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RISK AND RETURN IN THE MEXICAN REAL ESTATE MARKET: A MULTIFACTOR PRICING MODEL FOR FIBRAS

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Abstract

A multifactor linear asset pricing model is estimated in order to check whether Fideicomisos de Infraestructura y Bienes Raíces (FIBRAs) are exposed to stock market risk factors. Statistical procedures are used for estimating risk factor portfolios from a large sample of stocks listed in the Bolsa Mexicana de Valores. The pricing model for FIBRAs is estimated via a GMM-based asset pricing test in order to avoid imposing strong distributional restrictions on the data. Results show that all of the FIBRAs in the sample are exposed to at least one stock market risk factor. A test for the validity of the linear specification fails to reject that the proposed model prices FIBRAs correctly.

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1 Introduction

Empirical studies on the Mexican real estate market are sparse, although real estate arguably constitutes an important part of the country's stock of fixed capital. One of the main reasons for the existence of this literature gap is undoubtedly related to the availability and quality of data on real estate asset prices. However, the recent emergence of the Fideicomisos de Infraestructura y Bienes Raíces (FIBRAs), investment vehicles similar in nature to Real Estate Investment Trusts (REITs), provides a still unexploited opportunity to assess the real estate market from a financial perspective. As Zietz, Sirmans, and Friday (2003) correctly state, REITs "allow investors to hold portfolios of highly illiquid real estate assets while simultaneously enjoying traditional stock market liquidity and marketability advantages." For us researchers, the Initial Public Offering (IPO) of FIBRA Uno in 2011 brought along a stream of real estate data of unprecedented magnitude. This study aims at taking advantage of the availability of daily prices on real estate-related securities in order to assess the risk and return relationship in the Mexican real estate market.

According to Liu (2010), REITs were created by the U.S. Congress in 1960 and are characterized by a series of regulatory requirements that have to do with distribution of income among shareholders, holding of real estate assets and ownership schemes. In a brochure-like document easily accessible online[1], the BMV Group describes the main characteristics of the Mexican analogue to REITs: FIBRAs. FIBRAs are publicly traded investment vehicles designed to finance the acquisition or development of real estate assets. Their main regulatory characteristics are the requirement that at least 70% of their wealth must be invested in real estate assets, and the obligation to distribute 95% of their annual income among shareholders. There are essentially two ways of becoming a shareholder: the investor can contribute to the social capital of the FIBRA either with previously held real estate assets or by buying publicly traded shares on the Bolsa Mexicana de Valores. As of June, 2016, there exist 10 FIBRAs listed in the BMV market.[2] However, due to the fact that one FIBRA has only been in the market for roughly one year, this study will only consider 9 of them.

The main goal of this work is to estimate a multifactor pricing model for FIBRAs in order to study the relationship between risk and return in this branch of the real estate market. In particular, I plan to answer the question of whether the returns on FIBRAs can be explained by the movements of stock market-wide risk factors. It is my intention that the results of this work serve as a precedent for further investigation about common real estate risk factors and as a tool for the assessment

[1] The referred document can be consulted in `https://www.bmv.com.mx/docs-pub/MI_EMPRESA_EN_BOLSA/CTEN_MINGE/Fibras.pdf`

[2] A list of all publicly traded FIBRAs is available in the appendix.

of risk and portfolio analysis. The approach I take builds on the Euler equations derived from an investor's utility maximization problem and develops an expected return-beta representation of asset returns that is easily testable empirically. The econometric strategy is a two-step process: first I construct stock market risk factors through statistical techniques, and afterwards I use them to estimate a linear pricing model for FIBRAs.

The epitome of linear multifactor asset pricing models is the Arbitrage Pricing Theory (APT) of Ross (1976), and thus much of the literature on linear specifications for empirical asset pricing has focused on tests of this model. Early examples are the works of Roll and Ross (1980), and Chen, Roll, and Ross (1986), who perform empirical tests of the APT by using, respectively, statistically estimated and macroeconomic factors. In the specific context of REITs, there is a relatively vast literature on factor pricing models that started with the evaluation of both the Capital Asset Pricing Model (CAPM) and the APT in Titman and Warga (1986), and Chan, Hendershott and Sanders (1990). Of remarkable importance are the several works of Ling and Naranjo describing the behaviour of REIT returns through the use macroeconomic factors. See, for example, Ling and Naranjo (1997,1999), and Ling, Naranjo and Ryngaert (2000).

Other important study related to the pricing of REITs, and in particular to their response to stock market risk factors, is Peterson and Hsieh (1997), which concludes that there is a significant relationship between the volatility in the stock market and the risk premia of REITs. This last result creates a precedent for the strategy of this investigation. Methodologically, this work resembles the studies of Chen, Hsieh, and Jordan (1997), and Lizieri, Satchel, and Zhang (2007). These two articles employ, among others, the same statistical approaches I use for the estimation of the risk factors. Of course, the literature on REITs goes well beyond factor pricing models; general references on other issues discussed in the literature can be found in the excellent surveys of Liu (2010), and Zietz, Sirmans, and Friday (2003).

With respect to the Mexican real estate market, most of the recent works have focused on local housing issues. Worthy of mentioning are the works of Sobrino (2014) and Isunza (2010), who study housing demand and policy for the case of Mexico City. However, the empirical academic literature on FIBRAs is close to inexistent. Hopefully, this work will forego many others on the subject.

In the next section I state and detail the theoretical asset pricing framework upon which I base my empirical strategy. Section three describes the econometric methodology; I briefly go through the steps for the estimation of the risk factors, and then outline the specific empirical tests performed. The fourth section discusses data-related issues and displays the results of factor construction and the outcome of the estimation of the factor pricing model. Section 5 concludes.

2 The model

I now introduce the theoretical framework upon which the asset pricing tests will be performed. The development of the model is based upon Harvey and Kirby (1995) and Cochrane (2005). Following Harvey and Kirby (1995), this model starts from an endowment economy in which each agent chooses optimal holdings of assets. Within this setup, it is well known that the first order conditions for each investor's problem are the so-called *basic pricing equations*[3]

$$p_t^j = \mathbb{E}_t[m_{t+1}x_{t+1}^j] \tag{1}$$

where p_t^j is the price of security j at time t, x_{t+1}^j is security j's payoff at time $t+1$, m_{t+1} is the stochastic discount factor, henceforth SDF, and $\mathbb{E}_t[\cdot]$ is the expectation conditional on the agent's information set available at time t.

For the purpose of this paper, it is much more convenient to think of the traded assets in terms of their returns.[4] Returns are payoffs in terms of the price paid for them: $R_{t+1}^j \equiv \frac{x_{t+1}^j}{p_t^j}$. Equation (1) can be stated in such terms by diving both sides by p_t^j:

$$\mathbb{E}_t[m_{t+1}R_{t+1}^j] = 1 \tag{2}$$

Notice that, since p_t^j is an element of the information set at time t, it makes sense to multiply any term inside the conditional expectation operator by $\frac{1}{p_t^j}$. It follows from equation (2) that a return can be thought of as a payoff of unit price.

In order to develop a framework consistent with linear factor models, it is necessary to assume a particular expression for the SDF. Such functional form may well be some linear function of K underlying risk factors common to all assets. Let $\mathbf{f}_{t+1}$ be the $Kx1$ vector of factor realizations at time $t+1$, then the stochastic discount factor can be expressed as $m_{t+1} = a + \mathbf{b}'\mathbf{f}_{t+1}$, where $a \in \mathbb{R}$ and $\mathbf{b} \in \mathbb{R}^K$. Incorporating this particular SDF in Equation (2) and computing unconditional expectations yields:

$$\mathbb{E}[(a + \mathbf{b}'\mathbf{f}_{t+1})R_{t+1}^j] = 1$$

[3]Cochrane (2005) uses this particular terminology.

