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A DYNAMIC MATCHING AND SEARCH MODEL FOR DECEASED-DONOR KIDNEY TRANSPLANTATION

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Contents

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Abstract This paper deals with a critical feature of the complex real-life assignment problem of allocation of kidneys for transplantation: patient choice. We set up a stochastic game and introduce search theory tools to find equilibrium closed forms. To the best of our knowledge, it leads to two novel results in the related literature. First, we obtain various general comparative statics results for patient optimal behavior. The most relevant one is that with patient autonomy, an increase in organ supply does not necessarily improve the performance of the system in terms of social welfare, instead it can exacerbate organ wastage. Second, our modelling approach allows the introduction of patients' health conditions using an exogenous markovian taste shock. For a given organ quality, the less sick the patient, the greater the utility that she receives from the transplant, capturing in this way a well known stylized fact in the medical literature on post-transplant life expectancy. In this manner, we offer a new framework to study the problem of organ allocation procured from deceased donors.

Keywords: Dynamic matching, Waiting List, Organ allocation, Search theory.

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1 Introduction

Around 68 percent of the total global activity in organ transplantation corresponds to kidney transplants.[1] It is the best treatment for End-Stage Renal Disease (ESRD) compared to the alternative treatment, dialysis (peritoneal or hemodialysis), where renal functions are substituted by medical procedures.

In Mexico, the National Center for Transplant (CENATRA), a decentralized agency of the Department of Health, is responsible for the development of the National Transplant System (SNT). According to information provided by the CENATRA, there were 9,140 patients registered in the waiting list for a kidney transplant by the end of 2012, 2,364 of them were new patients; 5,176 patients who had been waiting two or more years; actually 1723 of them had been waited five or more years. In the same year, only 696 kidney transplants from deceased donors were conducted. In this context, the Department of Health has set as a priority to increase deceased donation, for three reasons: 1. To deal with this current scant supply of organs, 2. Because of the rise in the incidence of chronic degenerative diseases associated with renal failure, and 3. Because kidney transplantation has a lower cost compared to alternative therapeutics.[2]

In general, the over demand of organs is a characteristic of transplant markets, and patients' choice can exacerbate wastage of this scarce resource. When an organ is procured, there are many compatible candidates to receive the transplant. The organ is offered to candidates according to some prioritization rule. Prioritization rules attend to a complex set of principles such as fairness, medical efficiency, urgency and merit, as well as legal considerations (Calderón y Elbittar [3]). Any time a patient is offered an organ, she has the right to pass and wait for a kidney of better quality (for details on organ quality, see appendix A7). While searching for a recipient who does accept the offer, organs accumulate

[1] Global Observatory on Donation and Transplantation, (GODT). Organ Donation and Transplantation Activities. 2011. Retrieved from: http://www.transplant-observatory.org/Pages/home.aspx

[2] Programa de Acción Especifico 2007-2012. Transplantes. More information available at http://www.cenatra.salud.gob.mx/

cold ischemia time, which deteriorates the tissue. After some critical time, the organ becomes unsuitable for transplantation. Thus, patients' choice is a critical feature of this market, that should be taken into account when designing allocation policies.

Some figures from the US underline the problem: in 2000, nearly half the kidneys clinically acceptable were refused by the first patient to whom organs were offered, because of reasons such as kidney's size and weight, and age of the donor, among others (Su and Zenios [19]). Besides, according to data retrieved from Organ Procurement Transplant Network (OPTN), in 2011, 18 percent of the kidneys in conditions to be transplanted were finally discarded because they exceeded the maximum acceptable accumulated ischemia time.[3]

To capture this problem of organ allocation, we set up a model of dynamic assignment with waiting list, and study patients' choice. Specifically, our model consists of patients and organs sequentially entering the market. Organs are heterogeneous with random quality, and each one arriving is offered sequentially to patients in the queue, according to a queue discipline. Patients are homogeneous and have preferences over organs, which depend both on organ quality and their health state. A patient can either accept or pass the offer. If she accepts, the match is done and the patient is withdrawn of the queue. Otherwise, she stays in the waiting list. Then, this problem is a dynamic, stochastic discrete choice problem; it falls into the class of optimal stopping problems. Nevertheless, it has a special feature: strategic dependence, which arises from the sequentiality of the offers.

Due to these features of the problem, we incorporate in our approach the search theoretic formulation of optimal stopping problems. In our knowledge, ours is the first paper in the Market Design literature that explicitly introduces this theoretic framework. The economics of search is a prominent and well-stablished component of economic theory, with a broad application field.[4] Specifically, we introduce in our model two nice properties lodged in the

[3]Anual Data Report: Kidney. Retrieved from http://srtr.transplant.hrsa.gov.

[4]For a recent and exhaustive review of Search Theory, see McCall and McCall [11]. Search theory is a powerful theoretical framework to analyze resource allocation in environments with trading frictions. Its most prominent application is the study of labor markets and prices formation. It also has been applied to studies of issues in monetary theory, housing market, financial economics, urban economics and marriage markets (Shi [15]).

Basic Search Model (BSM): reservation (wage) property and myopic property. The former refers to the offers that are acceptable (specifically, those up to a critical value) whereas the latter tell us that this critical value is obtained comparing the return from one additional search with the return of stopping.

We employ such properties in our approach, with the purpose to: 1. obtain closed analytic forms for equilibrium, 2. do extensive comparative statics analysis, and 3. introduce in an intuitive way the evolution of patients' health.

We derive closed-forms expressions for equilibrium that are novel in the literature on patients' choice in kidney allocation. These expressions capture the strategic dependence which arises from the seniority of patients leading the waiting list, and allows us to obtain strong comparative statics conclusions on patients' choice with perfect information. The most relevant one is that with patient autonomy, an increase in organ supply does not necessarily improve the performance of the market in terms of social welfare, instead it can intensify organ wastage: some changes in the organ distribution can do patients more selective. Due to ischemia time is a temporal restriction and First Come First Served is the queue discipline, only patients in front of the list receive an offer. As they become more selective, the organ wastage may be exacerbate. There are two channels through these kind of changes occurs: first, with patient autonomy, an increase of the highest organ quality offered in the market makes the patients more selective, because it boosts the probability of receiving a better offer continuing the search one additional period. Second, under a rise in the variability in organ quality offers, by means of a mean-preserving spread of organ distribution, the higher incidence of high quality offers compensates the loss to continue the search one additional period, while the higher frequency of bad quality offers is not as bad, because patients always can remain in dialysis.

Finally, patients' health evolution in the transplant waiting list has been an awkward feature. We propose to address this critical and relevant point by introducing a Markov chain for health evolution. Each state of the health of the patient modifies the utility she receives from a transplant of an organ, in a multiplicative way. For an organ of a given quality, the less sick the patient, the greater the patient's utility from the transplant. In terms of post-transplant life expectancy, the less sick the patient, the greater the post-transplant added years of life, capturing in this way a well known fact in the medical literature. In this way, we offer a new framework to study the problem of organ allocation procured from deceased donors.

1.1 Related Literature

The branch of matching theory focused on allocation and exchange of indivisible goods has been extensively studied and applied to real-life problems.[5] Although there is a broader literature about the allocation of resources in one-sided and two sided-markets, it is primarily concerned with static environments. Only recently the dynamics of allocation and exchange problems have started to capture the attention of researchers. For example, on-campus housing allocation (Kurino [7]), kidney exchange (Ünver [21]), house allocation with overlapping generations (Bloch and Cantala [4]), public housing assignment (Leshno [9]) and school choice (Pereyra [13]).

Leshno [9] is, in our knowledge, the first paper that studies dynamic matching problems with waiting lists. Inspired by the problem of public housing allocation, the author sets up a model with two types of objects and agents, one type of agents prefer one type of object. Social welfare is maximized when agents and objects are matched appropriately, so he derives the queue policy that minimizes the misallocation probability between agents and objects. The author does abstraction of fairness, which is a crucial point in the case of organ allocation.

Kurt et al. [8] offer a stochastic-game modelling of kidney transplant with health evolution. The authors model transplant timing decisions of autonomous patients when their health is evolving. In the context of multi-way kidney live-donor exchange, timing of decisions

[5]There is a broader matching literature related to live-donor kidney exchange in static contexts. An excellent review is Sönmez et al. [14]

is relevant because exchange of organs only can take place if they are done simultaneously. In contrast, our paper deals with deceased donation, where strategic dependence arises from the fact that once procured any organ, it is sequentially offered to patients in the waiting list. Moreover, they model health evolution of patients as transitions between three states: pre-dialysis, dialysis (or waiting for a transplant) or death. Conversely, we propose to model health as evolving through dialysis, so that patient's preferences could be modified while waiting for an organ. In the same scenario of live-donor exchange, Ünver [21] studies the maximal number of exchanges that can be conducted in a dynamic evolving agent pool, from a central authority point of view, considering incentive compatibility.

In the Operation Research literature, there are various papers which study organ allocation.[6] Su and Zenios [20] is the closest to our model. The authors develop a queueing modelling to examine how the queue discipline bounds the impact of patient choice on organ wastage. They consider two different queue disciplines, First Come First Served (FCFS), and Last Come First Served (LCFS). They found that with patient autonomy, FCFS exacerbates organ wastage while LCFS provides optimality in organ use. Moreover, they prove that allowing candidate choice, social planner actions are ineffective in FCFS, while patients autonomy is efficient under LCFS, so optimal planner action is not taking anyone. While the authors consider organ and patient arrival processes, we abstract them from the analysis. Like us, they use a dynamic programming formulation of the patients' maximization problem in a given position waiting list, but their modelling approach is incompatible with the derivation of closed forms for equilibrium. On the contrary, we do derive these in a very generalized way, being able to do broader comparative statics.

Su and Zenios [2005] develop a model to study the conflict between patient choice and social welfare, in a static environment with a given number of organs and patients in a pool, so there is no waiting list. They build a first-best policy defined as the one that maximizes

[6] The seminal paper of Zenios [22] models the transplant waiting list as a queuing model with reneging with multiple classes of patients, to study the real-life differences in waiting time among different groups of patients. Zenios, et al. [23] and Bertimas et al. [2] study the trade-off between clinical efficiency and fairness.

the social welfare when patients accept all organ offer, and a second best, where the optimal policy is subject to a patient's incentive compatibility constraint. By means of a numerical study, a comparison between these two policies shows that patient choice leads to losses in social welfare if there is high variability in organ types.

