. But if $\gamma = \frac{A_f n}{B_0\Phi(n-1)}$, then the lower bound is still 0 which is lower than $\widehat{V_{gc_0}}$. ■

This proposition shows that violence may decrease even when traffickers and State incentives increases. Therefore, results in the text are weaker.

Chapter 3

Conflict and Narcotrafic: A Sequential Coalition Formation Game

3.1 Introduction

Mexico is a transit country for drugs. Mexico's drug traffickers buy most of the drugs from producer countries; then they distribute to consumer countries, especially the U.S. market. Drugs routes pass through Mexico to the US-Mexico border.

At the beginning of the twenty-first century, Mexico involves the army in drug trafficking fight. In 2006, the Mexican president, Felipe Calderon declares heavier efforts in what he called a "Drug War". After this announcement, intentional homicides raises and traffickers cartels start to fragment.

There an open debate if this policy increases violence rates by itself, or by the contrary, as a result of the policy, drug cartels fragmented and behave more violent. However, expenditure orientated to law enforcement, in prohibitionist environments, is positively correlated with an increase on intentional homicides, in the US (J Miron, 1994). Skaperdas (2010) makes a review between conflict and violence, in different environments. Many papers show a positive relationship between crime and drug use, but they are silent on the relationship between State efforts and crimes by the organized crime, for example UNODC (2014).

Grossman and Mejía, (2007) proposes an analytical model to analyze prohibition drugs policies[1]. In their sequential model, drug producers, and the State engage in three conflicts: a) to control land; b) to destroy illicit crops; and c) to traffic drugs. A generalization of this model is used to evaluate anti-drug policy in Colombia under the Plan Colombia; Mejía, (2008); Mejía and Restrepo (2008).

Grossman-Mejía model is useful because it also allows to evaluate effects on violence; Mejía at. al. (2011). Other papers assess the relationship between Mexico's drug policy and its impact on violence and drug trafficking; Dell, (2011), Rios, (2010). These papers assume that drug traffickers compete between them and the State to monopolize crime in the territories where they operate. However, these studies do not consider that traffickers may behave strategically, and avoid competition between them and only have a conflict with the State.

[1]Becker, at al, (2004), proposes a model to study the effects of prohibitionist policy on illegal goods with an inelastic elasticity. They also compare the effects of no prohibitionist policies outcomes

Guerrero, (2011); and Rios, (2012); mention that Mexican drug cartels use to cooperate with smaller organizations in trafficking activities. In journalist chronicles, this argument is very persistent. For instance Hernandez, (2012), mention that there was a truce and alliance attempt between the main mafia lords in Mexico, in 2007.

In this chapter, I develop a similar model as the Grossman-Mejía model. I consider that there is a State that wants to reduce trafficking activities. The State receives international aid, and it may punish if it does not reduce drug traffic. Also, there are traffickers that need routes and try to avoid State enforcement. Drug Dealers compete for trafficking routes with the State and other traffickers. This model allows endogenous coalition formation between traffickers. Members that belong to the same coalition does not fight against each other, but they fight against the State and others coalition.

I show that an increase in international resources or punishment reduces trafficking activities but increase violence and State's cost, if the coalition structure does not change. This result is the expected from prohibitionist policy. However, if the coalition structure changes most of the results are uncertain.

In the model, drug prices are assumed exogenous. If the coalition structure is constant, whenever drug prices increase, drug trafficking activities increase; but violence and State's cost effect are uncertain. However, when traffickers may change the coalition structure, results are unpredictable.

Mexico's drug fight data from 2000 to 2012 shows that international aid increases in 2002, and in 2008-2010 period. It also shows, that Mexico's defense and security expenditure, decreases from 2000 to 2006, and it was until 2009 when there was a significant increment. Intentional homicides in Mexico highly increase in 2008.

Cocaine U.S. price decreases from 2000 to 2007 after this year there was about 50% increase in 2008 with respect to 2006, UNODC (2014). The price remains almost constant at this level from 2009 to 2012.

Journalist works (e.g. Blancornelas, (2002); Osorno, (2009); (2012); Ravelo, (2011); and Hernandez, (2010)) mention that there were four major conflicts between Mexico's cartels in 2003, 2004, 2008 and 2010, and it was an agreement attempt in 2007. This data may confirm some conclusions of the model.

This chapter structure is the following. In Section 2, I present the model that is a three stages sequential game. In Section 3 and 4 I obtain the interior equilibrium solution and I perform a comparative statistics analysis when the coalition structure does not change. In section 5, I obtain the solution when the State does not have enough incentives to fight trafficking, in the third stage. I provide the solution from the game, in section 6, I also compare the total agreement solution and the no agreement solution as benchmark scenarios in this section. In Section 7, a numerical example is provided. Finally in Section 8, I conclude.

3.2 Model

The model is a three stages sequential game[2]. The players are an outsider which is a nature player; n drug traffickers; and the State S. The outsider gives incentives to the State to fight drug trafficking. The outsider selects a fraction $1 - \Omega$ of resources that it gives to the State to fight trafficking. The outsider also select a level of punishment h. In the second stage, traffickers may form coalitions between them. In the third stage, coalitions and the State fight for a drug traffic network.

In the second stage, the n traffickers may form coalitions according to the Bloch's game. That is a sequential game, where players in order forms coalitions, I explain it with more detail in the second stage section. After this stage m coalitions are formed. This is one of the main difference with the previous chapter, because this protocol allows to form any kind of coalition, in other words it allows that symmetric and asymmetric coalitions can be formed.

In the third stage, the m coalitions and the State fight in a contest for a drug traffic network $W > 0$. This Network allows to traffic drug to the consumer country and obtain a potential profit of $C > 0$. Traffickers need it as an input. The State wants to destroy the network.

The main difference between this chapter and the previous one is that the coalition formation

[2]Indeed there is at least $n + 1$ stages in the game, where n is the number of drug traffickers. However, I describe the game as one of three stages, for simplicity

protocol change, which allows to form any kind of coalition, i.e. symmetric and asymmetric coalitions can be formed. The second difference is that in this chapter, the main attention is on changes in the observer policy parameters, i.e. the fraction $1 - \Omega$ and h. I also analyze what happen when the State does not invest.

I obtain similar results as in the previous chapter because traffickers may choose different coalition structures. Also, I obtain results when the State does not invest, which represent the case that the State is weak. However as there can be a lot of coalition structures and traffickers may change between them, the condition when the coalition structure change is not so obvious as in the previous one.

3.2.1 Timing

The timing of the model is as follows: 1) In the first stage the observer selects the punishment and resources that he gives to the State[3]. 2) In the second stage, drug traffickers decide if they will form coalitions among themselves to fight other cartels and the State for the drug network. 3) In the third stage, given the number of cartels that were formed previously, simultaneously the State and cartels decide the resources that will ”invest” in the conflict to control a fraction a traffic network.

3.2.2 First Stage

In the first stage, the outsider which is a nature player selects a fraction $1 - \Omega$ from a uniform distribution $U[0, 1]$. The outsider gives resources to the State to fight trafficking, the conflict of the State is described in the third stage. The outsider gives an amount of resources $Z(1 - \Omega)$ to the State, where Z is the total expenditure in the conflict from the State. The outsider also select a level of punishment h from a uniform distribution $U[0, \bar{h}]$, where $\bar{h}$ is the maximum punishment

[3]For example under ”Plan Merida” U.S. government offered a financial and military help to the Mexican State with two years in advance.

for the State[4].

The outsider establishes a stick and carrots strategy, in order to incentivize the State to fight trafficking. The carrot are the resources that he gives to the State, and the stick is the level of punishment h that the State receives if it does not reduce the drug traffic.

The fraction of resources $1-\Omega$ and the level of punishment h is known by all players in the next stages of the game.

3.2.3 Second Stage

In the second stage, the set of n drugs traffickers, who are identical in all relevant aspects, decide to form coalitions. Coalition structures are a partition from this set. Coalitions are formed with Bloch's coalition game[5], (Bloch 1996). This game has multiple stages. In the model I consider all this stages but here I described them as a single stage to abbreviate the exposition. Then it does not have any difference.

Bloch's game gives a Coalition structure $\Pi = \{A_1, A_2, \ldots, A_m\}$. Note that drug traffickers are identical, therefore the main difference between coalitions is the number of members. Then a coalition structure can be rewritten as $\pi = \{a_1, a_2, \ldots, a_m\}$, where $a_j = |A_j|$ the cardinality of the set A_j[6].

I assume drug traffickers are completely rational, then they form cartels or stay alone in order to maximize their payoffs in the third stage.

[4]The otusider can not punish the State with a fine that it is equal to the GNP for the State, then the punishment has an upper bound

[5]Bloch's game is a sequential game of at least n stages. Where n is the number of players. In the game, the set players are ordered with a random device. In the first stage, the first player proposes a coalition A_1 of $m_1 \leq n$ members. The m_1 players decides to accept or refuse in the order assigned, if all accept coalition A_1, it is formed and the m_1 players are retired from the game and it continues with the next player in the list, who proposes a coalition A_2 of $m_2 \leq n - m_1$ members. If one of them rejects, then the rejecter becomes the initiator of the next round. The game continues in this way until the last player has a coalition. Note if m is equal to 1 then a singleton is formed and the game continues.

[6]Two different coalition structure in Bloch's game, Π and Π', may have the same coalition structure π. For example, let $\mathfrak{N} = \{1, 2, 3\}$, two different coalition structure under Bloch's game are $\Pi = \{12, 3\}$ and $\Pi' = \{13, 2\}$, however they have the same coalition structure $\pi = \{2, 1\}$.

3.2.4 Third stage

In the third stage, cartels and the State fight for a drug trafficking network $W > 0$, given the coalition structure π. This network represents: routes to the consumer country; corrupt agents; territories where drugs are processed, refined; etc.

Trafficking network is a perfect divisible private input good for cartels. The size of the networks depends on observable characteristics. For example, territories where drug trafficking cartels has presence.

The State and cartels fight for a network fraction. Once cartels obtain a network fraction they are able to sell drugs to the consumer country and they are the only ones that can use it, they obtain a profit of $C > 0$ from drug traffic. The State wants to destroy the network, in order to reduce the traffic and the supply of drugs.

The conflict between cartels and the State is modeled by a standard Contest Success Function (CSF)[7],

$$p_j = \frac{\Phi_j X_j}{Z + \sum_{j=1}^{m} \Phi_j X_j}, \tag{3.1}$$

where X_j is the resources valued in dollars that coalition j invest in the conflict. This X_j represents the aggregate coalition effort, formally $X_j = \sum_{i \in a_j} x_i$. Z is the resources valued in dollars that the State invest in the conflict. $\Phi_j \equiv \Phi(a_j)$ is a function that represents the effectiveness in the conflict of cartel j, this function depends on the group size a_j.

I assume there is *superadditivity technology*[8]. In other words, traffickers cooperation allows to share information between them, then they become at least as effective as they were alone, this implies that $\Phi(a_j)$ is nondecreasing with the number of cartel members, a_j. As well as Grossman and Mejia (2008), I assume that *drug traffickers are more effective in the conflict that the State*, because they can use guerrilla strategies, then $\Phi(1) \geq 1$.

[7]Usually a CSF define that $p_j = \frac{1}{n}$ when no one invest. However, if not one invest, player i can invest a little and obtain all the fraction network. See Skaperdas (1996) for an axiomatization in CSF.

[8]See Skaperdas (1998); and Garfinkel and Skaperdas (2006) for a review on superadditivity.

I define the *intensity of the conflict* as

$$I \equiv Z + \sum_{j=1}^{m} \Phi_j X_j, \tag{3.2}$$

the *intensity of the conflict* are the resources that the State and traffickers effectively[9] spend in the conflict. *Intensity* increases with the resources spend in the conflict, as well as traffickers effectiveness.

There is a tradeoff between traffickers group size and expenditure in the conflict. When traffickers choose a coalition with greater group size they become more effective by the *superadditivity* assumption, but it reduces the number of coalitions.

The State control a fraction network given by $p_s = 1 - \sum_{j=1}^{m} p_j$, in other words, the fraction that the State controls is

$$p_s = \frac{Z}{Z + \sum_j \Phi_j X_j}. \tag{3.3}$$

3.2.5 Cartel Payoffs

At the end of the third stage, payoffs are revealed. Cartels payoffs are gained from drug sales less conflict cost. Given the coalition structure π, payoffs for each drug trafficker i that belongs to cartel j is given by

$$R_{i \in a_j} = \rho C p_j W - x_i, \tag{3.4}$$

where $\rho \in [0, 1]$ is a sharing rule between alliance members, in this (model) the sharing rule is constant for all cartel members[10], i.e. $\rho = \frac{1}{a_j}$, C is the potential annual profits in dollars from drugs sales. When $a_j = 1$ a singleton is formed, she does not share profits.

[9] Traffickers effective expenditure is $\Phi_j X_j$

[10] See Corchón and Dahm (2010) for a discussion on sharing rules

3.2.6 State payoffs

Following Grossman and Mejia, (2008). The interested outsider wants to reduce the supply in the consumer country. He establishes a stick and carrot strategy to incentive the State. If the State does not fight traffic, it will be labeled as ”Narco-State” and will be penalized with a cost of h, by the outsider. The carrot is resources, training, intelligence reports, equipment, etc. All Resources are valuated in dollars and they represent a fraction, $1 - \Omega$ dollars, of the State conflict cost.

Assume that without the State intervention the number of drugs, that could be trafficked, are λW, where λ represents the potential drug that could be trafficked in the network W. Potential trafficking with the State intervention is

$$D = \lambda \sum_{j=1}^{m} p_j W \tag{3.5}$$

Where $D \leq \lambda W$. The probability that the State be punished and labeled as a ”Narco State” is $\frac{D}{\lambda W}$, and the expected cost to the State from label as a ”Narco-State” is given by $\frac{hD}{\lambda W}$.