[4]Returns are more convenient than prices because of their statistical properties, and in particular because time series of returns are often stationary. See, for example, Cochrane (2005), p. 8-9, and Brooks (2008), p. 7.

which can be expressed in terms of the covariance between the stochastic discount factor and the return:

$$\text{cov}(a+\mathbf{b}'\mathbf{f}_{t+1}\,,R^j_{t+1})+\mathbb{E}[a+\mathbf{b}'\mathbf{f}_{t+1}]\,\mathbb{E}[R^j_{t+1}]=1$$

After some algebraical manipulation, the previous expression becomes:

$$\mathbb{E}[R^j_{t+1}]=\frac{1}{a+\mathbf{b}'\mathbb{E}[\mathbf{f}_{t+1}]}-\frac{\mathbf{b}'\text{cov}(\mathbf{f}_{t+1}\,,R^j_{t+1})}{a+\mathbf{b}'\mathbb{E}[\mathbf{f}_{t+1}]}$$

I intend to express the basic return pricing equation (2) as an Expected Return-Beta representation of the linear factor model. In order to do this, I denote $\mathbf{\Omega}_K\equiv\mathbb{E}[(\mathbf{f}_{t+1}-\mathbb{E}[\mathbf{f}_{t+1}])(\mathbf{f}_{t+1}-\mathbb{E}[\mathbf{f}_{t+1}])']$.[5] The former equation can equivalently be written as

$$\mathbb{E}[R^j_{t+1}]=\gamma+\boldsymbol{\lambda}'\boldsymbol{\beta}_j \tag{3}$$

where $\gamma\equiv\frac{1}{a+\mathbf{b}'\mathbb{E}[\mathbf{f}_{t+1}]}$ and $\boldsymbol{\lambda}\equiv-\gamma\,\mathbf{\Omega}_K\,\mathbf{b}$. Note that $\boldsymbol{\beta}_j=\mathbf{\Omega}_K^{-1}\text{cov}(\mathbf{f}_{t+1}\,,R^j_{t+1})$ is a vector whose elements are the K slope coefficients of a time-series regression of the return of asset j on the factors.

Each of the K betas, i.e. each term in $\boldsymbol{\beta}_j$, can be understood as a measure of exposure to the correspondent risk factors. In order to illustrate the intuition of the beta pricing model in equation (3), one can think of a single term in the sum, say $\lambda_a\beta_{j,a}$, with some $a\in\{1,...,K\}$. The beta pricing model interpretation of $\lambda_a\beta_{j,a}$ is "for each unit of exposure β to risk factor a, you must provide investors with an expected return premium λ_a." (Cochrane, 2005, p. 78)

It may be the case that there is a risk-free asset in the economy, whose rate of return at time $t+1$ is denoted R^f_{t+1}. It is straightforward to say that, since the asset is risk-free, it doesn't have any exposure to any of the K risk factors, i.e. $\boldsymbol{\beta}_f=\mathbf{0}$. Applying equation (3) to this risk-free rate yields $\mathbb{E}[R^f_{t+1}]=R^f_{t+1}=\gamma+\boldsymbol{\lambda}'\mathbf{0}=\gamma$. Thus, in an economy with a risk-free rate, the beta pricing model can be stated in terms of excess returns -the difference between any given return and the risk-free rate- using the following expression:

$$\mathbb{E}[R^j_{t+1}-R^f_{t+1}]=\boldsymbol{\lambda}'\boldsymbol{\beta}_j \tag{4}$$

So far, I have said nothing about the nature or particular characteristics of the risk factors $\mathbf{f}$. As mentioned in Cochrane (2005), the factors in many factor models, including the widely known

[5] $\mathbf{\Omega}_K$ is, evidently, the variance-covariance matrix of the factors.

Capital Asset Pricing Model (CAPM), are expressed in terms of returns or excess returns. When the risk factors are themselves returns of portfolios or assets, the beta pricing model for an economy with a risk-free rate implies, for a given factor f^k, $\mathbb{E}[f^k_{t+1} - R^f_{t+1}] = \boldsymbol{\lambda}'\boldsymbol{\beta}_k$, with $k \in \{1,...,K\}$. But, since the betas are the time-series regression coefficients of the return -in this case the factor f^k- on the K factors, it is straightforward that each factor will have a β of 1 on itself and 0 on the other factors. By letting λ_k be the k^{th} element of $\boldsymbol{\lambda}$, the model yields

$$\mathbb{E}[f^k_{t+1} - R^f_{t+1}] = \lambda_k$$

This result shows that the expected return premia associated to each unit of exposure to the risk factors are non other than the excess returns of the factor mimicking portfolios:[6] each element of $\boldsymbol{\lambda}$ is the excess return on a portfolio that has a full loading ($\beta = 1$) on the corresponding risk factor and a zero beta on every other factor.

The previous expressions can be used to state the beta pricing model in a way that is most convenient to the objectives of the task at hand in this work. Let R^{ej}_{t+1} and f^{ek}_{t+1} denote the excess returns at time $t+1$ on the j^{th} asset and the k^{th} factor, respectively. Furthermore, let $\mathbf{f}^e_{t+1}$ be the $Kx1$ vector whose k^{th} element is f^{ek}_{t+1}. Then, equation (4) implies

$$\mathbb{E}[R^{ej}_t] = \boldsymbol{\beta}'_j \mathbb{E}[\mathbf{f}^e_t] \tag{5}$$

As the last equation shows, the beta pricing model predicts a very intuitive behaviour for risky assets: that, in equilibrium -remember that the starting point of our theoretical development were the first order conditions for the investor's optimization problem-, expected excess returns on a given risky asset are explained by the factors' risk premia -the excess returns on the factor portfolios- and the asset's exposure to each one of the risk factors -the partial correlations between the asset and the risk factors-.

Now that I have presented a theoretical framework for linear factor models, I can move forward in the task of studying the returns on FIBRAs. Equation (5) seems to suggest that all that is needed to study asset returns is the set of relevant risk factors. But the theoretical model provides no information whatsoever on the existence or the characteristics of these variables. Fortunately, there are certain statistical techniques that can be used to estimate proxies for the factors present in asset returns. The following section addresses these matters and specifies the empirical procedure that is to be followed in order to apply the pricing model to the returns on FIBRAs.

[6] The term factor mimicking portfolio refers to a portfolio, i.e. a linear combinations of assets, that is perfectly correlated with a particular risk factor.

3 Empirical methodology

The objective of this paper is to analyse the real estate market from the point of view of its assets' returns. The theoretical asset pricing model presented in the previous section provides one way to approach the empirical study of financial markets and, in particular, of the Mexican REITs (FIBRAs). Even a cursory look at equation (5) gives the idea that, in order to empirically test the model, one needs statistical information about the risk factors and, of course, the asset returns. It should be clear by now that the dataset used in this study must include observations of the returns on FIBRAs, and of the risk factors. Returns on FIBRAs can be easily obtained from the time series of their market prices, but the process of identifying the proper risk factors is somewhat trickier.

In the first part of this section, I describe two ways in which proxies for the risk factors can be derived empirically from a large sample of asset returns: Maximum Likelihood Factor Analysis, and Principal Component Analysis. Afterwards, I go through the details of the empirical methodology that I use in order to determine if the returns on FIBRAs are in fact exposed to these risk factors, and if the asset pricing framework described earlier is an adequate tool for the analysis of the FIBRAs market.

