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REAL BALANCE EFFECTS AND MONETARY POLICY UNDER INFORMALITY

HUMBERTO MARTÍNEZ GARCÍA

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ASESOR:

STEPHEN MCKNIGHT

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Declaration of Authorship

I, Humberto MARTÍNEZ GARCÍA, declare that this thesis titled, 'Real Balance Effects and Monetary Policy Under Informality' and the work presented in it is my own. I confirm that this work submitted for assessment is my own and is expressed in my own words. Any uses made within it of the works of other authors in any form (e.g., ideas, equations, figures, text, tables, programs) are properly acknowledged at any point of their use. A list of the references employed is included.

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"It's the wrong question to ask if something is a good model or a bad model; something could be a good model for one question and a bad model for another question simultaneously."

Peter A. Diamond

Abstract

A two sector formal-informal economy with real balance effects is modeled. The informal sector behaves perfectly competitive and the formal sector features a monopolistic competitive goods market and a search and matching labor market. In this context the model is calibrated to match the Mexican economy and the dynamic responses to five shocks are analyzed. It is found that real balance effects have substantial implications after a formal productivity shock and mixed implications after demand and monetary shocks. At the same time, it is found that the transmission of informal productivity and monetary policy shocks have no relevant differences with and without real balances effects.

The robustness of the model is tested via a sensitivity analysis which suggests that for empirically plausibly calibrations, real balance effects will always exist. Importantly, a key feature of the model is the size of the informal sector. The greater its size, the greater and positive real balance effects are on the sector and the greater and negative become real balance effects on the formal sector. Therefore, these findings strongly suggest that real balance effects matter under informality and should be taken into account by the central bank when deciding its monetary policy.

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Symbols

χ	degree of non-separability between consumption and real money balances
σ	intertemporal elasticity of substitution in consumption
σ_m	intertemporal elasticity of substitution in real balances
σ_n	inverse of the labor supply elasticity
η_c	consumption elasticity of money demand
η_r	interest rate semi-elasticity of money demand
υ	elasticity of $u_m(\cdot)$ w.r.t. changes in the level of real expenditure
β	discount factor
ϑ	matching elasticity
δ	rate of destruction of formal jobs
τ	substitutability between formal and informal goods
α	consumption preference for formal goods
ω	proportion of formal firms that can't change prices at period t
$\mathfrak{B}$	bargaining power of formal workers
c	real vacancy cost
μ_π	the inflation response coefficient of the central bank
μ_y	the output response coefficient of the central bank
κ	output elasticity of real marginal cost
$\digamma$	share of formal employment
ς	formal consumption-output ratio
Θ	$\equiv \frac{\overline{u_c} w^i}{\overline{v_n}}$
Ξ	$\equiv \frac{\overline{u_c} w^f}{\overline{u_c} w^f - \overline{v_n}}$
$\overline{u_c}$	partial derivative of $u(\cdot)$ w.r.t. consumption evaluated at the steady state
$\overline{u_{cc}}$	partial derivative of $u_c(\cdot)$ w.r.t. consumption evaluated at the steady state
$\overline{u_{cm}}$	partial derivative of $u_c(\cdot)$ w.r.t. real balances evaluated at the steady state
$\overline{u_m}$	partial derivative of $u(\cdot)$ w.r.t. real balances evaluated at the steady state
$\overline{u_{mc}}$	partial derivative of $u_m(\cdot)$ w.r.t. consumption evaluated at the steady state

$\overline{u_{mm}}$	partial derivative of $u_m(\cdot)$ w.r.t. real balances evaluated at the steady state
$\overline{v_n}$	derivative of $v(\cdot)$ w.r.t. employment evaluated at the steady state
$\overline{v_{nn}}$	derivative of $v_n(\cdot)$ w.r.t. employment evaluated at the steady state

Dedicated to my parents, Guillermina García & Felipe Martínez

Chapter 1

Introduction

Informality is a central problem in developing economies since it accounts for a rather relevant part of the aggregate output in these countries. Leyva and Urrutia (2018) document, for example, that on average in emerging economies the informal sector represents between 34 and 42% of the GDP, while the same figure for developed economies accounts roughly for 15% of their GDP. Therefore, the informal sector in developing economies represents between two and three times the size of the sector in developed economies.

It should be clear that when a sector in the economy represents almost half of its GDP, factors that affect this sector will have repercussions on the aggregate economy. Additionally, it is very likely that such a big sector influences the properties of business cycles in developing economies. Indeed, Leyva and Urrutia (2018) also showed that fluctuations in developing economies are quite different from those apparent in a developed economy.

Despite that fact, it was not until very recently that economists started to focus their attention on the developing world, and specifically on informality. Furthermore, the literature on the topic formerly had been mainly descriptive and qualitative. However, there have been recent efforts to formalize economic models of business cycles with an informal sector to see its implications. Some of these attempts comes from Samir Bandaogo (2018) and Batini et al. (2011) who modeled two sector economies to look for implications for monetary policy. Meghir et al. (2015) and Castillo B. and Montoro (2012) who using search and matching models get some interesting insights about the interactions between formal and informal labor in the market.

When it comes to Mexico, the situation is not better in terms of available literature of business cycles with an informal sector. Luckily some have considered the Mexican economy interesting enough to write about its informal sector. One paper with many interesting empirical findings was Fernández and Meza (2015). These authors, taking Mexico as a representative developing economy, found the following stylized facts

- The share of the informal employment in Mexico is large
- Informal employment in Mexico is strongly countercyclical
- Informal employment in Mexico is a lagging indicator of the cycle
- The countercyclical nature of informal employment is independent from the phase of the cycle
- Informal employment is more volatile than formal employment.
- Self-employment in Mexico is strongly correlated with most of the other proxies of informality.

However, money is the big absent in those models of monetary policy. But as have been pointed out for many authors previously "money matters", and matters a lot when it comes to monetary policy. For example, McKnight and Mihailov (2015) showed in a MIUF model that money can worse the indeterminacy problem of monetary policy. This paper is an example of the literature on real balance effects, which recently have recover some attention. The term real balance effects refers to the effects of real money holdings on the propensity to consume (Brückner and Schabert, 2006) and it's been a central topic in monetary theory from years, thanks to Pigou.

Real balance effects had been forgot in part because of the influential I*nterest and Prices* by Michael Woodford. But some efforts to reincorporate them into the literature had been done. For example, Schabert and Stoltenberg (2005), Stoltenberg (2012), Ireland (2005) demonstrated that the mere existence of real balance effects, despite its magnitude, have relevant implications for determinacy and other aspects of monetary policy.

But, as far as I know, no paper has been written about the role of real balance effects under informality. That's why I want to address this main question together with two other. Given that, as have been said, the size of the informal sector represents a big part of the GDP in developing economies, does this size plays a role in the transmission of monetary policy and real balance effects? Additionally I wonder what are the implications for the transmission mechanism of monetary policy when the central bank follows a formal-only rule.

The first question seems relevant to me because if it were the case that real balance effects have important implications for monetary policy, then these should be taken into account by the central bank when deciding its monetary policy. The second question justifies itself whereas the third one seems interesting for me because if the central bank is in possibility of following a formal-only rule, or even if the material conditions does not let to follow aggregate measures of the economy for absence of data from the informal sector, a formal-only rule could be a possibility for the central bank to set its monetary policy.

So, in the present work a two sector formal-informal closed economy is modeled in which the informal sector behaves perfectly competitive so that prices and wages are flexible. The formal sector in turn features a monopolistic competitive goods market so that firms set have market power and can set prices. They set prices according to Calvo (1983) while the wage is decided via a process of Nash Wage Bargaining (Nash, 1950) between the formal workers and the formal firms. The more bargaining power the formal workers have the higher the formal wage they will be able to gain.

Additionally, the formal labor market introduces search and matching frictions in the spirit of Mortensen and Pissarides (1994) which yields formal contracts by attracting informal workers to the formal sector. The model economy also features real balances effects via non-separability of the utility function.

Following Fernández and Meza (2015), unemployment is not modeled consistent with the low unemployment rate of the Mexican economy. In this context the monetary authority has to do a choice between a typical monetary policy rule á la Taylor (1993) or a formal only rule which reacts only to the formal side of the economy. Afterwards the model is calibrated to match the Mexican economy and some exogenous shocks are applied to it to see the dynamics it shows: a formal productivity shock, an informal productivity shock, a demand shock, a monetary shock and a monetary policy shock.

The main results can be summarized as follows. First of all, real balance effects are very relevant when a formal productivity shock hits the economy. In face of a monetary shock, real balance effects are only relevant for inflation measures and interest rates. When the model is shocked by a demand perturbation, no quantitatively relevant effects are found, but there are one qualitative result which seems relevant: all IRFs are more smooth under real balance effects. Finally, in face of a monetary policy shock or a informality productivity shock, no relevant effects are caused by real balances.

The reason for those differences comes from the channel through which monetary policy affects employment and consumption under real balance effects. As in McKnight and Mihailov (2015), there are two main channels. The typical aggregate demand channel and one which works through the money demand. Two aspects were found crucial in the behavior of the IRFs. One is the size of the informal sector, the greater it is the greater and positive are real balance effects on this sector while at the same time the greater and negative are real balance effects on formal sector. The other aspect is the parametrization. I argue that for empirically plausible calibrations, real balance effects always exist and therefore should be taken into account by the central bank in deciding its monetary policy. Therefore the analysis of the dynamics strongly suggests that real balances effects matters under informality.

In the reminder of this work the model is specified in Chapter 2, the log-linear version and its calibration are described in Chapter 3, Chapter 4 shows the dynamics of the model and Chapter 5 concludes.

Chapter 2

A Model of Closed Economy with Informality and Real Balance Effects

A two sector formal-informal closed economy is presented here. The informal sector is assumed to be perfect competitive. The formal sector features monopolistic competition and frictions in the labor market. Concretely, formal firms are assumed to set prices according to Calvo (1983) while the wage is decided via a process of Nash Wage Bargaining (Nash, 1950). Additionally, the formal labor market behaves according to a search and matching process in the spirit of (Mortensen and Pissarides, 1994) which yields formal contracts by attracting informal workers to the formal sector.

2.1 Labor Market Dynamics

The economy modeled here has a labor market in which formal and informal labor coexist. Therefore in each period t total labor N_t equals the sum of formal L_t^f and informal L_t^i labor

$$N_t = L_t^f + L_t^i. \tag{2.1}$$

It is assumed that the informal sector behaves in a classical perfect competitive way, therefore it always adjusts in order to keep the informal wage equal to the marginal product of informal labor. However, the existence of a formal sector provides an opportunity for informal workers to look for a higher wage in this other sector. Therefore, there is an interaction between the informal and formal sector in which informal workers are searching formal jobs, and formal firms wants to hire informal workers. For this to be the case, it is assumed that period by period occurs that formal firms have open vacancies and informal workers move in to fill some of them.

It is a key property of the model that vacant jobs do not match instantaneously with the searching workers and then a relationship for these interactions is needed.

The matching function $\mathfrak{M}(\bullet)$ relates vacancies available V_t and informal labor L_t^i and yields a $\mathfrak{M}_t$ number of hirings or successful matches. In this model such relationship is specified as follows

$$\mathfrak{M}_t = \mathfrak{M}(V_t, L_t^i) = \mathfrak{m} V_t^{\vartheta} {L_t^i}^{1-\vartheta} \tag{2.2}$$

where $\vartheta \in (0,1)$ is the matching elasticity and $\mathfrak{m} > 0$ is the matching efficiency coefficient. It is assumed that $\mathfrak{M}_t(\bullet)$ is monotonically rising in both arguments, concave, and homogeneous of degree one.

Now, for the process of searching and matching it is of crucial importance to define the relationship between the number of informal workers and the vacancy positions in formal firms since both firms and workers are interested on it. This relationship gives a measure of the labor market tightness and is defined as

$$\theta_t = \frac{V_t}{L_t^i}. \tag{2.3}$$

Given the tightness of the formal labor market θ_t and the number of hirings in the formal sector $\mathfrak{M}_t$ both firms and workers have information about the probability of filling a vacancy post and finding a job, correspondingly. For the firms, the probability of filling a vacancy is

$$\frac{\mathfrak{M}_t}{V_t} \equiv q(\theta_t) = \mathfrak{m}\theta_t^{\vartheta-1} \tag{2.4}$$

while for the workers, the probability of finding a job in the formal sector is given by

$$\frac{\mathfrak{M}_t}{L_t^i} = \theta_t q(\theta_t) = \mathfrak{m}\theta_t^{\vartheta}. \tag{2.5}$$

Let $\delta \in (0,1)$ be the rate at which formal jobs are destroyed, then the law of motion for the next period formal labor is given by

$$L_{t+1}^f = (1-\delta)L_t^f + \theta_t q(\theta_t) L_t^i. \tag{2.6}$$

2.2 Households

2.2.1 Utility maximization

The representative household chooses real consumption C_t, real money balances $m_t \equiv \frac{M_t}{P_t}$ and total labor supply N_t to maximize its expected discounted utility

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, m_t, N_t; \xi_t), \tag{2.7}$$

where E_0 is the expectations operator conditional on information available at the initial period 0. $\beta \in (0,1)$ is the discount factor and ξ_t is a vector of shocks (to consumption and money which will be specified later), subject to (2.1), (2.6), and the period budget constraint

$$P_t C_t + B_t + M_t \leq R_{t-1} B_{t-1} + W_t^f L_t^f + W_t^i L_t^i + M_{t-1} + \Gamma_t + \Pi_t \tag{2.8}$$

where P_t is the price of the consumption good, M_{t-1} and B_{t-1} are the quantity of money holdings and units of nominal bonds (which pay the gross nominal interest rate R_{t-1}) carried into period t, W_t^f y W_t^i are the formal and informal wages, Γ_t is a lump-sum nominal transfer from the central bank, and Π_t denotes the nominal profits from firm ownership.

Now, following McKnight and Mihailov (2015) let the period utility function be non-separable between consumption and real money balances but additively separable with respect to labor

$$U(C_t, m_t, N_t) \equiv u(C_t, m_t; \xi_t) - v(N_t), \tag{2.9}$$

where $u(\cdot)$ is assumed to be concave and strictly increasing in each argument, with C and m being normal goods, and $v(\cdot)$ is increasing and convex.