Su and Zenios [2006] studies the role of patient choice under information asymmetries from a mechanism design approach. Patients have private information related with degrees of risk represented by their type. They propose a multi-queue system, where each patient chooses what queue-class to join in order to maximize her expected utility. Moreover, they consider two different mechanisms. One in which the social planner seeks to maximize clinical efficiency, defined as the sum of expected utility of the candidates. Other in which the social planner aim to enhance equity defined by the Rawlsian max-min criterion. As a result they found that in the first mechanism, private information is profitable to high-risk candidates, while in the second one it is the case with low-risk candidates.

The remainder of the paper is organized as follows. Section 2 introduces the model of the market with waiting list like a stochastic game. In Section 3 we prove the existence and uniqueness of equilibrium, and we derive an analytic expression for it. Also, we state results on comparative statics. Specifically, we study the effect on patients' quality selectiveness of changes in different characteristics of the system: dialysis quality, patients' valuation of the future as well as characteristics of organ distribution. The next section is devoted to introduce the evolution of patients' health, and to find the equilibrium in this new scenario. Some concluding remarks and possible extensions are mentioned in Section 5. Omitted proofs are formally stated in the Appendix.

2 The Model

Consider a matching problem where agents/patients and items/objects/organs arrive sequentially. Agents have preferences over objects. Objects have types which stand for item's quality. There is a waiting list that agents join in order of arrival. Priority is assigned by seniority in the waiting list. Consumption is not compulsory, so whenever a patient is offered an object, she can either pass or accept (the agent confronts the decision of accepting the organ she is being offered or waiting for a better quality one). There is no sanction for declining an offer. When a patient passes, the organ is offered to the patient next in the queue.

We model this situation as an stochastic game, and analyze the equilibrium behavior of patients in such a game. There are several reasons for which we follow this approach. The most relevant one is that, except from the agent in the first position in the waiting list, there is strategic dependence in the agents' behavior. In fact, for any patient who is in the second position or any position behind this, her payoff depends not exclusively on her decision but on the decisions taken by other agents ahead her in the queue.

In addition, in the usual solution concept of stochastic games, Markov Perfect Equilibrium, the payoffs and strategies which are disposable to agents are markovian, in the sense that it is usually defined. We return to this property below. It refers to the notion that the transition from one state of the game to another depends both on the strategies taken by players in the previous period as well as on the state in such period, but not on the entire history of the game, as it is usual in dynamic games. It has two advantages. First, markovian strategies transform the dynamic programming problem into a stationary one, which facilitates the analysis considerably [7]. Second, several authors are aware that this markovian characteristic of patient's decision making capture real-life medical practice (Kurt et al. [8]).

Finally, in stochastic games agents care for life-time utility rather than for a one-period payoff, which is more than reasonable in the context of organ transplantation, where the tissue quality is highly correlated with graft survival and post-transplant life expectancy.

As mentioned above, our aim is to focus on the behavior of the patients in the waiting list. We make two kinds of abstraction to simplify the problem. The first one is a set of

[7]Research on equilibrium search usually assume a steady-state model, turning the problem also into a stationary one (Smith [16]).

abstractions that are hereafter maintained; general assumptions to put attention in agent's choice. The second one is a collection of assumptions to clarify the exposition and which are removed in a subsequent section. These are: agents and organs live forever, and patients' health evolution do not evolve in the waiting list. Furthermore, the former set of general assumptions are:

- Patients are perfectly compatible with organs. We abstract issues of medical compatibility, such as blood compatibility and Human Leucocyte Antigen.
- Time is discrete.
- Patients and organs arrive one per period, i.e. we abstract arrival process of patients and organs.
- Perfect information is available. In particular, patients know the organ's types distribution.
- Patients are homogeneous in the sense that they have the same preferences over objects, as well as the same valuation of the future, i.e. all patients are equally impatient.
- We do not consider geographical factors.
- It is an infinite horizon decision problem.
- Patients are forward-looking. It means that patients choose their actions with basis on their expected payoff.

2.1 Model: building blocks of the game

We refer to organs as the objects which have to be matched with agents. In each period, an organ of **type** θ^t is randomly and independently chosen from an exogenously given cumulative distribution function $F(\theta)$ truncated over $\Theta = [\underline{\theta}, \overline{\theta}]$.[8] Note that this implies

[8]For simplicity, in the next section we assume, without loss of generality, $\underline{\theta} = 0$.

that for any T, the sequence of random variables $\{\theta^i\}_{i=t}^{t+T}$ are independently and identically distributed, so this sequence is trivially a Markov Process. In fact, for all $A \subseteq \Theta$,

$$Prob[\theta^{t+1} \in A|\theta^t = \theta] = Prob[\theta^{t+1} \in A].$$

The **set of agents** is $\mathcal{N} = \{p^1, p^2, ...\}$. A **state** of the game $\mathcal{G}$ in period t is given by an ordered list $x^t = (p_1^{g(t,1)}, p_2^{g(t,2)}, ..., p_i^{g(t,i)}, ..., p_k^{g(t,k)}, \theta^t)$, where the first k elements stand for patients in the waiting list at period t, while θ^t stands for the type of the organ which is being offered in this period. The correspondence $g : \mathbb{N} \times \mathbb{N} \mapsto \mathbb{N}$ identifies the patient who occupies **position** i in the waiting list at period t. The subindex i is the position of the patient $g(t,i)$ at time t. Notice that the set $S_t = \{p_1^{g(t,1)}, p_2^{g(t,2)}, ..., p_k^{g(t,k)}\}$ is a poset[9] under the order relation $p_i^{g(t,i)} \sqsubseteq p_j^{g(t,j)} \Leftrightarrow g(t,i) \leq g(t,j)$ or under de relation order $p_i^{g(t,i)} \sqsubseteq' p_j^{(t,j)} \Leftrightarrow i \leq j$. Moreover, both orders yields to the same ordering over this set.[10] Both order relations, $\sqsubseteq$ and $\sqsubseteq'$ over the set of patients, captures the arrival order to the market, so $p_i^{g(t,i)} \sqsubseteq' p_j^{(t,j)}$ means that the patient $p^{g(t,i)}$ join the market earlier than the patient $p^{g(t,j)}$, so the former is ahead the later in the queue. Hereafter we refer to the patient i, as the patient which occupies position i in the waiting list, unless the distinction between position and patient is necessary. Once we introduce symmetry assumptions about strategies, this distinction will become irrelevant. The **set of states** of the queue is given by $\mathcal{X} = (2^{\mathcal{N}}, \sqsubseteq) \times \Theta$.

When a patient receives an offer, she must decide whether to accept (a) or decline (d) the organ. Otherwise, he cannot make any decision. Formally,

Definition 1 (Actions) *Let $A_i(x^t)$ the set of actions available to the candidate i in the state x^t,*

$$A_i(x^t) = \begin{cases} \{a, d\} & \text{if for all } j < i, a_j^t = d, \\ \emptyset & \text{otherwise.} \end{cases} \tag{1}$$

[9]A partial ordered set or poset formalizes the idea of an ordering. A partial order is a relation reflexive, transitive and antisymmetric. A poset is a set with a partial order.

[10]Formally, there exists an order isomorphism from $(S_t, \sqsubseteq)$ to $(S_t, \sqsubseteq')$.

Let a_i^t any action chosen by the player i in state x^t, so $a_i^t \in A_i(x^t)$. We define $a^t = (a_1^t, a_2^t, ..., a_k^t)$ as an action profile at time t. The history of the game at time t, $h^t = (x^0, a^0, x^1, a^1, ..., x^{t-1}, a^{t-1}, x^t)$ is common knowledge.

In a dynamic game, a *strategy* for a player i is a set of functions which maps any history h^t into the set of actions available in state x^t to player i, $A_i(x^t)$. However, we focus on *markovian strategies*. Intuitively, markovian strategies captures the idea that the past influences the present only through the state of the game; the only decision-relevant information to the agent is contained in the state of the market in the decision period, and agents do not care about the way in which they arrive to such state.

Definition 2 (Markovian Strategies (Fudenberg y Tirole [6])) *For a given player i, a strategy σ_i is said to be markovian if for each time t and t' and histories h^t and $\widehat{h}^{t'}$ such that $x^t = \widehat{x}^{t'}$, then $\sigma_i(h^t) = \sigma_i(\widehat{h}^{t'})$.*

Note that with markovian strategies, σ_i specifies an action for each state x rather than for any history h^t. This assumption reduces significatively the strategic space.

At the end of each period, each patient obtains a reward, which is related to the quality of life. If the agent gets an offer and accepts it, the reward is equal to the accepted organ type, and she is withdrawn from the game. In other cases, either because she declines the offer or because she cannot take any action, she remains in the queue for the next period and receives a payoff δ, which corresponds to stay in dialysis for a period. Formally,

Definition 3 (Period Payoffs) *Let $u_i(a^t, x^t) : \prod_i A_i \times \mathcal{X} \mapsto \mathbb{R}^+$,*

$$u_i(a^t, x^t) = \begin{cases} \delta, & \textit{if for some } j \leq i, a_j^t = a, \vee, a_i = d, \\ \theta^t, & \textit{if for all } j \leq i, a_j = d, \wedge, a_i = c. \end{cases} \tag{2}$$

Using the definition of markovian strategies, we obtain the following result.