3.1 Estimation of the risk factors

The asset pricing model developed in section 2 states that, under certain conditions, excess returns of risky assets can be explained by the risk premia of K risk factors and the risk exposures of each particular asset to such variables. But, what exactly are those risk factors? The empirical literature on multifactor linear asset pricing models has employed two different approaches for the identification of the factors. One of them consists in finding theoretical arguments that justify the use of macroeconomic variables that allegedly capture economy-wide systematic risks, as stated in Campbell, Lo, and MacKinlay (1997). The use of macroeconomic factors, in spite of their intuitive and often simple interpretation, implies a number of technical difficulties that hinders the usefulness and validity of models based upon such factors.[7]

The alternative approach relies on the statistical estimation of the common risk factors in a large sample of assets. Lizieri, Satchel and Zhang (2007) list some of the advantages of the use of statistical factors, e.g. availability of data at higher-frequencies and the possibility of guaranteeing statistical independence among the estimated variables. Their main downside is that, other than

[7] Some disadvantages are related to statistical or data availability, such as potential statistical dependence among the macrovariables or the fact that most macroeconomic data is available only in low-frequency schemes. Other drawbacks have to do with the lack of a proper theoretical framework for the selection of the macroeconomic variables. Cf. Lizieri, Satchel and Zhang (2007), and Chen, Hsieh, and Jordan (1997).

being linear combinations of asset returns, the interpretation of the factors is an arcane task.

In the particular case of the FIBRAs market, one of the largest hindrances for testing factor models using macroeconomic variables as risk factors is the fact that most of them are only available at monthly or quarterly frequencies, which considerably shortens the length of the time series that can be used for econometric analysis. For this reason, the fact that the FIBRAs market is only a few years old makes statistical factor estimation much more convenient than any macroeconomic-variable specification.[8] Hence, in this work, the variables used for the empirical testing of the pricing model in equation (5) are common risk factors from the Mexican stock market constructed via two statistical techniques: Maximum Likelihood Factor Analysis and Principal Component Analysis.

The crucial assumption is that random returns on the set of N assets follow some kind of factor structure. Thus, their return generating process can be expressed by

$$\begin{aligned} \mathbf{R}_t &= \mathbf{E} + \mathbf{B}\mathbf{f}_t + \boldsymbol{\varepsilon}_t \\ \mathbb{E}[\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t' | \mathbf{f}_t] &\equiv \boldsymbol{\Sigma} \end{aligned} \tag{6}$$

Where $\mathbf{R}_t$ is the $Nx1$ vector of asset returns, $\mathbf{E}$ is a vector whose elements are the N expected returns, $\mathbf{B}$ is the NxK matrix of factor sensitivities, $\mathbf{f}_t$ is the $Kx1$ vector of factor realizations at time t, and $\boldsymbol{\varepsilon}_t$ is the $Nx1$ vector of contemporaneous model disturbances. Σ is defined as the covariance matrix of model disturbances.

Both techniques have been employed in previous research concerning REITs. For example, in a study of REITs in the US, Chen, Hsieh and Jordan (1997) compare the performance of factors obtained through several methods, including Maximum Likelihood Factor Analysis. On the other hand, Lizieri, Satchell and Zhang (2007) estimate the underlying risk factors in the REITs market using both independent and principal component analysis. As to which of these approaches is better suited for finite samples, a definite answer has not yet been produced. (Campbell, Lo and McKinlay, 1997, p. 238-239).

3.1.1 Maximum Likelihood Factor Analysis

Maximum Likelihood Factor Analysis has been used several times in the context of linear factor models. One particular and most famous reference is the work of Roll and Ross (1980). The procedure of factor estimation and testing is thoroughly described in such article, and this part of the

[8]This allows me to work with daily prices of FIBRAs and, consequently, obtain longer time-series of returns.

paper is based upon it and chapter 6 of Campbell, Lo and McKinlay (1997).

Factor Analysis involves two steps: first, the estimation of factor sensitivity matrix **B** and disturbance covariance matrix Σ, and second, the construction of measures of factor realizations. The first step involves maximum likelihood estimation, while the second consists essentially of several Generalized Least Squares regressions (one for each time period in the sample).

As usual, and especially because of the maximum likelihood nature of this method, some assumptions are needed. In particular, returns are required to follow a *strict* factor structure, i.e. it is assumed that K factors account for all the cross covariance of asset returns and, thus, idiosyncratic disturbances are uncorrelated among assets. Thus, $\boldsymbol{\Sigma}$ becomes, by construction, a diagonal matrix, henceforth $\boldsymbol{D}$.

The covariance matrix of returns can therefore be expressed as

$$\boldsymbol{\Omega} = \boldsymbol{B}\boldsymbol{\Omega}_K\boldsymbol{B}' + \boldsymbol{D} \tag{7}$$

where $\boldsymbol{\Omega}_K = \mathbb{E}[(\mathbf{f}_t - \mathbb{E}[\mathbf{f}_t])(\mathbf{f}_t - \mathbb{E}[\mathbf{f}_t])']$ is the factor variance-covariance matrix. Factors are unknown, and thus potential rotational indeterminacies are an issue.[9] In order to eschew, or at least allay such ambiguities, factors can be restricted to be orthogonal to each other. Furthermore, since the procedure is scale-free, imposing unit variance is innocuous. Once this adaptations have been made, the variance-covariance matrix of the factors becomes the *KxK* identity matrix, and the original expression turns into

$$\boldsymbol{\Omega} = \mathbf{BB'} + \boldsymbol{D} \tag{8}$$

making **B** unique up to any orthogonal transformation.[10] Roll and Ross (1980), referring to the space spanned by the factor loadings **B**, state that "Orthogonal transforms leave that space unchanged, altering only the directions of the defining basis vectors, the column vectors of the loadings." Any conclusion of this work is, thus, unaltered by this milder rotational indeterminacy.

Estimates of **B** and $\boldsymbol{D}$ are then obtained through maximum likelihood estimation. The distributional properties imposed are joint normality and temporal independence of asset returns. Roll & Ross (1980) use numerical techniques proposed in Jöreskog (1967) and Jöreskog and Sörbom (1978), which allows a faster convergence to the maximum likelihood estimators. This completes

[9] Take any nonsingular matrix $\boldsymbol{G}$. Then, the covariance matrix of the returns can equivalently be written as $\boldsymbol{\Omega} = (\mathbf{B}\boldsymbol{G}^{-1})(\boldsymbol{G}\boldsymbol{\Omega}_K\boldsymbol{G}')(\mathbf{B}\boldsymbol{G}^{-1})' + \boldsymbol{D}$, and the maximum likelihood process could yield estimators of $\mathbf{B}\boldsymbol{G}^{-1}$ instead of **B**.

[10] Now, ambiguity stems only from matrices $\boldsymbol{O}$ such that $\boldsymbol{O}\boldsymbol{O}' = \mathbb{I}$.

the first step of the factor analysis procedure.