The State uses the resources from the outsider to minimize costs. The State cost is the addition of expected loss from being labeled as a ”Narco-State” plus the cost of the conflict with drug cartels. The State payoff is

$$S = \frac{-hD}{\lambda W} - \Omega Z$$

using equation 3.5, this expression is equivalent to

$$S = -h \sum_{j}^{m} p_j - \Omega Z \tag{3.6}$$

In the last expression is assumed that the State will not receive a revenue from the fraction network that it controls.

Miron (1999), find that law enforcement expenditure has a correlation with murders in the US. In that paper, the author explains that agents may use violence to obtain profits from the black market. For this reason, I assume that State and cartel expenditure positively affects violence level, and define violence as

$$V = Z + \sum_{j}^{m} X_j. \tag{3.7}$$

When the State or traffickers resources increase, violence does. Coalitions have an important effect in the expenditure in violence levels. Intuitively, when players cooperate they spend less money in the conflict than when they do not cooperate. Therefore, violence is lower in the first than in the second case.

3.3 Main Results

A solution for the third stage is the expenditure vector $\{Z, \mathbf{X_j}, x_{i\in a_j}\}$ and the fraction network vector that, $\{p_s, \mathbf{p_j}\}$, given the coalition structure, π.

In this stage, the fraction network that cartel j controls is obtained from the First Order Conditions (FOC), from 3.4 and 3.6,

$$p_j = 1 - \psi \frac{a_j}{\Phi_j}, \tag{3.8}$$

where $\psi = \frac{\frac{h}{\Omega} m}{\frac{h}{\Omega}\sum_j \frac{a_j}{\Phi_j} + CW}$ is a fraction which relates the State incentives, given by the fraction $\frac{h}{\Omega}$; the traffickers incentives, given by CW; and the cartel structure. From 3.8 the cartel j obtains a fraction that depends on the incentives to the State and the inverse average effectiveness, $\frac{a_j}{\Phi_j}$. Proposition 3.3.1 characterize the condition for an interior solution. If this condition is not satisfied cartel j is not formed and their members obtain a zero payoff.

Proposition 3.3.1. *Cartels get a fraction of the network if and only if*

$$\frac{h}{\Omega}m\frac{a_j}{\Phi_j} < \frac{h}{\Omega}\sum_j \frac{a_j}{\Phi_j} + CW \tag{3.9}$$

Moreover cartels that are more efficient in the average obtain a higher network fraction.

Proof in appendix 2

Proposition 3.3.1 shows that cartels do not participate in the conflict if they are not effective enough, with respect to the average. Cartels that are formed are more effective than the State by assumption.

The second part of the proposition shows that cartels that are more efficient, in the average, will have a higher fraction network. In other words its effective expenditure, $\Phi_j X_j$, will be higher than inefficient cartels.

Finally, this proposition shows that there are more cartels when traffickers incentives, CW, are large.

There is at least two coalition structures that can be always formed, the degenerated coalition structure[11], i.e. $\pi = \{1, 1, \ldots, 1\}$, and the "Grand cartel" coalition structure, i.e. $\pi = \{n\}$.

Remark 3.3.1. On symmetric coalition structures, i.e. $\pi = \{a, a, \ldots, a\}$ where a is constant, cartels obtain the same fraction network.

Remark 3.3.2. A negative fraction is not allowed. If cartel j has a negative fraction its optimal decision is not invest and obtain 0 from the game.

The State fraction is

$$p_s = 1 - \psi\frac{CW\Omega}{h}. \tag{3.10}$$

State fraction should be no negative positive, then the second term in the expression should be less

[11] Also known as singleton structure

than one if it is higher than one the State does not invest and obtain a fraction of zero. This gives a condition on State incentives, such that the State fights trafficking,

Proposition 3.3.2. *If the State incentives are large, then it obtains a positive fraction*

$$h \geq \frac{m-1}{\sum_j \frac{a_j}{\Phi_j}} CW\Omega, \tag{3.11}$$

moreover, the State always fights against the grand cartel[12].

Proof in appendix 2

Proposition 3.3.2 shows how large need to be the incentives in order for the State to fight trafficking. The State fights traffic when punishment or international help, $1-\Omega$, are big. The inequality depends on potential revenues from the trafficking network; the coalition structure; and the average effectiveness, the last two are unobserved for policy makers.

Remark 3.3.3. A sufficient condition for the State to fight trafficking is that punishment is bigger than potential trafficking profits from the most efficient cartel[13].

From 3.8 and 3.10 is easy to obtain the *intensity of the conflict* in parameters as

$$I = \psi CW, \tag{3.12}$$

intensity is always positive. As expected, intensity increases as cartel and State incentives do. From the definition of CSF, equation 3.1, and using 3.8 and 3.12 cartel j expenditure is

$$X_j = \frac{p_j}{\Phi_j}(Z + \sum_{j=1}^{m} \Phi_j X_j) = \frac{\psi CW}{\Phi_j}\left(1 - \psi \frac{a_j}{\Phi_j}\right), \tag{3.13}$$

from remark 2 cartel j expenditure is equal than zero when the fraction network is. Individual expenditure is

[12] In the grand cartel $m = 1$.

[13] Because $\sum_j \frac{a_j}{\Phi_j} \leq m \max\{\frac{a_j}{\Phi_j}\}$, therefore $h \geq \max\{\frac{\Phi_j}{a_j}\}CW$, the maximum is for all j but it is omitted to simplify notation.

$$x_{i\in a_j} = \frac{\psi CW}{a_j\Phi_j}\left(1-\psi\frac{a_j}{\Phi_j}\right). \tag{3.14}$$

From 3.10 and 3.12 the State expenditure in the conflict

$$Z = p_s(Z+\sum_j \Phi_j X_j) = \psi CW\left(1-\psi\frac{CW\Omega}{h}\right). \tag{3.15}$$

When State fraction is positive, then State expenditure is. From proposition 3.3.2 the State fights trafficking when incentives are large, otherwise the State does not participate. The State always fights the grand cartel.

Traffickers individual profits 3.4 are

$$R_{i\in a_j} = \frac{CWp_j}{a_j} - \frac{p_j}{a_j\Phi_j}(Z+\sum_{j=1}^{m}\Phi_j X_j) = \frac{CW}{a_j}\left(1-\frac{\psi}{\Phi_j}\right)\left(1-\psi\frac{a_j}{\Phi_j}\right), \tag{3.16}$$

when fraction network, that belongs to cartel a_j is positive, individual revenues are positive[14]. From the above expression is easy to see that individual profits are decreasing with respect to ψ.

In equation 3.16 the size of the coalition has two opposite effects. On the one hand, as the coalition size increases the individual profits decrease, because cartel revenues are shared with more members. On the other hand, as coalition size increases the effectiveness increases and consequently the individual profits increase too. Notice that bigger cartels, may not have greater individual profits then smaller cartels, as the revenues are shared with a bigger number of members.

In symmetric coalition structures, $\pi_j = \{a, a, \ldots, a\}$, all cartels participate and they obtain same individual profits,

$$R_{i\in a_j} = \frac{CW}{a}\left(\frac{CW}{\frac{h}{\Omega}\frac{ma}{\Phi}+CW}\right)\left(\frac{\frac{h}{\Omega}\frac{m(a-1)}{\Phi}+CW}{\frac{h}{\Omega}\frac{ma}{\Phi}+CW}\right).$$

[14] If $I\frac{a_j}{\Phi_j}\leq 1$, then $\frac{I}{\Phi_j}\leq 1$.

Drugs traffic is obtained using equation 3.8 in 3.5,

$$D = \lambda W \psi \frac{CW\Omega}{h}. \tag{3.17}$$

Finally violence is obtained in terms of 3.10, 3.8 and 2.5

$$V = I\left(p_s + \sum_j \frac{p_j}{\Phi_j}\right) = \psi CW\left(1 + \sum_{j=1}^{m} \frac{1}{\Phi_j} - \psi\left(\frac{CW\Omega}{h} + \sum_{j=1}^{m} \frac{a_j}{\Phi_j^2}\right)\right), \tag{3.18}$$

violence is lower than intensity on the conflict[15].

3.4 Parameters Change

In this section I do a comparative static with changes in State and traffickers incentives in the third stage, In other words I assume that the second stage of the game does not exist and traffickers stay in the same coalition even if incentives change. The paper from Grossman and Mejía (2008) has a similar analysis, but they consider that only the singleton structure can be formed. The results on this section are similar to theirs, but in section 4.6 I show that this result may not be straightforward.

I consider what happen when international resources, punishment, potential profits and the network size change and the coalition structure is constant. The first two variables are the policy variables, the last two are the market parameters. Table 3.1 shows changes on variables with respect to the parameters. The results on the table are proved in appendix 3.

3.4.1 Policy Parameters Change

From table 3.1, if the State incentives increases, i.e. the punishment increases or the fraction Ω decreases, then the State expenditure and its fraction of the network increase. The fraction of the

[15] Notice that $p_s + \sum_{j=1}^{m} p_j = 1$, then $p_s + \sum_j \frac{p_j}{\Phi_j} \leq 1$ because $\Phi_j \geq 1$.

Table 3.1: Results of changes in parameters

Variable	h	Ω	C	W
ψ	+	-	-	-
p_j	-	+	+	+
p_s	+	-	-	-
X_j	?	?	+	+
Z	+	-	?	?
I	+	-	+	+
$R_{i\in a_j}$	-	+	+	+
D	-	+	+	+
V	+	-	?	?

network that cartel j obtain, the drug traffic and the profits for cartels decrease. Finally the intensity and the violence increase.

The previous results can be interpreted as, the State controls a bigger fraction of the network, which reduces drug traffic. Drug cartels engage in a conflict with a stronger enemy, then its expenditure may increase or decrease[16]. However, they lose some of their fraction network, even if their expenditure increases. As a consequence, individual profits decrease, because fraction network decreases more than the traffickers expenditure[17]. As it is expected the intensity in the conflict increases, because the State raises its expenditure more than cartels *effective expenditure* decreases. This implies that violence expands, too.

3.4.2 Market Parameters Change

Suppose that traffickers incentives increase, i.e. there is an increase on C or W. In this case, cartel j fraction increases. As a result drug traffic, individual profits, and intensity on the conflict increase. However the State expenditure may increase or decrease as well as violence.

If traffickers incentives increase, then cartel j expenditure increases because the potential profits from the network are higher. Therefore cartel j obtains a higher fraction, which lead to a higher

[16]If the average effectiveness is $\frac{\Phi_j}{a_j} \geq 2\psi$, then cartel expenditure increases. In symmetric structures with two or more coalitions, the expenditure decreases.

[17]When cartel expenditure decreases. If the expenditure increase then the result is straight forward.

individual profits. State expenditure may increase or decrease[18], but its network fraction decreases. As a consequence drug traffic increases. Intensity on the conflict increases, because cartels spend more and they are more effective than the State. However, violence may decrease because the State expenditure may decrease more than the increase on traffickers expenditure.

These results are summarized in the following proposition

Proposition 3.4.1. *Suppose that traffickers are not allowed to change the coalition. If State incentives increase, then drug traffic and individual revenues decrease. As a collateral effect violence increases.*

Suppose that traffickers are not allowed to change the coalition. If traffickers incentives increase, then drug traffic and individual revenues increase. However, violence rates may increase or decrease.

Details are in appendix 3

Results from proposition 3.4.1 are the expected from prohibition policy, but they are based on the assumption that the coalition structure is constant. Changes in the coalition structure, are analyzed in section 3.6.

3.5 Low Punishment

In this section I consider the case of low punishment, i.e. $h \leq \frac{m-1}{\sum_j \frac{a_j}{\Phi_j}} CW\Omega$, then the State does not have enough incentives to fight trafficking. This completes the solution on the third stage. As the state does not participate the CSF is

$$p_j = \frac{\Phi_j X_j}{\sum_{j=1}^{m} \Phi_j X_j}. \tag{3.19}$$

In the third stage, drug traffickers maximize profits on equation 3.4 given the coalition structure π and this new CSF. From FOC the traffickers' network fraction is

[18] If $\frac{h}{\Omega} \geq \frac{2m-1}{\sum_j^m \frac{a_j}{\Phi_j}} CW$ then State expenditure increases.

$$p_j = 1 - \frac{(m-1)\frac{a_j}{\Phi_j}}{\sum_{j=1}^{m} \frac{a_j}{\Phi_j}}, \tag{3.20}$$

cartel j fraction is positive only if $(m-1)\frac{a_j}{\Phi_j} \leq \sum_{j=1}^{m} \frac{a_j}{\Phi_j}$. As well as in the previous case, more effective cartels obtain a higher fraction.

Intensity on the conflict is

$$\sum_{j=1}^{m} \Phi_j X_j = \frac{CW(m-1)}{\sum_{j=1}^{m} \frac{a_j}{\Phi_j}}, \tag{3.21}$$

intensity depends on the potential profits from the network, and the coalition structure. Cartel j spends

$$X_j = \left(1 - \frac{(m-1)\frac{a_j}{\Phi_j}}{\sum_{j=1}^{m} \frac{a_j}{\Phi_j}}\right) \frac{CW(m-1)}{\Phi_j \sum_{j=1}^{m} \frac{a_j}{\Phi_j}}, \tag{3.22}$$

in the conflict. This expression indicates that more effective cartels, respect to the average effectiveness, spend more in the conflict.