Proposition 4 (Transition Probability) *A transition probability for this game, is a function* $\mathcal{P} : \mathcal{X} \times A \times \mathcal{X} \mapsto [0, 1]$ *such that* $\mathcal{P}(x^{t+1}|x^t; \sigma(x^t))$ *is the probability of transition from state* x^t *to state* x^{t+1} *after the joint action* $\sigma(x^t)$*, given by:*

$$\mathcal{P}\left[x^{t+1}|x^t = \left(p_1^{g(t,1)}, p_2^{g(t,2)}, ..., p_{i-1}^{g(t,i-1)}, \mathbf{p_i^{g(t,i)}}, p_{i+1}^{g(t,i+1)}, ..., p_k^{g(t,k)}, \theta^t\right); \sigma(x^t)\right] =$$

$$\begin{cases} 1, & \textit{if } x^{t+1} = \left(p_1^{g(t+1,1)}, p_2^{g(t+1,2)}, ..., \mathbf{p_i^{g(t+1,i)}}, ..., p_{k+1}^{g(t+1,k+1)}, \theta^{t+1}\right) \wedge \forall\ h \leq k, \left(\sigma_h(x^t) = d \wedge g(t+1,h) = g(t,h)\right), \\ 1, & \textit{if } x^{t+1} = \left(p_1^{g(t+1,1)}, ..., \mathbf{p_{i-1}^{g(t+1,i-1)}}, p_i^{g(t+1,i)}, ..., p_k^{g(t+1,k)}, \theta^{t+1}\right) \wedge \sigma_1(x^t) = a \wedge \forall\ h < k,\ g(t+1,h) = g(t,h+1), \\ 1, & \textit{if } x^{t+1} = \left(p_1^{g(t+1,1)}, ..., p_{i-1}^{g(t+1,i-1)}, p_i^{g(t,i)}, ..., p_k^{g(t+1,k)}, \theta^{t+1}\right) \wedge \sigma_i(x^t) = a \wedge \forall\ h < i, \left(\sigma_h(x^t) = d \wedge g(t+1,h) = g(t,h)\right) \\ & \qquad \wedge \forall\ h|i < h < k, g(t+1,h) = g(t,h+1), \\ 0, & \textit{otherwise} \end{cases}$$

$$\forall\ \theta^t, \theta^{t+1} \in \Theta. \quad (3)$$

Proof. Recall that at the start of any period an organ and a candidate arrives on the market, and the later joins the last position in the waiting list. Consider the state x^t where an organ of type θ^t is being offered and there are k patients in the queue. If nobody in the market accepts the organ ($\forall h \leq k, \sigma_h(x^t) = d$), with probability one all patients remain in the same position for the next period, so $g(t+1, h) = g(t, h)$ for all $h \leq k$, and the new patient $g(t+1, k+1)$ joins the queue in the last position, $k+1$. Now consider the patient $g(t, i)$ who occupies the position $i \neq 1$ in period t. If the patient leading the waiting list accepts ($\sigma_1(x^t) = a$), with probability one she moves to the position $i-1$ in the next period, so $g(t+1, i-1) = g(t, i)$. Finally, if all patients ahead her decline, and she accepts, she is withdrawn from the market, all patients ahead her remains in the same position ($g(t+1, h) = g(t, h)$ for all h such that $h < i$), and candidates behind her move one position ahead, that is $g(t+1, h) = g(t, h+1)$ for all $i < h < k$. In both cases the arriving agent $g(t+1, k)$ joins position k in the queue. Given that the distribution of organ types is exogenously given, it does not affect transition probabilities across states. ■

Patients **discount** future with a factor $\beta \in (0, 1)$, so the greater the β the greater the valuation of the future, and the lesser the agent's impatience. Finally, there is an **initial state** $x^0 = \emptyset$.

This market with waiting list is represented as a stochastic game

$$\mathcal{G} = \left(\mathcal{N}, \mathcal{X}, \{A_i\}_{i=1}^{\infty}, \{u_i\}_{i=1}^{\infty}, \mathcal{P}, \beta\right).$$

The game proceed as follows. At period 1, an agent and an item arrive; the item is offered to the agent, who can either accept it or pass. If she accepts she is withdrawn from the market, and obtain a payoff equal to the organ type, each period, for the duration of her life. If she passes, she obtains a payoff of δ corresponding to remain in dialysis and wait for a better organ. In every period an agent and an organ arrive, and the organ is sequentially offered to agents, accordingly to $\sqsubseteq$. Agents choose their actions σ_i in order to maximize the expected value of their life-time utility,

$$U_i^{\infty}(\sigma_i, \sigma_{-i})(x^0) = \sum_{t=1}^{\infty} \beta^{t-1} u_i(x^t, \sigma_i(x^t), \sigma_{-i}(x^t)), \tag{4}$$

i.e. the sequential optimization problem for each agent is given by:

$$\max_{\sigma_i(x^t)} \mathbf{E}\left[\sum_{t=1}^{\infty} \beta^{t-1} u_i(x^t, \sigma_i(x^t), \sigma_{-i}(x^t))\right] \tag{5}$$

where expectations are taken over transition probabilities as defined in proposition 4.

3 Equilibrium

Next, we state the equilibrium concept that is used to analyse patients' behavior.

Definition 5 (Perfect Markov Equilibrium (Fudenberg y Tirole [6])) *A strategic profile $\sigma^* = (\sigma_i^*, \sigma_{-i}^*)$ where for all i, σ_i^* is markovian strategy, is a PME of this game, if for all i*

$$\mathbb{E}\left[U_i^{\infty}(\sigma_i^*, \sigma_{-i}^*)(x^0)\right] \geq \mathbb{E}\left[U_i^{\infty}(\sigma_i, \sigma_{-i}^*)(x^0)\right]. \tag{6}$$

It is important to notice that, in any period and for any candidate excepting the first one, she is able to make a choice if and only if all agents ahead her pass the organ which is being offered. So she cares only about the decisions taken by those agents who arrived before her on the market and who remain in it. Moreover, by assumption agents are homogeneous and forward-looking, so they only differ in their position in the queue. It means that when any patient has a given position in the waiting list, she confronts the same strategic considerations. Provided that, in a given period t, the relevant-state variables for any patient is her position in the waiting list i, and the type of the organ which is being offered in such period θ^t. In addition, considering that we restrict our attention to markovian strategies, it seems reasonable to assume that whenever two patients reach any given position, in some state follow the same strategy. We formalize these intuitions in the following two statements:

Definition 6 (Payoff-Relevant State Variables) *For any patient $g(t,i)$ who has the position i in the state of the market x^t for some t, the payoff-relevant state variables are $s^t = (i, \theta^t)$.*

Assumption 7 (Symmetry) *Let $g(t,i)$ and $g(t',i)$ be two patients who occupy the same position i in states x^t and $x^{t'}$. Then, they follow thereafter the same strategy. The sequence of such symmetric strategies, is the strategic profile $\{\gamma_i\}_{i=1}^{\infty}$ such that any agent follow the action prescribed by this profile whenever she occupies position i when the type of organ offered int the period t is θ^t.*

Notice that $\gamma : \mathbb{N} \times \Theta \mapsto \{a, d\} \bigcup \emptyset$. In the following definition and lemma, we restate the period payoffs and the transition probabilities given respectively by definition 3 and proposition 4 in terms of the relevant state variables as stated in definition 6 and when assumption 7 holds.

Definition 8 (Period Payoffs (restated)) *When assumption 7 holds, the period utility function $u : \prod_{j=1}^{i} A_j \times \mathbb{N} \times \Theta \mapsto \mathbb{R}^+$ in terms of the payoff-relevant state variables is given* *by:*

$$u\left(\{\gamma_j\}_{j=1}^{i},(i,\theta^t)\right)=\begin{cases}\delta, & \textit{if for some } j<i, \gamma_j(\theta^t)=a,\vee,\gamma_i(\theta^t)=d,\\ \theta^t, & \textit{if for all } j<i, \gamma_j(\theta^t)=d,\wedge,\gamma_i(\theta^t)=a.\end{cases} \tag{7}$$

Proposition 9 (Law of motion) *The transition probability $\mathcal{P}'(\cdot)$ for a patient in position i in the queue at period t and who remains in the market in period $t+1$, is:*

$$\mathcal{P}'(s^{t+1}|(i,\theta^t))=\begin{cases}1, & \textit{if } s^{t+1}=(i-1,\theta^{t+1}) \textit{ and for some } j<i,\ \gamma_j(\theta^t)=a,\\ 1, & \textit{if } s^{t+1}=(i,\theta^{t+1}) \textit{ and for all } j\leq i,\ \gamma_j(\theta^t)=d,\\ 0, & \textit{otherwise.}\end{cases} \tag{8}$$

Proof. It follows directly from proposition 4, taking into consideration those states where patient in position i in the state of the game x^t remains in the market in the state x^{t+1}. ■

The sequential formulation of the optimization problem (5) is cumbersome. In the next proposition, it is restated in the more tractable recursive form. Here we incorporate the tools to solve optimal stopping problems developed in the core of search theory, that are well known in the literature.[11] We modify the Basic Search Model (BSM) to adapt it to our strategic context. In fact, when a candidate receives an organ offer, she confronts the decision to stop (accepts) or continue the search for a better organ one additional period. In our context, additional considerations arise due to the strategic dependence of the decisions made by the preceding patients. This dependence modifies the support of the distribution of organs which are offered to a candidate different to the one who leads the waiting list.

Proposition 10 *The Bellman equations associated with the sequential problem (5)-(6), are*

$$V(1,\theta^t)=\max_{a,d}\left\{\frac{\theta^t}{1-\beta},\delta+\beta E[V(1,\theta^{t+1})]\right\} \tag{9}$$

[11]Our analysis of the optimal stopping problem is based on Adda and Cooper [1], Ljungqvist and Sargent [10], McCall and McCall [11] and Stokey and Lucas [17].

for $i = 1$, *and*

$$V(i,\theta^t) = \begin{cases} \delta + \beta E[V(i-1,\theta^{t+1})], & \text{if for some } j < i,\ \gamma_j(\theta^t) = a, \\ \max_{a,d}\left\{\frac{\theta^t}{1-\beta}, \delta + \beta E[V(i,\theta^{t+1})]\right\}, & \text{if } \forall j, \text{ such that } 1 < j < i,\ \gamma_j(\theta^t) = d, \end{cases} \tag{10}$$

for $i > 1$. *The law of motion is given by the transition probability as defined in proposition 9.*

Proof. Recall that δ stands for the period payoff of being in dialysis for a period, β is the discount factor, (i,θ^t) are the state variables of the patient which occupies a position i in the queue at period t, when an organ of type θ^t, randomly chosen from its distribution $F(\theta)$, is being offered. If any patient j ahead of her accepts, then the patient i does not receive an offer in this period, she receives a payoff δ of being in dialysis and advances to the position $i-1$ in the waiting list, so in this case her total expected payoff is

$$\delta + \beta E[V(i-1,\theta^{t+1})]$$

On the other hand, if she actually receives an organ offer, then she has two possibilities: to accept (or to stop the search), receiving an adjusted lifetime payoff

$$\frac{\theta^t}{1-\beta},$$

being removed from the market, or to decline, staying for a longer period on dialysis, and continuing the search in the next period, and remaining in the same position in the waiting list, in which case her total expected reward is

$$\delta + \beta E[V(i,\theta^{t+1})]$$

■

Proposition 11 (Existence of Equilibrum) *There is a unique function V satisfying the dynamic programming problem stated in proposition 10.*