Once the estimators $\hat{\mathbf{B}}$ and $\hat{\boldsymbol{D}}$ are at hand, one can easily substitute them for the true parameters in equation (6), which yields $(\mathbf{R}_t - \mathbf{E}) = \hat{\mathbf{B}}\mathbf{f}_t + \boldsymbol{\varepsilon}_t$ and $\mathbb{E}[\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'|\mathbf{f}_t] = \hat{\boldsymbol{D}}$. From a regression point of view, $\hat{\boldsymbol{D}}$ is an estimate of the error covariance matrix, and it permits the estimation of the unknown vector $\mathbf{f}_t$ through Feasible Generalized Least Squares (FGLS). Factor realizations for each time period can thus be obtained through an FGLS regression of the (demeaned) asset returns on the factor loadings:

$$\hat{\mathbf{f}}_t = (\hat{\mathbf{B}}'\hat{\boldsymbol{D}}^{-1}\hat{\mathbf{B}})^{-1}\hat{\mathbf{B}}'\hat{\boldsymbol{D}}^{-1}(\mathbf{R}_t - \mathbf{E}) \tag{9}$$

The asset pricing model in equation (5) considers factors in the form of returns. Since the risk factors obtained through (9) are linear combinations of asset returns, they can, with some modifications, be interpreted as portfolio returns. This can be achieved by forcing the coefficients of this linear combination, i.e. the weights of each asset on the risk factors, to sum 1. Using matrix notation, a closed-form expression for the vector of K factor portfolio returns, which I denote $\mathbf{f}_t^P$ is

$$\mathbf{f}_t^P = \boldsymbol{W}(\hat{\mathbf{B}}'\hat{\boldsymbol{D}}^{-1}\hat{\mathbf{B}})^{-1}\hat{\mathbf{B}}'\hat{\boldsymbol{D}}^{-1}\mathbf{R}_t \tag{10}$$

where $\boldsymbol{W}$ is a K-dimensional diagonal matrix whose k^{th} diagonal element is the sum of the weights of each asset on factor k. That is, $\boldsymbol{W}_{kk} \equiv ([(\hat{\mathbf{B}}'\hat{\boldsymbol{D}}^{-1}\hat{\mathbf{B}})^{-1}\hat{\mathbf{B}}'\hat{\boldsymbol{D}}^{-1}]\mathbf{1})_k$. The factor portfolio returns obtained in this manner can be used directly in the asset pricing model described in section 2.

There remains the question of the number of relevant factors. Notice that in describing the Maximum Likelihood Factor Analysis procedure, I made the (implicit) assumption that the exact number (K) of factors was known. Fortunately, maximum likelihood provides a very straightforward statistical test for the true value of K. By restricting equation (6) to consider exactly one factor, a likelihood ratio test statistic can be constructed in which the unrestricted model considers N factors. Under the null hypothesis that the single factor specification is correct, the test statistic has a distribution χ^2 with $\frac{1}{2}[(N-1)^2 - (N+1)]$ degrees of freedom. If H_0 is rejected, another test statistic should be constructed, now restricting the model to include 2 factors. The testing procedure should go on until the null hypothesis that the k-factor specification is correct cannot be rejected.

3.1.2 Principal Component Analysis

Principal Component Analysis (PCA) has been widely used in the financial literature.[11] The main reason for its widespread application is the result obtained by Chamberlain and Rothschild (1983), which relaxes the requirements of the factor model by eliminating the need for a *strict* factor structure. All that is needed for the estimation of the latent factors is that asset returns follow an *approximate* factor structure as defined in Chamberlain and Rothschild (1983). The main empirical implication of this weaker assumption is that the idiosyncratic disturbance terms $\boldsymbol{\varepsilon}_t$ need not be iid, i.e. the K common risk factors need not explain all of the joint variation in asset returns.

Principal Component Analysis is a statistical method whose applications transcend financial economics, and therefore a thorough development of it can be found in several editorial sources, like Dunteman (1989) or Jackson (1991). Here, however, my focus lies on its potential implementation in financial econometrics and, thus, the succinct description that follows is based almost entirely in Campbell, Lo and McKinlay (1997).

The goal of PCA is to reduce the number of variables being considered by gathering as much information (variance) as possible in some small number (K) of orthogonal components. As stated above, Chamberlain and Rothschild (1983) showed that, under reasonable preconditions, these principal components can be used as the risk factors in a linear asset pricing model. The first principal component is the linear combination of the asset returns that has maximum variance, i.e. $\mathbf{R}_t'\mathbf{x}_1^*$, where

$$\mathbf{x}_1^* = \arg\max \ \mathbf{x}_1'\hat{\boldsymbol{\Omega}}\mathbf{x}_1 \qquad s.t. \quad \mathbf{x}_1'\mathbf{x}_1 = 1$$

This $\mathbf{x}_1^*$ is non other than the eigenvector associated with the largest eigenvalue of $\hat{\boldsymbol{\Omega}}$, which is exactly why this approach is compatible with the specification of Chamberlain and Rothschild (1983). The second factor is obtained in an analogous manner, with the additional restriction of orthogonality with the first principal component. The procedure is repeated until K principal components have been computed.

As with Maximum Likelihood Factor Analysis, the factors obtained via PCA are subject to a portfolio interpretation. This is done, once more, by forcing the asset weights of each principal components to sum 1. Multiplying each eigenvector $\mathbf{x}_i^*$ by $\frac{1}{\mathbf{1}'\mathbf{x}_i^*}$ provides the desired factor portfolio returns. As stated in Lizieri, Satchel and Zhang (2007), there are several available criteria for the selection of the proper number of factors K. One of the most common is to determine an arbitrary

[11] Besides its use in the REITs literature referred above, slight modifications of PCA have been applied to macroeconomic financial forecasting and linear factor models. See, for example, Stock and Watson (2002), and Connor and Korajczyk (1986, 1988, 1993).

proportion of variance to be explained by the principal components, say 70% or 80%. That is, principal components are computed one by one until the cumulative proportion of total sample variance explained by the first K components reaches the arbitrarily predetermined proportion.

3.2 Empirical asset pricing tests

The factors obtained through Maximum Likelihood Factor Analysis and Principal Component Analysis can be used as arguments in an empirical test of the linear asset pricing model described by equation (5). Such model states that excess asset returns are equal to the product of the factor risk premia and the asset betas, which are measures of risk exposure. Factor risk premia, as shown in section 2, are excess returns on the factor portfolios. In this section, I describe the empirical strategy used for testing whether the asset pricing model is an adequate specification for the FIBRAs market.

The empirical strategy I follow was first proposed in MacKinlay and Richardson (1991) as a way of correcting previous asset pricing tests by allowing model disturbances to be heteroskedastic and autocorrelated. The test in MacKinlay and Richarson (1991) exploits the time-series specification for testing linear factor models first developed by Black, Jensen, and Scholes (1972). Both of the referred articles test a single factor asset pricing model. Nonetheless, as stated in Jagganathan, Skoulakis, and Wang (2002), a generalization to the case of multiple factors is quite straightforward. This description of the empirical test follows closely MacKinlay and Richarson (1991), and makes the pertinent changes in order to test the model with K risk factors.

For the FIBRAs market, the conditions implied by equation (5) can be seen as a set of restrictions on regression equations:

$$
\begin{aligned}
R_t^{ej} &= \alpha_j + \boldsymbol{\beta}'_j \mathbf{f}_t^e + \varepsilon_t^j \\
\mathbb{E}[\varepsilon_t^j] &= 0 \\
\mathbb{E}[\varepsilon_t^j \mathbf{f}_t^e] &= \mathbf{0} \\
\alpha_j &= 0
\end{aligned}
\tag{11}
$$

Notice that both $\mathbb{E}[\varepsilon_t^j] = 0$ and $\mathbb{E}[\varepsilon_t^j \mathbf{f}_t^e] = \mathbf{0}$ are the usual regression conditions from least squares projection theory. However, the last line in system of equations (11), $\alpha_j = 0$, imposes testable restrictions on the data. On the other hand, the elements of $\boldsymbol{\beta}_j$ provide information on whether the FIBRAs are significantly exposed to stock market risk factors.