Individual profits are

$$R_{i \in a_j} = \frac{CW}{a_j} \left(1 - \frac{(m-1)\frac{a_j}{\Phi_j}}{\sum_{j=1}^{m} \frac{a_j}{\Phi_j}}\right) \left(1 - \frac{m-1}{\Phi_j \sum_{j=1}^{m} \frac{a_j}{\Phi_j}}\right). \tag{3.23}$$

Individual profits increase with traffickers incentives. There is the same tradeoff between profits and the group size as in section 3. If the group size is greater, cartel j effectiveness is higher, but cartel j profits are shared with more members.

Finally, violence is

$$V = \frac{CW(m-1)}{\left(\sum_{j=1}^{m} \frac{a_j}{\Phi_j}\right)^2} \left(\sum_{j=1}^{m} \frac{a_j}{\Phi_j} \sum_{j=1}^{m} \frac{1}{\Phi_j} - (m-1) \sum_{j=1}^{m} \frac{a_j}{\Phi_j^2}\right). \tag{3.24}$$

Violence increases with the number of coalitions, as expected. Notice, that in this case the State does not fight trafficking it receives the punishment then payoff for the State are $S = -h$ and drugs traffic is $D = \lambda W$.

3.6 Coalition Structure Choice: Degenerated and Grand Cartel Structures.

In the second stage, traffickers decide the number of members which they want to collude, according to the Bloch game which is described in section 3.2. They compare the payoffs in the third stage, given each coalition structure π, and chooses the one that maximize their profits. Coalition structures are a finite set, then a unique solution exist, see Bloch (1996).

Remember that in the Bloch's game on section 4.2 the coalition structure π only has the number of players in each coalition, then the coalition structures $\Pi = \{1, 23\}$ and $\Pi = \{13, 2\}$ has the same coalition structure $\pi = \{1, 2\}$, this not make a difference because traffickers are identical then the payoffs only differ in the number of members in each coalition, In other words the game is symmetric.

The following examples[19]. Suppose that the number of players are three, then there is three coalition structure, $\pi_{GC} = \{3\}$; $\pi_C = \{1, 2\}$ I call this structure the cartel structure; and $\pi_G = \{1, 1, 1\}$[20]. I define the payoff vector as $R^l = \{R_1, R_2, R_3\}$ where $l = \{GC, C, G\}$ and R_i with $i = \{1, 2, 3\}$ is the individual payoff for player i.

1. **Grand cartel choice:** Assume that the payoffs are $\mathbf{R^{GC}} = \{\mathbf{4}, \mathbf{4}, \mathbf{4}\}$; $\mathbf{R^1} = \{\mathbf{5}, \mathbf{1}, \mathbf{1}\}$; and $\mathbf{R^G} = \{\mathbf{2}, \mathbf{2}, \mathbf{2}\}$. In this case, player 1 chooses the grand cartel and the other players accept. Player 1 does not choose a singleton coalition because the other two players becomes singletons and all players are worse than in the grand cartel.

2. **Degenerated structure choice:** Assume that payoffs are as in example 1, but $\mathbf{R^{GC}} =$

[19] These examples are not exhaustive.

[20] The coalition structure $\pi = \{2, 1\}$ is not considered since it has the same payoff as π_C

$\{\mathbf{3}, \mathbf{3}, \mathbf{3}\}$ and $\mathbf{R^G} = \{\mathbf{4}, \mathbf{4}, \mathbf{4}\}$. In this case players does not have incentives to cooperate, because if they become singletons they obtain a higher payoff that if they stay in a coalition, then player 1 chooses a singleton as well as player 2, and the degenerated structure is formed.

3. **Cartel structure choice :** Assume that payoffs are as in example 1, but $\mathbf{R^1} = \{\mathbf{5}, \mathbf{3}, \mathbf{3}\}$. In this case, player 1 chooses a singleton since he knows that the two-member coalition has a higher payoff than degenerated structure. Player 2, proposes a two-member coalition to player 3 because she does not have incentives to deviate a form the degenerated structure, then π_C is formed.

4. **Last player is excluded :** Assume that payoffs are as in example 1, but $\mathbf{R^1} = \{\mathbf{0}, \mathbf{5}, \mathbf{5}\}$. In this case, player 1 proposes a two members coalition to player 2, whose does not have incentives to reject. Player 3 cannot block this action then he becomes a singleton and the coalition structure, π_C, is formed in the equilibrium.

Remark : Notice that the degenerated structure can be formed, always.

These examples show that the last player receives the worst payoff and players requires to be non myopic[21].

Bloch's game equilibrium solution is found by backward induction. If player $n-1$[22], has not a coalition, he proposes a coalition of two members to player n, if his payoffs are higher than being a singleton. If player $n-2$ has not a coalition, she proposes a coalition of three members if her payoffs are higher than being a singleton, or if players $n-2$ and $n-1$ cannot obtain higher payoffs when they excludes the last player, and so on.

In general, given coalitions $a_1, a_2, \ldots, a_{l-1}$ that were formed, player $j-th$[23] proposes a coalition a_l of $n-j$ members and the coalition structure $\pi = \{a_1, a_2, \ldots, a_{l-1}, a_l\}$ is formed, if

1. His individual profits are bigger in the a_l cartel than in the associated degenerated structure, $R^{\pi \cup \{a_l\}}_{j \in a_l} \geq R^{\pi \cup \{1,\ldots,1\}}_{j \in 1}$.

[21] See Ray and Vohra (2014) and (2015).
[22] Player n has not be chosen before.
[23] He does not belong to a previous coalition.

2. There is not another coalition structure $\pi* = \{a_1, a_2, \ldots, a_{l-1}, a_l*, \ldots, a_m\}$, with fewer members $a_l* < n - j$, such that

   (a) Player $j - th$ obtain higher payoffs in a_l*, than in the a_l cartel,

   $$R^{\pi*}_{j \in a_l*} > R^{\pi}_{j \in a_l}.$$

   (b) The remaining set of players obtain higher payoffs in $\pi*$, than in the associated degenerated structure. In other words, the subset of players $\{n - j - a_l*\}$ obtains payoffs

   $$R^{\pi*}_{i \in a_j} \geq R^{\{a_1, a_2, \ldots, a_{l-1}\} \cup \{a_l*\} \cup \{1, 1, \ldots, 1\}}_{i \in 1},$$

   for all $i \in \{n - a_1\}$.

   (c) The remaining players cannot form a different coalition structure π', where they are at least equal than in $\pi*$, and player $j - th$ is worst. That is, in the subset of players $\{n - j - a_l*\}$,

   $$R^{\pi'}_{i \in a_j} \geq R^{\pi*}_{i \in a_j},$$

   for all the players in the remaining subset, but n, and

   $$R^{\pi'}_{j \in a_l*} < R^{\pi}_{j \in a_l},$$

   for player $j - th$.

Condition *(c)* rules out situations where players can form another coalition that makes the set of players belonging to a_l* worst. For example, suppose that there are ten players, $R_{GC} = 3$ and $R_G = 1$. Also, assume that $\pi = \{4, 6\}$ has payoffs of $\mathbf{R}^{\pi} = \{5, \ldots, 5, 2, 2, \ldots, 2\}$, which are greater than the grand cartel and degenerated structure, respectively. Player 1 may propose a coalition of four members. However, player 5 and player 8 may propose a coalition of three members each, i.e. they form the coalition $\pi' = \{4, 3, 3\}$ with payoffs of $\mathbf{R}^{\pi'} = \{0, \ldots, 0, 5, \ldots, 5\}$. The remaining six players are better, and the payoff of 0 is worse than the grand cartel, then player 1 never will choose a four-member alliance.

Above, the grand cartel and the degenerated structure has a very important role. These structures also represents the total cooperation and total competition[24] cases, respectively. As these coalitions represent limit cases have a special interest.

3.6.1 Degenerated Coalition Structure

Let that the alliance structure be given by $\pi_G = \{1, 1, \ldots, 1\}$. In other words, it is the case of full competition, players not cooperate and $m = n$. Each trafficker controls the same fraction network and spend the same in the conflict. Then, they obtain the same individual revenues,

$$R_G = CW\left[\frac{CW\Phi(1)}{\frac{h}{\Omega}n + CW\Phi(1)}\right]^2. \tag{3.25}$$

Total drugs that are trafficked are

$$D_G = \lambda W\frac{CW\Phi(1)}{\frac{h}{\Omega}n + CW\Phi(1)}n. \tag{3.26}$$

Finally violence is

$$V_G = \frac{\frac{h}{\Omega}CWn\Phi(1)}{\frac{h}{\Omega}n + CW\Phi(1)}\left(\frac{\frac{h}{\Omega}n - CW\left(n\Phi(1) - n - \Phi(1)\right)}{\frac{h}{\Omega}n + CW\Phi(1)}\right). \tag{3.27}$$

3.6.2 Grand cartel Coalition Structure

Now let that the coalition structure be given by $\pi_{GC} = \{n\}$, this case represent full cooperation between traffickers, where $m = 1$ and $a_j = n$. In this case, there is only one cartel, and all the

[24] Most of the papers assume this case.

players share the profits, individual profits are

$$R_{i\in GC}=\frac{CW}{n}\left(1-\frac{\frac{h}{\Omega}}{\frac{h}{\Omega}n+CW\Phi(n)}\right)\left(\frac{CW\Phi(n)}{\frac{h}{\Omega}n+CW\Phi(n)}\right). \tag{3.28}$$

Drugs that are trafficked,

$$D_{GC}=\lambda W\left(\frac{CW\Phi(n)}{\frac{h}{\Omega}n+CW\Phi(n)}\right), \tag{3.29}$$

finally violence in the grand cartel is

$$V_{GC}=\frac{\frac{h}{\Omega}CW\Phi(n)}{\frac{h}{\Omega}n+CW\Phi(n)}\left(\frac{\frac{h}{\Omega}n+CW}{\frac{h}{\Omega}n+CW\Phi(n)}\right). \tag{3.30}$$

3.6.3 Total Agreement and No Agreement: High State Incentives

In the second stage, traffickers choose the coalition structure. Consider the case of the equation 3.11 in proposition 3.3.2, which is satisfied.

The traffickers payoffs only differ on the size of the coalition. Then players adopt the same strategy at each node of the sequential game in the second stage[25]. Bloch (1996), shows that the sequential equilibrium in this kind of games exists, is not empty and unique. Therefore traffickers form only one coalition structure in the second stage.

I do not analyze the set of sequential equilibriums, because it has many coalitions that can be formed and comparative statics require specific examples. Instead, In this section I analyze two coalition that can be formed with any number of players n, that is the grand cartel and the degenerated structure, as an example of what happen when the coalition structure change. This analysis gives intuition of what happen in the general case when a traffickers decide to change from one coalition structure to other with a bigger number of coalitions. However, the grand cartel and the

[25] Bloch (1996), defined this kind of games as symmetric games.

degenerated structure below may not be the equilibrium outcome. The equilibrium otucomes in the case of three agents are obtained in the next section where a numerical example is developed.

Traffickers prefer the grand cartel if the following condition is satisfied.

Proposition 3.6.1. *Traffickers prefer the grand cartel to the degenerated structure if*

$$\frac{h}{\Omega}\frac{n-1}{n} \geq CW\Phi(n)\left[\left(\frac{\frac{h}{\Omega}\frac{n}{\Phi(n)}+CW}{\frac{h}{\Omega}\frac{n}{\Phi(1)}+CW}\right)^2 - \frac{1}{n}\right]$$

Proof in appendix 2

Above proposition establish that, if State incentives are large[26], then traffickers prefer collusion and they form the grand cartel because they expect a strong enemy.

If superadditivity technology is constant, i.e. $\Phi(1) = \Phi(n) = \Phi$, then the condition simplifies to

$$\frac{h}{\Omega} \geq CW\Phi.$$

This condition satisfies equation 3.11 in proposition 3.3.2.

Proposition 3.6.2. *If grand cartel efficiency is less than the number of players, then drug traffic and violence rates are higher in the degenerated structure than in the grand cartel structure.*

Proof in appendix 2

Propositions 3.6.1 and 3.6.2 mention that, if equation 3.11 is satisfied, whenever State incentives are large, then traffickers cooperate which implies that traffic and violence rates are low. Therefore, If State incentives increase, such that traffickers change from a competitive behavior to a cooperative behavior, then drugs traffic decreases. Violence may decrease, also.

[26]In other words, h is big and/or Ω is small.

3.6.4 Total Agreement and No Agreement: Low State Incentives

Here I compare the grand cartel and the degenerated structures, but I assume that equation 3.11 is not satisfied. Remember that the State always fights the grand cartel, but it does not fight traffickers in the degenerated structure when incentives are low. In this case, traffickers may fight among them because the State does not participate and they do not share their profits.

Proposition 3.6.3. *If equation 3.11 is not satisfied, then traffickers prefer the grand cartel to the degenerated structure when*

$$(n-1)C^2W^2\Phi(n)^2 \geq \left(\frac{h}{\Omega}n\right)^2 + \frac{h}{\Omega}CWn\Phi(n)\left[3-n\right].$$

Proof in appendix 2

This condition is satisfied when State incentives are very low, i.e. h is small and Ω goes to one, but as State incentives increase this condition may not be true. In other words, as $Bn \to (n-1)A\Phi(1)$ players have more incentives to behave competitively.

Notice that the second term, in brackets, in the right hand side is negative for four or more players. Also, notice that $(n-1)CW\Phi(n) \geq (n-1)CW\Phi(1) > \frac{h}{\Omega}n$. When the number of players is big and grand cartel effectiveness is big, the most probable outcome is that the grand cartel dominates the degenerated structure in the low incentives case.

Proposition 3.6.4. *If equation 3.11 is not satisfied, whenever the grand cartel efficiency is lower than the number of players, then violence in degenerated structure is higher than violence in grand cartel.*

Proof in appendix 2

Propositions 3.6.3 and 3.6.4 tells that, If State incentives are very low, then traffickers behave cooperatively. When State incentives increase, traffickers may change their behavior and be competitive, as a result, violence and traffic[27] escalate.