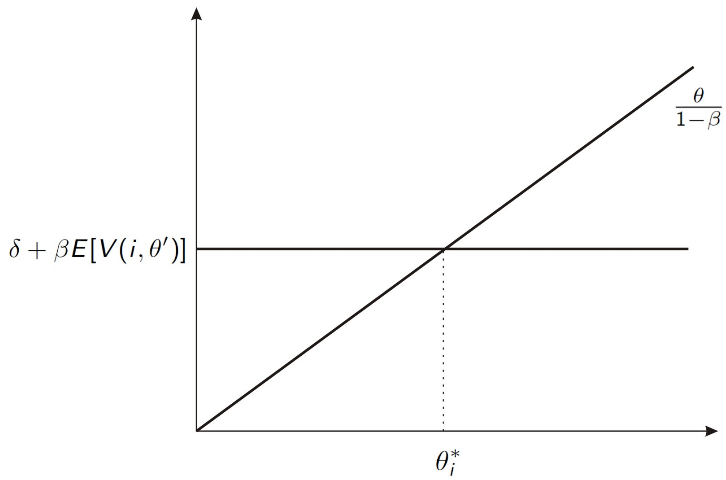
Proof sketch. We use the Contraction Mapping Theorem. First, we show that the Bellman operator T associated to (9)-(10) is a contraction mapping, proving that T satisfies the Blackwell sufficient conditions of monotonicity and discounting. Being T a contraction, the Contraction Mapping Theorem assures that there is a unique fixed point V such that $TV = V$. (see details in Appendix A1) ■

Proposition 12 (Uniqueness of Equilibrium) *There is a unique sequence of policy functions (unique strategy profile) $\{\gamma_i^*\}_{i=1}^{\infty}$ which solves the Dynamic Programming problem stated by proposition 10. Moreover, this rule takes the reservation form*

$$\gamma_i^*(\theta^t) = \begin{cases} d & \text{if } \theta^t < \theta_i^* \\ a & \text{if } \theta^t \geq \theta_i^*, \end{cases} \tag{11}$$

where θ_i^* is the organ's quality reservation type.

Proof. Consider the Dynamic Programming problem stated in proposition 10. Notice that when any candidate $i \geq 1$ confronts a decision, the value function can take the form $V(i, \theta^t) = \frac{\theta^t}{1-\beta}$, or the form $V(i, \theta^t) = \delta + \beta E[V(i, \theta^{t+1})]$. In the first case, $V(\cdot)$ is strictly increasing in θ^t while in the second case $V(\cdot)$ is constant. The following figure depicts $V(\cdot)$.



It is clear that if $\theta^t < \theta_i^*$ then it is optimal to decline the offer θ^t and continue the search, while if $\theta^t > \theta_i^*$ the optimal choice is to accept and to stop the search. The facts that $\frac{\theta^t}{1-\beta}$ is strictly increasing, while $\delta + \beta E[V(i, \theta^{t+1})]$ is constant, guaranties that there exist a unique θ_i^* which satisfies

$$\frac{\theta_i^*}{1-\beta} = \delta + \beta E[V(i, \theta^{t+1})].$$

■

Theorem 13 (Optimal Reservation Thresholds) *The sequence of optimal reservation thresholds $\{\theta_i^*\}_{i=1}^{\infty}$ for the problem 10-9 is given by*

$$\theta_1^* = \delta(1-\beta) + \beta \left[\theta_1^* F(\theta_1^*) + \int_{\theta_1^*}^{\overline{\theta}} \theta dF(\theta)\right] \tag{12}$$

for $i = 1$; and,

$$\theta_i^* = \delta(1-\beta) + \beta \left\{(1 - F(\theta_{i-1}^*))\theta_{i-1}^* + \left[F(\theta_i^*)\theta_i^* + \int_{\theta_i^*}^{\theta_{i-1}^*} \theta dF(\theta)\right]\right\} \tag{13}$$

for $i > 1$.

Proof sketch. For the reservation organ type in any given position i, it has to be the case that $\delta + \beta E[V(i, \theta^{t+1})] = \frac{\theta_i^*}{1-\beta}$. To develop expectations, we introduce the fact that the candidate in position i receives a transplant offer of an organ of type θ^{t+1} if for all $j < i$ $\theta_i^* > \theta^{t+1}$. By assuming that $\theta_1^* \geq \theta_2^* \geq ...$, i receives an offer in period $t+1$ with probability $F(\theta_{i-1}^*)$, in which case she obtains the expected utility $\mathbf{E}\left[V(i,\theta')|\theta < \theta_{i-1}^*\right]$, where expectation is taken over the distribution of organ types conditional on receiving an offer. Moreover, she does not receive an offer in $t+1$ with probability $1 - F(\theta_{i-1}^*)$, receiving a continuation payoff $\delta + \beta E[V(i,\theta^{t+1})]$. (See details in appendix A2.) ■

By definition 5, the strategic profile characterized by the sequence of thresholds $\{\theta_i^*\}_{i=1}^{\infty}$ is a Markov Perfect Equilibrium of the game $\mathcal{G}$. Notice that for candidates behind first position, the optimal threshold (13) incorporates the fact that in a given period, the candidate only receives an offer in the next period conditional to all patients ahead her passing the offer, i.e., the organ distribution which the patient will confront if she decides to continue the search for a new period is $F(\theta|\theta < \theta_{i-1}^*)$, which has a reduced domain; this fact makes the candidate less selective when she receives an offer.

3.1 Comparative Analysis

In the next theorems, we establish some results on comparative statics.

Theorem 14 (Comparative Statics I) *Let $f(\theta)$ the density function associated to $F(\theta)$. If $f(\cdot)$ is differentiable on $(0,\overline{\theta})$, Then, for all $i \geq 1$,*

$$\frac{d\theta_i^*}{d\delta} > 0 \tag{14}$$

$$\frac{d\theta_i^*}{d\overline{\theta}} > 0 \tag{15}$$

$$\frac{d\theta_i^*}{d\beta} \geq 0. \tag{16}$$

Proof sketch. The proof uses an inductive argument and the well-known Leibniz's Rule to differentiate under the integral sign (See details in appendix A3). ■

In words, the higher the utility that a patient receives by staying in dialysis for a period or the higher the maximal organ quality that could be offered in the market, the higher the incentives to wait for a better organ. In addition, as the candidate becomes more patient (values more the future), she is ready to wait for a better organ offer.

Theorem 15 (Comparative Statics II-Riskiness of Organ Distribution) *Consider two distributions $F(\cdot)$ and $G(\cdot)$ for θ, with support over $[0, \overline{\theta}]$, such that:*

1. $\int_0^{\overline{\theta}} \theta dF(\theta) = \int_0^{\overline{\theta}} \theta dG(\theta)$ *and,*
2. *For all* $t \in [0, \overline{\theta}]$, $\int_0^t (G(\theta) - F(\theta))\, d\theta \geq 0$,

i.e., $F(\cdot)$ second order dominates or is less risky than $G(\cdot)$ (Sargent and Ljugqvist [10]). Let $\theta^*_{i,F}, \theta^*_{i,G}$ the optimal reservation thresholds for a patient in position i under the organs distributions $F(\cdot)$ and $G(\cdot)$, respectively, as they are given by theorem 13. Then, for $i = 1$,

$$\theta^*_{1,F} \leq \theta^*_{1,G}. \tag{17}$$

Proof. See appendix A5. ■

Notice that this theorem, and the result (15) of theorem 14, have a relevant implication for the design of organ allocation policies: from a social planner point of view, not all increases in organ supply are desirable under First Come First Served queue discipline, which is the gold standard of fairness. Moreover, if the rise in supply increases the highest organ quality offered in the market, as well as if the rise in supply increase the variability of organs without change the mean quality offered, patients will become more selective and valuable transplant opportunities will be missed. For simplicity, let us to consider the case of an increase in the highest organ quality. Suppose that an organ can be offered T times, by the restriction imposed by ischemia cold time. The probability that an organ randomly drawn be wasted is $Prob[\theta \leq \theta^*_T] = F(\theta^*_T)$, which is nondecreasing in θ^*_T. In theorem 14 we prove that an increase in the highest organ quality increases θ^*_T, so the rise in the highest quality increases

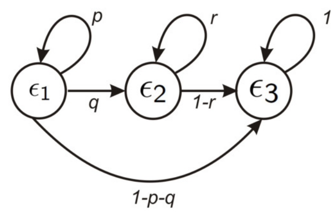


Figure 1: Markov chain for patient's health evolution

the probability of an organ to be wasted. Intuitively, the increase in the highest organ quality boosts the probability of receiving a better offer if the search continues one additional period, so there are incentives to do so.

Similarly, with a rise in the variability of quality offers, by means of a mean-preserving spread of organ distribution, the higher incidence of high-quality organ offers compensates the loss to continue the search one additional period, while the higher frequency of bad quality offers is not as bad, because patients always can remain in dialysis, or do not exercise the option of transplant. [12]

4 Introducing Patients' Health Evolution

In this section, we propose a formal approach to introduce patient's health conditions in a tractable way. We model it as an exogenously given three-state absorbing Markov chain, which drives the evolution of the health condition of any patient in this market. The states of the chain are $\epsilon_1, \epsilon_2, \epsilon_3 \in [0, 1]$ and are such that $\epsilon_1 \geq \epsilon_2 \geq \epsilon_3 = 0$.

In this absorbing Markov chain, the state ϵ_1 stands for the health condition of a patient which have just joined the market, i.e. those patients whose Chronic Kidney Disease have just reached the End-Stage Kidney Disease (ESKD), so they are less sick than those patients whose health condition has reached the state ϵ_2, while ϵ_3 is an absorbing state which stands for decease.[13]

[12] In the context of search theory, Ljungqvist and Sargent originally interpreted the risk increase of the distribution in terms of option pricing theory. The reader must bear in mind that according to the Black-Scholes formula, the value of an option is increasing in the variance of the underlying asset.

[13] It may seem quite restrictive to model the health evolution by three stages, but it is enough to capture the

Definition 16 (Health State Transition Matrix) *The transition probabilities between health states are given by the transition matrix:*

$$\begin{pmatrix} & \epsilon_1 & \epsilon_2 & \epsilon_3 \\ \epsilon_1 & p & q & 1-p-q \\ \epsilon_2 & 0 & r & 1-r \\ \epsilon_3 & 0 & 0 & 1 \end{pmatrix}$$

where $p, q, r \in [0, 1]$ *and,* $p + q > r$*.*

The condition $p+q > r$ guarantees the intuitive idea that, for a given agent, the probability of being dead in the next period when she is less sick (ϵ_1) in the present, is lower than the probability of dying in the next period when she is sicker (ϵ_2) in the present.