The Lagrangean for this problem is

$$\begin{aligned}\mathscr{L} =& E_0 \sum_{t=0}^{\infty} \{\beta^t [u(C_t, m_t; \xi_t) - v(L_t^f + L_t^i) + \\ &+ \lambda_t [R_{t-1} B_{t-1} + W_t^f L_t^f + W_t^i L_t^i + M_{t-1} + \Gamma_t + \Pi_t - P_t C_t - B_t - M_t] + \\ &+ \Psi_t [(1-\delta) L_t^f + \theta_t q(\theta_t) L_t^i - L_{t+1}^f]]\}.\end{aligned} \tag{2.10}$$

The first order conditions are then

$$C_t: \quad \beta^t u_c(C_t, m_t; \xi_t) - \beta^t \lambda_t P_t = 0 \tag{2.11}$$

$$L_t^i: \quad -\beta^t v_n(N_t) + \beta^t \lambda_t W_t^i + \beta^t \Psi_t \theta_t q(\theta_t) = 0 \tag{2.12}$$

$$L_{t+1}^f: \quad -\beta^t \Psi_t - \beta^{t+1} E_t[v_n(N_{t+1})] + \beta^{t+1} E_t[\lambda_{t+1} W_{t+1}^f] + (1-\delta)\beta^{t+1} E_t[\Psi_{t+1}] = 0 \tag{2.13}$$

$$m_t: \quad \beta^t u_m(C_t, m_t; \xi_t) - \beta^t \lambda_t P_t + \beta^{t+1} E_t[\lambda_{t+1}] P_t = 0 \tag{2.14}$$

$$B_t: \quad -\beta^t \lambda_t + \beta^{t+1} E_t[\lambda_{t+1}] R_t = 0. \tag{2.15}$$

Using (2.11) and (2.15) the following Euler equation is obtained

$$\beta E_t \left[\frac{u_c(C_{t+1}, m_{t+1}; \xi_{t+1})}{P_{t+1}} \right] = \frac{1}{R_t} \left[\frac{u_c(C_t, m_t; \xi_t)}{P_t} \right], \tag{2.16}$$

with (2.11) and (2.14) a money demand function can be defined

$$\frac{u_m(C_t, m_t; \xi_t)}{u_c(C_t, m_t; \xi_t)} = \frac{R_t - 1}{R_t}, \tag{2.17}$$

from (2.11) and (2.12) a relationship for the optimal labor supply condition

$$v_n(N_t) = \frac{W_t^i}{P_t} u_c(C_t, m_t; \xi_t) + \Psi_t \theta_t q(\theta_t), \tag{2.18}$$

where the shadow price of a worker in the formal sector

$$\Psi_t = \beta E_t \left[\frac{W_{t+1}^f}{P_{t+1}} u_c(C_{t+1}, m_{t+1}; \xi_{t+1}) - v_n(N_{t+1}) + (1-\delta)\Psi_{t+1} \right]. \tag{2.19}$$

2.2.2 Consumption and price indexes

C_t is a constant elasticity of substitution (CES) composite consumption index *à la* Dixit-Stiglitz defined as

$$C_t \equiv [\alpha^{\frac{1}{\tau}}(C_t^f)^{\frac{\tau-1}{\tau}} + (1-\alpha)^{\frac{1}{\tau}}(C_t^i)^{\frac{\tau-1}{\tau}}] \tag{2.20}$$

where $\tau > 0$ measures the substitutability between formal and informal goods from the point view of the consumer and $\alpha \in (0,1)$ so that households are always consuming a positive proportion of both formal and informal goods. As will be clearer later on, one can see α as representing the proportion that aggregate formal consumption represents in aggregate total consumption.

Specifically households consume C_t^f and C_t^i which in turn are also composite consumption indexes

$$C_t^f \equiv \left[\int_0^1 C_t^f(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{2.21}$$

$$C_t^i \equiv C_t^i(k), \tag{2.22}$$

where $C_t^f(j)$ are the j different varieties of individual formal goods, while $C_t^i(k)$ is the only kind of informal good. $\varepsilon > 1$ governs the elasticity of substitution across varieties in formal goods.

Given any level of expenditure for the individual formal and informal goods respectively, the representative household minimizes the cost of achieving them. Therefore, following the usual cost minimization method the price indexes P_t^f and P_t^i are derived and defined as

$$P_t^f \equiv \left[\int_0^1 P_t^f(j)^{1-\varepsilon} dj\right]^{\frac{1}{1-\varepsilon}}, \tag{2.23}$$

$$P_t^i = P_t^i(k), \tag{2.24}$$

where $P_t^f(j)$ and $P_t^i(k)$ are the prices of the j varieties of formal goods and the only k type of informal good. Notice that, since all informal goods are the same, all of them have the same price $P_t^f(k)$, (*i.e.* there is no price dispersion among informal goods).

From the same cost minimization problems, the following demands for individual goods are implied

$$C_t^f(j) = \left[\frac{P_t^f(j)}{P_t^f}\right]^{-\varepsilon} C_t^f, \tag{2.25}$$

$$C_t^i(k) = C_t^i. \tag{2.26}$$

But the representative household must also allocate optimally its expenditures between formal and informal composite goods indexes. So it chooses sequences of C_t^f and C_t^i in order to minimize

$$P_t^f C_t^f + P_t^i C_t^i \tag{2.27}$$

subject to (2.20).

The first order conditions for this problem are

$$C_t^f: \quad P_t^f - \psi_t C_t^{-1} \alpha^{\frac{1}{\tau}} (C_t^f)^{-\frac{1}{\tau}} = 0, \tag{2.28}$$

$$C_t^i: \quad P_t^i - \psi_t C_t^{-1} (1-\alpha)^{\frac{1}{\tau}} (C_t^i)^{-\frac{1}{\tau}} = 0, \tag{2.29}$$

where ψ_t is the Lagrange multiplier associated with the expenditure restriction. Solving equations (2.28) and (2.29) for C_t^f and C_t^i respectively, and plugging those into the restriction (2.20) yields the overall price index of the modeled economy as defined by

$$P_t \equiv [\alpha (P_t^f)^{1-\tau} + (1-\alpha)(P_t^i)^{1-\tau}]^{\frac{1}{1-\tau}}. \tag{2.30}$$

Finally, using (2.28), (2.29), and (2.30) the following demand functions for C_t^f and C_t^i are obtained

$$C_t^f = \alpha \left[\frac{P_t^f}{P_t} \right]^{-\tau} C_t, \tag{2.31}$$

$$C_t^i = (1-\alpha) \left[\frac{P_t^i}{P_t} \right]^{-\tau} C_t. \tag{2.32}$$

2.3 Firms

2.3.1 Informal firms

By the competitive assumption, the informal firms are price takers. Each k informal firm produces an homogeneous good $Y_t^i(k)$ using only informal labor $L_t^i(k)$ as input whose productivity is $A_t^i > 0$. Then, the production function of the k firm is

$$Y_t^i(k) = A_t^i L_t^i(k), \tag{2.33}$$

where it is assumed that $A_t^i < A_t^f$ reflecting the fact that, at least in this model, informal firms are less productive than formal ones and $a_t^i \equiv \log(A_t^i)$ follows an AR(1) process of the form

$$a_t^i = \rho_a^i a_{t-1}^i + \varepsilon_t^i, \tag{2.34}$$

where $\rho_a^i \in (0,1)$ and $\{\varepsilon_t^i\}$ is a sequence which follows a white noise process such that $\varepsilon_t^i \sim \mathscr{N}(0,(\sigma_a^i)^2)$.

Selling the homogeneous produced good, each k informal firm maximizes its benefits subject to any level of production according to (2.33) which implies that the informal nominal wage W_t^i is the following

$$W_t^i = P_t^i A_t^i. \tag{2.35}$$

2.3.2 Formal firms

Since the formal sector is assumed to be imperfectly competitive then each of the j formal firms produces a differentiated good $Y_t^f(j)$ using only formal labor $L_t^f(j)$ as their input as described by

$$Y_t^f(j) = A_t^f L_t^f(j), \tag{2.36}$$

where A_t^f represents the productivity of firms in formal sector and $a_t^i \equiv \log(A_t^i)$ follows an AR(1) process of the form

$$a_t^f = \rho_a^f a_{t-1}^f + \varepsilon_t^f, \tag{2.37}$$

where $\rho_a^f \in (0,1)$ and $\{\varepsilon_t^f\}$ is a sequence which follows a white noise process such that $\varepsilon_t^f \sim \mathcal{N}(0,(\sigma_a^f)^2)$.

Each formal firm minimizes the real cost of producing any level of output (2.36) by choosing the optimal quantity of formal labor which implies the following real marginal mc_t cost for each formal firm

$$mc_t = \frac{W_t^f}{A_t^f P_t} \tag{2.38}$$

so that real profits for the jth firm can be written as follows

$$\Pi_t^f(j) = \frac{P_t^f(j)}{P_t^f} Y_t^f(j) - mc_t Y_t^f(j). \tag{2.39}$$

Now, by the differentiated products assumption it is the case that formal firms have some market power and can set prices. When faced with the problem of setting an optimal price these firms must take into account the effect of the chosen price through the entire path of future profits and not just on those for today. This considerations are summarized by the following stochastic discount factor

$$\Delta_{t,t+s} \equiv \beta^s \left(\frac{u_c(C_{t+s}, m_{t+s}; \xi_{t+s})}{u_c(C_t, m_t; \xi_t)} \right). \tag{2.40}$$

To continue, it is convenient to follow Calvo (1983) and assume that in each period a fraction ω of formal firms are not able to change price and has to stick to the previous period price. This implies that there is a probability ω^s that a formal firm will be stuck with the price set in period t by s periods. Then the pricing decision of the jth formal firm is

$$\max_{\{P_t^f(j)\}} E_t \left\{ \sum_{s=0}^{\infty} \omega^s \Delta_{t,t+s} \left[\frac{P_t^f(j)}{P_{t+s}^f} Y_{t+s|t}^f(j) - mc_{t+s|t} Y_{t+s|t}^f(j) \right] \right\} \tag{2.41}$$

subject to

$$Y_t^f(j) = \left[\frac{P_t^f(j)}{P_t^f} \right]^{-\varepsilon} C_t^f \tag{2.42}$$

where (2.42) is the demand faced by the jth formal firm and the subscript $t+s|t$ indicates the s periods that have occurred since the last time t that the firm was able to change its prices.

Solving the previous maximization problem implies the following first order condition

$$E_t \left\{ \sum_{s=0}^{\infty} \omega^s \Delta_{t,t+s} \left[(1-\varepsilon) \frac{P_t^{f*}(j)}{P_{t+s}^f} + \varepsilon mc_{t+s|t} \right] \left(\frac{P_t^{f*}(j)}{P_{t+s}^f} \right)^{-\varepsilon} \frac{1}{P_{t+s}^{f*}} C_{t+s}^f \right\} = 0, \tag{2.43}$$

which in turn implies the following optimal price-setting condition

$$P_t^{f*} = \mathscr{M} \frac{E_t[\sum_t^{\infty} (\omega\beta)^s u_c(C_{t+s}, m_{t+s}; \xi_{t+s}) (P_{t+s}^f)^{\varepsilon} C_{t+s}^f mc_{t+s|t}]}{E_t[\sum_t^{\infty} (\omega\beta)^s u_c(C_{t+s}, m_{t+s}; \xi_{t+s}) (P_{t+s}^f)^{\varepsilon-1} C_{t+s}^f]}, \tag{2.44}$$

where $\mathscr{M} \equiv \frac{\varepsilon}{\varepsilon-1}$ is a positive markup and P_t^{f*} is the optimal price set by any jth formal firm which changes prices at period t.

Finally, the fact that just a fraction ω of formal firms change its prices at any given time t yields an easy way to reformulate the price index for formal firms (2.23) as a weighted average of those who changed its price during the current period and the previous period price index of the formal firms

$$P_t^f = [(1-\omega)(P_t^{f*})^{1-\varepsilon} + \omega(P_{t-1}^f)^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}}. \tag{2.45}$$

2.3.2.1 Wage resolution

In this model there is no unemployment because focus is in the interaction between formal and informal labor. For workers in the informal sector to be there represents a value V_t^i given by

$$V_t^i = W_t^i + E_t[\Delta_{t,t+1}\{\theta_t q(\theta_t) V_{t+1}^f + [1-\theta_t q(\theta_t)] V_{t+1}^i\}] \tag{2.46}$$

where $\Delta_{t,t+1} \equiv \beta \frac{u_c(C_{t+1}, m_{t+1}; \xi_{t+1})}{u_c(C_t, m_t, ; \xi_t)}$ is the stochastic discount factor and V_t^f is in turn the value that a formal job represents also for the informal sector workers and is given by

$$V_t^f = W_t^f + E_t[\Delta_{t,t+1}\{\delta V_{t+1}^i + (1-\delta) V_{t+1}^f\}]. \tag{2.47}$$

On the other hand, to keep an open vacancy represents for the formal firm the following value

$$V_t^v = -cP_t + E_t[\Delta_{t,t+1}\{q(\theta_t) V_{t+1}^e + [1-q(\theta_t)] V_{t+1}^v\}] \tag{2.48}$$

where V_t^v is the value of an open vacancy, cP_t is a *constant* cost to keep such vacancy open, and V_t^e is the value that a new worker means for the formal firm and is specified as follows

$$V_t^e = P_t^f A_t^f - W_t^f + E_t[\Delta_{t,t+1}\{\delta V_{t+1}^v + (1-\delta)V_{t+1}^e\}]. \tag{2.49}$$

For an informal worker to accept a job in the formal sector it must happen that

$$V_t^f \geq V_t^i \tag{2.50}$$

and the formal firm will only hires the worker if it happens that

$$V_t^e \geq V_t^v. \tag{2.51}$$

Notice also that firms will keep posting vacancies until it yields no more profit to them, therefore vacancies will grow until the free entry condition holds

$$V_t^v = 0. \tag{2.52}$$

Notice that if at least one of the equations (2.50) and (2.51) hold with strict inequality, then there is a positive surplus obtained by a successful vacancy filled. This surplus will be divided between formal workers and formal firms as specified by the following weighted product

$$(V_t^f - V_t^i)^{\mathfrak{B}}(V_t^e - V_t^v)^{1-\mathfrak{B}} \tag{2.53}$$

where $\mathfrak{B} \in (0,1)$ is the bargaining power of formal workers. Therefore, the Nash Bargaining Problem is to maximize (2.53) subject to equations (2.50), (2.51), and (2.52) by choosing the formal wage W_t^f. The first order condition for this problem is

$$\mathfrak{B}(V_t^f - V_t^i)^{\mathfrak{B}-1}(V_t^e)^{1-\mathfrak{B}} - (1-\mathfrak{B})(V_t^e)^{-\mathfrak{B}}(V_t^f - V_t^i) = 0 \tag{2.54}$$

which implies that

$$\mathfrak{B}(V_t^e) = (1-\mathfrak{B})(V_t^f - V_t^i). \tag{2.55}$$

In order to derive an equation for the formal wage, it is convenient to define the total surplus that arises from the match S_t as follows

$$S_t = V_t^e + V_t^f - V_t^i, \tag{2.56}$$

which, for this case can be obtained by using (2.46), (2.47), (2.49) into (2.56). This yields

$$S_t = P_t^f Y_t^f - W_t^i + E_t\{\Delta_{t,t+1}[(1-\delta)[V_{t+1}^e + V_{t+1}^f - V_{t+1}^i] - \theta_t q(\theta_t)[V_{t+1}^f - V_{t+1}^i]]\}. \tag{2.57}$$

In order to proceed, some other results are needed. First notice that from (2.55) and (2.56) the following two equations are implied

$$\mathfrak{B} S_t = V_t^f - V_t^i, \tag{2.58}$$

$$(1-\mathfrak{B})S_t = V_t^e. \tag{2.59}$$

Second, the no vacancy condition (2.52) implies from equation (2.48) that

$$V_{t+1}^e = \frac{cP_t}{\Delta_{t,t+1} q(\theta_t)}, \tag{2.60}$$

then, by equation (2.59)

$$(1-\mathfrak{B})S_{t+1} = \frac{cP_t}{\Delta_{t,t+1} q(\theta_t)}. \tag{2.61}$$

Then using (2.56) and (2.58) forwarded one period, and (2.61) into (2.57) yields

$$S_t = P_t^f Y_t^f - W_t^i + E_t\left\{\frac{cP_t[1-\delta-\theta_t q(\theta_t)\mathfrak{B}]}{(1-\mathfrak{B})q(\theta_t)}\right\} \tag{2.62}$$

On the other hand, by plugging (2.46) and (2.47) into (2.58) the following expression is obtained

$$\mathfrak{B} S_t = W_t^f - W_t^i + E_t\{\Delta_{t,t+1}[1-\delta-\theta_t q(\theta_t)]\mathfrak{B} S_{t+1}\}. \tag{2.63}$$

Finally, plugging equations (2.61) and (2.62) into (2.63), and solving for the formal wage the following is obtained

$$W_t^f = (1-\mathfrak{B})W_t^i + \mathfrak{B}(P_t^f A_t^f + cP_t\theta_t), \tag{2.64}$$

so that the current formal wage is a linear combination between the informal wage W_t^i, which turns out to be the reservation wage for informal workers looking for a place into the formal sector, and the value of the formal production plus the expected (saved) cost of keeping the vacancy open. Notice that when formal workers have no bargaining power at all $\mathfrak{B} = 0$ then they are payed exactly equal than in the informal sector. In contrast, when they have full bargaining power $\mathfrak{B} = 1$ then they take all the surplus from the formal firm as part of their wage.