The straightforward strategy for testing $\alpha_j = 0$ is running time-series regressions for each FIBRA in the sample in order to check if the intercept estimates α_j are indeed statistically different from 0. Traditional ordinary least squares (OLS) distribution theory may lead to incorrect statistical conclusions because of potential problems that may arise due to the unknown distribution of the error terms. These issues can be dealt with by adopting the Generalized Method of Moments (GMM) approach described in Jagganathan, Skoulakis, and Wang (2002) and in McKinlay and Richardson (1991), along with a heteroskedasticity and autocorrelation consistent (HAC) covariance matrix for the parameters as proposed by Newey and West (1987). The use of this techniques allow me to adequately perform statistical inference without imposing unrealistic distributional properties upon the data. [12]

The GMM test involves estimating all time-series regressions at once. Using matrix notation, as in Cochrane (2005), the parameters of interest are the elements of $\boldsymbol{\alpha}$ and $\mathbf{B}$ in the system of equations

$$\mathbf{R}_t^e = \boldsymbol{\alpha} + \mathbf{B}\mathbf{f}_t^e + \boldsymbol{\varepsilon}_t \tag{12}$$

where $\mathbf{R}_t^e$, $\boldsymbol{\alpha}$ and $\boldsymbol{\varepsilon}_t$ are a $9x1$ vectors of, respectively, FIBRAs excess returns, regression constants and error terms, and $\mathbf{B}$ is a $9xK$ matrix whose j^{th} row is equal to $\boldsymbol{\beta}_j'$. The moment conditions $\mathbb{E}[\mathbf{h}_t(\alpha, \mathbf{B})] = \mathbf{0}$ used for GMM estimation in this linear case are

$$\begin{aligned} \mathbb{E}[\boldsymbol{\varepsilon}_t] &= \mathbf{0} \\ \mathbb{E}[\boldsymbol{\varepsilon}_t \otimes \mathbf{f}_t^e] &= \mathbf{0} \end{aligned} \tag{13}$$

where $\otimes$ is the Kronecker product. The system consists of 9x$(1+K)$ equations and 9x$(1+K)$ unknowns, and exactly identifies the parameters of interest. Furthermore, the sample analogues $\mathbf{g}_T(\boldsymbol{\alpha}, \mathbf{B})$ of the moment conditions in (11) coincide with OLS estimation, and the estimators of $\boldsymbol{\alpha}$ and $\mathbf{B}$ are equivalent to those produced by OLS. Hansen (1982) showed that the GMM estimator of $\boldsymbol{\delta} \equiv (\boldsymbol{\alpha}', \boldsymbol{\beta}_1', \cdots, \boldsymbol{\beta}_9')'$ is asymptotically normally distributed with mean $\boldsymbol{\delta}$ and variance $(\mathbf{D}_0'\mathbf{S}_0^{-1}\mathbf{D}_0)^{-1}$, where

$$\mathbf{D}_0 = \mathbb{E}\left[\frac{\partial \mathbf{h}_t}{\partial \boldsymbol{\delta}'}(\boldsymbol{\delta})\right] \qquad \text{and} \qquad \mathbf{S}_0 = \sum_{r=-\infty}^{\infty} \mathbb{E}[\mathbf{h}_t(\boldsymbol{\delta})(\mathbf{h}_{t-r}(\boldsymbol{\delta}))']$$

However, both population matrices $\mathbf{D}_0$ and $\mathbf{S}_0$ are unknown and must be estimated. Chaussé (2010) suggests that the HAC matrix proposed by Newey and West (1987) can be used as an estimator of

[12] GMM requires only that FIBRAs asset returns be stationary and ergodic with finite fourth moments. (MacKinlay and Richardson, 1991)

the asymptotic variance-covariance matrix $(\mathbf{D}_0'\mathbf{S}_0^{-1}\mathbf{D}_0)^{-1}$. The HAC matrix is, in its general form,

$$\hat{\boldsymbol{\Omega}}_\delta = \sum_{r=-(T-1)}^{T-1} k_h(r)\hat{\boldsymbol{\Gamma}}_r(\hat{\boldsymbol{\delta}}) \tag{14}$$

where $k_h(\cdot)$ is a kernel density function with bandwith h, $\hat{\boldsymbol{\Gamma}}_r(\hat{\boldsymbol{\delta}}) = \frac{1}{T}\sum_{t=1}^{T}\mathbf{h}_t(\hat{\boldsymbol{\delta}})\mathbf{h}_{t+r}(\hat{\boldsymbol{\delta}})'$ and $\hat{\boldsymbol{\delta}}$ is the GMM estimator of $\boldsymbol{\delta}$. In this work, I use a Quadratic Spectral kernel density and select the bandwidth according to Andrews (1991).

The asymptotic joint normality of the GMM estimators for δ allows us to construct standard t-statistics -with HAC standard errors- that permit to assess the statistical significance of each of the individual elements of α and $\mathbf{B}$. Rejection of the null hypothesis that the elements of $\mathbf{B}$ are individually 0 means that the correspondent FIBRA is significantly exposed to the respective risk factor. On the other hand, if some α_j were statistically different from 0, the validity of the asset pricing model would be rejected for the corresponding asset. The purpose of this paper is, nonetheless, to check whether the pricing model is adequate for all the FIBRAs in the market. The relevant null hypothesis is that the intercept coefficients in $\boldsymbol{\alpha}$ are jointly equal to 0. Constructing a test statistic for this H_0 is quite straightforward: asymptotic normality implies that, under the null hypothesis, $\hat{\boldsymbol{\alpha}}'\text{var}(\hat{\boldsymbol{\alpha}})^{-1}\hat{\boldsymbol{\alpha}} \sim \chi_9^2$. The estimate for $\text{var}(\hat{\boldsymbol{\alpha}})$ is the 9-dimensional square sub-matrix at the top-left corner of $\hat{\boldsymbol{\Omega}}_\delta$.

The details of every empirical technique employed in order to carry out the analysis of FIBRAs were presented in this section. Summarizing, the first step towards testing the pertinence of the linear asset pricing model for the study of FIBRAs is to extract common risk factors from a large sample of assets. This can be achieved through both Maximum Likelihood Factor Estimation and Principal Component Analysis. Afterwards, factor portfolio returns are computed and applied as arguments in the asset pricing model of FIBRAs. Because of the unknown distributional properties of asset returns, a GMM based empirical test is performed. Statistical significance of the slope coefficients in the time-series regressions suggests that FIBRAs are exposed to common risk factors in asset markets, while the intercept coefficients test the validity of the linear factor pricing framework for this particular market.

4 Data and estimation results

The asset pricing model and the way in which it can be empirically tested have been thoroughly described in the previous parts of this work. This section goes through the details of the actual implementation of the techniques presented above. I start by describing the dataset and show some basic statistics of the returns on FIBRAs. Subsequently, I narrate the factor estimation procedure and analyse the main characteristics of the factor portfolio returns. In the last part of this section, results of the Generalized Method of Moments estimation are shown, along with the main test statistic, that which offers information about the validity of the asset pricing specification.

4.1 Data

The dataset used in this work includes time-series of the daily closing prices of 9 out of the 10 FIBRAs currently listed in the Mexican stock market. I decided not to include FIBRA HD, the youngest of all the FIBRAs, because it has been in the market only for a relatively short time.[13] The sample period covers all trading days between January 1st, 2015 and March 14th, 2016. For the estimation of the risk factors, I used data on the daily closing prices of 106 stocks listed in the Bolsa Mexicana de Valores, those with available data for all the days in the sample period. Logarithmic returns were computed for each of the series, resulting in 312 daily returns for each of the variables in the dataset.[14] Prices of stocks and FIBRAs were obtained from Yahoo!, and abnormal data points were cross-checked with data from Bloomberg and the BMV.