Notice that although in any period there is a small probability that any patient be alive in the next period, this probability decreases with the number of periods a given agent has been in the market. This simple Markov chain captures the fact that in the long run patients die. Intuitively, in this new context, the threshold is expected to be lower than the one given by theorem 13, because there is uncertainty about her health condition in the future. Any agent confronts a more complicated decision when she receives an offer, because she can decline it and continue the search for a better organ, but the incentives to do it are undermined by the risk of expiring in the next period.

In this new setup, in addition to her position in the waiting list and the quality of the offered organ in a given period, any patient has to take into account her health condition when she chooses to stop or continue the search. So the relevant-payoff state variables in the period t, are now (i, θ^t, ϵ), where as before, i and θ^t stands for position and organ quality, while $\epsilon \in \{\epsilon_1, \epsilon_2, \epsilon_3\}$ is the health condition of the patient in position i. Assuming that symmetry holds as before, the next statement defines the way in which health conditions

basic facts, and streamline the analysis considerably. For further applications, the states must be established based on medical criteria, taking into account the slow evolution of the health of patients with ESKD and generating a discretization over such evolution.

affects the preferences, and therefore, the utility function of a patient.

Definition 17 (Health-State Period Payoffs) *Let $\{\gamma_j\}_{j=1}^{\infty}$, the sequence of symmetric strategies such that $\gamma_j : \mathbb{N} \times \theta \times \{\epsilon_1, \epsilon_2, \epsilon_3\} \mapsto \{a, d\} \times \emptyset$. The period payoff function ϑ : $\prod_{j=1}^{i} A_j \times \mathbb{N} \times \Theta \times \{\epsilon_1, \epsilon_2, \epsilon_3\} \mapsto \mathbb{R}^+$, is*

$$\vartheta\left(\{\gamma_j\}_{j=1}^{i}, (i, \theta^t, \epsilon)\right) = \begin{cases} \epsilon\delta, & \textit{if for some } j < i, \gamma_j(\theta^t, \cdot) = a, \vee, \gamma_i(\theta^t, \epsilon) = d, \\ \epsilon\theta^t, & \textit{if for all } j < i, \gamma_j(\theta^t, \cdot) = d, \wedge, \gamma_i(\theta^t, \epsilon) = a. \end{cases}$$

From there on, we consider in detail the choice of the patient in front of the queue. Note that the dynamic optimization problem is analogous to the one stated in proposition 10, but in this case the expectation is taken not only over the distribution of the organs, but also over the probabilities given by Markov chain. Consequently, it is natural that existence and uniqueness are preserved. Formally stated, the recursive form of the optimization problem for the candidate in the first position is given by,

$$V(1, \theta^t, \epsilon) = \max\left\{\frac{\epsilon\theta^t}{1-\beta}, \epsilon\delta + \beta E_{\epsilon'|\epsilon}[V(1, \theta^{t+1}, \epsilon')]\right\} \tag{18}$$

Given that there are now two health states (formally three, but $\epsilon_3 = 0$ is trivial), then we have now two optimal thresholds for the first position, which characterizes, as before, the optimal strategies: one $\theta_1^*(\epsilon_1)$ is the minimum organ quality that a patient in the position 1 accepts when her health condition is ϵ_1, while $\theta_1^*(\epsilon_2)$ is the minimum quality which a patient in position 1 accepts being in the more sick state ϵ_2.

Proposition 18 *When the health condition of the patient leading the waiting list is governed by the Markov chain stated in definition 16, her optimal stopping thresholds are given by:*

$$\theta_1^*(\epsilon_2) = \delta(1-\beta) + \beta r\left\{\theta_1^*(\epsilon_2)F(\theta_1^*(\epsilon_2)) + \int_{\theta_1^*(\epsilon_2)}^{\overline{\theta}} \theta dF(\theta)\right\} \tag{19}$$

$$\theta_1^*(\epsilon_1) = \delta(1-\beta) + \beta p \left[\theta_1^*(\epsilon_1) F(\theta_1^*(\epsilon_1)) + \int_{\theta_1^*(\epsilon_1)}^{\overline{\theta}} \theta dF(\theta) \right] + \frac{\epsilon_2}{\epsilon_1} \frac{q}{r} \left[\theta_1^*(\epsilon_2) - \delta(1-\beta) \right] \quad (20)$$

Proof sketch. the proof uses indifference arguments, similar to those used in the proof of theorem 13 (See details in appendix A5). ■

There are several remarkable facts. First, in eq. (19), the expression is analogous to the one given in theorem 13, with a discount factor of the future βr rather than β. Intuitively, the sicker the patient, the higher the probability of being dead in the next period $(1-r)$, the lower the discount factor of the future (the higher the agent's impatience), and, by theorem 14, the lower the selectiveness of the patient. Second, when $q = 0$ or when $\epsilon_2 = 0$, the Markov chain degenerates into two states (alive or death) so the expression (20) reduces to (19): if the probability of passing by intermediate health condition is 0, or if the probability of being dead in the next period is q, the agent makes her decisions as she does when she is sicker. Third, the term $\frac{\epsilon_2}{\epsilon_1}$ captures that, the closer the utility of the patient across being-alive states, the lesser the loss of the patient by transit from state ϵ_1 to ϵ_2. Finally, we go back to recall that the fraction $\frac{q}{r}$ is defined as,

$$\frac{q}{r} = \frac{Prob[\epsilon' = \epsilon_2 | \epsilon = \epsilon_1]}{Prob[\epsilon' = \epsilon_2 | \epsilon = \epsilon_2]}$$

so it corresponds to the probability of transiting to a sicker health state being healthier in the present, pondered by the probability of not dying when she arrives to such state. Clearly, the term $\frac{\epsilon_2}{\epsilon_1}\frac{q}{r}$ captures the dynamics of the patient's health conditions. When r is lower than q, the relative measure of probabilities is greater (and so the probability of being dead in future periods), and the risk of continuing the search rather than stopping it when the patient is less sick increases, implying a lower threshold for this state.

4.1 Simulation

It follows from equations (19) and (20) that $\theta_1^*(\epsilon_2) = \lambda(\beta, \delta, r, \overline{\theta})$ and $\theta_1^*(\epsilon_1) = \varphi(\beta, \delta, p, q, r, \overline{\theta}, \epsilon_1, \epsilon_2)$. To illustrate how the transition probabilities as well as the relative valuation across states affect patient's choice when she is less sick, we have run a simulation of the thresholds given by proposition 18 assuming $\theta \sim N(2, 0.5)$, $\delta = 1$, $\beta = 0.5$ and $\Theta = [0, 4]$, which is consistent with the KDRI quality index. The results for the health state ϵ_1 are shown in the Appendix A6. The plot must be read as follows: for given values of the probabilities of remain in the same state in the next period (p and q), each subplot corresponds to a contour plot of the optimal threshold given by the equation (20), taking as independent variables the ratio $\frac{\epsilon_2}{\epsilon_1}$, i.e. the relation of utilities across states,[14] and the probability of death being sicker, r. The lighter the color, the higher the minimum acceptable organ quality in the healthier state.

For a given probability p, the higher the probability of being alive in the next period $p + q$, the more selective the candidates. Moreover, such selectiveness increases as $\frac{\epsilon_2}{\epsilon_1}$ tends to 1, i.e. when the utility across states tends to be the same. For constant values of p, q, the impact of r is marginal, only relevant when $r \to 1$ and for those relations between ϵ_1 and ϵ_2 such that $\epsilon_2 \simeq \epsilon_1$. For a constant value of $\frac{\epsilon_2}{\epsilon_1}$, when the probability of dying in the sicker state is lower (r is greater), a marginal increase in r boosts the certainty of the healthy patient of remain alive, increasing the minimum quality she accepts when she is less sick.

5 Concluding Remarks

In this paper, we propose a new modelling approach to study the problem of patient choice in waiting systems for organ transplant. Our proposal is based on the theoretical developments of search theory as well as in the novel literature in dynamic matching problems. By incorporating these tools in our sequential and stochastic environment, we have

[14] In the simulation we assume that $\frac{\epsilon_2}{\epsilon_1} \in (0, 1)$, so $\epsilon_1 \geq \epsilon_2$.

proved existence and uniqueness of equilibrium, derived equilibrium closed forms, and done comparative statics in a very general and intuitive way.

In our opinion, the model developed is appealing because it simplifies considerably the derivation of general analytic results. Future tasks are to incorporate in our framework previous results mentioned above, to introduce arrival processes for organs and patients as well as to derive equilibrium closed forms for any patient in the waiting list with evolving health conditions. The last task implies the use of techniques of non-stationary dynamic programming, while the two first are interesting challenges to the generalization power of our approach. From a market design perspective, however, we are far to obtaining an acceptable organ allocation system. Following Su and Zenios [18], it "remains one of the modern medicine's most difficult problems".

Some of our results lead to recommendations for public policy designers. With patient autonomy, an increase in organ supply *does not necessarily* imply a better performance of the market in terms of social welfare defined as the number of transplants conducted. On the contrary, without the right design of allocation policies, it may result in an increase of organ wastage. Given the restriction of ischemia time, the following question arises: is it viable to design an allocation policy which (taking into account the selectiveness of patients leading the waiting list) offers lower quality organs to those patients whose positions in the waiting list make them willing to accept?