To finish this part, it will be important to be aware of the fact that real marginal cost (2.38) can be restated using (2.64) as

$$mc_t = \frac{1}{A_t^f}\left[(1-\mathfrak{B})\frac{W_t^i}{P_t} + \mathfrak{B}\left(\frac{P_t^f}{P_t}A_t^f + c\theta_t\right)\right] \tag{2.65}$$

2.4 Central bank

To close the model an specification for the monetary policy should be made. In the context of an informal economy it is possible that central banks respond only to formal inflation because of the lack of data about informal economy. But it is also possible that the central bank can react to general (aggregate) inflation due to measuring the informal sector. For this reason two possible expressions for the monetary policy are going to be used and compared

$$R_t = R\left(\frac{P_t}{P_{t-1}}\right)^{\mu_\pi}\left(\frac{Y_t}{Y}\right)^{\mu_y}\mathfrak{X}_t \tag{2.66}$$

or

$$R_t = R\left(\frac{P_t^f}{P_{t-1}^f}\right)^{\mu_\pi}\left(\frac{Y_t^f}{Y^f}\right)^{\mu_y}\mathfrak{X}_t \tag{2.67}$$

where $\xi_t^r \equiv log\mathfrak{X}$ follows a zero mean AR(1) process, Y is the steady state value of output, R_t is the interest rate under influence of the central bank and R is the steady state value of it, $\mu_\pi \geq 0$ and $\mu_y \geq 0$ are the weights that central bank puts on inflation and output respectively. Additionally it is assumed that the Taylor principle is satisfied so that $\mu_\pi + \mu_y > 1$.

2.5 Market clearing

The market clearing satisfies the following six equations, which corresponds to the labor market clearing

$$N_t = L_t^i + L_t^f, \tag{2.68}$$

the formal goods market

$$Y_t^f = C_t^f + cV_t, \tag{2.69}$$

the informal goods market

$$Y_t^i = C_t^i, \tag{2.70}$$

the aggregate production

$$Y_t = Y_t^f + Y_t^i, \tag{2.71}$$

the money market

$$\Gamma_t = M_t - M_{t-1}, \tag{2.72}$$

and the bond market

$$B_t = 0. \tag{2.73}$$

2.6 Equilibrium

Definition 2.1 (Rational-expectations Equilibrium)**.** Given an initial allocation of B_0, M_0, and L_0^f, and the exogenous sequences $\{a_t^f\}_{t=0}^{\infty}$, $\{a_t^i\}_{t=0}^{\infty}$, $\{\xi_t^c\}_{t=0}^{\infty}$, $\{\xi_t^m\}_{t=0}^{\infty}$, and $\{\xi_t^r\}_{t=0}^{\infty}$, a rational expectations equilibrium is the set of 16 sequences of quantities

$$\{N_t, B_t, M_t, \Psi_t, L_t^i, L_t^f, C_t, C_t^f, C_t^i, Y_t, Y_t^f, Y_t^i, \theta_t, V_t, \Gamma_t, \mathfrak{M}_t\}_{t=0}^{\infty},$$

and 7 sequences of prices

$$\{P_t, P_t^f, P_t^i, W_t^f, W_t^i, mc_t, R_t\}_{t=0}^{\infty},$$

which satisfy the following 23 conditions

(i) the optimality conditions of the representative household (2.16)-(2.19),

(ii) the variety formal production function (2.36), formal price-setting rules (2.44) and (2.45), and real marginal cost (2.65),

(iii) the informal production function (2.33) and the informal wage setting rule (2.35),

(iv) the matching function (2.2), the law of motion of labor (2.6) and the formal wage-setting rule (2.64),

(v) the aggregate price index (2.30) and the consumption demand conditions (2.31) and (2.32),

(vi) all markets clear (2.68)-(2.73),

(vii) one of the two monetary policy rules is satisfied (2.66) or (2.67).

□

Chapter 3

Log-linear Model and Calibration

3.1 Log-linearization

The model is log-linearized around a zero inflation steady state. In what follows each hated variable will represent a log deviation from their steady state value as specified by the following definition

$$\widehat{x} \equiv \log(x_t) - \log(x) \tag{3.1}$$

where x_t is the value of the variable at time t and x (without time subscript) is their steady state value.

For reference the steady state equilibrium is found in the Appendix A. But it is important to note that the assumption of zero inflation steady state implies that price levels are all equal at the steady state, *i.e.*

$$P = P^f = P^i \tag{3.2}$$

recalling that the absence of subscripts represent their respective steady state values. This is a reasonable assumption for a closed economy as the modeled here.

3.1.1 Household's optimality conditions

The log-linear version of the euler equation (2.16) yields the IS equation of the model

$$\widehat{C}_t = E_t[\widehat{C_{t+1}}] - \sigma\left[\widehat{R}_t - E_t[\widehat{\pi_{t+1}}] + \chi(E_t[\widehat{m_{t+1}} - \widehat{m_t}])\right] + (\xi_t^c - E_t[\xi_{t+1}^c]) \tag{3.3}$$

where $\sigma \equiv -\frac{\overline{u_c}}{\overline{u_{cc}}C} > 0$ is the intertemporal elasticity of substitution in consumption, $\chi \equiv \frac{\overline{u_{cm}}m}{\overline{u_c}}$ is the degree of non-separability between consumption and real money balances. It is assumed that money and consumption are complements in utility (*i.e.* $u_{cm} \geq 0$) and therefore $\chi \geq 0$. Notice that when $\chi = 0$ the IS equation is not affected by real balances. By this reason, this will be the benchmark case to which compare the effects of real balances apparent when $\chi \neq 0$. The deviation of *gross inflation* $\pi_t \equiv \frac{P_t}{P_{t-1}}$ from its steady state value is $\widehat{\pi}_t = \log(\pi_t) - \log(\pi) = \widehat{P}_t - \widehat{P_{t-1}}$.[1] Finally, as in Kurozumi (2006) the IS is affected by consumption shocks ξ_t^c and ξ_{t+1}^c which are assumed to be zero mean AR(1) processes.

Log-linearizing the money demand equation (2.17) yields the LM equation

$$\widehat{m_t} = \eta_c\widehat{C}_t - \eta_r\widehat{R}_t + \xi_t^m \tag{3.4}$$

where on the one hand $\eta_c > 0$ is the consumption elasticity of money demand defined as

$$\eta_c \equiv \frac{\sigma^{-1} + \upsilon}{\chi + \sigma_m^{-1}}$$

where $\upsilon \equiv \frac{\overline{u_{mc}}C}{\overline{u_m}} \geq 0$ represents the elasticity of u_m with respect to changes in the level of real expenditure and $\sigma_m \equiv -\frac{\overline{u_m}}{\overline{u_{mm}}m} > 0$ is the intertemporal elasticity of substitution in real money balances.

On the other hand $\eta_r > 0$ is the interest-rate semi-elasticity of money demand defined as

$$\eta_r \equiv \left(\frac{\beta}{1-\beta}\right)\left(\frac{1}{\chi + \sigma_m^{-1}}\right),$$

and again as in (Kurozumi, 2006) a monetary shock ξ_t^m is considered which is assumed to be a zero mean AR(1) process.

From log-linearization of the optimal labor supply condition (2.18) the following equation for deviations in employment is obtained

$$\sigma_n\widehat{N}_t = \Theta\left[\chi\widehat{m_t} + \widehat{w}_t^i - \sigma^{-1}(\widehat{C}_t - \xi_t^c)\right] + (1-\Theta)\left[\widehat{\Psi}_t + \vartheta\widehat{\theta}_t\right] \tag{3.5}$$

[1] To see this notice that a zero inflation steady states implies that the *gross inflation* at steady state π equals one, and therefore $\log(\pi) = 0$.

where $\sigma_n \equiv \frac{\overline{v_{nn}}N}{\overline{v_n}} > 0$ is the inverse intertemporal elasticity of substitution in employment, $\overline{v_n}, \overline{v_{nn}}$ are the first second derivatives of the $v(N)$ function valuated at steady state, $\Theta \equiv \frac{\overline{u_c} w^i}{\overline{v_n}} > 0$ and w^i is the steady state value of the *real* informal wage which in turn is defined as $w_t^i \equiv \frac{W_t^i}{P_t}$.

By the same method, the equation for the shadow price of a formal worker (2.19) yields the following

$$\widehat{\Psi_t} = [1-\beta(1-\delta)]\left[\Xi\left(\chi E_t[\widehat{m_{t+1}}] - \sigma^{-1}E_t[\widehat{C_{t+1}}] + E_t[\widehat{w_{t+1}^f}]\right) + (1-\Xi)\sigma_n E_t[\widehat{N_{t+1}}]\right] + \beta(1-\delta)E_t[\widehat{\Psi_{t+1}}] \tag{3.6}$$

where $\Xi \equiv \frac{\overline{u_c} w^f}{\overline{u_c} w^f - \overline{v_n}}$ and $w_t^f \equiv \frac{W_t^f}{P_t}$ is the real formal wage.

3.1.2 Aggregate inflation and consumption demands

From log-linearizing the aggregate price index (2.30) the aggregate inflation is obtained

$$\widehat{\pi_t} = \alpha\widehat{\pi_t^f} + (1-\alpha)\widehat{\pi_t^i} \tag{3.7}$$

where $\widehat{\pi_t^f} = \widehat{P_t^f} - \widehat{P_{t-1}^f}$ and analogously $\widehat{\pi_t^i} = \widehat{P_t^i} - \widehat{P_{t-1}^i}$. Notice also that here equation (3.2) was used.

By the same method applied over the demand functions (2.31) and (2.32) the two following equations are derived

$$\widehat{C_t^f} = \tau\left(\widehat{P_t} - \widehat{P_t^f}\right) + \widehat{C_t} \tag{3.8}$$

$$\widehat{C_t^i} = \tau\left(\widehat{P_t} - \widehat{P_t^i}\right) + \widehat{C_t}. \tag{3.9}$$

3.1.3 Firms production and prices

The log-linear versions of firms production functions (2.33) and (2.36) are the following

$$\widehat{Y_t^f} = \widehat{L_t^f} + a_t^f \tag{3.10}$$

$$\widehat{Y_t^i} = \widehat{L_t^i} + a_t^i \tag{3.11}$$

while from the log-linear forms of (2.44) and (2.45) the following formal New Keynesian Phillips equation is obtained

$$\widehat{\pi_t^f} = \kappa\widehat{mc_t} + \beta E_t[\widehat{\pi_{t+1}^f}] \tag{3.12}$$

where $\kappa \equiv \frac{(1-\omega)(1-\omega\beta)}{\omega}$ and the real marginal cost is derived from (2.65) as

$$\widehat{mc_t} = \widehat{w_t^f} - a_t^f. \tag{3.13}$$

3.1.4 Labor market

Log-linearizing the matching function (2.2) together with equation (2.4) yields

$$\widehat{\theta}_t = \widehat{V}_t - \widehat{L}_t^i, \tag{3.14}$$

and from the the law of motion of next period formal labor (2.6)

$$\widehat{L_{t+1}^f} = \delta(\vartheta\widehat{\theta}_t + \widehat{L}_t^i) + (1-\delta)\widehat{L_t^f}, \tag{3.15}$$

and informal real wage (2.35)

$$\widehat{w}_t^i = \widehat{P}_t^i - \widehat{P}_t + a_t^i, \tag{3.16}$$

while the real formal wage, which comes from dividing (2.64) by P_t and log-linearizing it

$$\widehat{w_t^f} = \frac{1}{w^f}\left[(1-\mathfrak{B})\widehat{w}_t^i + \mathfrak{B}\left(\widehat{P_t^f} - \widehat{P}_t + a_t^f + c\theta\widehat{\theta}_t\right)\right] \tag{3.17}$$

where (3.2) is used once again, in particular the implication that the steady state value of real informal wage $w^i = 1$.[2]

3.1.5 Monetary policy rules

The log-linear version of the two possible monetary policy rules (2.66) and (2.67) are the following

[2] To see this, notice that equation (2.35) evaluated at the steady state implies it.

$$\widehat{R}_t = \mu_\pi \widehat{\pi}_t + \mu_y \widehat{Y}_t + \xi_t^r \tag{3.18}$$

$$\widehat{R}_t = \mu_\pi \widehat{\pi_t^f} + \mu_y \widehat{Y_t^f} + \xi_t^r. \tag{3.19}$$

3.1.6 Market clearing conditions

Finally, the log-linear versions of the market clearing conditions (2.68) - (2.71) are the following[3]

$$\widehat{N}_t = \digamma \widehat{L_t^f} + (1-\digamma)\widehat{L}_t^i, \tag{3.20}$$

where $\digamma \equiv \frac{L^f}{N}$,

$$\widehat{Y_t^f} = \varsigma \widehat{C_t^f} + (1-\varsigma)\widehat{V}_t, \tag{3.21}$$

where $\varsigma \equiv \frac{C^f}{Y^f}$,

$$\widehat{Y}_t^i = \widehat{C}_t^i, \tag{3.22}$$

$$\widehat{Y}_t = \Phi \widehat{Y_t^f} + (1-\Phi)\widehat{Y}_t^i, \tag{3.23}$$

where $\Phi \equiv \frac{Y^f}{Y}$.

3.2 The full log-linearized model

The following 19 equations plus one of the two monetary policy rules (for a total of 20 equations) specify the full log-linearized model.

$$\widehat{C}_t = E_t[\widehat{C_{t+1}}] - \sigma\left[\widehat{R}_t - E_t[\widehat{\pi_{t+1}}] + \chi(E_t[\widehat{m_{t+1}} - \widehat{m}_t])\right] + (\xi_t^c - E_t[\xi_{t+1}^c]) \tag{3.24}$$

$$\widehat{m}_t = \eta_c \widehat{C}_t - \eta_r \widehat{R}_t + \xi_t^m \tag{3.25}$$

[3]Notice that the conditions (2.72) (2.73) are no longer needed.

$$\sigma_n \widehat{N_t} = \Theta \left[\chi \widehat{m_t} + \widehat{w_t^i} - \sigma^{-1} (\widehat{C_t} - \xi_t^c) \right] + (1 - \Theta) \left[\widehat{\Psi_t} + \vartheta \widehat{\theta_t} \right] \tag{3.26}$$

$$\widehat{\Psi_t} = [1 - \beta(1 - \delta)] \left[\Xi \left(\chi E_t[\widehat{m_{t+1}}] - \sigma^{-1} E_t[\widehat{C_{t+1}}] + E_t[\widehat{w_{t+1}^f}] \right) + (1 - \Xi) \sigma_n E_t[\widehat{N_{t+1}}] \right] + \beta(1 - \delta) E_t[\widehat{\Psi_{t+1}}] \tag{3.27}$$

$$\widehat{\pi_t} = \alpha \widehat{\pi_t^f} + (1 - \alpha) \widehat{\pi_t^i} \tag{3.28}$$

$$\widehat{C_t^f} = \tau \left(\widehat{P_t} - \widehat{P_t^f} \right) + \widehat{C_t} \tag{3.29}$$

$$\widehat{C_t^i} = \tau \left(\widehat{P_t} - \widehat{P_t^i} \right) + \widehat{C_t} \tag{3.30}$$

$$\widehat{Y_t^f} = \widehat{L_t^f} + a_t^f \tag{3.31}$$

$$\widehat{Y_t^i} = \widehat{L_t^i} + a_t^i \tag{3.32}$$

$$\widehat{\pi_t^f} = \kappa \widehat{mc_t} + \beta E_t[\widehat{\pi_{t+1}^f}] \tag{3.33}$$

$$\widehat{mc_t} = \widehat{w_t^f} - a_t^f \tag{3.34}$$

$$\widehat{\theta_t} = \widehat{V_t} - \widehat{L_t^i} \tag{3.35}$$

$$\widehat{L_{t+1}^f} = \delta(\vartheta \widehat{\theta_t} + \widehat{L_t^i}) + (1 - \delta) \widehat{L_t^f} \tag{3.36}$$

$$\widehat{w_t^i} = \widehat{P_t^i} - \widehat{P_t} + a_t^i \tag{3.37}$$

$$\widehat{w_t^f} = \frac{1}{w^f} \left[(1 - \mathfrak{B}) \widehat{w_t^i} + \mathfrak{B} \left(\widehat{P_t^f} - \widehat{P_t} + a_t^f + c\theta \widehat{\theta_t} \right) \right] \tag{3.38}$$

$$\widehat{N_t} = \digamma \widehat{L_t^f} + (1 - \digamma) \widehat{L_t^i} \tag{3.39}$$

$$\widehat{Y_t^f} = \varsigma \widehat{C_t^f} + (1 - \varsigma) \widehat{V_t} \tag{3.40}$$

$$\widehat{Y_t^i} = \widehat{C_t^i} \tag{3.41}$$

$$\widehat{Y_t} = \Phi \widehat{Y_t^f} + (1 - \Phi) \widehat{Y_t^i} \tag{3.42}$$

$$\widehat{R_t} = \mu_\pi \widehat{\pi_t} + \mu_y \widehat{Y_t} + \xi_t^r \tag{3.43}$$

$$\widehat{R_t} = \mu_\pi \widehat{\pi_t^f} + \mu_y \widehat{Y_t^f} + \xi_t^r \tag{3.44}$$