[27] If $(n-1)A\Phi(1) > Bn$, drugs traffic in the degenerated structure is $D_G = \lambda W$, always. This traffic level is higher than D_{GC}.

3.6.5 Remarks

Propositions 3.6.1 to 3.6.4 tells that, if State incentives are very low, then players prefer to behave cooperatively. When State incentives increase, such that $\frac{h}{\Omega} < CW\Phi(n)$, traffickers change their behavior and become competitive. As a result, drugs traffic and violence rates expand. Finally, if State incentives are such that $\frac{h}{\Omega} > CW\Phi(n)$, traffickers behavior changes again and they prefer cooperate, which reduces traffic and violence.

In this section, I show that increments in State incentives are not always a good idea because players may become more competitive. This expands drug traffic and violence rates.

As I said above, grand cartel and degenerated structure are not the only structures that may form, but this analysis gives an idea of what happen in the general solution.

3.7 Numerical Example

In this section, I develop an example the main objective is to show the results when traffickers are allowed to change between coalition structures. In this example, there are three drug traffickers. In the second stage drug traffickers decide to form one of the following coalition structures: the degenerated coalition structure $\pi_G = \{1, 1, 1\}$; the cartel coalition structure $\pi_C = \{2, 1\}$; and the grand cartel coalition structure $\pi_{GC} = \{3\}$. Let the punishment be $h = 0.36$ and the technology, $\Phi(a_j) = 1.5 * a_j$[28].

Figure 3.1 shows: individual traffickers profits; State's expenditure; Total violence; and Drug trafficking, at different levels of State's proportion resources, i.e. Ω, and at each coalition structure. Green, blue and red lines represent grand cartel, degenerated and cartel coalition structures, respectively. Continuous lines indicated the equilibrium path where traffickers chose a particular coalition structure. Notice that the Degenerated coalition structure is never an equilibrium, for this example.

[28]Note that $\Phi(a_j)$ is increasing with respect to the number of members in the coalition, and the average effectiveness, i.e. $\frac{\Phi(a_j)}{a_j}$, is constant.

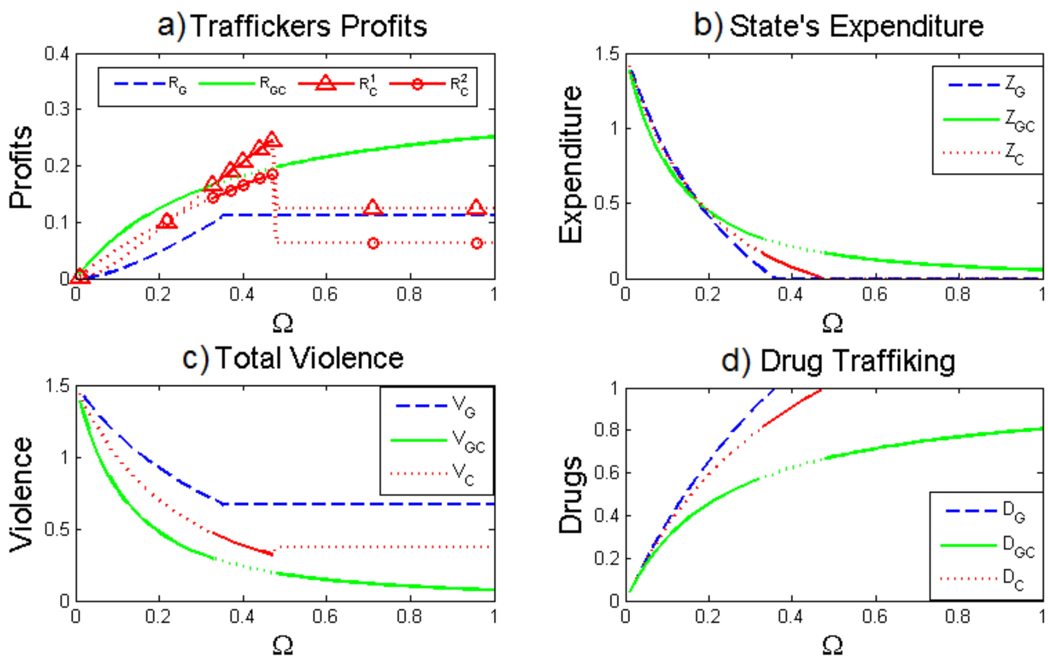

Figure 3.1: Variables changes when State's resources proportion changes

Figure 3.1 has two red lines, the circle (o) line represents profits for the two members coalition, and triangles (△) represents profits for the singleton, on the cartel structure. In this figure, cartel structure is chosen for a State's fraction resources between 0.33 and 0.48, while grand cartel structure is chosen for all other fractions, $\Omega < 0.33$ and $\Omega > 0.48$. This implies that there is two coalition structure change at $\Omega = 0.33$ and $\Omega = 0.48$.

Figure 3.1 illustrate results on propositions 3.6.1 to 3.6.4. When international resources are small traffickers prefer a full cooperation agreement. When external resources are in the middle, traffickers fight among them and the full cooperation agreement is broken. Finally when resources are big, traffickers prefer the full cooperation agreement again, because the State is too strong.

This figure 3.1 shows that when the Ω fraction increases such that traffickers change from the grand cartel to cartel structure, then State expenditure decreases, but drug trafficking and violence have a big increment because of the coalition structure change. It also shows the opposite result when traffickers change from the grand cartel to the cartel structure.

When an increase on parameters do not involve a coalition structure changes, then the results are as described in section 3.4. State's expenditure and violence increase, whereas drug trafficking and traffickers profits decrease.

3.8 Conclusion

In this chapter I develop a model with endogenous coalition formation with contest success functions. I use Bloch's sequential for coalition formation. With this game any coalition structure can be formed in the sequential equilibrium.

This model allows to analyze drug fight policy. I show that an increment in State incentives reduces drug traffic, but increases violence rates, whenever the coalition structure is constant. However, when the coalition structure may change, small increments in State incentives, have uncertain results. In this case, there are two effects, on one hand as the State incentives increase, traffickers prefer collusion. On the other hand, State cost increases, then traffickers may prefer smaller coalitions because State's cost is higher. In other words traffickers fragment to deal with a weaker enemy. As a final result violence expands. This result is unexpected by the policy.

When State incentives increase significantly traffickers collude, then drug traffic decreases nonlinear, but violence may increase because the State is more intensive in the conflict.

In chapter 2, Mexico's drug fight data shows that State and traffickers incentives increase after 2008. This data also shows that Mexico's expenditure in national security increases in 2009; cartels fragment from 2008 to 2010; and homicides change rates are highly positive after 2008. Model predictions are consistent with those facts. However, cocaine supply indicators for the U.S. market, increase from 2000 to 2008, and decrease from 2008 to 2012, UNODC (2013). The model expects the contrary effect. This may be due because traffickers diversify the drug markets and strong assumptions on the model as the *exogenous prices*.

Future works need to relax the exogenous drug price assumption because drug policies may affect drug prices. Then, an increase in State's incentives may increase deviation profitability. Therefore, violence and drug trafficking activities may increase as a result of the policy.

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3.10 Appendix 1

In this section, I describe the solution to the third stage of the game when the State participate in the conflict. In other words, given the coalition π which are the optimal State and traffickers expenditure.

Given the coalition structure π, in the third stage the FOC of 3.4 are

$$\frac{CW\Phi_j}{a_j}(1-p_j) = Z + \sum_{j=1}^{m} \Phi_j X_j. \tag{3.31}$$

Here I use that $\frac{Z+\sum_{k\neq j}^{m} \Phi_k X_k}{(Z+\sum_{j=1}^{m} \Phi_j X_j)} = 1-p_j$. Consider two different coalition k and j then

$$a_j(1-p_k) = \frac{a_k}{\Phi_k}\Phi_j(1-p_j),$$

adding over j and using $\sum_{j=1}^{m} a_j = n$, p_k can be expressed as

$$n(1-p_k) = \frac{a_k}{\Phi_k}\sum_j \Phi_j(1-p_j),$$

adding this expression over k, and renaming the k index to j

$$\frac{n}{\sum_{j=1}^{m}\frac{a_j}{\Phi_j}}(m-\sum_{j=1}^{m} p_j) = \sum_{j=1}^{m}\Phi_j(1-p_j). \tag{3.32}$$

From the State problem, the FOC of 3.6 is

$$\frac{h}{\Omega}\sum_{j}^{m} p_j = Z + \sum_{j=1}^{m}\Phi_j X_j, \tag{3.33}$$

using equation 3.31 above,

$$\frac{h}{\Omega}a_j\sum_{j}^{m} p_j = CW\Phi_j(1-p_j). \tag{3.34}$$

Adding over j and using 3.32 in the right hand side

$$\frac{h}{\Omega}n\sum_{j}^{m}p_j = CW\frac{n}{\sum_{j=1}^{m}\frac{a_j}{\Phi_j}}(m - \sum_{j=1}^{m}p_j).$$

From above expression, the total network that traffickers control is

$$\sum_{j}^{m}p_j = \psi\frac{CW\Omega}{h}, \tag{3.35}$$

where $\psi = \frac{\frac{h}{\Omega}m}{\frac{h}{\Omega}\sum_j\frac{a_j}{\Phi_j}+CW}$.

Remember that, $\sum_{j=1}^{m}p_j = 1 - p_s$, then from 3.35 is easy to obtain the State fraction network 3.10. Cartel fraction network 3.8, is obtained from equations 3.34 and 3.35.

From equation 3.33 it is clear that the intensity on the conflict 3.12, depends on the total fraction that traffickers control.

Finally, notice that Cartel expenditure; State expenditure; Individual traffickers profits; Drugs traffic; and Violence are expressed in terms of 3.8, 3.10 and 3.12.

3.11 Appendix 2

In this section, I provide the proofs to propositions in the text. I define $A \equiv CW$ as the traffickers incentives and $B \equiv \frac{h}{\Omega}$ as the State incentives in order to simplify the results.

Proof to proposition 1.

From 3.8 is easy to see that the fraction that cartel j obtain is positive only if $\psi\frac{a_j}{\Phi_j}$ is less than one. The other direction is straightforward.

For the second part of the proposition, notice that cartel fraction network depends inversely on the average ineffectiveness. Let cartel k to be the most effective in the average, then $\frac{a_k}{\Phi_k} = \min_j\{\frac{a_j}{\Phi_j}\}$. This implies that $Bm\frac{a_k}{\Phi_k} < Bm\frac{a_k}{\Phi_k} + A \leq B\sum_{j=1}^{m}\frac{a_j}{\Phi_j} + A$. Therefore,

$$\frac{Bm\frac{a_k}{\Phi_k}}{B\sum_{j=1}^{m}\frac{a_j}{\Phi_j}+A} < \frac{Bm\frac{a_j}{\Phi_j}}{B\sum_{j=1}^{m}\frac{a_j}{\Phi_j}+A}.$$

For any $j \neq k$. ■

Proof to proposition 2.

The State fraction is positive when $\psi\frac{A}{B} \leq 1$. Inequality in proposition 3.3.2 is obtained simplifying last expression. When $m = 1$ any positive punishment satisfies the inequality, which is the second part of the proposition. ■

Proof to proposition 4.

Grand cartel individual profits are bigger than degenerated structure profits if

$$\frac{1}{n}\left(\frac{(B\frac{n-1}{\Phi(n)}+A)A}{(B\frac{n}{\Phi(n)}+A)^2}\right) \geq \left[\frac{A}{B\frac{n}{\Phi(1)}+A}\right]^2,$$

rearranging terms

$$\frac{B}{\Phi(n)}\frac{n-1}{n}+\frac{A}{n} \geq A\left(\frac{B\frac{n}{\Phi(n)}+A}{B\frac{n}{\Phi(1)}+A}\right)^2,$$

and the result in the proposition is straighted forward. ■

Proof to proposition 5. The first part of proposition 3.6.2 establishes that $D_G - D_{GC} > 0$, from equations 3.26 and 3.29 it is sufficient to show that

$$Bn\left(n\Phi(1) - \Phi(n)\right) + A\Phi(1)\Phi(n)(n-1) > 0.$$

If $\Phi(n) < n$, then this condition is satisfied, always.

The second part of the preposition establishes that $V_G - V_{GC} > 0$. From 3.27 and proposition 3.3.2 $V_G > \frac{B}{\Phi(1)}$. Proposition 3.4.1 mentions that violence increases with State incentives. Then it is sufficient to show that $\frac{B}{\Phi(1)} - V_{GC} > 0$. From equation 3.30 last condition requires,

$$Bn\left(Bn + 2A\Phi(n) - A\Phi(1)\Phi(n)\right) + A^2\left(\Phi(n)^2 - \Phi(n)\Phi(1)\right) > 0.$$

Using condition in proposition 3.3.2 and the assumption that $\Phi(n) \geq \Phi(1)$, then $Bn + 2A\Phi(n) > A\Phi(1)(n-1) + 2A\Phi(n) > A\Phi(1)(n+1)$. Hence the above expression is greater or equal to,

$$A\Phi(1)Bn\left((n+1) - \Phi(n)\right) + A^2\left(\Phi(n)^2 - \Phi(n)\Phi(1)\right).$$

This expression is positive because $\Phi(1) \leq \Phi(n) < n$. ■

Proof to proposition 3.6.3. It is required to compare the grand cartel and degenerated profits, in the low punishment case, equations 3.28 and 3.23 respectively. Degenerated profits are $\frac{A}{n^2}$, then the condition is

$$\frac{A^2\Phi(n)}{n}\left(\frac{B(n-1) + A\Phi(n)}{\left(Bn + A\Phi(n)\right)^2}\right) > \frac{A}{n^2},$$

Simplifying this expression I obtain the condition in proposition 3.6.3. ■

Proof to proposition 3.6.4.