To compute excess returns, both for the FIBRAs and the factor portfolios, the 28-day CETEs return is considered as the risk-free asset. Daily data on this (annualized) rate of return was downloaded directly from the statistical site of Banco de México. For the purpose of this work, annualized rates had to be properly transformed to daily returns. In order to do so, I adopted the convention of dividing the annualized rate by 360 days. It is worthy of mention that public auctions of these risk-free securities take place only once a week, and that the return of the CETEs in days in which no auction occurred corresponds to a shorter maturity -sometimes as short as 22 days-. In spite of this disclaimer, this rate of return can still be considered as risk-free, since 28-day CETEs are available for the investors everyday, and their payoff is known at the current day regardless of whether the transaction takes place 22 or 28 days prior to maturity.

Table 1 shows the mean and standard deviation of the log-returns of each FIBRA in the dataset. The mean returns of all the FIBRAs in the sample are very close to zero. The only FIBRA that earned a positive mean return during the sample period is FIBRA Monterrey. On the other hand,

[13]FIBRA HD had its initial public offering in June, 2015. Including it in the empirical asset pricing model would have considerably shortened the length of the time-series and hampered the capacity for statistical inference.

[14]Logarithmic returns imply, among other computational advantages, continuous compounding. Besides, they provide a sensible approximation to arithmetic returns when the holding period -and thus the return itself- is small (as is the case for daily returns). This stems from the fact that $ln(x) \approx x - 1$ when x is close to 1.

Table 1: Returns on FIBRAs

FIBRA	*Mean*	*S.D.*
Danhos	-0.01	1.07
Hotel	-0.14	1.66
Inn	-0.05	1.01
Macquarie	-0.04	1.24
Monterrey	0.03	0.82
Prologis	-0.01	1.16
Shop	-0.03	1.12
Terrafina	-0.01	1.05
Uno	-0.04	1.29

Note:Values are expressed in percentage points.

FIBRA Hotel presented the worst performance in terms of returns: a net loss of 0.14%. The standard deviation of returns can be thought of as a proxy of the riskiness of each asset, in the sense that it states how much a return usually diverges from its mean. Under such risk measure, FIBRA Monterrey is once more the FIBRA with the most favourable behaviour. Every other FIBRA departs, on average, more than 1% from its mean and, notably, Hotel has a sample standard deviation of 1.66%, which makes it the riskiest of all the FIBRAs considered in our sample. The sample correlations between each pair of FIBRAs are shown in Table 2. Remarkably, Monterrey's correlation with any of the FIBRAs is comparatively low: its highest sample correlation is 0.13. On the other hand, Shop's correlation with every FIBRA, excluding Monterrey,lies between 0.14 and 0.44. Monterrey's apparent atypical behaviour could be explained by the fact that it is the youngest of all the FIBRAs included in the dataset.

Table 2: FIBRAs correlation matrix

FIBRA	Danhos	Hotel	Inn	Macquarie	Monterrey	Prologis	Shop	Terrafina	Uno
Danhos	1.00								
Hotel	0.15	1.00							
Inn	0.05	0.07	1.00						
Macquarie	0.12	0.24	0.08	1.00					
Monterrey	0.04	0.05	0.13	-0.07	1.00				
Prologis	0.08	0.26	0.06	0.33	0.12	1.00			
Shop	0.19	0.44	0.14	0.21	0.06	0.30	1.00		
Terrafina	0.14	0.12	0.13	0.33	0.01	0.28	0.16	1.00	
Uno	0.10	0.19	0.18	0.11	0.06	0.16	0.14	0.25	1.00

4.2 Results: Factor estimation

The risk factors used for the pricing of FIBRAs were extracted from the stock market. Many of the stock prices in the dataset showed little or no variation at all during the sample period, and thus were not useful for the purpose of identifying risk factors. I restricted the sample to those firms whose stock price changed in at least one tenth of the days in the sample period. Each of the 87 stock series that survived this criterion is listed on table A3 in the appendix. Stationarity tests were conducted prior to the implementation of the factor analytical procedures. The null hypothesis of the existence of a unit root was rejected at all confidence levels for every series of stock returns in the sample, according to both Phillips-Perron and Augmented Dickey-Fuller tests.[15]

Table 3: Tests for the number of factors

K	$\chi 2$	P-value
1	4522.22	0.00
2	4285.40	0.00
3	4084.13	0.00
4	3887.16	0.00
5	3730.12	0.00
6	3593.05	0.00
7	3455.54	0.00
8	3321.88	0.00
9	3197.43	0.00
10	3080.56	0.02
11	2963.09	0.05
12	2852.49	0.12

Likelihood ratio tests as described in section 3.1.1 were performed in order to determine the adequate number of risk factors to be estimated through Maximum Likelihood Factor Analysis. The test statistics and P-values of these tests are displayed in Table 3. The null hypothesis that exactly K factors are present in the stock market was rejected of all K lower than 12, although eleven factors were enough at the 5 percent level. Hence, the MLFA procedure was performed in order to extract 12 common risk factors from the sample of stocks in the dataset and factor portfolios were constructed. As stated above, economic interpretation of factors obtained through statistical methods is not an easy task. For the task at hand, it suffices to say that our risk factors are the 12 linear combinations of the 87 series listed on table A3 that best describe the behaviour of the entire sample of stocks. Table 4 shows basic statistics of the factor portfolio returns. Mean

[15] As an additional check, the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test was performed on every stock return series. The null hypothesis of stationarity was rejected for only one of the 87 stock return series.

portfolio returns lie in the range of -0.56% - 0.21%, while standard deviations of factor portfolio returns are between 1.22 and 7.15 percent. When compared to returns on FIBRAs, factor portfolios seem considerably riskier. This comes as no surprise since the objective of MLFA is to identify and estimate risk factors, i.e. to construct variables that explain the variation of all assets in the sample.

Table 4: Risk factors obtained via MLFA

Risk factor	*Mean*	*S.D.*
1	0.14	1.22
2	-0.19	2.19
3	0.21	5.78
4	-0.37	5.21
5	0.14	2.62
6	-0.07	5.49
7	0.20	7.15
8	0.00	1.85
9	-0.10	2.42
10	-0.02	1.60
11	-0.13	2.78
12	-0.56	5.05

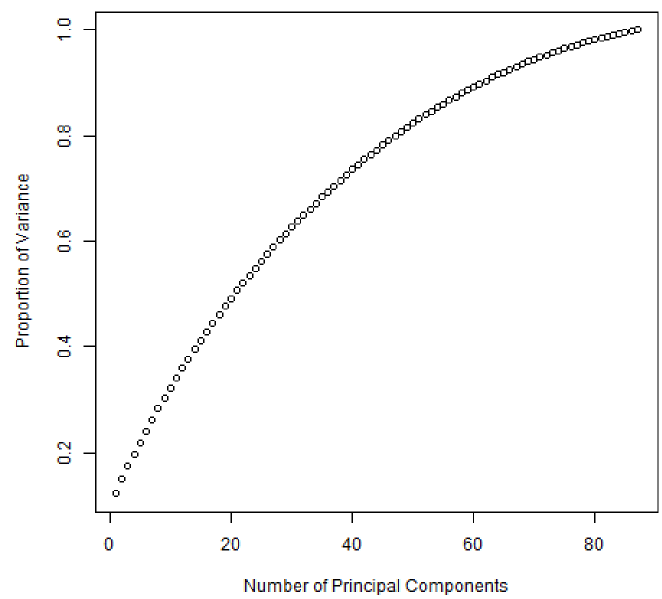
Note:Values are expressed in percentage points.

Principal Component Analysis was performed on the same 87 stocks. Figure 1 shows the cumulative proportion of total sample variance as a function of the number of (ordered) principal components. The graphic shows that the amount of explained variance increases at a relatively slow rate. The first 10 principal components together explain only about 32% of the total variance, and up to 33 principal components are needed to accumulate two thirds of the sample variance. It is reasonable to conclude that the nature of the stock data does not allow a considerable reduction of the number of relevant variables. The use of so many risk factors in the pricing model for FIBRAs would represent a considerable loss of parsimony in the results, and thus I decided base all subsequent analysis exclusively on the factors estimated through MLFA.