3.3 Baseline Calibration

To calibrate the model each period is assumed to last one quarter. Since the primary interest of this work is the implications of real balance effects, which are known to be little, then following McKnight and Mihailov (2015) I set $\chi = 0$ for the baseline model and $\chi = 0.03$ to compare with. The values of $\beta = 0.99$ while the preference for consumption of formal goods $\alpha = 0.72$ is consistent with the last ten years of data from Mexico.[4] An intertemporal elasticity of substitution in consumption of $\sigma = 2$ together with a price stickiness index $\omega = 0.75$ follows McKnight (2018). A 2.5 value for the inverse elasticity of substitution in employment, which implies a 0.4 Frisch elasticity of labor supply, is according to the median estimate for this parameter by Whalen and Reichling (2017). For the substitutability between formal and informal goods I set a value of 1.7, very close to that of 1.5 used by Batini et al. (2011). A unitary consumption elasticity of money demand η_c and a interest rate semi-elasticity of money demand of 28 are in line with McKnight and Mihailov (2015).[5]

The formal sector productivity persistence ρ_a^f is set to 0.92 consistent with estimates by Leyva and Urrutia (2018). Here I assume an equal persistent shock for informal productivity sector so that ρ_a^i is also set to 0.92.[6] For the consumption shock and the monetary shock the values are similar to those found in the standard literature, in particular $\rho_c = 0.6$ and $\rho_m = 0.5$.

For the formal labor market, Leyva and Urrutia (2018) calculated a quarterly separation rate δ of 8.8% with Mexican data from the ENOE survey; they also found that a vacancy post cost of 0.03 is consistent with some relevant empirical correlations in the data. Here I take a value of 0.02 for the vacancy post c to be consistent with the requirements of this model and at the same time keep it very close to that of these authors. To set matching elasticity and matching efficiency I follow recent estimates for both of this parameters. In particular, it has been found for several authors that matching efficiency $\mathfrak{m}$ has been constantly decaying and is around 0.3 (or 30%),[7] and then 0.29 is a pretty good approximation to those estimates. Also, a matching elasticity ϑ of 0.4 is consistent with the same group of estimations.

Now, a special attention must be paid to bargaining power of workers in the formal sector. Until very recently this parameter was under a lot of controversy for the very different estimates available for it. However, a recent paper by Ciccarone et al. (2013) demonstrates that, under their theoretical grounds, values between 0.34 and 0.39 are plausible.[8] Therefore I set a $\mathfrak{B}$ of 0.4 as a good approximation of their analytical suggestions.

For the parameters of the policy rules considered, I follow Moura and de Carvalho (2010) estimates for the Mexican economy. They empirically found response values between 1.07 and 1.32 for the annual output gap and between 2.33 and 7.07 for the annualized inflation. This figures

[4]This is obtained as the ratio of formal sector consumption with respect to total consumption in Mexico, with data from INEGI.

[5]This values were also adopted previously by Woodford (2003) among others, therefore those are standard values.

[6]This is a fair assumption given the fact that the estimates of Leyva and Urrutia (2018) were made for the aggregate productivity, and then, the persistence of 0.92 should be a weighted average between the formal and informal productivity persistence. Therefore, given that 0.92 is a very high persistence parameter, it would be strange that any of the two sectors productivity persistence would be far away that benchmark value.

[7]See for example Barnichon and Figura (2011) who estimated a matching elasticity of 0.39 and a matching efficiency of 0.21 for the period after the financial crisis.

[8]This values for the bargaining power are obtained by the authors under a plausible assumption: workers are more loss averse than firms and therefore are more likely to accept a contract by less than a *fair* (50% - 50%) share of the economic surplus which results from the matching process.

then are divided by factor of 4 to get the plausible values.[9] Then the parameter response values for inflation and output gap should be $0.5825 \leq \mu_\pi \leq 1.7675$ and $0.2675 \leq \mu_y \leq 0.33$ respectively. Given this, the baseline calibration considers $\mu_\pi = 1.7675$ and $\mu_y = 0.2675$.[10] Table 3.1 summarizes the calibration.

[9]The reason is because the periods in the economy modeled here are quarters instead of years. See for example Galí (2015).

[10]Notice that this values are close to those used often in standard literature, see for example Gali (2011) and Galí (2015) where $\mu_\pi = 1.5$ and $\mu_y = 0.125$ are employed.

Parameter	Value	Decription	Source
χ	0-0.03	Degree of non-separability between consumption and real money balances	McKnight and Mihailov (2015)
σ	2	Intertemporal elasticity of substitution in consumption	McKnight (2018)
σ_n	2.5	Inverse intertemporal elasticity of substitution in employment	Whalen and Reichling (2017)
η_c	1	Consumption elasticity of money demand	McKnight and Mihailov (2015)
η_r	28	Interest rate semi-elasticity of money demand	McKnight and Mihailov (2015)
β	0.99	Discount factor	Standard
τ	1.7	Substitutability between formal and informal goods	Batini et al. (2011)
α	0.72	Consumption preference for formal goods	Own calculations
ω	0.75	Proportion of formal firms that can't change prices at period t	Standard
ϑ	0.4	Matching elasticity	Barnichon and Figura (2011)
$\mathfrak{m}$	0.29	Matching efficiency	Barnichon and Figura (2011)
δ	0.088	Rate of destruction of formal jobs	Leyva and Urrutia (2018)
$\mathfrak{B}$	0.4	Bargaining power of formal workers	Ciccarone et al. (2013)
c	0.02	Real vacancy cost	Leyva and Urrutia (2018)
ρ_a^f	0.92	Formal productivity shock persistence	Leyva and Urrutia (2018)
ρ_a^i	0.92	Informal productivity shock persistence	Leyva and Urrutia (2018)
ρ_c	0.6	Consumption shock persistence	Standard
ρ_m	0.5	Monetary shock persistence	Standard
μ_π	1.7675	Weight that the central bank puts on inflation	Moura and de Carvalho (2010)
μ_y	0.2675	Weight that the central bank puts on output	Moura and de Carvalho (2010)

TABLE 3.1: Baseline Calibration

Chapter 4

Model Dynamics

4.1 Propagation mechanism before exogenous shocks

In this part the dynamic responses before several kind of shocks to the model economy are analyzed: a formal productivity shock, an informal productivity one, a demand shock, a monetary shock and a monetary policy shock. For each of the shocks four specifications of the model are considered depending on two factors, the first one is whether the model includes real balance effects and the second is whether the central bank reacts to aggregate measures of the economy or to formal only measures. For reference, the model without real balance effects and with an aggregate monetary policy rule is named the Model I and will be the benchmark model, since this is usually the way in which models are structured in the standard literature (see table 4.1 below).

		Real Balance Effects	
		No	Yes
Monetary Policy Rule	Aggregate	Model I (Benchmark)	Model II
	Formal Only	Model III	Model IV

TABLE 4.1: Models for the dynamics analysis

Under the classification of Table 4.1 two relevant comparisons can be made. Model I vs Model II to understand the implications of the presence of real balance effects. And Model II vs Model IV to look for the different propagation mechanisms of monetary policy when an aggregate rule or an formal only rule is used.[1] Then, in what follows, for each of the shocks considered here some words will be said about each of the comparisons.

[1] Two additional comparisons can be made using Model III but this add almost nothing to the ones considered for analysis.

4.1.1 Formal productivity shock

4.1.1.1 Transmission mechanism of the benchmark model

From now on, the dynamics of the benchmark model appears inside each figure with the solid plot line. As can be seen in Figure 4.1 a formal productivity shock of 100 basis points (1 percent point) affects directly formal production by the same amount. This initial effect increases aggregate output and simultaneously an initial spillover low effect on the informal production of 0.1 percentage points.[2]

When formal production increases, formal firms can set lower prices than before because now they produce more with the same inputs. Also, with the lower prices the consumption of formal goods begins to increase at the expense of informal consumption which diminishes. Then, the increase in formal production and formal consumption also increase formal sector revenues, part of which are used to open new vacancies to keep satisfying the increasing demand for formal goods. To attract people in the labor market to this new open vacancies, formal firms offer higher formal wages than before. Then workers who initially were in the informal sector looks for a place in the new formal vacancies. This lowers informal employment lowering also informal production.

The lost jobs in the informal sector causes the marginal productivity of informal labor to rise with a corresponding rise on their wages, which is of a lower magnitude than the rise in the formal sector. This rise keeps workers producing on informal sector to keep it of lowering more.

The previous dynamics yields a decrease in aggregate inflation which leads the central bank to increase money balances in order to lower the nominal interest rate. The monetary authority keeps doing this until economy returns to their steady state values, which occurs some 5 years after the initial formal productivity shock. This is because, as discussed in the calibration section, formal productivity is strongly persistent over time.

4.1.1.2 Comparison of transmission mechanism of II vs Model I

Given an aggregate monetary policy rule, the presence of real balance effects in Model II compared to the benchmark model implies some key differences (see Figure 4.1). The initial formal productivity shock of 100 basis points rises the formal production a bit more than without real balance effects. This apparent little effect rises the demand of formal employment. The spillover effect on the informal firms is greater than without the presence of real balance effects and then informal output decreases in a lower way than without them. Therefore, the informal wage

[2] To avoid a difficult understanding of the IRFs I plotted all of them beginning since period 2, because some endogenous variables are solved until then.

should rise relatively more than before to keep those workers in the sector. The previous effects rise both formal and informal consumption with respect to the model without real balance effects. Notice that these differences do not yield a change in the formal sector wage because the increase in formal productivity is exactly the same. Therefore all the increase in production is equivalent to the increase in consumption, which keeps prices to the same level than before. As a consequence, the central bank should respond with the same amount of decrease in the nominal interest rate.

Then in short, with the presence of real balance effects production, consumption employment and the informal wage, are all greater than without those effects, while prices, formal wages and nominal interest rates respond in the same way than in the absence of the effects.

4.1.1.3 Comparison of transmission mechanism of Model IV vs Model II

Under the presence of real balance effects, as in this case, central banks following a formal only monetary policy rule, reacts stronger as a consequence of the formal productivity shock than those with a typical aggregate rule. This is because aggregate inflation is a weighted average of the formal and informal sector inflations, and since the formal sector is greater than the informal one, a coefficient response with the same intensity to the formal sector only traduces in a stronger response over the aggregates in the economy. This is reflected in the IRFs of Figure 4.1.

As a consequence, when compared model IV vs model II, the initial exogenous productivity shock causes an increase of lower measure than that with an aggregate rule over the aggregate economy and also over the formal sector. Only the informal sector rises relatively more than with the aggregate rule, because central bank is not responding to this sector of the economy. Indeed, informal employment also is relatively higher here.

At the same time, formal and informal wages are relatively lower, and then, in order to sell its growing production given the not so great increase in employment, firms must set lower prices than in the benchmark economy. Therefore inflation is more negative than before and the central bank should react with a more negative nominal interest rate to compensate for this.

4.1.2 Informal productivity shock

4.1.2.1 Transmission mechanism of the benchmark model

As can be seen in the Figure 4.2, an informal sector productivity shock of 100 basis points generates an increase more than proportional in the informal output. This increase in production requires an increase in the demand for labor for this sector, therefore informal employment

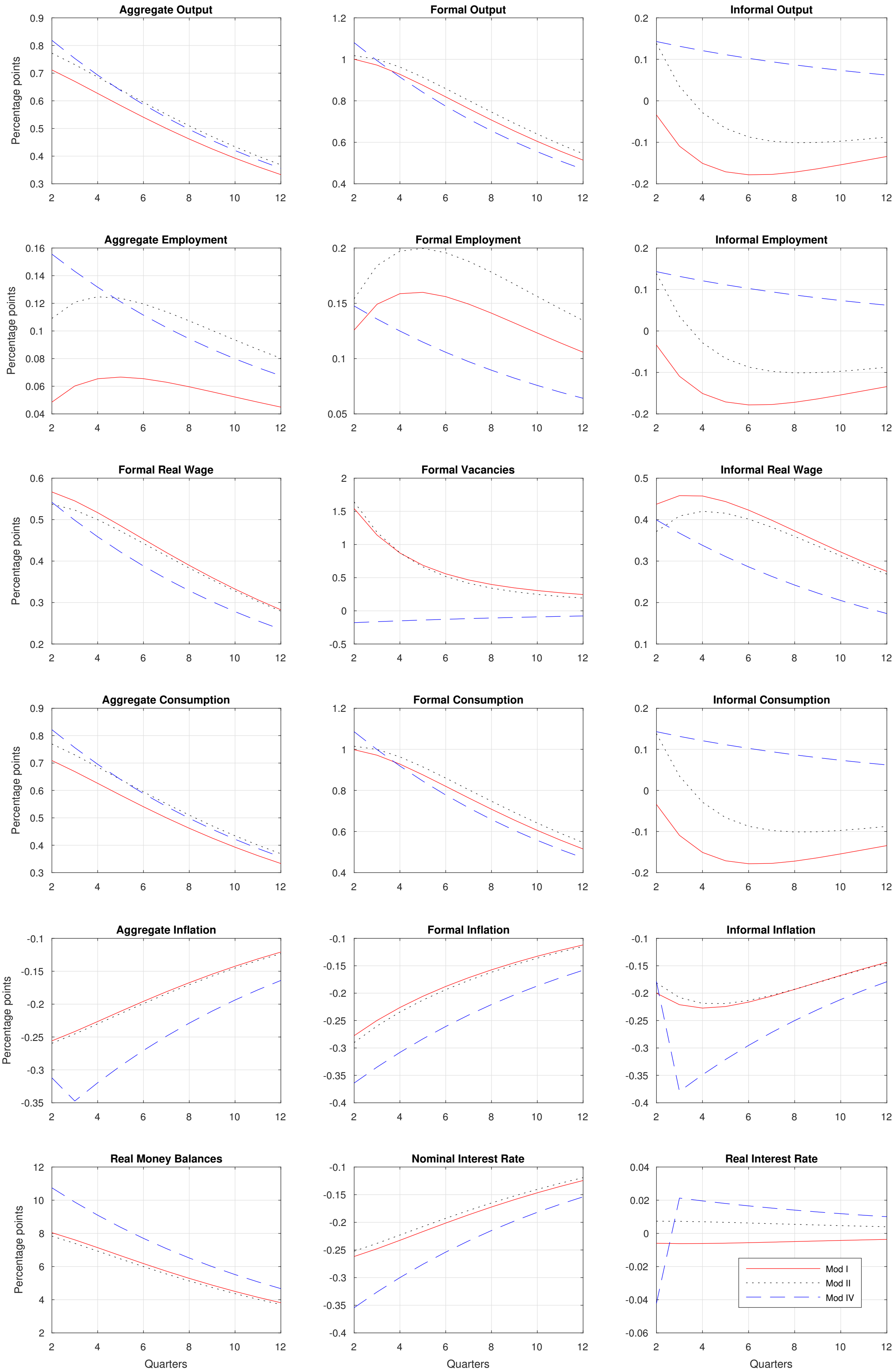

FIGURE 4.1: Dynamic Responses to a Formal Productivity Shock

Notes: Mod I = Benchmark, Mod II = Real Balance Effects and an aggregate monetary policy rule, Mod IV = Real Balance Effects and a formal only monetary policy rule.

increases at expense of formal employment. But the increase of informal production also decreases informal goods prices. Therefore the demand for informal goods increase at expense of formal ones. Therefore, aggregate production increases rather modestly.