Appendix

A1. Proof of Proposition 11

Proof. Note that (9), the first part of proposition 10, is a standard search problem (McCall and McCall [11]). The Bellman operator associated to the problem stated in proposition (10) is:

$$(Tf)(1,\theta^t) = \max_{a,d} \left\{ \frac{\theta^t}{1-\beta}, \delta + \beta E[f(1,\theta^{t+1})] \right\}$$

for $i = 1$, and

$$(Tf)(i, \theta^t) = \begin{cases} \delta + \beta E[f(i-1, \theta^{t+1})], & \text{if for some } j < i, \ \gamma_j(\theta^t) = a, \\ \max_{a,d} \left\{ \frac{\theta^t}{1-\beta}, \delta + \beta E[f(i, \theta^{t+1})] \right\}, & \text{if for all } j < i, \ \gamma_j(\theta^t) = d \end{cases}$$

for $i > 1$. Theorem 3.3. from Stokey and Lucas [17] states Blackwell's sufficient conditions for a contraction. Given that $\theta \in [0, \overline{\theta}]$, $\frac{\theta^t}{1-\beta}$ and $\delta + \beta E[f(i, \theta^{t+1})]$ are bounded, so T maps the space of bounded functions into itself. To prove monotonicity, let us to consider two functions f, g such that $f(1, \theta) < g(1, \theta)$ for all θ. So, the Bellman operator

$$\begin{aligned} (Tf)(1, \theta^t) &= \max_{a,d} \left\{ \frac{\theta^t}{1-\beta}, \delta + \beta E[f(1, \theta^{t+1})] \right\} \\ &\leq \max_{a,d} \left\{ \frac{\theta^t}{1-\beta}, \delta + \beta E[g(1, \theta^{t+1})] \right\} \\ &= (Tg)(1, \theta^t). \end{aligned}$$

To prove discounting

$$\begin{aligned} T(f+a)(1, \theta^t) &= \max_{a,d} \left\{ \frac{\theta^t}{1-\beta}, \delta + \beta E[f(1, \theta^{t+1}) + a] \right\} \\ &\leq \max_{a,d} \left\{ \frac{\theta^t}{1-\beta}, \delta + \beta E[f(1, \theta^{t+1})] \right\} + \beta a \\ &= (Tf)(1, \theta^t) + \beta a. \end{aligned}$$

Now consider the expression (10). Recall the associated operator:

$$(Tf)(i, \theta^t) = \begin{cases} \delta + \beta E[f(i-1, \theta^{t+1})], & \text{if for some } j < i, \ \gamma_j(\theta^t) = a, \\ \max_{a,d} \left\{ \frac{\theta^t}{1-\beta}, \delta + \beta E[f(i, \theta^{t+1})] \right\}, & \text{if for all } j < i, \ \gamma_j(\theta^t) = d \end{cases}$$

And, analogously to the first part of this proof, consider other function such that $f(i, \theta) \leq g(i, \theta)$ for all (i, θ). If some patient $j < i$ accepts, then, it is straightforward monotonicity

$$\begin{aligned} (Tf)(i, \theta^t) &= \delta + \beta E[f(i-1, \theta^{t+1})] \\ &\leq \delta + \beta E[g(i-1, \theta^{t+1})] \\ &= (Tg)(i, \theta^t) \end{aligned}$$

and discounting:

$$\begin{aligned} T(f+a)(i, \theta^t) &= \delta + \beta E[f(i-1, \theta^{t+1}) + a] \\ &\leq \delta + \beta E[f(i-1, \theta^{t+1})] + \beta a \\ &= (Tf)(i, \theta^t) + \beta a. \end{aligned}$$

If all patients j, $1 < j < i$ pass the offer, then the proof is analogous to the proof for $i = 1$.
■

A2. Proof of Theorem 13

Proof. For the patient 1, indifference is given by:

$$\frac{\theta_1^*}{1-\beta} = \delta + \beta E[V(1, \theta^{t+1})] \tag{21}$$

$$\frac{\theta_1^*}{1-\beta} = \delta + \beta \left[\left(\delta + \beta E[V(1,\theta')]\right) F(\theta_1^*) + \int_{\theta_1 *}^{\overline{\theta}} \frac{\theta}{1-\beta} dF(\theta) \right]$$

using eq. (21),

$$\frac{\theta_1^*}{1-\beta} = \delta + \beta \left[\frac{\theta_1^*}{1-\beta} F(\theta_1^*) + \int_{\theta_1^*}^{\overline{\theta}} \frac{\theta}{1-\beta} dF(\theta) \right].$$

For the patient in position i, the critical value is given by:

$$\frac{\theta_i^*}{1-\beta} = \delta + \beta E[V(i, \theta^{t+1})] \tag{22}$$

Now, we calculate $E[V(i, \theta^{t+1})]$. The patient i receives an offer in the next period with probability $F(\theta_{i-1}^*)$. If she receives an offer, she confronts the same optimal stopping problem problem. If does not receive an offer, she receives a payoff of δ for this period, advancing one position in the waiting list. Formally:

$$E[V(i,\theta')] = \left(1 - F(\theta_{i-1}^*)\right) \left(\delta + \beta E[V(i-1, \theta')]\right) + F(\theta_{i-1}^*) \mathbf{E}\left[V(i,\theta') | \theta < \theta_{i-1}^*\right]$$

$$E[V(i,\theta')] = \left(1 - F(\theta_{i-1}^*)\right) \left(\delta + \beta E[V(i-1, \theta')]\right) + \left[F(\theta_i^*) \frac{\theta_i^*}{1-\beta} + \int_{\theta_i^*}^{\theta_{i-1}^*} \frac{\theta}{1-\beta} dF(\theta) \right]$$

and evaluating (22) for $i-1$ and replacing in the last expression, we obtain

$$E[V(i,\theta')] = \left(1 - F(\theta_{i-1}^*)\right) \frac{\theta_{i-1}^*}{1-\beta} + \left[F(\theta_i^*) \frac{\theta_i^*}{1-\beta} + \int_{\theta_i^*}^{\theta_{i-1}^*} \frac{\theta}{1-\beta} dF(\theta) \right].$$

Replacing this expression in (22) we finally obtain:

$$\frac{\theta_i^*}{1-\beta} = \delta + \beta \left\{ \left(1 - F(\theta_{i-1}^*)\right) \frac{\theta_{i-1}^*}{1-\beta} + \left[F(\theta_i^*) \frac{\theta_i^*}{1-\beta} + \int_{\theta_i^*}^{\theta_{i-1}^*} \frac{\theta}{1-\beta} dF(\theta) \right] \right\}$$

■

A3. Proof of Theorem 14

We use mathematical induction to verify this theorem. We show that (14), (15) and (16) hold for $i = 1$, assume that these expressions hold for $i - 1$ and then prove that hold for i.

(14). We consider $i = 1$, and derive implicity the optimal strategy respect to δ:

$$\frac{d\theta_1^*}{d\delta} = 1 - \beta + \beta \left[F(\theta_1^*)\frac{d\theta_1^*}{d\delta} + \theta_1^* \frac{dF(\theta_1^*)}{d\delta} + \frac{d}{d\delta}\left(\int_{\theta_1^*}^{\overline{\theta}} \theta f(\theta) d\theta \right) \right]$$

By assumption, $f(\theta)$ is differentiable, so we can apply the Leibniz's Rule in the last term of the right hand. In addition, by the chain rule $\frac{dF(\theta_1^*)}{d\delta} = \frac{dF(\theta_1^*)}{d\theta_1^*}\frac{d\theta_1^*}{d\delta} = f(\theta_1^*)\frac{d\theta_1^*}{d\delta}$, resulting in

$$\begin{aligned}
\frac{d\theta_1^*}{d\delta} &= 1 - \beta + \beta \left[F(\theta_1^*)\frac{d\theta_1^*}{d\delta} + \theta_1^* f(\theta_1^*)\frac{\theta_1^*}{d\delta} - \theta_1^* f(\theta_1^*)\frac{\theta_1^*}{d\delta} \right] \\
&= 1 - \beta + \beta F(\theta_1^*)\frac{d\theta_1^*}{d\delta} \\
&= \frac{1-\beta}{1-\beta F(\theta_1^*)} \\
&> 0,
\end{aligned}$$

where the inequality follows from the fact that $\beta \in (0,1)$ and $F(\cdot) \leq 1$. Now we consider the case of i, and derive implicitly the optimal strategy with respect to δ. It yields to

$$\begin{aligned}
\frac{d\theta_i^*}{d\delta} = 1 - \beta + \beta &\left[-\frac{dF(\theta_{i-1}^*)}{d\delta}\theta_{i-1}^* + \left(1 - F(\theta_{i-1}^*)\right)\frac{d\theta_{i-1}^*}{d\beta} + F(\theta_i^*)\frac{d\theta_i^*}{d\delta} + \theta_i^* \frac{dF(\theta_i^*)}{d\delta} \right. \\
&\left. + \frac{d}{d\delta}\left(\int_{\theta_i^*}^{\theta_{i-1}^*} \theta f(\theta) d\theta \right) \right]
\end{aligned}$$

Now, by applying the Leibniz's Rule in the last term of the right hand, and substituting $\frac{dF(\theta_k^*)}{d\delta} = f(\theta_k^*)\frac{d\theta_k^*}{d\delta}$ for $k = i-1, i$, we obtain

$$\begin{aligned}
\frac{d\theta_i^*}{d\delta} &= 1 - \beta + \beta \left[-f(\theta_{i-1}^*)\frac{d\theta_{i-1}^*}{d\delta}\theta_{i-1}^* + \left(1 - F(\theta_{i-1}^*)\right)\frac{d\theta_{i-1}^*}{d\delta} + F(\theta_i^*)\frac{d\theta_i^*}{d\delta} + \theta_i^* f(\theta_i^*)\frac{d\theta_i^*}{d\delta} \right. \\
&\left. + \theta_{i-1}^* f(\theta_{i-1}^*)\frac{d\theta_{i-1}^*}{d\delta} - \theta_i^* f(\theta_i^*)\frac{d\theta_i^*}{d\delta} \right] \\
&= 1 - \beta + \beta \left[\left(1 - F(\theta_{i-1}^*)\right)\frac{d\theta_{i-1}^*}{d\delta} + F(\theta_i^*)\frac{d\theta_i^*}{d\delta} \right] \\
&= \frac{1}{1-\beta F(\theta_i^*)}\left[(1-\beta) + \beta\left(1 - F(\theta_{i-1}^*)\right)\frac{d\theta_{i-1}^*}{d\delta} \right] \\
&> 0.
\end{aligned}$$

The last step follows by using the inductive hypothesis $\frac{d\theta_{i-1}^*}{d\delta} > 0$ and the facts that $\beta \in (0,1)$ and $F(\cdot) \leq 1$. ■

(15). As before, we start by obtaining the derivative of optimal strategy for $i = 1$, in this case respect to $\overline{\theta}$. We again use the Leibniz´s and chain rules, which yields to:

$$\begin{aligned}\frac{d\theta_1^*}{d\overline{\theta}} =& \beta\left[F(\theta_1^*)\frac{d\theta_1^*}{d\overline{\theta}} + \overline{\theta}f(\overline{\theta})\right]\\ =& \frac{\beta\overline{\theta}f(\overline{\theta})}{1-\beta F(\theta_i^*)}\\ >& \, 0.\end{aligned}$$