4.3 Multifactor pricing of FIBRAs

The main objective of this work is to apply the linear asset pricing model in equation (5), $\mathbb{E}[R_t^{ej}] = \boldsymbol{\beta}'_j \mathbb{E}[\mathbf{f}_t^e]$, to the study of risk and return in the FIBRAs market. The main research question is whether FIBRAs are exposed to risk factors pervasive in the financial market. In this study, I iden-

Figure 1: Cumulative proportion of variance explained by PC



tified such common risk factors by using MLFA to construct portfolios of stocks that concentrate the variation of stock market returns. A GMM-based approach was adopted in order to be able to estimate standard errors consistent with potential issues of heteroskedasticity and/or autocorrelation, as explained in section 3.2. Nine time-series regressions of FIBRAs excess returns on factor portfolios excess returns were carried on simultaneously. The regression equations are

$$R_t^{ej} = \alpha_j + \boldsymbol{\beta}'_j \mathbf{f}_t^e + \varepsilon_t^j$$

The GMM estimates of $\boldsymbol{\beta}_j$ are measures of the exposure of FIBRAs returns to the risk factors, and the parameters α_j are indicative of whether the linear pricing specification constitutes an adequate approach to the FIBRAs market. Table 5 displays the output of the estimation, and Table 6 provides a succinct summary of the results and shows the computed test statistic for the null hypothesis that the model prices FIBRAs correctly, i.e. that the alphas are jointly zero.

The results show that all FIBRAs are exposed to at least one stock market risk factors, with a level of confidence of 90%. Factors 1, 2, and 3, are significantly present in many of the FIBRAs. FIBRA Hotel and FIBRA shop are the more responsive to stock market risk factors, followed closely by Prologis. On the other hand, the estimation shows that Monterrey is only exposed to one of the factors. Oddly, the risk factor to which Monterrey is exposed does not have any significant presence on any of the other FIBRAs. Regarding to the validity of the linear asset pricing specification, a remarkable result is that none of the elements of $\boldsymbol{\alpha}$ are statistically different from 0 individually. Furthermore, the computed test statistic for the null hypothesis that the intercepts are jointly zero is quite distant from the rejection regions. The statistic and the critical values for the χ^2 distribution with 9 degrees of freedom are displayed in the bottom panel of Table 6.

As an additional check, I used the same Generalized Method of Moments procedure to estimate a model with the returns of the main Mexican stock market index, the Índice de Precios y Cotizaciones (IPC), as the single risk factor. This alternative specification is equivalent of an application of the traditional CAPM to the FIBRAs market. Results of this estimation show that 7 out of the 9 FIBRAs are significantly exposed to IPC-risk. This single-factor model indicates that FIBRA Macquarie is the most risky security in the sample, i.e. the one that is most responsive to the IPC-risk factor. Once again, FIBRA Danhos and FIBRA Monterrey show the smallest asset betas, both insignificant at the 10% level. Although this single factor model is computationally simpler, the fact that 2 FIBRAs are not significantly exposed to this single factor suggests that a multifactor specification like the one performed in this study constitutes a robuster approach to the risk and return relationship in the market for FIBRAs. Furthermore, the intercept in the regression for FIBRA Hotel is statistically different from zero at the 10% level, which casts further doubts on the appropriateness of the single-factor model. Table A4 in the appendix displays detailed results of this estimation.

In short, a linear multifactor pricing model like the one evaluated here constitutes an adequate approach to the assessment of risk and return in the FIBRAs market. All of the FIBRAs in the sample are responsive, although at different degrees, to stock market risk factors. The riskiest securities in the FIBRAs market, i.e. the ones most responsive to risk factors, are Hotel and Shop. On the other hand, FIBRA Monterrey behaves quite differently from the other FIBRAs: besides the fact that that Monterrey had a higher return and a lower standard deviation than other REITs, it is the only FIBRA that responds to the eleventh stock market risk factor. As stated above, this abnormal behaviour could stem from the relative youth of Monterrey in the market. However, the explanation of such peculiar characteristics transcends the reach of this work.

Table 5: GMM estimation results

	Danhos	Hotel	Inn	Macquarie	Monterrey	Prologis	Shop	Terrafina	Uno
α	0	-0.0014	-0.0003	-0.0005	0.0001	-0.0001	-0.0006	-0.0002	-0.0003
	(0.0007)	(0.001)	(0.0007)	(0.0007)	(0.0005)	(0.0007)	(0.0006)	(0.0006)	(0.0009)
β_1	0.1316*	0.4758**	0.0851	0.3293**	0.0287	0.2314**	0.3815**	0.1727**	0.1633**
	(0.0729)	(0.12)	(0.0784)	(0.0741)	(0.0661)	(0.0799)	(0.0912)	(0.0661)	(0.0691)
β_2	0.0556	0.1621**	0.1035**	0.1907**	0.0124	0.145**	0.1301**	0.1224**	0.1876**
	(0.0476)	(0.0658)	(0.0365)	(0.0469)	(0.0305)	(0.0447)	(0.05)	(0.0404)	(0.0487)
β_3	0.002	0.0489**	-0.0009	0.0442**	0.0099	0.0443**	0.0323**	0.052**	0.087**
	(0.0128)	(0.0239)	(0.0126)	(0.0159)	(0.0129)	(0.0132)	(0.0137)	(0.0116)	(0.0227)
β_4	0.0283	0.0743**	0.0232*	0.0228	0.0088	0.0137	0.0524**	0.0159	0.01
	(0.0173)	(0.0271)	(0.0137)	(0.0178)	(0.011)	(0.0138)	(0.0215)	(0.0137)	(0.0164)
β_5	0.0203	-0.0309	0.0314	0.0047	0.0102	0.0295	0.0371	0.0452	0.0236
	(0.0297)	(0.0535)	(0.0286)	(0.0346)	(0.0188)	(0.0323)	(0.029)	(0.0292)	(0.039)
β_6	0.0106	0.0535**	-0.0012	0.0147	0.0052	0.041**	0.0253*	0.0253*	0.0019
	(0.0171)	(0.0215)	(0.0139)	(0.0183)	(0.0099)	(0.0168)	(0.0138)	(0.0142)	(0.0162)
β_7	-0.0102	0.0012	0.0129	0.0064	0.0009	-0.0013	-0.001	0.0002	-0.0088
	(0.0109)	(0.018)	(0.0118)	(0.0137)	(0.0095)	(0.012)	(0.0133)	(0.0112)	(0.0142)
β_8	0.0729*	0.1217**	0.0656*	-0.0113	0.0213	0.0205	0.1054**	0.0158	0.0016
	(0.0439)	(0.0566)	(0.0357)	(0.0503)	(0.0265)	(0.0377)	(0.0421)	(0.0392)	(0.0506)
β_9	0.052	0.0361	0.0215	-0.0063	-0.0166	0.0059	-0.0014	0.0178	0.005
	(0.0368)	(0.0488)	(0.0312)	(0.0322)	(0.0207)	(0.0318)	(0.0384)	(0.0317)	(0.0346)
β_{10}	-0.0384	-0.0555	-0.0332	0.0176	0.0459	-0.0235	-0.0113	0.0441	-0.0183
	(0.0597)	(0.0821)	(0.0485)	(0.0585)	(0.0413)	(0.0535)	(0.0562)	(0.0564)	(0.0513)
β_{11}	-0.0379	0.0063	0.0011	0.0127	0.0399*	0.0339	-0.0031	0	0.0081
	(0.027)	(0.0357)	(0.0303)	(0.0245)	(0.0214)	(0.0274)	(0.024)	(0.0355)	(0.0326)
β_{12}	0.0219	0.0161	0.0131	0.0016	-0.0207	0.0168	-0.0149	0.0009	0.029
	(0.017)	(0.0244)	(0.0143)	(0.0184)	(0.014)	(0.0172)	(0.0185)	(0.017)	(0.0212)

Note: Heteroskedasticity and autocorrelation consistent standard errors are shown in parentheses.
*Statistically significant at the 10% level. **Statistically significant at the 5% level.