Despite the rise of employment in the informal sector, aggregate employment decreases because most workers were in the formal sector. In a try to reverse the situation, formal firms lower prices a bit but not enough to recover.

Since both, informal and formal sector prices decreased, aggregate inflation also decreased. To face this situation the central bank increases the money supply which leads the nominal interest rate to lower around 10 basis points and continue to lower it for the time that takes the economy to recover their steady state paths after the informal productivity shock.

4.1.2.2 Comparison of transmission mechanism of Model II vs Model II

As it is evidenced by Figure 4.2, there are no substantial differences between the benchmark model and the one with real balance effects when an aggregate monetary policy rule is taken as given in the presence of a positive formal productivity shock.

4.1.2.3 Comparison of transmission mechanism of Model IV vs Model II

Under the presence of real balance effects, the IRFs produced when a central bank follows a formal only monetary policy rule are quite different with respect of when an aggregate monetary policy rule is used.

As can be seen in Figure 4.2), the initial informal shock rises informal output less than under a standard aggregate rule and this leads the formal output to decrease lower and to increase aggregate output higher than before.

The relative lower increase in the informal production requires a lower increase of informal sector employment then reducing considerably the extent to which formal employment decreases. This dual effect creates one of the key differences between the two models here: instead of decreasing aggregate employment, it rises for at least the first 5 quarters. Therefore, informal firms are able to pay higher wages. In order to keep its workers, formal firms also rise wages. Because of this duple effect employees can buy more goods in both sectors, but they prefer informal goods because the rise in productivity make them more efficient and attractive goods. Therefore, informal consumption rises at the expense of formal consumption but in a lower extent than under an standard aggregate policy rule.

The increase in the consumption of informal goods increase its prices. This open the opportunity to formal goods to rise prices too and keep their revenues to lower more. Both effects leads to an increase in aggregate inflation with a correspondent response by the central bank rising the nominal interest rate which is in stark contrast to when the monetary authority follows a standard aggregate rule. It is important to note that all the effects over prices under a formal only rule differs (in an opposite way) to those where the central bank follows an aggregate monetary policy rule.

Summarizing, the effects on aggregate employment, informal real wages, prices and monetary policy from the central bank are all opposed when one considers a formal only monetary policy rule versus an aggregate standard rule.

4.1.3 Demand shock

4.1.3.1 Transmission mechanism of the benchmark model

The IRFs in Figure 4.3 shows the transmission of a positive exogenous demand shock of 100 basis points. The initial shock increases formal consumption at expense of informal consumption because there is a formal consumption bias by the agents of the model, represented by the size of α. The increase in consumption also increase formal production. Now formal firms are in possibility to open new vacancies while offering higher formal wages. Therefore, formal employment increases and consequently so does aggregate employment. Informal wages rise a little bit to keep some workers from leaving the sector, but despite these efforts the quantity of informal employment decreases.

Since the shock is a exogenous source of demand for both formal and informal goods, then prices rises in both sectors and therefore aggregate inflation also increases. In consequence, the central bank rises the nominal interest rate by lowering the money supply.

Notice that the effects of a demand shock are modest, compared to those of productivity shocks, and also lasts comparatively few quarters. This can be explained because people understands that an exogenous increase in consumption today must be in exchange of a decrease in future consumption. Therefore, the net effects are lowered.

4.1.3.2 Comparison of transmission mechanism of Model II vs Model I

Figure 4.3 shows that there is one, and only one, crucial difference between the benchmark model and the one with real balance effects in the presence of a exogenous consumption shock of 100 basis points when the central bank uses an aggregate monetary policy rule. The difference is that the IRFs of each endogenous variable is smoother where real balance effects are present. In other words, the effects of the shock all goes in the same direction than in the benchmark model but with a lower intensity.

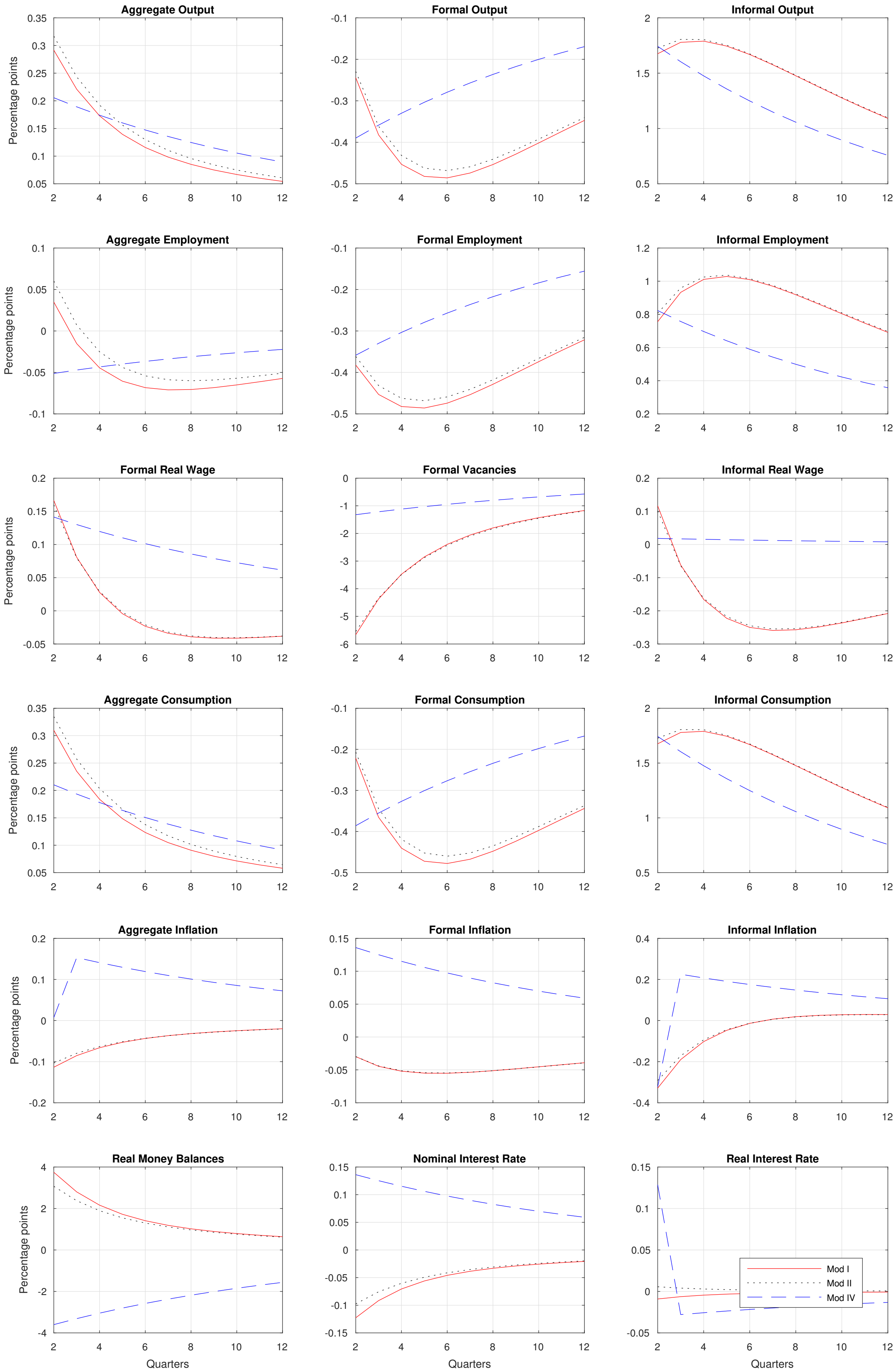

FIGURE 4.2: Dynamic Responses to a Informal Productivity Shock

Notes: Mod I = Benchmark, Mod II = Real Balance Effects and an aggregate monetary policy rule, Mod IV = Real Balance Effects and a formal only monetary policy rule.

Additionally, notice that the implications for the economy of a demand shocks are in contrast to the implications of productivity shocks. Specifically, as it is shown in Figures 4.1, 4.2, and 4.3, demand shocks do not matter so much whereas productivity shocks do matter.

4.1.3.3 Comparison of transmission mechanism of Model IV vs Model II

Given the presence of real balance effects, again using a formal only rule instead of an aggregate monetary rule, to face a demand shock just generates spikes in the initial quarters after the shock, but with a rapid return to steady sate values. The reminder of the effects are quite similar to the benchmark case.

4.1.4 Monetary shock

For this part of the analysis a couple of clarifications are needed previously. First of all, recall that real balances are defined as $m_t \equiv \frac{M_t}{P_t}$ for each period t. All the way through here there would not exist any doubt that when talking about real balance *effects*, the thing that one looks for is the role of m_t on the transmission mechanism of monetary policy. Now, when turning to analyze the IRFs generated by a monetary shock, what is happening is that for some exogenous reason, the demand for money (see equation (3.25)) is rising. Therefore, here the interest is to see what happens after this initial shock.

As it turns out, by looking at 4.4, the IRFs of the model without real balance *effects* are all in zero. Except for the money supply, which rises at the same extent that money demand which is also the same extent of the rise in the shock.

What is happening here? To make sense of this notice that

$$\widehat{m_t} = \widehat{M_t} - \widehat{P_t},$$

therefore, there is an exogenous rise in real balances m_t if and only if one of the following happens

1. $\widehat{M_t} > 0$ and $\widehat{P_t} = 0$,
2. $\widehat{M_t} > 0, \widehat{P_t} > 0$ and $\widehat{M_t} > \widehat{P_t}$,
3. $\widehat{M_t} < 0, \widehat{P_t} < 0$ and $|\widehat{P_t}| > |\widehat{M_t}|$.

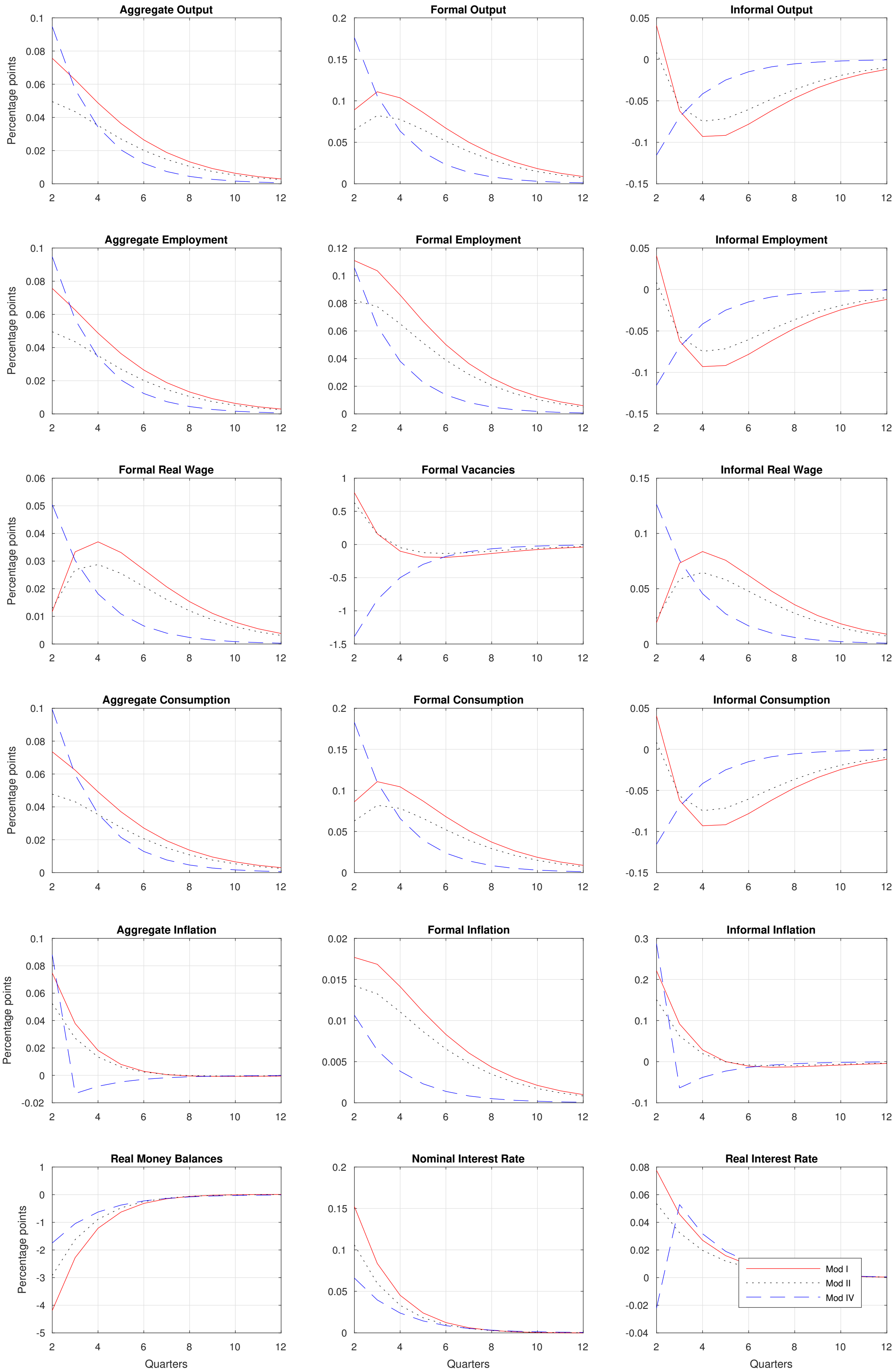

FIGURE 4.3: Dynamic Responses to a Demand Shock

Notes: Mod I = Benchmark, Mod II = Real Balance Effects and an aggregate monetary policy rule, Mod IV = Real Balance Effects and a formal only monetary policy rule.

One can see that, under this model, in absence of non-separability of utility function in consumption and money (*i.e.* when $\chi = 0$), the only way in which money can affect dynamics of the model is through (2.) or (3.), or in other words, it affects only if $\widehat{P}_t \neq 0$ is implied.

But, contrary to that possibility, what is happening here is that (1.) holds. When money demand increases by the positive monetary shock, money supply rises in exactly the same amount than real balances, and therefore the change in prices is null. Thus, this can explain why there are no deviations from steady state values in the IRFs of the monetary shock (Figure 4.4).[3] Despite that, one can do analysis on IRFs of models II and IV.

4.1.4.1 Comparison of transmission mechanism of Model IV vs Model II

As can be seen in the Figure 4.4, given the presence of real balances, when the central bank follows a formal only monetary policy rule, this generates IRFs with huge spikes in the first periods followed by a rapid decrease to the steady state values of the endogenous variables. On the contrary, when the monetary authority follows a conventional aggregate rule the responses to the monetary shock are quite smoother and lasts few more quarters.