This proposition establishes that if $(n-1)A\Phi(1) > Bn$, then $V_G - V_{GC} > 0$. From 3.24, violence in the degenerated structure is $V_G = \frac{n-1}{n}A$, from 3.30 is sufficient to show that

$$nAB\Phi(n)\left[(n-1)A\Phi(n) - Bn\right] + Bn\left[Bn(n-1) + 2(n-1)A\Phi(n) - Bn\Phi(n)\right] > 0.$$

by the low incentives condition, $(n-1)A\Phi(1) > Bn$, we know that $2(n-1)A\Phi(n) > 2Bn$.

The first term in the left hand side is positive by the low incentives condition, $(n-1)A\Phi(1) > Bn$, and this condition implies that $2(n-1)A\Phi(n) > 2Bn$, then the second term is positive, also. ■

3.12 Appendix 3: Parameters Changes

In this section, I develop the parameter change given in section 4 and details on proposition 3.4.1. I assume that equation 3.11 is satisfied[29] and the coalition structure, π, remains constant.

Below I use A and B as defined in appendix 2. The fraction ψ change with respect to A and B are

$$\frac{\partial \psi}{\partial A} = -\frac{Bm}{(B\sum_{j=1}^{m}\frac{a_j}{\Phi_j} + A)^2} = -\frac{\psi}{B\sum_{j=1}^{m}\frac{a_j}{\Phi_j} + A} < 0, \tag{3.36}$$

and

$$\frac{\partial \psi}{\partial B} = \frac{Am}{(B\sum_{j=1}^{m}\frac{a_j}{\Phi_j} + A)^2} = \frac{A\psi}{B\left(B\sum_{j=1}^{m}\frac{a_j}{\Phi_j} + A\right)} > 0, \tag{3.37}$$

respectively. This shows that as traffickers incentives increase, then the fraction ψ decreases. Analogously if State incentives increase, then ψ increases.

Cartel fraction network, 3.8, is inverse related with ψ. Therefore, changes on traffickers and State incentives has opposite directions on cartel fraction.

The State fraction 3.10 is $p_s = 1 - \psi\frac{A}{B}$, then the change with respect to A is

$$\frac{\partial p_s}{\partial A} = -\frac{\psi\sum_{j=1}^{m}\frac{a_j}{\Phi_j}}{B\sum_{j=1}^{m}\frac{a_j}{\Phi_j} + A} < 0, \tag{3.38}$$

and with respect to B is

$$\frac{\partial p_s}{\partial B} = \frac{A}{B}\frac{\psi\sum_{j=1}^{m}\frac{a_j}{\Phi_j}}{B\sum_{j=1}^{m}\frac{a_j}{\Phi_j} + A} > 0. \tag{3.39}$$

The fraction network on State control is decreasing with traffickers incentives and increasing with

[29] If punishment is low, i.e. condition in proposition 2.3.1 is not satisfied, then the interest variables does not depend on State incentives and the changes are obvious.

State incentives, as expected.

From 3.12 the intensity on the conflict is $I = A\psi$, then the change with respect to A,

$$\frac{\partial I}{\partial A} = \frac{B\sum_{j=1}^{m}\frac{a_j}{\Phi_j}}{B\sum_{j=1}^{m}\frac{a_j}{\Phi_j} + A}\psi > 0. \tag{3.40}$$

In other words, the intensity is increasing with traffickers incentives. It is easy to see that intensity is increasing with State incentives.

Cartel expenditure 3.13 is $X_j = \frac{I}{\Phi_j}\left(1 - \frac{a_j}{\Phi_j}\psi\right)$. Cartel expenditure is increasing with traffickers incentives because the intensity increases and the fraction ψ decreases. However, when the State incentives increase the effect is uncertain, to see this notice that the change on cartel expenditure with respect to B is

$$\frac{\partial X_j}{\partial B} = \frac{A}{\Phi_j}\left(1 - 2\psi\frac{a_j}{\Phi_j}\right)\frac{\partial \psi}{\partial B}, \tag{3.41}$$

the sign of the expression between brackets is uncertain. Moreover, if the average effectiveness is greater than twice the fraction ψ then cartel j expenditure is increasing with respect to State incentives. This result implies that if the cartel j is the most efficient, then it fights harder when State incentives increase.

Notice that, for all symmetric structures with more than one coalition, i.e. $\pi = a, a, \ldots, a$ and $m \geq 2$, then cartel expenditure change is negative, because the term in brackets becomes $\frac{-Bm\frac{a_j}{\Phi_j}+A}{Bm\frac{a_j}{\Phi_j}+A}$, and by proposition 3.3.2, this is negative. The change is still uncertain when $m = 1$, e.g. the grand cartel case.

State expenditure, 3.15, depends on the State fraction network, 3.10, and the intensity, 3.12. State expenditure change with respect to traffickers incentives is uncertain, as the following expression shows

$$\frac{\partial Z}{\partial A} = \frac{\psi B \sum_{j=1}^{m} \frac{a_j}{\Phi_j}}{B \sum_{j=1}^{m} \frac{a_j}{\Phi_j} + A} \left(1 - 2 \frac{Am}{B \sum_{j=1}^{m} \frac{a_j}{\Phi_j} + A} \right). \tag{3.42}$$

The term in brackets can be positive or negative. If the State incentives is $B \geq \frac{2m-1}{\sum_{j=1}^{m} \frac{a_j}{\Phi_j}} A$, then the State expenditure is increasing with A.

State expenditure is increasing with respect to State incentives because both the fraction and intensity increase.

Individual traffickers profits 3.4 depend on traffickers incentives A, the size of the coalition a_j and the fraction ψ. Individual profits are positively related with traffickers incentives because traffickers incentives directly increase revenues and decrease the fraction ψ. Conversely, individual profits are negatively related with State incentives, because the fraction ψ increases.

Violence 3.18, depends on State and total traffickers expenditure. Here changes, with respect to the incentives, are no so obvious because expenditure effects have opposite directions. The change on violence with respect to traffickers incentives, A, is

$$\frac{\partial V}{\partial A} = \frac{\psi}{B \sum_{j=1}^{m} \frac{a_j}{\Phi_j} + A} \left[\left(B - 2\psi A - \psi B \sum_{j=1}^{m} \frac{a_j}{\Phi_j^2} \right) \sum_{j=1}^{m} \frac{a_j}{\Phi_j} + \left(B \sum_{j=1}^{m} \frac{a_j}{\Phi_j} + \psi A \right) \sum_{j=1}^{m} \frac{1}{\Phi_j} \right]. \tag{3.43}$$

From the above equation violence change, with respect to traffickers incentives, is uncertain, because the sign on the brackets may be positive or negative. However, if $B \geq \frac{2m-1}{\sum_{j=1}^{m} \frac{a_j}{\Phi_j}} A$, then violence increases because State and traffickers expenditure do.

Violence change with respect to State incentives, B, is

$$\frac{\partial V}{\partial B} = \left(p_s + \sum_{j=1}^{m} \frac{p_j}{\Phi_j} \right) \frac{\partial I}{\partial B} + \frac{IAm}{(h \sum_{j=1}^{m} \frac{a_j}{\Phi_j} + A)^2} \left(\sum_{j=1}^{m} \frac{a_j}{\Phi_j} - \sum_{j=1}^{m} \frac{1}{\Phi_j} \right). \tag{3.44}$$

Violence change is positive in this case[30]. Therefore when State incentives increase violence does the same because State expenditure augment more than reductions on cartel expenditure.

Finally drugs traffic is given by 3.5, this expression can be rewritten as $D = \lambda W \frac{Am}{B\sum_j \frac{a_j}{\Phi_j}+A}$. It is obvious that drug traffic decreases as State incentives increase, as it is expected, because the State control a higher fraction network. The change on drugs with respect to the size of the network, W, is

$$\frac{\partial D}{\partial W} = \lambda \frac{Am}{B\sum_j \frac{a_j}{\Phi_j} + A} + \lambda W \frac{\partial}{\partial A} \frac{Am}{B\sum_j \frac{a_j}{\Phi_j} + A} \frac{\partial A}{\partial CW} > 0. \tag{3.45}$$

The change with respect to potential profits, C, is completely analogous. This shows that if the parameters, C and W, are nondecreasing, whenever traffickers incentives increase, then drugs traffic raises.

The change with respect to potential profits, C, follows from the chain rule, $\frac{\partial y}{\partial C} = \frac{\partial y}{\partial A}\frac{\partial A}{\partial C}$, where y is the interest variables[31]. In this case, the traffickers incentives with respect to potential profits are positive, $\frac{\partial A}{\partial C} > 0$. Therefore, the changes that were described previously has the same direction. Analogously, $\frac{\partial A}{\partial W} > 0$, then all the signs in the previous equations are identical.

State incentives are positively related with external resources, because when external resources augment the fraction Ω decreases and $\frac{\partial B}{\partial \Omega} = -\frac{h}{\Omega^2} < 0$. Analogously State incentives are positively related with punishment because $\frac{\partial B}{\partial h} > 0$. Then, the interest variables have the same direction that are described above.

3.13 Appendix 4: Low Punishment Results

When there is low punishment, the State does not participate in the conflict. In this case traffickers obtain all the network W and drug traffic is $D = \lambda W$. The FOC from the traffickers revenues, 3.4

[30] Notice that the second term in the right hand side is positive because $a_j \geq 1$.

[31] Fraction ψ; cartel fraction p_j; intensity I; cartel expenditure X_j; State expenditure Z; individual profits $R_{i\in a_j}$; and violence V.

using the CSF without the State intervention, equation 3.19, are

$$\frac{CW\Phi_j}{a_j}\left(1-p_j\right)=\sum_{j=1}^{m}\Phi_j X_j. \tag{3.46}$$

For all cartels the total effective expenditure, i.e. $\sum_{j=1}^{m}\Phi_j X_j$ is equal, then for two different cartels j and k are,

$$a_k\left(1-p_j\right)=\frac{a_j\Phi_k}{\Phi_j}\left(1-p_j\right), \tag{3.47}$$

This expression is a relationship between the fractions network that each cartel obtain in terms on inverse average effectiveness. Adding over k,

$$n\left(1-p_j\right)=\frac{a_j}{\Phi_j}\sum_{k=1}^{m}\Phi_k(1-p_k), \tag{3.48}$$

adding over j the following expression is obtained,

$$\sum_{k=1}^{m}\Phi_k(1-p_k)=\frac{(m-1)n}{\sum_{j=1}^{m}\frac{a_j}{\Phi_j}}. \tag{3.49}$$

From this equation in 3.48, the fraction network that cartel j controls, 3.20 in text, is obtained. The intensity in the conflict comes from 3.49 in traffickers FOC, 3.46. Cartel expenditure and individual traffickers profits are in terms on the cartel fraction network and the intensity in the conflict, equations 3.20 and 3.21 respectively.

Finally, violence is the total expenditure from traffickers, i.e. $\sum_{j=1}^{m}X_j$, which is equal to

$$V=\frac{CW(m-1)}{\sum_{j=1}^{m}\frac{a_j}{\Phi_j}}\sum_{j=1}^{m}\left(\frac{1}{\phi_j}-\frac{m\sum_{j=1}^{m}\frac{a_j}{\Phi_j^2}}{\sum_{j=1}^{m}\frac{a_j}{\Phi_j}}\right), \tag{3.50}$$

which leads to 3.24.

Chapter 4

Conflict Private and Communal Property[1].

[1]This chapter is a joint collaboration with Jaime Sempere.

Sanchez Pages (2006) shows that conflict leading to private property rights can be ex-ante Pareto superior to free access to a resource when the number of agents is large enough. In this chapter, instead, we analyze the possibility of appearance of private property as result of a game between private agents. We focus on the relationship between value of the resource, conflict, and the appearance of private property, and analyze its sensibility to changes in the population size.

We analyze the agents' incentives for obtaining private property in Grossman's (2001)[2] model on the appearance of property rights. As in Sanchez Pages (2007), "free access" is an agreement (i.e. coalition) between all agents to share collectively a valuable resource. Then private property appears as a rational deviation[3] from this agreement.

In this framework we show that the value of the resource has a non monotonic effect on the emergence of private property. More specifically, when the resource is sufficiently valuable, agents have an incentive to leave the free access agreement. However, if the value of the resource increases enough, deviations from the free access agreement lead to a very costly conflict so in order to avoid it, agents stick to the agreement. Therefore we show that private property of the resource is only sustainable for intermediate values of the resource.

Increases in the number of agents have also a non monotonic effect. We show that the set of parameter values for which "free access" agreement is unstable (i.e. private property appears as equilibrium) increases with the number of agents in a small enough economy, but it could get reduced when the number of agents is large enough.

Umbeck (1981) presents a theoretical investigation of how the initial distribution of property rights can arise starting from a situation of free access. Each agent can use labor time in violence to appropriate land or in getting gold. The marginal rate of substitution between land and labor in the production of gold is a measure of how much labor is willing to allocate to maintain the exclusivity of a marginal unit of land. The equilibrium allocation of land would be characterized to equal willingness to fight (and no conflict). In a symmetric model, the equal willingness to

[2]Grossman (2001) presents a General Equilibrium model in which by making an effort people can obtain private property from a common pool of a valuable resource. The resource appropriated is used, together with labor, for production activities. Then he characterizes what would be the equilibrium allocation of private efforts to obtain property.

[3]Our concept of stability is based on Bloch (1996). See Bolgomolnaia and Jackson (2002), Chwe (1994) and Ray and Vohra (2015) for a discussion on stability.

allocate labor to conflict implies an equal distribution of land.