For i, analogously to the preceding proof, we obtain the derivative of θ_i^* respect to $\overline{\theta}$ implicitly, from the corresponding expression. Since we can use the Leibniz Rule as well as $\frac{dF(\theta_k^*)}{d\overline{\theta}} = f(\theta_k^*)\frac{d\theta_k^*}{d\overline{\theta}}$ for $k = i-1, i$, it follows that

$$\begin{aligned}\frac{d\theta_i^*}{d\overline{\theta}} &= \beta\left[\left(1 - F(\theta_{i-1}^*)\right)\frac{d\theta_{i-1}^*}{d\overline{\theta}} + F(\theta_i^*)\frac{d\theta_i^*}{d\overline{\theta}}\right]\\ &= \frac{\beta\left(1-F(\theta_{i-1}^*)\right)}{1-\beta F(\theta_i^*)}\frac{d\theta_{i-1}^*}{d\delta}\\ &\geq 0,\end{aligned}$$

by the induction hypothesis. ■

(16). Considering $i = 1$, and deriving the optimal strategy respect to β:

$$\frac{d\theta_1^*}{d\beta} = -\delta + \theta_1^*F(\theta_1^*) + \int_{\theta_1^*}^{\overline{\theta}}\theta f(\theta)d\theta + \beta\left[F(\theta_1^*)\frac{d\theta_1^*}{d\beta} + \theta_1^*\frac{dF(\theta_1^*)}{d\beta} + \frac{d}{d\beta}\left(\int_{\theta_1^*}^{\overline{\theta}}\theta f(\theta)d\theta\right)\right]$$

By assumption, $f(\theta)$ is differentiable, we can apply the Leibniz's Rule in the last term of the right hand. In addition, by the chain rule $\frac{dF(\theta_1^*)}{d\beta} = \frac{dF(\theta_1^*)}{d\theta_1^*}\frac{d\theta_1^*}{d\beta} = f(\theta_1^*)\frac{d\theta_1^*}{d\beta}$, so

$$\begin{aligned}\frac{d\theta_1^*}{d\beta} &= -\delta + \theta_1^*F(\theta_1^*) + \int_{\theta_1^*}^{\overline{\theta}}\theta f(\theta)d\theta + \beta\left[F(\theta_1^*)\frac{d\theta_1^*}{d\beta} + \theta_1^*f(\theta_1^*)\frac{d\theta_1^*}{d\beta} - \theta_1^*f(\theta_1^*)\frac{d\theta_1^*}{d\beta}\right]\\ &= -\delta + \theta_1^*F(\theta_1^*) + \int_{\theta_1^*}^{\overline{\theta}}\theta f(\theta)d\theta + \beta F(\theta_1^*)\frac{d\theta_1^*}{d\beta}.\end{aligned}$$

Notice that, it follows from theorem 13 that:

$$\frac{\theta_1^* - \delta}{\beta} = -\delta + \theta_1^*F(\theta_1^*) + \int_{\theta_1^*}^{\overline{\theta}}\theta f(\theta)d\theta$$

so, collecting terms, our expression reduces to

$$\frac{d\theta_1^*}{d\beta} = \frac{\theta_1^* - \delta}{\beta\left(1 - \beta F(\theta_1^*)\right)}.$$

In the right hand term, $\beta\left(1-\beta F(\theta_1^*)\right) > 0$. Optimality of θ_1^* implies that $\theta_1^* \geq \delta$. To show it, suppose the opposite, i.e. $\theta_1^* < \delta$. It follows that $\frac{\theta_1^*}{1-\beta} < \frac{\delta}{1-\beta}$, a contradiction with the fact that choose θ_1^* as the reservation type is an optimal strategy. It results in:

$$\frac{d\theta_1^*}{d\beta} \geq 0.$$

Now, for the case of i, derive implicitly the expression for the optimal strategy, applying Leibniz's rule we obtain:

$$\frac{d\theta_i^*}{d\beta} = -\delta + \left(1 - F(\theta_{i-1}^*)\right)\theta_{i-1}^* + \theta_i^* F(\theta_i^*) + \int_{\theta_i^*}^{\theta_{i-1}^*} \theta f(\theta) d\theta + \beta \left[\left(1 - F(\theta_{i-1}^*)\right) \frac{d\theta_{i-1}^*}{d\beta} + F(\theta_i^*) \frac{d\theta_i^*}{d\beta} \right]$$

Collecting terms,

$$(1 - \beta F(\theta_i^*)) \frac{d\theta_i^*}{d\beta} = -\delta + \left(1 - F(\theta_{i-1}^*)\right)\theta_{i-1}^* + \theta_i^* F(\theta_i^*) + \int_{\theta_i^*}^{\theta_{i-1}^*} \theta f(\theta) d\theta + \beta \left(1 - F(\theta_{i-1}^*)\right) \frac{d\theta_{i-1}^*}{d\beta}$$

and, by the definition of θ_i^*,

$$(1 - \beta F(\theta_i^*)) \frac{d\theta_i^*}{d\beta} = \frac{\theta_i^* - \delta}{\beta} + \beta \left(1 - F(\theta_{i-1}^*)\right) \frac{d\theta_{i-1}^*}{d\beta}$$

The inductive assumption implies that $\frac{d\theta_{i-1}^*}{d\beta} \geq 0$, so the second term of the right hand is no negative. By optimality of θ_i^*, it has to be the case that $\theta_i^* \geq \delta$, so we have

$$\frac{d\theta_i^*}{d\beta} \geq 0,$$

completing the proof. ■

A4. Proof of Theorem 15

Proof. Under the organ distribution $F(\cdot)$, the optimal threshold for $i = 1$, given by theorem 13,

$$\theta_{1,F}^* = \delta(1-\beta) + \beta \left[\theta_{1,F}^* F(\theta_{1,F}^*) + \int_{\theta_{1,F}^*}^{\overline{\theta}} \theta dF(\theta) \right]$$

can be rewritten as

$$\frac{1}{\beta}\left(\theta_{1,F}^* - \delta(1-\beta)\right) = \int_0^{\theta_{1,F}^*} \theta_{1,F}^* dF(\theta) + \int_{\theta_{1,F}^*}^{\overline{\theta}} \theta dF(\theta)$$

i.e.,

$$\begin{aligned}\frac{1}{\beta}\left(\theta^*_{1,F} - \delta(1-\beta)\right) &= \int_0^{\theta^*_{1,F}} \theta^*_{1,F} dF(\theta) + \int_{\theta^*_{1,F}}^{\overline{\theta}} \theta dF(\theta) + \int_0^{\theta^*_{1,F}} \theta dF(\theta) - \int_0^{\theta^*_{1,F}} \theta dF(\theta) \\ &= \int_0^{\overline{\theta}} \theta dF(\theta) - \int_0^{\theta^*_{1,F}} \theta dF(\theta) + \int_0^{\theta^*_{1,F}} \theta^*_{1,F} dF(\theta) \\ &= \int_0^{\overline{\theta}} \theta dF(\theta) - \int_0^{\theta^*_{1,F}} (\theta - \theta^*_{1,F}) dF(\theta).\end{aligned}$$

Integrating by parts in the last term on the right side, with $dv = dF(\theta)$ and $u = \theta - \theta^*_{1,F}$, we obtain:

$$\begin{aligned}\frac{1}{\beta}\left(\theta^*_{1,F} - \delta(1-\beta)\right) &= \int_0^{\overline{\theta}} \theta dF(\theta) - \left[(\theta - \theta^*_{1,F})F(\theta)\Big|_0^{\theta^*_{1,F}} - \int_0^{\theta^*_{1,F}} F(\theta) d\theta \right] \\ &= \int_0^{\overline{\theta}} \theta dF(\theta) + \int_0^{\theta^*_{1,F}} F(\theta) d\theta.\end{aligned} \tag{23}$$

Similarly, when the organ distribution is G, the optimal thereshold $\theta^*_{1,G}$ satisfies,

$$\frac{1}{\beta}\left(\theta^*_{1,G} - \delta(1-\beta)\right) = \int_0^{\overline{\theta}} \theta dG(\theta) + \int_0^{\theta^*_{1,G}} G(\theta) d\theta. \tag{24}$$

We define the functions $h_G(t) = \int_0^{\overline{\theta}} \theta dG(\theta) + \int_0^t G(\theta) d\theta$, and $l(t) = \frac{1}{\beta}\left(t - \delta(1-\beta)\right)$. Notice that $l(\cdot)$ is injective and strictly increasing in t. By conditions 1 and 2, it has to be the case that, $h_G(t) \geq h_F(t)$ for all $t \in [0, \overline{\theta}]$. Thus, by the definition of $\theta^*_{1,F}$ and $\theta^*_{1,G}$ in (23)-(24), and because $l(\cdot)$ is strictly increasing, it follows that $l(\theta^*_{1,G}) \geq (\theta^*_{1,F})$, or,

$$\theta^*_{1,G} \geq \theta^*_{1,F}.$$

■

A5. Proof of Proposition 18

Proof. In any state, indifference is given by

$$\frac{\epsilon\theta}{1-\beta} = \epsilon\delta + \beta E_{\epsilon'|\epsilon}[V(1, \theta', \epsilon')] \tag{25}$$

We begin with the sicker state. In such case, indifference is given by:

$$\frac{\epsilon_2\theta}{1-\beta} = \epsilon_2\delta + \beta E_{\epsilon'|\epsilon_2}[V(1, \theta', \epsilon')] \tag{26}$$

where now expectations are conditional on being in the health status ϵ_2 in the present period.