4.1.5 Monetary policy shock

4.1.5.1 Transmission mechanism of the benchmark model

The initial exogenous positive shock of 25 basis points to the nominal interest rate causes a slightly decrease in aggregate consumption which is generated by a decrease in formal consumption because money now is unexpectedly more expensive, and then transfered to informal consumption because informal goods are cheaper. This changes in consumption are reflected into output and employment measures on the same directions. Additionally, the exogenous contractionary monetary policy shock also contracted prices, which leads to a decrease in aggregate inflation. Therefore, the central bank issues money to lower the nominal interest rate. Notice that the duration of the shock is short and its effects very little.

[3] However, notice that there is indeed a way to produce changes on IRFs by managing money, if it were not the case these models hardly can call themselves monetary policy models. To obtain effects from a change in the quantity of money, one needs to consider both a demand for money equation as the one in this model, and a money supply rule which will take the place of the Taylor rule. So, this is an explicit monetary policy rule (a fixed increase in the supply of money each period, for example) instead of just an exogenous increase of money demand. Galí (2015) explains how to implement a money rule very straightforward.

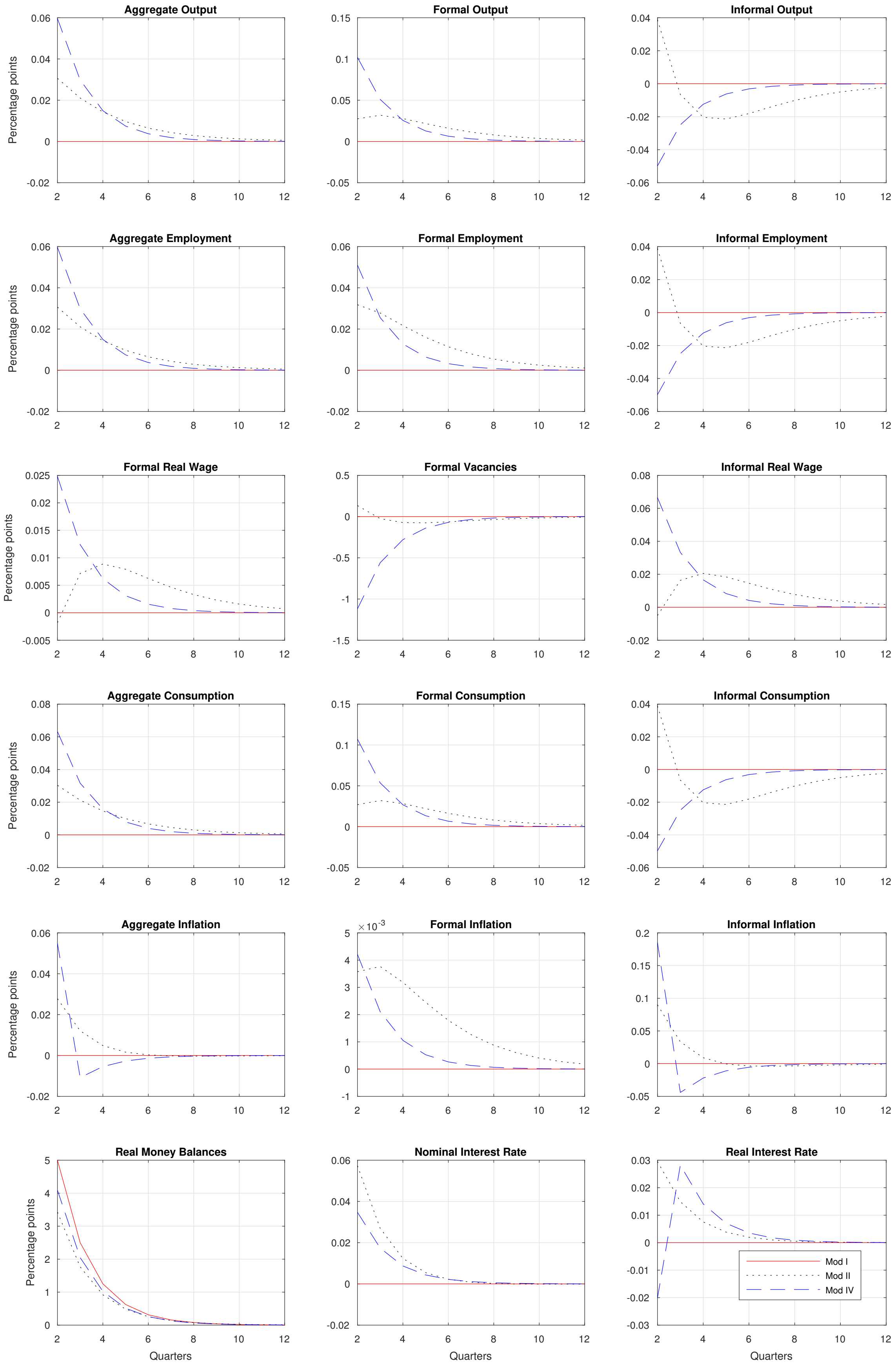

FIGURE 4.4: Dynamic Responses to a Monetary Shock

Notes: Mod I = Benchmark, Mod II = Real Balance Effects and an aggregate monetary policy rule, Mod IV = Real Balance Effects and a formal only monetary policy rule.

4.1.5.2 Comparison of transmission mechanism of Model II vs Model I

It is pretty clear from Figure 4.5 that, given the regime of an aggregate monetary policy rule, before a positive 25 basis points shock to the nominal interest rate there are no substantial differences between the IRFs with and without real balance effects.

4.1.5.3 Comparison of transmission mechanism of Model IV vs Model II

Recall that a monetary policy rule that responds only to formal measures is more tight than an aggregate rule. Therefore, as can be seen in Figure 4.5 the initial contraction in aggregate and formal measures is stronger than with the aggregate rule. Also the increase in informal production, employment and consumptions is lower than in the aggregate rule case. Therefore, even though there is also an initial decrease in prices of goods for the exogenous contraction in monetary policy, after just three quarters prices rise and generates positive inflations in both sectors because of the scarcity of goods caused by the contraction in production. To face the increase on aggregate inflation, the monetary authority rise even more the nominal interest rate to get back inflation to its steady state value.

In sum, there is a crucial difference between using an aggregate typical monetary policy rule versus a formal only rule when facing a monetary policy shock. The latter will lead to a even more contractionary policy whereas the former will be accommodative.

4.1.6 Summary of effects

It will be useful for future reference to summarize some of the key results of the section. First of all, real balance effects are very relevant when a formal productivity shock hits the economy. In face of a monetary shock, real balance effects are only relevant for inflation measures and interest rates. When the model is shocked by a demand perturbation, no quantitatively relevant effects are found, but there are one qualitative result which seems relevant: all IRFs are more smooth under real balance effects. Finally, in face of a monetary policy shock or a informality productivity shock, no relevant effects are caused by real balances.

Turning now to the effects caused by a formal only interest rate. It was found that in general, when compared with a typical aggregate rule, there are relevant differences in the propagation of shocks. Some of the most relevant differences appear when the economy is hit by a informal productivity shock. IRFs of prices and interest rates change in sign, and informal and formal sector measures grow less.

When the economy is hit by a formal productivity shock, informal output, employment and consumption IRFs are reversed in sign. The use of a formal only rule make this effects positive

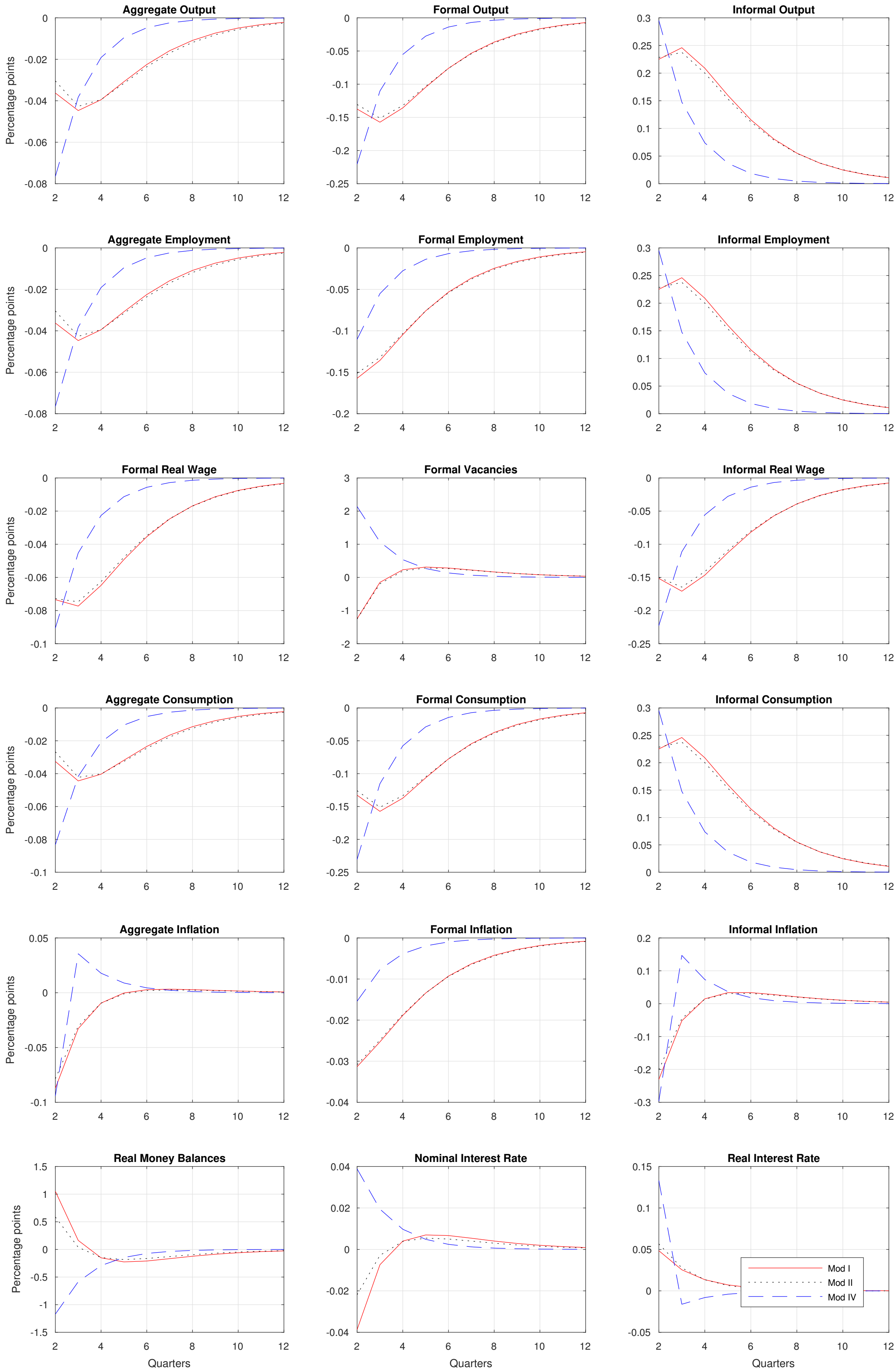

FIGURE 4.5: Dynamic Responses to a Monetary Policy Shock

Notes: Mod I = Benchmark, Mod II = Real Balance Effects and an aggregate monetary policy rule, Mod IV = Real Balance Effects and a formal only monetary policy rule.

on the informal sector, compared to negatives with an aggregate rule. Also, inflation are more negative and monetary policy tighter. Therefore, aggregate measures of output, employment and consumption grow less than with the aggregate rule.

If shocked by a exogenous perturbation on monetary policy, the use of a formal-only rule do have relevant implications. Measures of aggregate and informal inflation change in sign and the interest rate with them. Most endogenous variables initially show a stronger response than the model with aggregate rule but they return to their steady state values sooner. Finally, in the face of a demand and monetary shocks there are no relevant differences between the model with an aggregate rule and the one with a formal-only rule.

		Real Balance Effects	Formal Only Rule Effets
	a_t^f	Relevant effects on key variables	Relevant on some key variables
	a_t^i	No relevant effects at all	Many relevant changes
Shock	ξ_t^c	Smooths all IRFs of endogenous variables	Same sign as benchmark
	ξ_t^m	Relevant effects on inflation and interest rates	No relevant effects
	ξ_t^r	No relevant effects at all	Relevant on some key variables

TABLE 4.2: Summary of real balance effects and formal-only rule effects

4.2 Sensitivity analysis

In the Appendix B there are graphics for the sensitivity analysis for which nine structural parameters are considered. In few words, the graphs are interpreted as the difference in IRFs of the real balance effects model minus the benchmark model. So that, if the plot is near to zero, the real balance effects are no relevant. Additionally, if the plots for various values of the parameter are very near between them, then changes in the corresponding parameter are not so relevant.[4] It is important to stress that the analysis is done for each parameter, it is the same to say that keeping all other parameters at the benchmark level (see 3.1) only the studied parameter is varied.[5]

First of all, the analysis of sensitivity is done with respect to the benchmark model (compared with Model II) and with IRFs generated by a formal productivity shock. All graphics show responses to the same amount of shock and through the same 20 quarters in order to facilitate the analysis.

Given that, the very first important fact is that the results in the model are very robust to changes in σ, the intertemporal elasticity of substitution in consumption, as can be seen in Figure B.1

[4]The interpretation of the graphs for the sensitivity analysis is included also in the Appendix B.

[5]Of course, if a parameter of the model depends directly on one of the parameters studied in this section, then also it changes. For example, by varying ω also κ changes varies.

where model was tested changing this parameter from 1 to 5 without any noticeable difference. This feature is even more true for values of σ greater than 3.

At the same time, it is clear from the Figure B.2 that the model is very sensitive to changes in σ_n, the inverse elasticity of labor supply. Testing for values of the parameter ranging from 1 to 5 it turns out that the lower the value, the greater is the influence of real balance effects compared to the benchmark model. The more relevant differences are in output, both aggregate and by sector, employment and consumption. All of this variables are positively influenced by a lower value of σ_n. Additionally, for values of the parameter greater than 4, there are not any more relevant diminishes in their implications for real balance effects.

Through the analysis of Figure B.3 one can see that the greater the value of η_c, the consumption elasticity of money demand, the greater are the real balance effects. However, the values considered in the analysis are quite big compared to the literature standards. Even if it wasn't the case, it is possible to see that the changes in real balance effects caused by changes in this parameter are relatively low, for all the endogenous variables considered.

In the same line, the Figure B.4 indicates that a greater value of η_r implies greater real balance effects. Concretely, some specific values were tested between 4 and 40, showing that for values of the parameter close to zero, real balance effects are also close to zero. Unitary changes in the parameter have no big implications for real balance effects, approximately similar to those of η_c. It is noticeable that effects of informal sector caused for changes in this parameter are higher than those caused over the aggregate economy.

The sensibility analysis of τ in Figure B.5 is one of the most interesting. As can be seen, for values of τ ranging from from 1 to 5, the effects on aggregate measures of the economy are almost null (on aggregate output, employment, consumption), specially since the five quarter, and the same applies to inflation measures and nominal interest rate. But, the implications over sector measures are relevant. The most crucial is the contrary change in the direction in which real balance effects affects informal and formal measures in the economy, the greater is the substitutability between formal and informal goods τ, the greater are the real balance effects through informal sector and the lower are through the formal sector.

A look to the sensibility to price stickiness ω (Figure B.6) reveals another key feature of the model. When prices tends to be fully flexible (ω near zero), real balance effects disappear on both formal and informal wages. So, the greater the stickiness, the greater the real balance effects on wages. Also, as with τ, a greater value of the parameter generates stronger real balance effects on the informal sector (output, consumption and employment) while simultaneously lowering the effects on the formal sector. But, in contrast with τ there are relevant variation on aggregate measures of the economy. The more flexible prices the greater real balance effects on aggregate output, employment and consumption are found.

When one focuses on the parameter of response to inflation of the monetary policy μ_π, one is able to see that changes on it generate really hard changes on the effects of real balances. First of all, the lower the response to inflation the greater the real balance effects, but particularly the nearer is the parameter to unity the harder are the real balance effects generated by it. Also, by lowering the parameter from 1.5 to 1.1 real balance effects on all measures of output, employment and consumption rise a lot (when compared with changes generated by the other parameters analyzed here), for example informal consumption rise in 0.3 percentage points just for the effect of real balances while aggregate output and consumption rise 0.1 percentage points by the same effect.