This research is related with the literature of conflict with coalition formation summarized in Bloch (2009), and the corresponding sections in Garfinkel and Skaperdas (2007). On the other hand, our research is complementary to the literature on the appearance of property rights (see Alchian and Demsetz, 1973, Demsetz, 1967, and Grossman, 2001, among others).

The rest of the chapter is organized as follows: Section 2 presents the basic model. Section 3 analyzes formation of property rights in a three agent economy. Section 4 analyzes coalition formation with exogenous responses. Section 5 presents some results when the process of coalition formation is endogenous. Section 6 analyzes the change in the equilibrium coalition structure when the number of agents changes. Finally section 7 presents some conclusions.

4.1 The Model

Assume that there is a valuable resource (i.e. a pool of "land") of size 1 in a n agent economy. Let $N = \{1, 2, \ldots, n\}$ denote the set of players. Players are identical. Each agent i has a stock of time of size 1 that can be used in production and appropriative activities. Agents can participate individually or collectively in these activities.

A coalition structure $\pi = [\{A_1, A_2, \ldots, A_k\}]$ is a partition of the set N. In other words, in a coalition structure each $A_m \subset N$, $A_m \cap \hat{A}_q = \emptyset$ and the union of all this coalitions $\cup_m^k A_m$ is equal to the set N.

Since all players are assumed to be identical, payoffs for each player are dependent on the group size rather than on the specific players that are in the group (i.e. the game is symmetric in the sense of Bloch, 1996). Let $a_m = |A_m|$ be the cardinality of the coalition m, that also denotes the size of that alliance. Therefore we characterize a coalition structure by $\pi = [\{a_1, a_2, \ldots, a_k\}]$ which only depends on the number of members that are in each alliance[4]. From now on, i denotes the player

[4]Note that with this notation there could be more than one partition of N that gives the same coalition structure. For example, suppose there are three players, $N = \{1, 2, 3\}$, two different partition of this set are $[\{12|3\}]$ and $[\{13|2\}]$, but both of them has the same coalition structure $\pi = [\{2, 1\}]$.

and m the alliance a_m to which she belongs.

Agents that participate individually have a group size of $a_m = 1$, and members that participate collectively has a group size $a_m > 1$. Each agent i that belongs to the alliance a_m divides her available time in production (l_{im}) and appropriative activities (e_{im}). For each agent, assume that $1 = l_{im}+e_{im}$ is satisfied. Then in a coalition a_m production time is the sum of individual production time from each member ($L_m = \sum_{i\in a_m} l_{im}$). Analogously, coalition appropriative effort is the sum of individual effort within the coalition ($E_m = \sum_{i\in a_m} e_{im}$).

We assume the particular following functional form for appropriated land for a given coalition m,

$$r_m = \begin{cases} \sum_{i\in m} e_{im}/(\sum_{m=1}^{k}\sum_{i\in m} e_{im}) & \text{if} \quad \sum_{m=1}^{k}\sum_{i\in m} e_{im} > 0 \\ \frac{1}{k} & \text{otherwise} \end{cases} \tag{4.1}$$

in which the amount appropriated depends on the relative coalition effort on appropriative activities. This functional form is a trivial extension of the Grossman's (2001) form to an economy with coalitions.[5]

We consider a sequential game of two stages. In the second stage, coalitions are formed. In the second stage, agents decide how much time to spent in appropriative e_{im} and productive l_{im} activities, given the coalition structure. The benefit that the alliance gets from productive activities is shared between the alliance members according to a proportional sharing rule $\frac{l_{im}}{L_m}$[6]. We assume that all agents in a coalition can freely use the common land and they get consumption in function of the labor they supply. The individual utility that agent $i \in m$ obtains is

$$U_{im} = \frac{l_{im}}{L_m} r_m^{\alpha} L_m^{1-\alpha},$$

Given a particular coalition structure π, players maximize their individual utility subject to the time constraint. Each agent solves

[5]This is a simple form of what the literature knows as a Contest Success Function, and a particular simple form of the one analyzed in Skaperdas (1996).

[6]This is the typical assumption when agents exploit a common property resource. See, for instance, Miceli and Lueck (2001).

$$max_{e_{im},l_{im}|\pi} \frac{l_{im}}{L_m} \left(\frac{E_m}{\sum_m^k E_m} \right)^{\alpha} L_m^{1-\alpha} \ s.t. \quad e_{im} + l_{im} = 1. \tag{4.2}$$

The marginal rate of substitution obtained from this maximization problem is[7]

$$\frac{L_q - \alpha l_{iq}}{\alpha l_{iq} L_q \frac{1-r_q}{r_q} \frac{1}{\sum_m^k E_m}} = 1. \tag{4.3}$$

From the solution of the maximization of problem (4.2) we obtain the following lemma.

Lemma 4.1.1. *In equilibrium, players that belong to the same alliance make the same appropriative effort, e_{im} and offer the same productive labor supply l_{im}.*

Proof: See the appendix.

From now on we omit the subindex i for $e_{im} = e_m$ and $l_{im} = l_m$ since it only depends on the alliance that each player belongs. Lemma 1 also implies that $L_m = a_m l_m$ and $E_m = a_m e_m$. Then if two different alliances p and q have the same group size the level of efforts will be the same.

We can also show that, given a coalition structure, a Nash equilibrium in efforts exists.

Lemma 4.1.2. *Given the coalition structure π, for $0 < \alpha < 1$,there is a Nash equilibrium $\mathbf{e}^* = (e_1^*, e_2^*, \ldots, e_k^*)$ and $\mathbf{l}^* = (l_1^*, l_2^*, \ldots, l_k*)$ corresponding to the second stage of the game.*

Proof: See the appendix.

4.2 A Three Agent Economy.

We fully work the process of coalition formation in a three agent economy in order to illustrate one of the main insights of the model. In the examples we use the letters $a, b, c, \ldots$ to denote the players and make explicit what are the coalitions.

[7] See proof of 4.1.1 in the appendix.

4.2.1 Efforts and Utilities for Each Coalition Structure

The possible coalition structures are $[\{a,b,c\}]$, $[\{a,bc\}]$[8], and $[\{abc\}]$. The first (degenerate coalition structure) occurs when the tree agents make individual appropriation efforts. The second coalition structure occurs when an agent makes individual appropriation efforts and the other two make collective appropriation efforts. The third coalition is the grand coalition implies a free access agreement between the three agents. In the first and second cases private property arises and the valuable resource is divided in parts from which agents can exclude non coalition members.

We start computing utilities corresponding to the grand coalition [$\{abc\}$]. In this case, appropriation efforts are zero for each agent and the resource is shared and exploited among the three agents. Then, given the parameter values, each agent i has a payoff of

$$U_i^{[\{abc\}]} = U_G = (1/3)^{\alpha}. \tag{4.4}$$

For the coalition structure [$\{a,b,c\}$], from Grossman's (2001) all players receives the same payoff

$$U_i^{[\{a,b,c\}]} = U_d = (\frac{1}{3})^{\alpha}(\frac{3(1-\alpha)}{3-\alpha})^{1-\alpha}. \tag{4.5}$$

Consider now the coalition structure [$\{a,bc\}$]. The singleton remaining agent, s, decides the appropriating effort by maximizing consumption. Therefore its reaction function solves equation.

$$\frac{dU_a^{[\{a,bc\}]}}{de_a} = \frac{dU_s}{de_s} = \alpha(\frac{l_s}{r_s})^{1-\alpha}(\frac{\sum_{j=1}^{2} e_j}{(\sum_{j=1}^{3} e_j)^2}) - (1-\alpha)(\frac{r_s}{l_s})^{\alpha} = 0. \tag{4.6}$$

For an agent remaining in a coalition exploiting and defending collectively the land against the deviant (i.e. for $i \in \{bc\}$), the first order condition is:

$$\frac{dU_i^{[\{a,bc\}]}}{de_i} = \frac{dU_2}{de_2} = \frac{l_2}{L_2}[\alpha(\frac{L_2}{r_2})^{1-\alpha}\frac{e_s}{(\sum_{j=1}^{3} e_j)^2} - (1-\alpha)(\frac{L_2}{r_2})^{-\alpha}] - \frac{L_2 - l_2}{L_c^2} r_2^{\alpha} L_2^{1-\alpha} = 0 \tag{4.7}$$

[8]We omit the coalitions $[\{b,ac\}]$, $[\{c,ab\}]$ because the coalition is the same.

Table 4.1 presents solutions of the equation system (4.6) and (4.7) for explicit values of α. Finally, table 4.2 presents the explicit values of α (first column) and corresponding utility levels for the two different coalitions in the $[\{2,1\}]$ coalition structure (second and third columns), the grand coalition (fourth column), and the degenerated coalition structure (fifth column)[9]. We have the following remarks about table 4.2.

Table 4.1: Appropriative Effort Values

α	e_2	e_s
0.1	0.025	0.052
0.3	0.082	0.173
0.348	0.0978	0.2066
0.4	0.114	0.243
0.5	0.148	0.323
0.549	0.1659	0.3669
0.6	0.184	0.414
0.7	0.224	0.519
0.9	0.314	0.798

Table 4.2: Comparison of Utilities for Different Coalition Structures

α	U_2	U_s	U_G	U_d
0.1	0.849	0.889	0.895	0.84
0.3	0.616	0.715	0.719	0.603
0.348	0.572	0.682	0.682	0.559
0.4	0.526	0.649	0.644	0.516
0.5	0.451	0.594	0.577	0.447
0.549	0.418	0.571	0.547	0.418
0.6	0.387	0.550	0.517	0.392
0.7	0.332	0.519	0.463	0.349
0.9	0.246	0.505	0.372	0.306

Remark 4.2.1. A particularly interesting value $\alpha = 0.348$ is obtained as a solution of the system of three equations (4.6) and (4.7) and $U_G = U_s$ in α. It is easy to show (see table 4.2) that for numerical values $\alpha < 0.348$, U_G is larger than U_s. However for $\alpha > 0.348$ this inequality is reversed. The immediate consequence is that for low values of α it does not pay for agent s to deviate from the grand coalition.

[9] The solution from the system of equations for general values of α can be obtained by using Lemma 3 presented in section 6.

Remark 4.2.2. Another interesting value is $\alpha = 0.549$ which is obtained as the solution of the system of three equations (4.6) and (4.7) and $U_2 = U_d$ in α. It is easy to show (see table 4.2) that for α less than 0.549, $U_2 > U_d$, and that for α greater than 0.549, $U_2 < U_d$. Therefore, for large enough α, doing private appropriation efforts is better than doing efforts in a coalition, when an agent deviates from the free access agreement.

Remark 4.2.3. A trivial observation is that the grand coalition structure is always better for each agent than the degenerate coalition structure (i.e. $U_G > U_d$) for every value of α.

4.2.2 The Sequential Equilibrium

We find equilibrium coalition structures as the result of a game of sequential coalition formation[10]. Following Bloch (1996), in a symmetric game, a perfect equilibrium coalition structure can be reached as the outcome of a finite game of choice of coalition sizes. In the Bloch's game, an exogenous protocol sets an order on agents. The first player proposes a coalition size. All the prospective members of the coalition respond in turn to the offer. If all the agents accept the offer, the cooperative agreement takes effect and they leave the game. If one of the agents rejects the offer, the proposed coalition is not formed and the agent that rejected the offer becomes the initiator in the next round.

Remark 4.2.4. *Private property is sustainable as a perfect equilibrium coalition structure for intermediate values of α (i.e $0.348 < \alpha < 0.549$). For the rest of the values of α the grand coalition is the only perfect equilibrium coalition structure.*

Proof of the remark: For α small (i.e. $\alpha < 0.348$) no individual agent has incentives to deviate from the grand coalition as $U_2 < U_d < U_s < U_G$, so the grand coalition is the only stable coalition structure.

Consider now intermediate values of α (i.e $0.348 < \alpha < 0.549$). In the 3 agent economy the Bloch's protocol could choose randomly any player. Without loss of generality assume that it is player s. As $U_s > U_G$, and $U_2 > U_d$ (so, upon its deviation, the other two players would stick

[10]Following a bargaining protocol as proposed by Bloch (1996) and Ray and Vohra (1999).

together in a complementary coalition), player s would rationally offer a coalition of size one that is accepted and the coalition is formed. In the second stage, one of the remaining players (1 or 2) offers a coalition of size two that is accepted by the other agent (as $U_2 > U_d$ for the corresponding values of α) and the coalition is formed. Then the coalition structure [$\{$2,1$\}$] arises.

Assume now that we are in the region $\alpha > 0.549$. In the first stage player s will not offer a coalition size of one as it knows that $U_2 < U_d$ and upon its deviation from the grand coalition, the rest of players will become singletons (and $U_G > U_d$). Therefore the grand coalition is an equilibrium structure. ■

As α is an index of the value of the resource (the share of the resource in production), the conclusion of this section is that private property is only sustainable as a perfect equilibrium coalition structure for intermediate values of the resource. Unless the resource is sufficiently valuable, agents do not have incentives to deviate from the free access agreement. If the value of the resource increases enough, deviations from the free access agreement are too costly in terms of conflict.

4.3 Coalition Formation with Exogenous Responses.

We say that a coalition structure is stable if no member can unilaterally o collectively deviate. Testing stability is difficult as we would need to specify the responses of the other members of a coalition once a member o group of members deviated.

Hart and Kurz (1983) present two models of stability[11] that assume two types of responses of members of a coalition once a member deviates. Each model corresponding to a coalition game, and in each one stability is based on the strong equilibria concept. The first one called the γ game and corresponds to the case in which each agent chooses the coalition to which she wants to belong, and a coalition forms if all its members have chosen to form it. The players not belonging to these unanimous consent coalitions become singletons. This means that if a player leaves a given coalition, the rest of the players become singletons (the coalition breaks). As Hart and Kurz claim, this game is supported by the view of coalitions as the result of an unanimous agreement

[11] Also analyzed in Bloch, 2012, for the particular case of contests by coalitions

among all its members to act together. Then, if one of the players leave, the agreement breaks down.