So,

$$\frac{\epsilon_2\theta_1^*(\epsilon_2)}{1-\beta} = \epsilon_2\delta + \beta r\left\{\left(\epsilon_2\delta + \beta E_{\epsilon'|\epsilon_2}[V(1,\theta',\epsilon')]\right)F(\theta_1^*(\epsilon_2)) + \int_{\theta_1^*(\epsilon_2)}^{\bar{\theta}}\frac{\epsilon_2\theta}{1-\beta}dF(\theta)\right\} \tag{27}$$

and, by eq. (26) it reduces to

$$\frac{\epsilon_2\theta_1^*(\epsilon_2)}{1-\beta} = \epsilon_2\delta + \beta r\left\{\frac{\epsilon_2\theta_1^*(\epsilon_2)}{1-\beta}F(\theta_1^*(\epsilon_2)) + \int_{\theta_1^*(\epsilon_2)}^{\bar{\theta}}\frac{\epsilon_2\theta}{1-\beta}dF(\theta)\right\} \tag{28}$$

so,

$$\frac{\theta_1^*(\epsilon_2)}{1-\beta} = \delta + \beta r\left\{\frac{\theta_1^*(\epsilon_2)}{1-\beta}F(\theta_1^*(\epsilon_2)) + \int_{\theta_1^*(\epsilon_2)}^{\bar{\theta}}\frac{\theta}{1-\beta}dF(\theta)\right\} \tag{29}$$

Next, we consider the optimal reservation type for ϵ_1, $\theta_1^*(\epsilon_1)$. In such case, indifference is given by:

$$\frac{\epsilon_1\theta_1^*(\epsilon_1)}{1-\beta} = \epsilon_1\delta + \beta E_{\epsilon'|\epsilon_1}[V(1,\theta',\epsilon')] \tag{30}$$

Recall that the expectation is taken over both, organ's type distribution and health conditions. Being in the present in the better health state, with probability p the patient will continue in this condition in the next period, so the total discounted reward from period $t+1$ from now on, will be, if she continues the search:

$$\epsilon_1\delta + \beta E_{\epsilon'|\epsilon_1}[V[1,\theta^{t+1},\epsilon']]$$

And, if stops,

$$\frac{\epsilon_1\theta^t}{1-\beta}$$

On the other hand, with probability q, she will be in state ϵ_2 in the next period, and the total discounted reward in such case is:

$$\epsilon_2\delta + \beta E_{\epsilon'|\epsilon_2}[V[1,\theta',\epsilon']]$$

So, replacing these expressions in (30):

$$\begin{aligned}\frac{\epsilon_1\theta_1^*(\epsilon_1)}{1-\beta} =& \epsilon_1\delta + \beta\left\{p\left[\left(\epsilon_1\delta + \beta E_{\epsilon'|\epsilon_1}[V[1,\theta',\epsilon']]\right)F(\theta_1^*(\epsilon_1)) + \int_{\theta_1^*(\epsilon_1)}^{\bar{\theta}}\frac{\epsilon_1\theta}{1-\beta}dF(\theta)\right]\right.\\ &\left. + q\left[\left(\epsilon_2\delta + \beta E_{\epsilon'|\epsilon_2}[V[1,\theta',\epsilon']]\right)F(\theta_2^*(\epsilon_2)) + \int_{\theta_2^*(\epsilon_2)}^{\bar{\theta}}\frac{\epsilon_2\theta}{1-\beta}dF(\theta)\right]\right\}\end{aligned}$$

and using the definitions for reservation thresholds, (26)-(30),

$$\begin{aligned}\frac{\epsilon_1\theta_1^*(\epsilon_1)}{1-\beta} =&\epsilon_1\delta+\beta\left\{p\left[\frac{\epsilon_1\theta_1^*(\epsilon_1)}{1-\beta}F(\theta_1^*(\epsilon_1))+\int_{\theta_1^*(\epsilon_1)}^{\overline{\theta}}\frac{\epsilon_1\theta}{1-\beta}dF(\theta)\right]\right.\\ &\left.+q\left[\frac{\epsilon_2\theta_1^*(\epsilon_2)}{1-\beta}F(\theta_2^*(\epsilon_2))+\int_{\theta_2^*(\epsilon_2)}^{\overline{\theta}}\frac{\epsilon_2\theta}{1-\beta}dF(\theta)\right]\right\}\end{aligned}$$

and from (29)

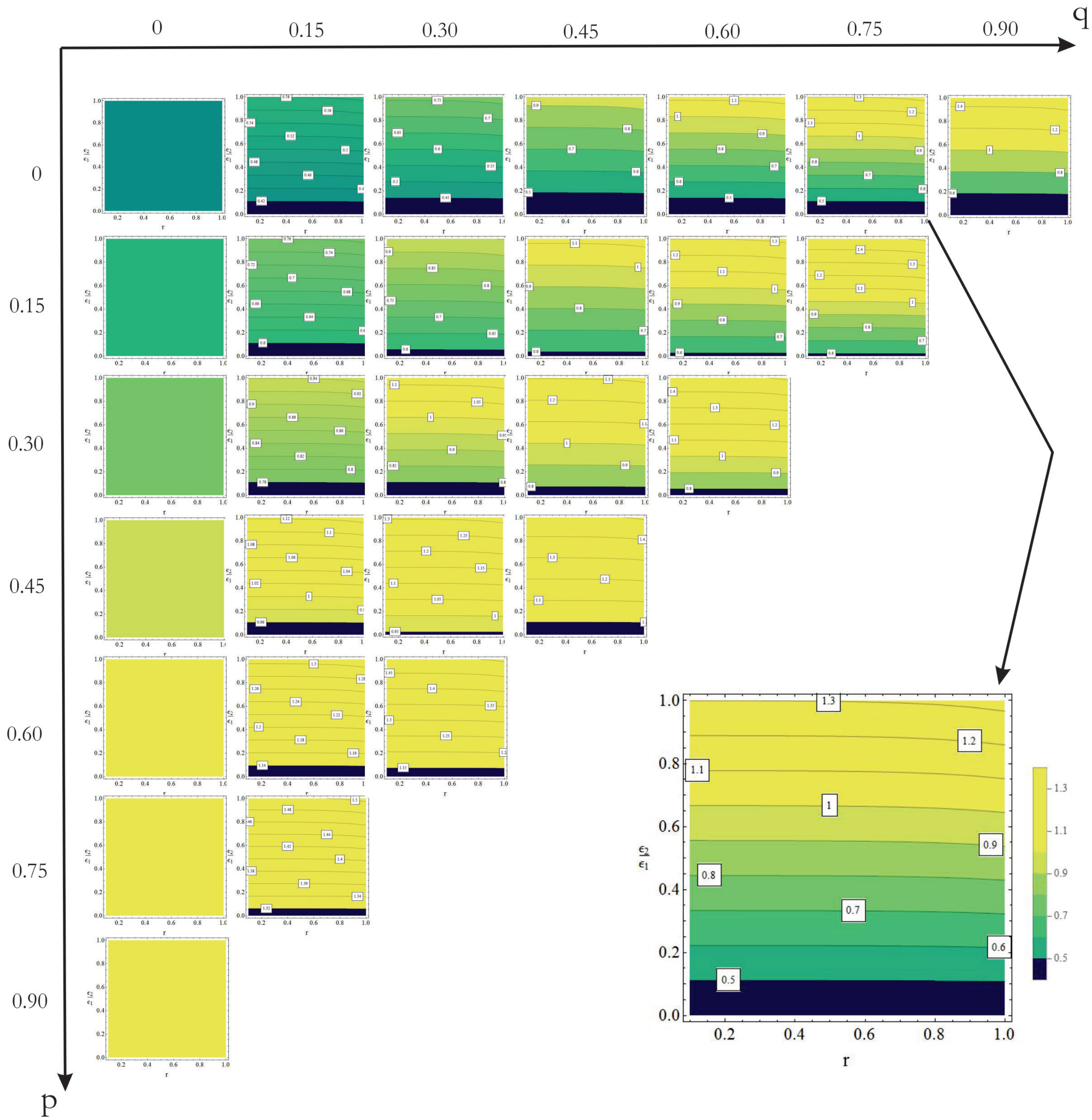
$$\frac{\epsilon_2\theta_1^*(\epsilon_2)}{1-\beta}F(\theta_2^*(\epsilon_2))+\int_{\theta_2^*(\epsilon_2)}^{\overline{\theta}}\frac{\epsilon_2\theta}{1-\beta}dF(\theta)=\frac{\epsilon_2}{\beta r}\left[\frac{\theta_1^*(\epsilon_2)}{1-\beta}-\delta\right], \tag{31}$$

so, finally,

$$\frac{\theta_1^*(\epsilon_1)}{1-\beta} =\delta+\beta p\left[\frac{\theta_1^*(\epsilon_1)}{1-\beta}F(\theta_1^*(\epsilon_1))+\int_{\theta_1^*(\epsilon_1)}^{\overline{\theta}}\frac{\theta}{1-\beta}dF(\theta)\right]+\frac{\epsilon_2}{\epsilon_1}\frac{q}{r}\left[\frac{\theta_1^*(\epsilon_2)}{1-\beta}-\delta\right].$$

■

A6. Simulation



A7. A Note on Organ Quality

Organ quality is a fundamental variable in the context of kidney transplant. The first attempt to classify qualities of deceased donor kidneys dates to 2002 (Panduranga and Ojo [12]). It groups organs in two sets: Standard-Criteria Donor (SCD) and Expanded-Criteria Donor (ECD). The distinction is based on variables that increased the risk of graft failure, so an ECD kidney has 70 percent risk of need for dialysis a week after transplantation, compared to a SCD kidney. The ideal SCD is a man aged 35 with no history of diabetes or hypertension who died in a vehicle accident. Projected average added-life is 10 years for a SCD while only 5.1 years for an ECD kidney (Panduranga and Ojo [12]). However, an ECD recipient has improved survival compared with patients that remain in dialysis, and higher health-related quality of life (QOL).

A parallel classification was developed considering the mechanism of cessation of vital functions: Donation after Brain Death (DBD) describes a "patient who had primary brain death in whom cardiac circulation and respiration remain intact or are maintained by medical measures" (Panduraga and Ojo [12], 1828). Donation after Cardiac Death stands for a donor who does not meet the DBD and is classified in two subcategories: controlled DCD and uncontrolled DCD. There is a 42 to 51 percent of risk for graft failure in a DCD compared to 24 percent of a BDC, although there is no significative difference in 5 years patient survival.

The combination of both criteria leads to the most used method to sort kidneys. Nevertheless, OPTN and UNOS (United Network for Organ Sharing) developed recently the Kidney Donor Profile Index (KDPI), a continuous index of quality for organs from deceased donors. It is based on the Kidney Donor Risk Index (KDRI), an estimate of the relative risk of graft failure compared to the median, which incorporates 10 donor factors: age, height, weight, ethnicity, history of hypertension, history of diabetes, cause of death, serum creatine, Hepatitis C and VIH status, and DCD status.[15]

[15] OPTN-UNOS. A Guide to Calculating and Interpreting the Kidney Donor Profile Index (KDPI) retrieved from http://optn.transplant.hrsa.gov

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