But also, the sensitivity of the model to the output response coefficient of the central bank μ_y is huge. As can be seen in Figure B.8 there are three main results that turn out when varying μ_y. The first is that in a consistent way, the greater the parameter, the greater are real balance effects in almost all endogenous variables. The ones on which that doesn't happen are formal output, employment, vacancies and consumption, and the actions of the central bank (interest rates and money balances). Also, when the parameter is set to zero, real balance effects virtually disappear on many of the endogenous variables, indeed these reduce to the zone of 0.01 percentage points on almost all endogenous variables and in some of them reduces to zero (see, for example, inflation measures).

Finally, to see the role of the size of the informal economy some analysis of sensitivity on α is done. Given the structure of the model, $(1-\alpha)$ represents the proportion of the informal sector in the aggregate economy. One can see in Figure B.9 that this size plays a crucial role on the effects of real balances. To begin with, notice that for values of α greater than 0.5 there are changes in real balance effects but they are not so big in aggregate measures, nor in the formal sector. Indeed the greatest effect is in the informal sector, where varying α from 0.6 to 0.7 (reducing the size of the informal economy) implies a rise in real balance effects of 0.1 on informal output, employment and consumption. Now, when α is 0.5 then real balance effects are almost nullified on formal employment, output and consumption. Finally, when α falls below 0.5 then real balance effects rises a lot in all measures of the informal sector in their original direction (sign) while rising in contrary direction (sign) in the formal sector measures. So this change in α does not only increases real balance effects on both sectors, but also inverts the direction of the the effects on the formal sector. However, real balance effects on the aggregate output, employment and consumption falls together with the fall in the parameter; but a particular case is when the change is from 0.5 to 0.4, which implies virtually null changes in these aggregate measures. Therefore, one can conclude that the positive rise in real balance effects on the informal sector are compensated by the negative rise in real balance effects on the formal economy. Furthermore, the sign of the response of monetary policy by the central bank is also reverted when α is equal or below 0.5. All of these effects reflect the importance of the size of the informal economy for the effects of real balances.

In sum, the sensitivity analysis yields some important features of the model. First, that real balance effects are almost not affected by changes in parameters related with consumption (σ and η_c). Second, that some parameters can nullify real balance effects on some endogenous variables but no parameter can do it on all endogenous variables. This means that real balance effects are present in the model despite the different plausible parameterizations. Third, that lowering σ_n and increasing η_r shows a consistent and relevant increase in real balance effects on all endogenous variables. Fourth, that varying τ has only relevant real balance effects changes between the formal and informal sectors of the economy. Fifth, that modifying both of the coefficient responses of the central bank μ_π and μ_y have strong implications for the existence of real balance effects. Concretely, for some variables $\mu_y = 0$ nullifies real balance effects, while values greater than 4 for μ_π minimizes (a lot) real balance effects on many of the endogenous variables. And sixth, that the size of the informal economy $(1-\alpha)$ place a crucial role on the strength and direction of real balance effects.

4.3 Discussion: do real balance effects matter under informality?

The key to understand what is happening here is in the channels through which monetary policy acts. First of all, it must be clear that there is no presence of a wealth effect here. Indeed, as Woodford (2003) clearly states, in this kind of models there is no omission of any effect of real money balances upon aggregate demand that results from a wealth effect. Therefore, the effects of real balances apparent in the IRFs previously shown must come from another channel.

Following McKnight and Mihailov (2015), as their model also features real balance effects, there are here two channels through which monetary policy mechanism works.[6] The first channel is a conventional aggregate demand channel and the second operates via money demand, affecting the cost of production of formal and informal firms. To see that this is the case, notice that plugging equations (3.24) and (3.25) into equation (3.26) the following can be obtained [7]

$$\widehat{N}_t = \frac{\Theta}{\sigma_n}\left[\chi E_t[\eta_c\widehat{C_{t+1}} - \eta_r\widehat{R_{t+1}} + \xi^m_{t+1}] + \widehat{w}^i_t + \widehat{R}_t - E_t[\widehat{\pi_{t+1}}] - \sigma^{-1}E_t[\widehat{C_{t+1}} - \xi^c_{t+1}]\right] + other. \tag{4.1}$$

so that in equation (4.1) the size of real balance effects depends on four parameters, concretely the relevant term to see real balance effects on employment is

[6] They also use a third channel because their model was of open economy.

[7] The term called "other" is $\frac{1-\Theta}{\sigma_n}[\widehat{\Psi} + \vartheta\widehat{\theta}]$. It is omitted to simplify analysis because it is approximately zero under the baseline calibration (and many other plausible calibrations too).

$$\left(\frac{\Theta}{\sigma_n}\right)\chi E_t[\eta_c\widehat{C_{t+1}} - \eta_r\widehat{R_{t+1}} + \xi^m_{t+1}]. \tag{4.2}$$

Also, notice that something analogous for the IS equation can be done. Plugging equation (3.25) into (3.24) yields

$$\begin{aligned}\widehat{C}_t &= E_t[\widehat{C_{t+1}}]\\ &-\sigma\left[\widehat{R}_t - E_t[\widehat{\pi_{t+1}}] + \chi\eta_c E_t[\widehat{C_{t+1}} - \widehat{C}_t] - \chi\eta_r E_t[\widehat{R_{t+1}} - \widehat{R}_t] + \chi E_t[\xi^m_{t+1} - \xi^m_t]\right]\\ &+E_t[\xi^c_{t+1} - \xi^c_t],\end{aligned} \tag{4.3}$$

where one can see that there are three possible sources of real balance effects on consumption

$$(-)\sigma\chi\eta_c E_t[\widehat{C_{t+1}} - \widehat{C}_t], \tag{4.4}$$

$$\sigma\chi\eta_r E_t[\widehat{R_{t+1}} - \widehat{R}_t], \tag{4.5}$$

and

$$(-)\sigma\chi E_t[\xi^m_{t+1} - \xi^m_t]. \tag{4.6}$$

Finally, noticing that the monetary policy rule (3.43) can be restated

$$\widehat{R}_t = \mu_\pi(\alpha\widehat{\pi^f_t} + (1-\alpha)\widehat{\pi^i_t}) + \mu_y(\Phi\widehat{Y^f_t} + (1-\Phi)\widehat{Y^i_t}),$$

so that using equations (3.31), (3.32), (3.33), and (3.37), inside it, one obtains

$$\begin{aligned}\widehat{R}_t &= \mu_\pi\left[\kappa\sum_{i=0}^{\infty}\beta^i E_t[\widehat{mc_{t+i}}] + \left(\frac{1-\alpha}{\alpha}\right)\left(\widehat{w^i_t} - \widehat{w^i_{t-1}} + a^i_{t-1} - a^i_t\right)\right]\\ &+\mu_y\left[\Phi(\widehat{L^f_t} + a^f_t) + (1-\Phi)(\widehat{L^i_t} + a^i_t)\right].\end{aligned} \tag{4.7}$$

So that the transmission mechanism of monetary policy and the source of real balance effects are now exposed. Let's explain the logic behind it using the model with real balance effects under a formal productivity shock as an example.

When a formal productivity shock hits the model economy, it enters to the monetary policy rule of the central bank by two sides. One is inside the $i-th$ terms of the real marginal cost where the shock enters negatively. The other is inside the formal production as a positive term. The response of the central bank depends then on five key parameters, μ_y, μ_p, κ, β, and Φ. Given the high stickiness of the formal sector prices, κ is around 0.1, so that it hits inflation relatively light. However, the infinite sum term consists of all future shocks to formal productivity. Therefore, this number yields a considerable big negative effect on the interest rate. At the same time the formal output - aggregate output ratio Φ is around 0.7 and then, together with μ_y implies a considerable positive effect. But this effect is exceeded by the negative hit through marginal costs.

Now then, a considerable negative interest rate is set by the central bank. Since this is a deviation from the steady state value of the rate, it will be less negative each period until it returns to its steady state value. Therefore, one expects that future interest rates will be less negatives than current ones and then, via (4.5) there would be a positive effect of real balances in consumption. Additionally, since the interest rate is considerably negative, then through (4.2) there would also be a positive real balance effect on employment. Later on, this positive effects will be split between the formal and informal sector according to the respective parameters.

So, this framework can help to understand what happens with the formal productivity shock. But is it useful to understand the propagation and real balance effects caused by the other shocks considered before? Yes, it does.

Take for example the informal productivity shock. It hits directly into inflation through $\frac{1-\alpha}{\alpha}$. Since α is around 0.7, then the ratio is around 0.43, together with the coefficient response to inflation it is a considerable negative effect. On the other side, the term $(1-\Psi)$ is around 0.3 and then, together with the response to output coefficient implies a positive effect of around 0.1. The mix of effects led the central bank to set a negative nominal interest rate but not as negative as the one in the formal productivity shock. Therefore the real balance effects on consumption and employment are very low and probably outweighed by other effects.

The same reasoning helps in understanding why the other shocks are transmitted the way they do. So that some important insights are left by the previous analysis. First, it is very important to understand that the strongest source of real balance effects is on the production side of the economy. Thats why in the Figure 4.1 the greatest differences between the benchmark model and the model with real balance effects are in the labor market. Second, the role of the relative size of formal and informal sectors is crucial for results. This was already suggested by the sensitivity analysis but is further supported by equation (4.7) where it is clear the role of the relative weights of the sector (both α and Φ are important). Third, now it is easy to see what's the role of structural parameters. For example, η_c, η_r, σ_n, Θ determine the size of real balance effects on employment, whereas σ, η_c, η_r determine the real balance effects on consumption.

Fourth, the parameters that enters in the monetary policy rule directly and indirectly determines the proportion of real balance effects for each of the two sectors in the economy. Furthermore, some parameters as μ_y, κ, Φ and μ_π have the power to nullify the real balance effects on almost all endogenous variables.

Now then arises the question: do real balance effects matter under informality after all? The recent discussion could take some people to think that if a specific parametrization of the model can nullify real balance effects, then why should they care about it. I argue that they should and here is why. It is true that for some values of structural parameters one can find zones where real balance effects will be almost null. But this is not going to be the case for values of the parameters with empirical support. For example, as was showed in the sensitivity analysis a μ_y near to zero will take real balance effects to zero on several endogenous variables, but empirically it has been showed several times that this almost never happens.[8] Furthermore, almost the whole contemporaneous monetary theory and policy is built upon the consideration that central banks focuses not only on inflation but also on output. Therefore, a positive μ_y should be set in any monetary model at least that the parameter were the focus of the study.

In the same line, it's been shown in the sensitivity analysis that one of the most important parameters, if not the most, is the interest rate semi-elasticity of money demand η_r. For values under 4 for this parameter, real balance effects almost disappear in several crucial variables as formal output, labor, consumption, and wages. Galí (2015) sets $\eta_r = 4$ on its baseline calibration for example. However, as Inagaki (2009) has shown, the value of the semi-elasticity of money demand rises exponentially under environments with low interest rates. Using Japan as example, the author showed how the parameter has exceed orders of 100, 200, and more. Furthermore, he did the same analysis for the U.S. economy, and through all his sample (a quarter of century), η_r almost always was well above 4. Therefore, parameterizations with lower values for the interest rate semi-elasticity of money demand are not supported by the data.

Turning now to the price stickiness parameter ω. Also in the sensitivity analysis, and the corresponding Figure B.6, it is possible to see that for values of stickiness near to zero, *i.e.* near to fully flexible prices, some real balance effects virtually disappear. Such is the case for aggregate and formal inflation, the formal wage, formal consumption, production and labor. But values of ω near zero are not supported by the data. Indeed, Kehoe and Midrigan (2015) showed that the degree of price stickiness is around 59% (or an $\omega = 0.59$). In their study authors stress that even though prices at the micro-level change continuously, aggregate prices are sticky. So, again, parameterizations with ω near to zero have no support from the data.

[8] See for example Mcknight et al. (2016) and Moura and de Carvalho (2010). In both o these works the authors showed that central banks in Latin America are interested not only on inflation. Additionally, both papers found a positive and relevant value for the coefficient response to output of the central bank.

Therefore, under realistic parameterizations of the model real balance effects will always exist. And as such, they should be taken into account by the monetary authority when setting its monetary policy. This is specially true when it comes to a country in which the informal sector has a relevant size. It's been shown in the sensitivity analysis how the bigger the informal sector, the bigger the real balance effects with a positive sign on this sector and the bigger with a negative sign on the formal sector would be. Additionally, recall that the stronger real balance effects were shown to be in the informal sector (see Figure 4.1), together with formal and aggregate employment. And since employment is usually a recurrent issue in economies with a big informal sector, then it becomes even harder to ignore real balance effects. Because now it must be clear that real balance effects matter under informality, and it is specially true for the informal sector of the economy.

Chapter 5

Conclusions

Despite the widespread dominance of the so called New Keynesian models in the study of economic fluctuations, there are some considerable gaps in the literature when it comes to specific aspects of developing economies. One of these aspects is the presence of a relevant informal sector. In this work I've argued that the existence of an informal sector matters for the economy dynamics. Further I also argued that real balance effects are relevant under informality and that a formal-only monetary policy rule features some important differences compared with a typical policy rule.

Concretely, a New Keynesian model with two sectors has been developed to account for the referred dynamics. The informal sector was modeled in a classical perfectly competitive way, while the formal sector featured some frictions. Concretely, the goods market side of the formal sector was modeled as being immerse in monopolistic competition while the labor market of the sector was described through a search and matching approach where wages are solved via a Nash Bargaining procedure.

The two sector model was calibrated to the Mexican economy and its dynamics were analyzed when several kind of shocks hit it. Concretely, when the economy faced a positive formal productivity shock, consumption, employment and production, all rose more with real balance effects than without them. In face of a monetary shock, real balance effects are only relevant for inflation measures and interest rates.

When the model is shocked by a demand perturbation, no quantitatively relevant effects are found, but there are one qualitative result which seems relevant: all IRFs are smoother under real balance effects. Importantly, and in stark contrast, if the exogenous shock is a monetary policy or a informality productivity shock, no relevant real balance effects are found.

Turning now to the effects caused by a formal only interest rate. It was found that in general, when compared with a typical aggregate rule, there are relevant differences in the propagation of shocks. Some of the most relevant differences appear when the economy is hit by a informal productivity shock. IRFs of prices and interest rates change in sign, and informal and formal sector measures grow less. When the economy is hit by a formal productivity shock, informal output, employment and consumption IRFs are reversed in sign.

If the economy is shocked by a exogenous perturbation on monetary policy, the use of a formal-only causes measures of aggregate and informal inflation to change in sign and to the interest rate change with them. Most endogenous variables initially show a stronger response than the model with aggregate rule but they return to their steady state values sooner. Finally, in the face of a demand and monetary shocks there are no relevant differences between the model with an aggregate rule and the one with a formal-only rule.

To test for robustness of the results, a sensitivity analysis was done on some of the most relevant structural parameters. Two relevant results deserves mention. First, that some key parameters can virtually neutralize real balance effects on several endogenous variables, but not on all of them. Second, and in spite of the former, for empirically plausible calibrations, real balance effects will always exist in a model as the considered here.

Finally, it was shown that the dynamics of the model and specially the significance of real balance effects crucially depends on the size of the informal sector. Through the sensitivity analysis and also analytically, it was shown that the greater the size of the informal sector, the greater and positive are real balance effects on this sector but also the greater and negative are real balance effects on the formal sector.

All the above strongly suggests that real balance effects matter under informality and therefore should be taken into account by the monetary authority when deciding its monetary policy.

Appendix A

Steady State Equilibrium

The steady state of the model economy is assumed to be around a zero (gross) inflation steady state, so that all the time the steady state values for prices will be the same. Let's further assume that $P = P^f = P^i$ state values of formal and informal prices are the same, this is a fair assumption for a closed economy as this one.