The second is called the δ game and corresponds to the case in which each player chooses the largest set of players he is willing to be associated with in the same coalition. Coalitions are formed among all the players that choose to be in the same coalition. In Hart and Kurz's words "a coalition corresponds to an equivalence class, with respect to equality of strategies". This means that if a player leaves a given coalition, the rest of the members form one new coalition. As Hart and Kurz claim, this model is justified specially in large games in which the fact that a player leaves a coalition has no influence in the others agreement to act together.

We characterize stable coalition structure for each of the games proposed by Hart and Kurz.

Obviously, when deciding her appropriation effort the agent has to consider that the rest of the agents would also make appropriating efforts to keep some of the land. Otherwise the single agent would keep all the land. To analyze the appropriating efforts in the economy we have to compute the reaction functions of the deviating agent and also of the agents remaining in the coalition.

Proposition 4.3.1. *Assume that we are in the δ model (i.e. upon a deviation by one agent from the grand coalition the rest of agents remain in a complementary coalition). There is a finite $\bar{n}$ such that for any $n \geq \bar{n}$ the grand coalition is not stable for any* $0 < \alpha < 1$.

Proof: See appendix.

This proposition establish that if we are in the δ model defined by Hart, S. and Kurz, M. (1983) private property would be sustainable for a large enough number of agents.

Proposition 4.3.2. *Assume that we are in the γ model (i.e. upon a deviation by one agent from the grand coalition the rest of agents become singletons). The grand coalition is stable.*

proof: This result follows from the simple comparison between the individual utility. In case all agents form the grand coalition $U_i^{\{[GC]\}} = (\frac{1}{n})^{\alpha}$; and the individual utility in the case all agents make individual appropriative efforts $U_i^{\{[1,1,\ldots,1]\}} = (\frac{1}{n})^{\alpha}(\frac{n(1-\alpha)}{n-\alpha})^{1-\alpha}$. ■

In the case we are in the γ model, private property would not be sustainable in Grossman's model.

4.4 Endogenous Coalition Formation

In the general case with an arbitrary number of agents, closed form solutions for the strategies and utilities associated to each coalition structure are impossible to obtain. This is only possible for particular coalition structures. One of them is a coalition structure that divides the set of agents into two. The other is for symmetric coalition structures.

A closed form solution can be obtained when the alliance structure is $\pi = [\{c, s\}]$ where s is an integer, $1 \leq s \leq \frac{n}{2}$, and $c + s = n$.

Lemma 4.4.1. *The optimal effort level for the problem of s players in the coalition structure $\pi = [\{c, s\}]$ is given by*

$$e_s = \frac{2}{3}\sqrt{f(6+f)}cos\left(\frac{\theta}{3} + \frac{4\pi}{3}\right) - \frac{f}{3}, \tag{4.8}$$

where $f(n, s, \alpha) \equiv \frac{c\alpha}{(n-\alpha)(s-\alpha)}$ and $\theta(n, s, \alpha) \equiv\ cos^{-1}\left(-\frac{1}{2}\frac{f(18f+27\frac{\alpha}{s}+2f^2)}{\sqrt{(f(6+f)^3}}\right)$.

Proof: See the appendix.

A symmetric coalition structure is $\pi = \{a, a, \ldots, a\}$ where a is repeated k times and $a = \frac{n}{k}$ with $\frac{n}{k}$ an integer.

Lemma 4.4.2. *In a symmetric coalition structure every player obtains a payoff given by*

$$U_{i\in a_m} = \frac{1}{a}\left(\frac{1}{k}\right)^{\alpha}\left(\frac{ak(a-\alpha)}{ak-\alpha}\right)^{1-\alpha}. \tag{4.9}$$

Proof: See the appendix.

A symmetric coalition structure cannot be an equilibrium of the game of sequential coalition formation.

Proposition 4.4.1. *A symmetric coalition structure is strictly dominated by the grand coalition*

This result comes from the observation that $ak = n$

$$U_{i \in a_m} = \frac{1}{a}\left(\frac{1}{k}\right)^{\alpha}\left(\frac{ak(a-\alpha)}{ak-\alpha}\right)^{1-\alpha},$$

can be written as

$$\left(\frac{1}{n}\right)^{\alpha}\left(\frac{n(a-\alpha)}{a(n-\alpha)}\right)^{1-\alpha}$$

and $\left(\frac{n(a-\alpha)}{a(n-\alpha)}\right) < 1$. Therefore $U_{i \in a_m} < (\frac{1}{n})^{\alpha}$.

4.5 The Role of Changing the Number of Players

The closed form computation of coalitional equilibria with arbitrary number of agents is impossible as the number of coalitions to be considered is also arbitrary.

We analyze the role of changing the number of agents by computing equilibrium coalitions for economies with different number of agents. We detail the computation of equilibria when the number of agents are four and five. Three important conclusions are obtained. The first is that the non-monotonic effect of changing the α holds. The second is the appearance of new equilibrium coalition structures that would imply different private property regimes. The third is a non-monotonic effect of changing the number of agents on the size of the set of α for which the grand coalition is stable.

4.5.1 Four and Five Agent Example

In the four agent example the possible coalition structures are $[\{a, b, c, d\}]$, $[\{a, b, cd\}]$, $[\{ab, cd\}]$, $[\{a, bcd\}]$ and $[\{abcde\}]$. We omit the coalition structures that have associated the same payoff. In the first stage players compare five possible outcomes. Table 4.3 shows the utility levels for each coalition structure at different levels of α, U_a denotes the utility for player 1, U_b denotes the utility for player 2, and so on.

Remark 4.5.1. Private property is sustainable as a perfect equilibrium solution for low values of

Table 4.3: Comparison of Utilities for Different Coalition Structures Four Agents Case.

	Utility	α							
Coalition structure		0.05	0.073	0.074	0.4	0.492	0.493	0.6	0.9
$[\{a, bcd\}]$	U_a	0.9427	0.9176	0.9168	0.6524	0.6026	0.6023	0.5571	0.5174
	$U_b = U_c = U_d$	0.9068	0.8666	0.8652	0.4584	0.3828	0.3824	0.3102	0.1692
$[\{ab, cd\}]$	$U_a = U_b = U_c = U_d$	0.9218	0.8879	0.8867	0.5352	0.4679	0.4675	0.4028	0.2775
$[\{a, b, cd\}]$	$U_a = U_b$	0.9162	0.8801	0.8789	0.5213	0.4580	0.4576	0.4000	0.3211
	$U_c = U_d$	0.8995	0.8566	0.8551	0.4382	0.3653	0.3649	0.2970	0.1728
$[\{abcde\}]$	$U_a = U_b = U_c = U_d = U_e$	0.9330	0.9035	0.9025	0.5743	0.5053	0.5049	0.4353	0.2872
$[\{a, b, c, d\}]$	$U_a = U_b = U_c = U_d = U_e$	0.8993	0.8566	0.8551	0.4503	0.3828	0.3825	0.3220	0.2340

α, (i.e $\alpha \leq 0.493$). If $\alpha > 0.493$], then the perfect equilibrium coalition structure is the grand coalition, $\pi = [\{abcd\}]$.

Proof of the remark:

First notice that the grand coalition dominates two and three members coalition strategies. Hence player a never proposes a two or three-member coalition. In the following analysis we do not consider these strategies.

If $\alpha \leq 0.073$, then player a proposes to form the grand coalition and all players receive the same payoff. If she deviates and chooses to form a singleton, then player b forms a singleton also. Player c offers a two-member coalition to player d which is accepted because the payoffs for the two-member in $[\{a, b, cd\}]$ are bigger than those corresponding to the degenerated coalition structure. In the coalition structure in $[\{a, b, cd\}]$ player a receives $U_a^{[\{a,b,cd\}]}$ which is lower than the payoff corresponding to the grand coalition.

If $0.74 \geq \alpha \leq 0.492$, player a chooses to form a singleton. Player b proposes a three-member coalition to players c and d, which is accepted. Notice that Player b never proposes a two-member coalition because the payoffs are lower than in the three-member coalition. If player b deviates to a one-member coalition, then player c and d become singletons, and all players would receive the degenerated coalition structure payoff which is lower than $U_b^{[\{a,bcd\}]}$. Therefore player b would not deviate (and symmetrically, neither c nor d) and, as player a receives her best payoffs, she does not deviate.

If $0.493 \geq \alpha$, then player a chooses the grand coalition again which is accepted by all players. Notice that for these values of α the degenerated structure dominates coalitions with three and two members if a singleton is formed. Therefore, if player a deviates and chooses to play alone, then players b to d do the same and all of them receive the degenerated payoff. Hence player a does not deviates. ■

The five agent case is analyzed similarly. Table 4.4 shows the perfect equilibrium coalitions structures for given values of α in the five agent example. There are only four equilibrium coalition structures.

Table 4.4: Comparison of Utilities for Different Coalition Structures Five Agents Case.

	Utility	α								
Coalition structure		0.05	0.075	0.081	0.4	0.469	0.5	0.566	0.7	0.9
$[\{a, bcde\}]$	U_a	0.9427	0.9158	0.9095	0.6540	0.6163	0.6012	0.5727	0.5309	0.5228
	$U_b = U_c = U_d = U_e$	0.8956	0.8476	0.8364	0.4128	0.3539	0.3301	0.2845	0.2095	0.1298
$[\{ab, cde\}]$	$U_a = U_b$	0.9219	0.8854	0.8769	0.5375	0.4869	0.4662	0.4258	0.3578	0.2842
	$U_c = U_d = U_e$	0.9070	0.8639	0.8539	0.4651	0.4093	0.3866	0.3428	0.2696	0.1908
$[\{a, bc, de\}]$	U_a	0.9163	0.8776	0.8687	0.5260	0.4788	0.4600	0.4246	0.3708	0.3393
	$U_b = U_c = U_d = U_e$	0.8996	0.8536	0.8429	0.4416	0.3866	0.3644	0.3221	0.2528	0.1808
$[\{a, b, cde\}]$	$U_a = U_b$	0.9162	0.8775	0.8685	0.5226	0.4744	0.4552	0.4187	0.3623	0.3247
	$U_c = U_d = U_e$	0.8862	0.8344	0.8224	0.3839	0.3261	0.3032	0.2596	0.1897	0.1192
$[\{a, b, c, de\}]$	$U_a = U_b = U_c$	0.8994	0.8535	0.8429	0.4525	0.4012	0.3809	0.3428	0.2841	0.2402
	$U_d = U_e$	0.8849	0.8327	0.8207	0.3872	0.3314	0.3092	0.2675	0.2012	0.1353
$[\{abcde\}]$	$U_a = U_b = U_c = U_d = U_e$	0.9227	0.8863	0.8778	0.5253	0.4701	0.4472	0.4021	0.3241	0.2349
$[\{a, b, c, d, e\}]$	$U_a = U_b = U_c = U_d = U_e$	0.8872	0.8362	0.8245	0.4065	0.3539	0.3333	0.2949	0.2363	0.1903

Remark 4.5.2. Private property is sustainable as a perfect equilibrium solution for intermediate values of α, (i.e $0.081 \leq \alpha \leq 0.469$). If $\alpha \in (0.469, 0.566]$, then the perfect equilibrium coalition structure is $\pi = [\{ab, cde\}]$. Finally for the rest of values of α the grand coalition is the perfect equilibrium coalition structure.

Proof of the remark: If $\alpha \leq 0.081$, then player a does not have incentives to deviate from the grand coalition. Player a never deviates and chooses a coalition of size two, three, or four; because the grand coalition gives a higher payoff. If player a deviates and chooses a coalition of size one, then player b and d offer a size two coalition which is accepted by player c and e, respectively, because $U_b^{[\{a,bc,de\}]} > U_b^{[\{a,b,c,de\}]}$ and $U_b^{[\{a,bc,de\}]} > U_b^{[\{a,b,c,d,e\}]}$. Hence player a obtains a payoff of $U_a^{[\{a,bc,de\}]}$ which is worse than $U_a^{[\{abcde\}]}$. Player b does not choose a coalition of size one because he knows that the next three players would choose to form a singleton in that case. He never offers a three or four-member coalition because this gives lower payoffs than the grand coalition.

If $0.081 < \alpha \leq 0.469$, then player a prefers to be a singleton, and player b proposes a four-member coalition which is accepted. If player b deviates and proposes a one-member coalition, then the other three player choose a singleton, and all players obtain the degenerated payoff which is worse than $U_b^{[\{a,bcde\}]}$. If he proposes a two-member coalition to player c, then the proposal is rejected because players d and e become a singleton as $U_d^{[\{a,bc,d,e\}]} > U_d^{[\{a,bc,de\}]}$, and player b and c receives a lower payoff than $U_b^{[\{a,bcde\}]}$. He never proposes a three-member coalition because his payoff would be worse than the degenerated coalition structure payoffs. Player a does not have incentives to deviate as $U_a^{[\{a,bcde\}]}$ is the best payoff for her.

If $0.469 < \alpha \leq 0.566$, player a proposes a two-member alliance to player b who accepts, and player c proposes a three-member coalition which is accepted by players d and e. If player c deviates and chooses a singleton then players d and e prefer to be singletons, and they obtain $U_c^{[\{ab,c,d,e\}]} < U_c^{[\{ab,cde\}]}$. This player never proposes a two-member coalition because it gives a lower payoff as $U_c^{[\{ab,cd,e\}]} < U_c^{[\{ab,cde\}]}$. Player a never chooses a coalition of three, four or five members, because it gives lower payoffs in any structure that it is formed. If she deviates and chooses a singleton, then the remaining players become singletons too, and the degenerated structure is formed which gives worse payoffs than $\pi = [\{ab, cde\}]$. In the case that player a

chooses to form a singleton, player b never chooses a two, three or four members coalition because if it is accepted, the remaining players become singletons and then they obtain lower payoffs. The same argument applies for players c and d.