Table 6: Summary of estimation

Danhos	Hotel	Inn	Macquarie	Monterrey	Prologis	Shop	Terrafina	Uno
β_1	β_1^*	β_2^*	β_1^*	β_{11}	β_1^*	β_1^*	β_1^*	β_1^*
β_8	β_2^*	β_4	β_2^*		β_2^*	β_2^*	β_2^*	β_2^*
	β_3^*	β_8	β_3^*		β_3^*	β_3^*	β_3^*	β_3^*
	β_4^*				β_6^*	β_4^*	β_6	
	β_6^*					β_6		
	β_8^*					β_8^*		

Test Statistic

χ_9^2	90%	95%	99%
3.1476	14.68	16.91	21.66

Note: The top panel of this table shows the statistically significant parameters for each FIBRA at the 10% level. Parameters marked with * are also significant at the 5% level. The bottom panel shows the test statistic for the null hypothesis that the α coefficients are jointly equal to 0, along with the critical values of the distribution.

5 Final remarks

The recent appearance of FIBRAs has opened a window of opportunity for research in real estate-related topics. This work exploits the availability of daily, transaction-based information about the returns on real estate assets in order to investigate risk factors in the FIBRAs market. This investigation contributes to the literature on REITs by applying the GMM-based approach for empirical asset pricing proposed in MacKinlay and Richardson (1991). Furthermore, this work is a pioneer in the econometric study of FIBRAs and hopefully sets a precedent for future research on the subject.

Based upon a stochastic discount factor specification, I describe conditions under which expected excess returns are linear combinations of the factor risk premia. I then estimate risk factors from a large sample of Mexican stocks via Maximum Likelihood Factor Analysis. Risk factors were also estimated through Principal Component Analysis, but too many variables were needed to explain a reasonable proportion of the variance in the stock market. Consequently, only MLFA factor portfolios were used in the subsequent estimation of the linear pricing model for FIBRAs. I used a GMM-based strategy that generalizes the method postulated in MacKinlay and Richardson

(1991) to the case of multiple risk factors.

Results show that all FIBRAs are exposed to at least one stock market risk factor. The most responsive securities in our sample were FIBRAs Hotel and Shop. On the contrary, FIBRA Monterrey displayed a very weak correlation structure with the other assets in the market, and is significantly exposed to only one of the 12 estimated risk factors. One more peculiarity of FIBRA Monterrey is the fact that it is the youngest FIBRA in the sample. This might cause the atypical behaviour, but this work provides no useful information for determining the validity of such assertions. A test for the validity of the linear multifactor specification indicates that the null hypothesis that the model prices FIBRAs correctly on average cannot be rejected at any of the traditional significance levels.

The conclusions of this investigation should be assessed carefully. The most evident caveats have to do with the data, e.g. a very limited number of FIBRAs that does not permit the performance of cross-sectional asset pricing tests, and the relatively short time-series I was forced to consider because of the recent emergence of FIBRAs. In the future, once the market can provide enough data, it may be advisable to study multifactor models for FIBRAs using observable macroeconomic variables as the risk factors, or even statistical factors derived from other asset markets, like corporate debt or derivatives. Other issues are related to the factor estimation procedures, like the fact that risk factors were constructed via a maximum likelihood approach that imposes strong distributional assumptions on the data. Hence, this work should be taken as nothing more than what it is: a first attempt at the study of the implicit risk factors in this branch of the real estate market and an opportunity for future research regarding the specific characteristics of FIBRAs and their broader role in the real estate and financial markets.

Appendix

Table A1: Publicly traded FIBRAs

FIBRA	*Portfolio*	*IPO*
Uno	Diversified	March, 2011
Macquarie	Diversified	December, 2012
Hotel	Lodging	November, 2012
Inn	Lodging	March, 2013
Terrafina	Industrial	March, 2013
Shop	Commercial	July, 2013
Danhos	Diversified	October, 2013
ProLogis	Industrial	June, 2014
Monterrey	Office buildings	October, 2014
HD	Diversified	June, 2015

Source: Websites of BMV and the individual FIBRAs.

Table A2: Stationarity tests

FIBRA	*PP*	*ADF*	*KPSS*
Danhos	0.01	0.01	0.1
Hotel	0.01	0.01	0.1
Inn	0.01	0.01	0.1
Macquarie	0.01	0.01	0.1
Monterrey	0.01	0.01	0.1
Prologis	0.01	0.01	0.1
Shop	0.01	0.01	0.1
Terrafina	0.01	0.01	0.1
Uno	0.01	0.01	0.1

Note: Table shows P-values for three tests: Phillips-Perron (PP), Augmented Dickey-Fuller (ADF), and Kwiatkowski–Phillips–Schmidt–Shin (KPSS).

Table A3: Stock series used for factor estimation

ACTINVR	CEMEX	GCC	HILASAL	MEDICA	SIMEC
AEROMEX	CHDRAUI	GENTERA	HOTEL	MEGA	SORIANA
AGUA	CIDMEGA	GFAMSA	ICA	MEXCHEM	SPORT
ALFA	CMOCTEZ	GFINBUR	ICH	MFRISCO	TEAK
ALPEK	CMR	GFINTER	IDEAL	MONEX	TLEVISA
ALSEA	COMERCI	GFNORTE	IENOVA	OHLMEX	TMM
AMX	CREAL	GFREGIO	KIMBER	OMA	VALUEGF
ARA	CULTIBA	GISSA	KOF	PAPPEL	VASCONI
ASUR	CYDSASA	GMD	KUO	PINFRA	VESTA
AUTLAN	ELEKTRA	GMEXICO	LAB	POCHTEC	VITRO
AXTEL	FINDEP	GNP	LALA	RASSINI	VOLAR
AZTECA	FRAGUA	GRUMA	LAMOSA	RCENTRO	WALMEX
BACHOCO	GAP	GSANBOR	LIVEPOL	SANMEX	
BIMBO	GBM	HCITY	MASECA	SARE	

Note: Table shows all the 87 stock series used for factor estimation. Selection criteria are: 1) data availability over the whole sample period, and 2) price changes in at least one tenth of the days in the sample period.

Table A4: FIBRAs CAPM

	Danhos	Hotel	Inn	Macquarie	Monterrey	Prologis	Shop	Terrafina	Uno
α	-0.0002	-0.0015*	-0.0006	-0.0005	0.0002	-0.0002	-0.0004	-0.0002	-0.0005
	(0.0006)	(0.0009)	(0.0006)	(0.0006)	(0.0004)	(0.0006)	(0.0006)	(0.0005)	(0.0007)
β_{IPC}	0.1156	0.4628**	0.1778**	0.5199**	0.044	0.4716**	0.369**	0.3721**	0.4091**
	(0.0881)	(0.1526)	(0.0812)	(0.0941)	(0.0605)	(0.0979)	(0.1108)	(0.0776)	(0.0982)

Note: Heteroskedasticity and autocorrelation consistent standard errors are shown in parentheses.
*Statistically significant at the 10% level. **Statistically significant at the 5% level.

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