If $0.566 < \alpha \leq 1$ then the grand coalition is formed again. Player a never deviates and chooses a coalition of three or four members because it gives lower payoffs than the grand coalition. If she deviates and chooses to form a singleton, then all the players have more incentives to become singletons too, and they obtain the payoff corresponding to the degenerate coalition structure which is lower than the grand coalition payoff. If she deviates and chooses a two-member coalition then the offer is rejected, because if it is accepted then the remaining players prefer to be singletons since $U_c^{[\{ab,c,d,e\}]} > U_c^{[\{ab,cde\}]} > U_c^{[\{ab,cd,e\}]}$. Players' strategies are symmetric so the same arguments apply for the rest of the players. ■

The two remarks show that private property is sustainable as a perfect equilibrium for intermediate values of α. Another conclusion is that if the number of agents increases there are new equilibria that neither imply strictly private property nor common land. In this example, the grand coalition is a perfect equilibrium structure for a greater set of values of α, than in the tree agents example.

Table 4.5, shows the α values for which the grand coalition is a perfect equilibrium structure in the sequential coalition formation game, as the number of agents increases. We calculate the perfect equilibrium, from $n = 3$ to $n = 8$, as described previously. In the table, we can see that the values of α for which the grand coalition is a perfect equilibrium is non monotonic.

Table 4.5: Values of $\bar{\alpha}$, such that for $\alpha \geq \bar{\alpha}$, the Grand Coalition is a Perfect Equilibrium in a Sequential Coalition Formation Game

n	$\bar{\alpha}$
3	0.55
4	0.580
5	0.566
6	0.568
7	0.558
8	0.54

In this section, we conclude that as the number of players increases, private property is sustainable for intermediate values of α and there are new forms of property that are neither strictly private nor common property in the equilibrium[12].

4.6 Final Remarks

The chapter contributes to the literature on the foundations of private property rights by setting a model in which private property rights can emerge as an equilibrium allocation. The chapter analyzes conditions such that private property arises as equilibrium in the model by Grossman (2001). For private property to arise as coalitional equilibrium, the resource has to be valuable enough to incentive some agents to do private appropriation efforts on the resource. However, if the resource is too valuable then too many agents will be doing appropriation efforts and too much effort in conflict is wasted in equilibrium. This can make not worthwhile to attain private property rights on the resource for any agent.

Our (model) would imply that, as in Demsetz (1967), increases in the value of land lead to the appearance of private property. However, if land value increases too much then the appearance of private property is through too much conflict. The loss of resources can be large enough and could make the appearance of private property not desirable for any of the individuals. The implication is that the appearance of private property, apart from private gains, may also require the existence of institutions that reduce the amount of conflict. One of such institutions can be a superior authority. Others can be family links between the agents that reduce conflict.

4.7 References

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[12] Sánchez-Pagés (2007) concludes that in in the sequential coalition formation game there are a bigger set of equilibrium coalition structures. Our conclusion is similar.

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4.8 Appendix A.1

Proof to lemma 4.1.1: The Lagrangian of the problem for player i in alliance m is

$$L(e_{im}, l_{im}, \lambda) = \frac{l_{im}}{L_m}\left(\frac{E,}{\sum_{m=1}^{k} E_m}\right)^{\alpha} L_m^{1-\alpha} - \lambda(1 - e_{im} - l_{im}),$$

the first order conditions of this problem are

$$\frac{\alpha l_{im}}{L_m}\frac{1}{\sum_m^k E_m}(\sum_{-m}^{k} r_m) r_m^{\alpha-1} L_m^{1-\alpha} = \lambda, \tag{4.10}$$

and

$$\frac{r_m^{\alpha} L_m^{1-\alpha}}{L_m^2}(L_m - \alpha l_{im}) = \lambda. \tag{4.11}$$

WLOG suppose that agents i and j belongs to the same alliance , $i, j \in a_m$. In equilibrium, L_m and r_m are the same for i and j, then from 4.11, is easy to see that $l_{im} = l_{jm}$. This implies that $e_{im} = e_{jm}$, by the time constraint. ■

Proof to lemma 4.1.2: From 4.10 and 4.11, we obtain the marginal rate of substitution

$$\frac{L_q - \alpha l_{iq}}{\alpha l_{iq} L_q \frac{1-r_q}{r_q}\frac{1}{\sum_m^k E_m}} = 1. \tag{4.12}$$

From 4.3 and Lemma 4.1.1 we have

$$\frac{(a_m - \alpha)}{\alpha}\frac{e_m}{l_m} = \sum_{m=1}^{k} r_{-m}, \tag{4.13}$$

using the time constraint $l_m = 1 - e_m$ and $\sum_{m=1}^{k} r_{-m} = 1 - r_m$

$$e_m = \frac{1}{1 + \frac{(a_m - \alpha)}{\alpha(1-r_m)}}. \tag{4.14}$$

In the above expression, the right hand size is a function that depend on the appropriative effort level through $r_m(\mathbf{e})$. The CSF, i.e. $r_m(\mathbf{e})$, is an increasing convex function with respect to e_m.

Define $f(\mathbf{e}) \equiv \frac{\alpha(1-r_m(\mathbf{e}))}{\alpha(1-r_m(\mathbf{e}))+(a_m-\alpha)}$.

Claim: *The function $f(\mathbf{e})$ is bounded and twice differentiable with respect to e_m*

Proof of the claim

The CSF is a twice differentiable function, then $f(\mathbf{e})$ is a twice differentiable function with respect to e_m, the derivatives are

$$f(\mathbf{e})_{e_m} = \frac{-\alpha r_m(\mathbf{e}_{e_m}(a_m - \alpha)}{(\alpha(1-r_m) + (a_m - \alpha))^2} < 0,$$

and

$$f(\mathbf{e})_{e_m^2} = \frac{-\alpha^2(a_m - \alpha)^2 r_m(\mathbf{e}_{e_m^2}}{(\alpha(1-r_m) + (a_m - \alpha))^3} > 0.$$

Indeed, $f(\mathbf{e})$ is decreasing and concave. By definition, $a_m \geq 1$ and $r_m(\mathbf{e}) \leq 1$, then $f(\mathbf{e}) \in [0, 1]$ for any $\alpha \in (0, 1)$. ■

From above, $\mathbf{e} = f(\mathbf{e})$, has a fixed point. Hence $\mathbf{l}$ has a fix point, also. ■

Proof to lemma 4.4.1:

From equation 4.13, for a coalition structure $\pi[\{c, s\}]$, land fraction that each coalition obtain are

$$r_c = \frac{s - \alpha}{\alpha}\frac{e_s}{l_s}, \tag{4.15}$$

and

$$r_s = \frac{c-\alpha}{\alpha}\frac{e_c}{l_c}, \tag{4.16}$$

using that $r_c = \frac{ce_c}{ce_c+se_s}$ and $l_m = 1 - e_m$

$$e_c = \frac{\left(\frac{s-\alpha}{\alpha}\right)\frac{s}{c}e_s^2}{1-\frac{s}{\alpha}e_s}. \tag{4.17}$$

Notice that $0 \leq e_s \leq \frac{\alpha}{s}$. If $e_s > \frac{\alpha}{s}$ then $e_c = 0$ (By definition $e_i \geq 0$, then an effort level can not be negative), but this is not an equilibrium. Analogously $e_c < \frac{\alpha}{c}$. From 4.17 and $r_c + r_s = 1$, the following polynomial is obtained,

$$e_s^3 + f(n,s,\alpha)e_s^2 - 2f(n,s,\alpha)e_s + f(n,s,\alpha)h(s,\alpha) = 0, \tag{4.18}$$

where $f(n,s,\alpha) \equiv \frac{c\alpha}{(n-\alpha)(s-\alpha)}$ and $h(s,\alpha) \equiv \frac{\alpha}{s}$. By Descarte's rule, this polynomial has one negative real root and two or none positive real roots. Evaluating the polynomial at $e_s = \frac{s}{2\alpha}$, $e_s = \frac{s}{\alpha}$ and $e_s = 1$ the polynomial has three real roots, one of them in the interval $[\frac{s}{2\alpha}, \frac{s}{\alpha}]$, and the other in the interval $[\frac{s}{\alpha}, 1]$.

Define,

$$Q \equiv -f\frac{6+f}{9},$$

and

$$R \equiv -f\frac{18f + 27h + 2f^2}{54}.$$

Let $D \equiv Q^3 + R^2$ be the discriminant. If $D < 0$ all roots are real and unequal. In this case the discriminant is,

$$D = \frac{\left(\frac{c\alpha}{(n-\alpha)(s-\alpha)}\right)^2}{3^3}\frac{\alpha}{s}\left[\frac{3^3}{4}\frac{\alpha}{s} - \frac{(n-s)(8n-9\alpha)}{(n-\alpha)^2}\right]$$

Claim: *D is always negative*

Proof.

For any α, $n \geq 3$ and $s \geq 1$, we only need to show that

$$\frac{27}{4}\frac{\alpha}{s} < \frac{(n-s)(8n-9\alpha)}{(n-\alpha)^2},$$

the left hand side (LHS) and the right hand side (RHS) are increasing on α. Moreover the second derivative on the RHS is

$$\frac{\partial^2}{\partial\alpha^2}\frac{(n-s)(8n-9\alpha)}{(n-\alpha)^2} = \frac{6(n-s)(2n-3\alpha)}{(n-\alpha)^4} > 0.$$

Above expression implies that the RHS increases faster than the LHS. At $\alpha = 0$ the RHS is greater than the LHS. Therefore, the RHS is always greater than the LHS for any $\alpha \in (0,1)$.bf QED There are three real roots,

$$e_{s1} = 2\sqrt{-Q}cos\left(\frac{\theta}{3}\right) - \frac{f}{3},$$

$$e_{s2} = 2\sqrt{-Q}cos\left(\frac{\theta}{3} + \frac{2\pi}{3}\right) - \frac{f}{3},$$

and

$$e_{s3} = 2\sqrt{-Q}cos\left(\frac{\theta}{3} + \frac{4\pi}{3}\right) - \frac{f}{3},$$

Where $\theta \equiv cos^{-1}\left(\frac{R}{\sqrt{-Q^3}}\right)$. In this case, $D < 0$ and $R \leq 0$, then $-1 < \frac{R}{\sqrt{-Q^3}} < 0$. Therefore

$\theta \in \left(\frac{\pi}{2}, \pi\right)$, which implies that $\frac{\theta}{3} \in \left(\frac{\pi}{6}, \frac{\pi}{3}\right)$, $\frac{\theta+2\pi}{3} \in \left(\frac{5\pi}{6}, \pi\right)$, and $\frac{\theta+4\pi}{3} \in \left(\frac{3\pi}{2}, \frac{5\pi}{3}\right)$. From above e_{s2} is the negative root, and $0 \leq e_{s3} \leq e_{s1}$. Therefore e_{s3} is the root in the interval $[\frac{\alpha}{2s}, \frac{\alpha}{s}]$. ■

Proof to 4.4.2: From Lemma 4.1.1, in the main text, players that belong to the same coalition make the same appropriative effort and the same labor. Therefore, in a symmetric coalition structure every player make the same effort and obtain the same fraction of land. Then $e_m = e$ and

$$r_m = \frac{1}{k}. \tag{4.19}$$

Using equation (4.13), in the main text, the appropriative effort is

$$e = \frac{\alpha(k-1)}{ak-\alpha}. \tag{4.20}$$

The individual labor supply is $l_m = l = 1 - e$ and the labor supply from one coalition is

$$la = \frac{ak(a-\alpha)}{ak-\alpha}. \tag{4.21}$$

Using equations (4.19), (4.20) and (4.21) in the utility expression the equation (4.9) is obtained.

■

Proof to proposition 4.3.1:

We want to show that $U_s > U_g$ when $n \geq 9$ for any α with $s = 1$ and $c = n - 1$.

Claim A lower bound for the single deviator utility is $\left(\frac{1}{3}\right)^{\alpha}(1-\alpha)^{1-\alpha}$.

Proof.

From Lemma 4.4.1, $e_s \in [\frac{\alpha}{2}, \alpha)$, and $ce_c < \alpha$, given the coalition structure $\pi = [\{s, c\}]$. From equation (4.1) the fraction of land that the single deviator obtain is increasing with e_s. Therefore $r_s \geq \frac{1}{3}$ and $l_s \geq 1 - \alpha$.

From Lemma 4.4.1, $e_s 3$ is the solution from the maximization problem (4.2). Hence $U_s >$

$\left(\frac{1}{3}\right)^{\alpha}(1-\alpha)^{1-\alpha}$. ■

For this lower bound, $\bar{n}=9$, to see this notice that $\left(\frac{1}{3}\right)^{\alpha}(1-\alpha)^{1-\alpha} > U_g$ then

$$(1-\alpha)^{\frac{1-\alpha}{\alpha}} > \frac{3}{n}.$$

The McLaurin series of $(1-\alpha)^{\frac{1-\alpha}{\alpha}}$ is $\frac{1}{e}+\sum_{p=1}^{\infty}\frac{\frac{\partial^{(p)}}{\partial\alpha^{(p)}}\left[(1-\alpha)^{\frac{1-\alpha}{\alpha}}\right]_{\alpha=0}}{p!}(\alpha)$. Then $\frac{1}{e} > \frac{3}{n}$, if $n \geq 9$ at any α. Hence $U_s > U_g$ at least for $n \geq 9$. ■

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