Therefore, the system of steady state equations is the following of 19 equations and 20 endogenous variables. The condition established (equality of steady state prices) close the system to have 20 equations.

$$\theta = \frac{V}{L^i} \tag{A.1}$$

$$L^f = \left(\frac{1}{\delta}\right) \mathfrak{m}\theta^{\vartheta} L^i \tag{A.2}$$

$$\beta = \frac{1}{R} \tag{A.3}$$

$$\frac{\overline{u_m}}{\overline{u_c}} = \frac{R-1}{R} \tag{A.4}$$

$$\overline{v_n} = w^i \overline{u_c} + \Psi \mathfrak{m}\theta^{\vartheta} \tag{A.5}$$

$$\Psi = \frac{\beta}{1-\beta(1-\delta)} \left[w^f \overline{u_c} - \overline{v_n}\right] \tag{A.6}$$

$$P = \left[\alpha (P^f)^{1-\tau} + (1-\alpha)(P^i)^{1-\tau}\right]^{\frac{1}{1-\tau}} \tag{A.7}$$

$$C^f = \alpha \left(\frac{P^f}{P}\right)^{-\tau} C \tag{A.8}$$

$$C^i = (1-\alpha) \left(\frac{P^i}{P}\right)^{-\tau} C \tag{A.9}$$

$$Y^i = L^i \tag{A.10}$$

$$w^i = 1 \tag{A.11}$$

$$Y^f = L^f \tag{A.12}$$

$$mc = w^f \tag{A.13}$$

$$P^f = \mathscr{M} mc \tag{A.14}$$

$$w^f = (1-\mathfrak{B})w^i + \mathfrak{B}\left(\frac{P^f}{P} + c\theta\right) \tag{A.15}$$

$$N = L^i + L^f \tag{A.16}$$

$$Y^f = C^f + cV \tag{A.17}$$

$$Y^i = C^i \tag{A.18}$$

$$Y = Y^f + Y^i \tag{A.19}$$

However, the previous system has some partial derivatives evaluated at steady state values. Therefore in order to use it computationally some manipulation is needed. Using the definitions of χ, υ and σ the following equation, equivalent to (A.4), is obtained

$$m = \frac{CR}{(R-1)}\frac{\chi}{\upsilon}. \tag{A.20}$$

Likewise, plugging equation (A.5) inside (A.6) and using the same previous definitions the following expression for Ψ is derived

$$\Psi = \frac{\beta}{1-\beta(1-\delta-\mathfrak{m}\theta^{\vartheta})}\frac{CR}{(R-1)}[w^f - w^i]. \tag{A.21}$$

Therefore, the system reduces to 18 equations and 20 endogenous variables which can be solved with the equality price condition and normalizing total labor N to unity. This finally left a system of 19 equations and 19 endogenous variables suitable to be solved in dynare.

Appendix B

Sensitivity Figures

The upcoming graphs are obtained in the following way. For each of the parameters considered in the sensitivity analysis

1. For the Model I and Model II (with an aggregate typical monetary policy rule) I applied a formal productivity shock. This is because it's been shown that it features relevant real balance effects.
2. I saved the IRFs of this two models.
3. For each of the values of parameters I generated a series with the "residuals" result from a simple operation: $RESID = IRF_{withRBeffects} - IRF_{withoutRBeffects}$.

Since the interest here is to see what are the changes in real balance effects when changing the structural parameters, then the *RESID* series provides a simple way to see for each value of the parameter the wider difference and the closer difference between the two series. Therefore, the interpretation for graphs is straightforward: the nearer to zero the lesser real balance effects, the farther from zero the stronger the real balance effects are. Additionally, the graphs lets us see if changes in the differences are relevant or not, because the units of the y axis are percentage points still.

So, for example, the model is very sensitive before changes in μ_y but zero sensitive before changes in ω.

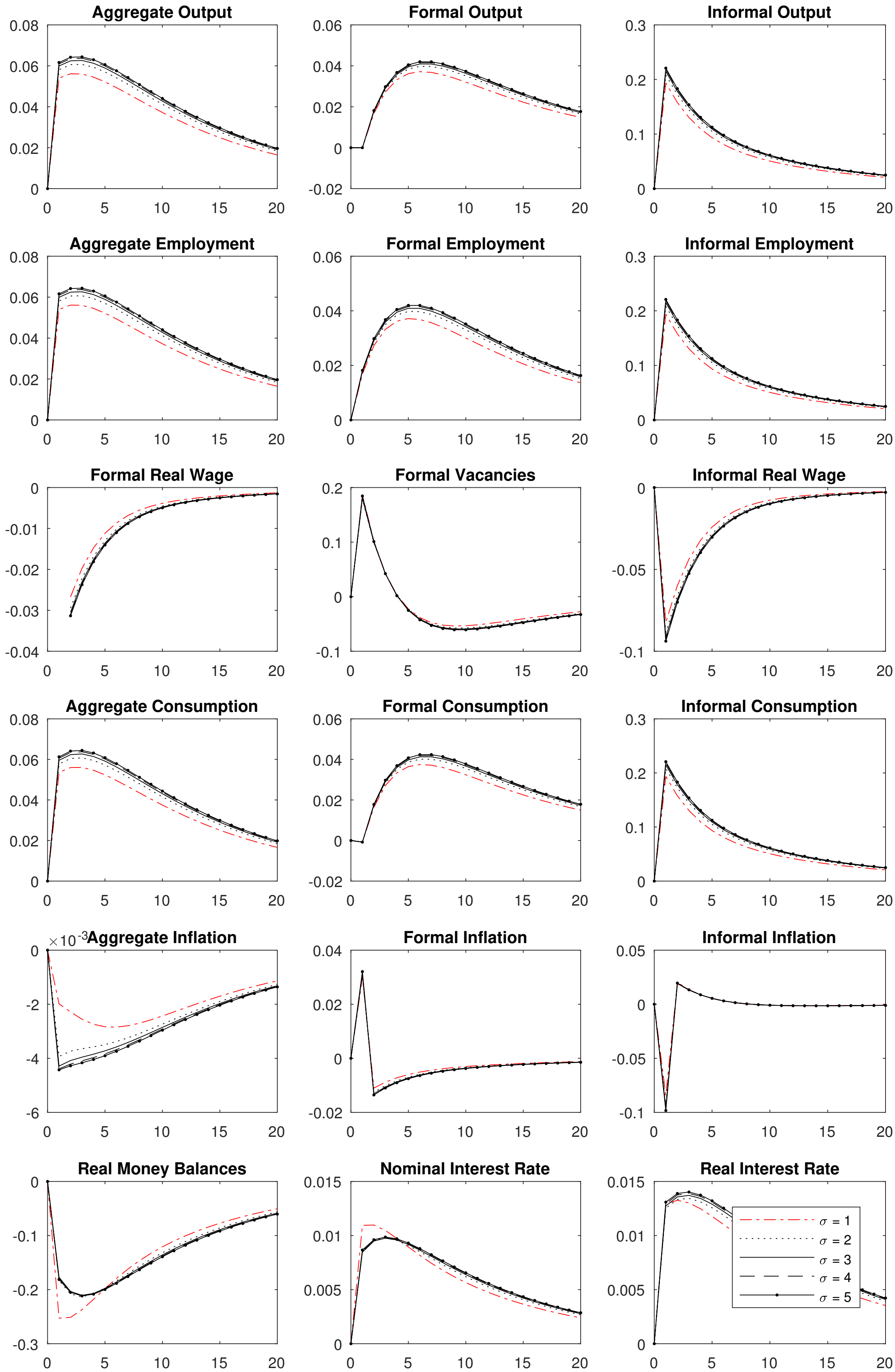

FIGURE B.1: Sensitivity of real balance effects before changes in σ

Notes: In the figure, the difference between the IRFs with and without real balance effects (benchmark model) are plotted.

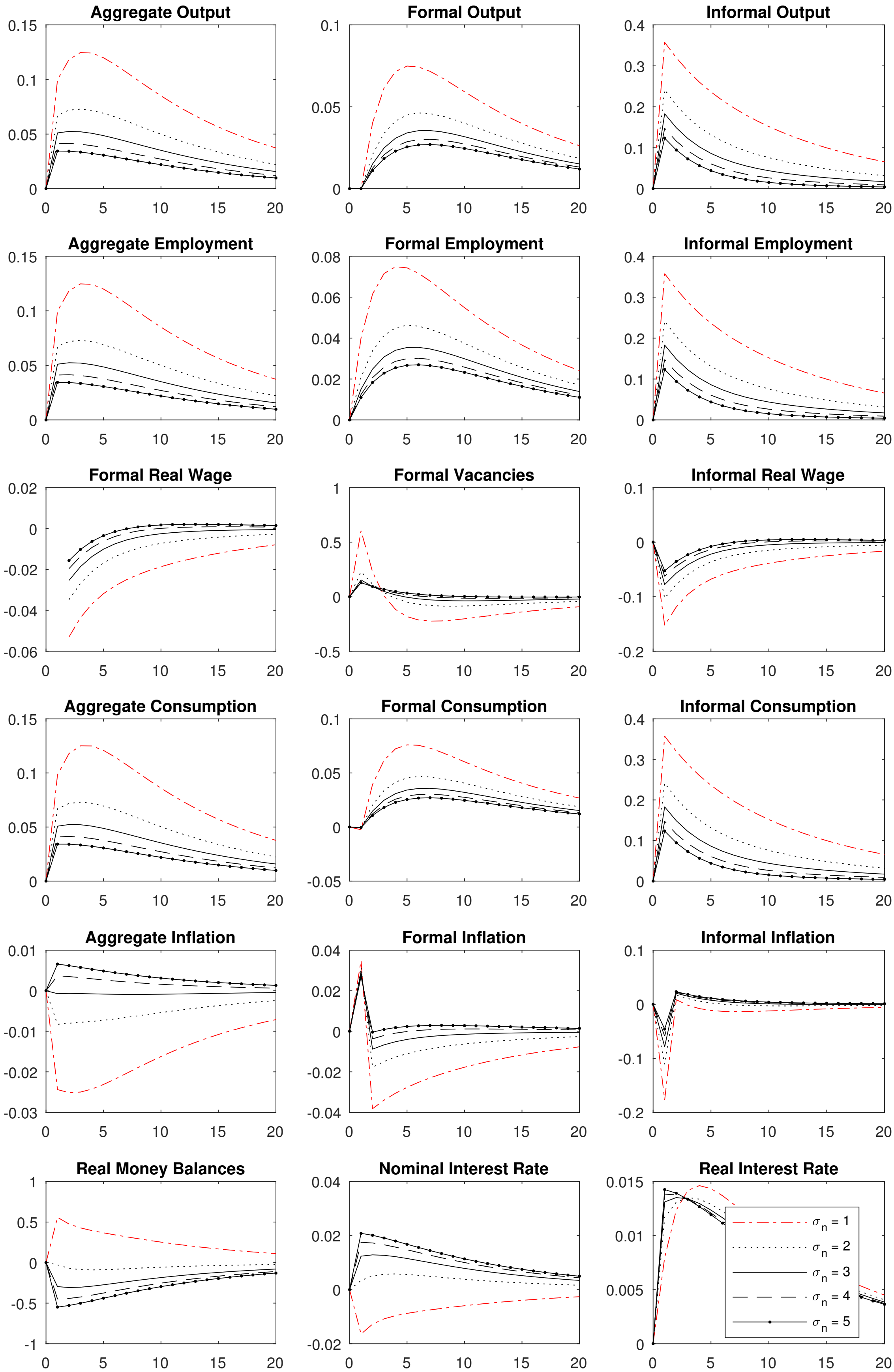

FIGURE B.2: Sensitivity of real balance effects before changes in σ_n

Notes: In the figure, the difference between the IRFs with and without real balance effects (benchmark model) are plotted.

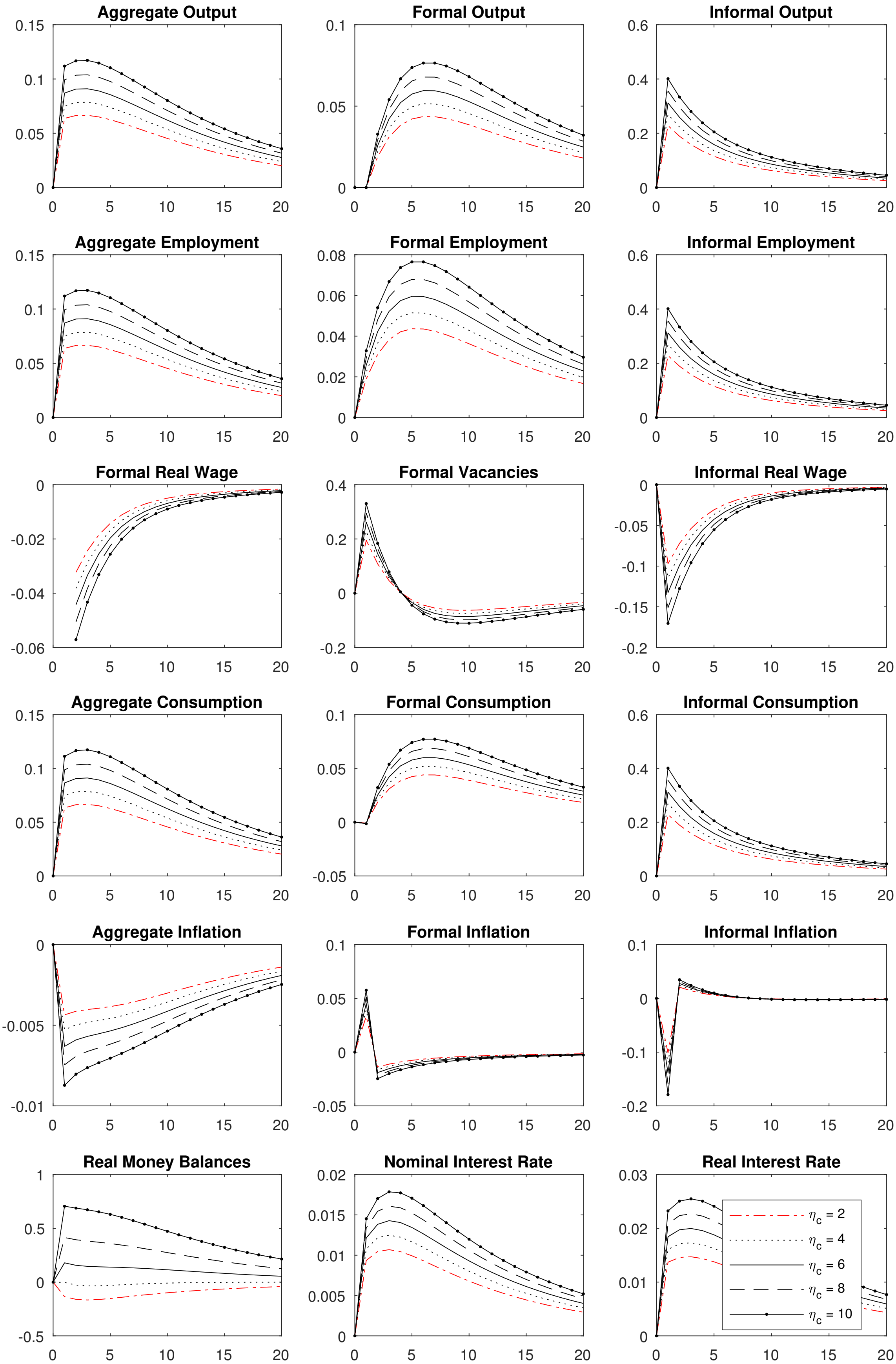

FIGURE B.3: Sensitivity of real balance effects before changes in η_c

Notes: In the figure, the difference between the IRFs with and without real balance effects (benchmark model) are plotted.

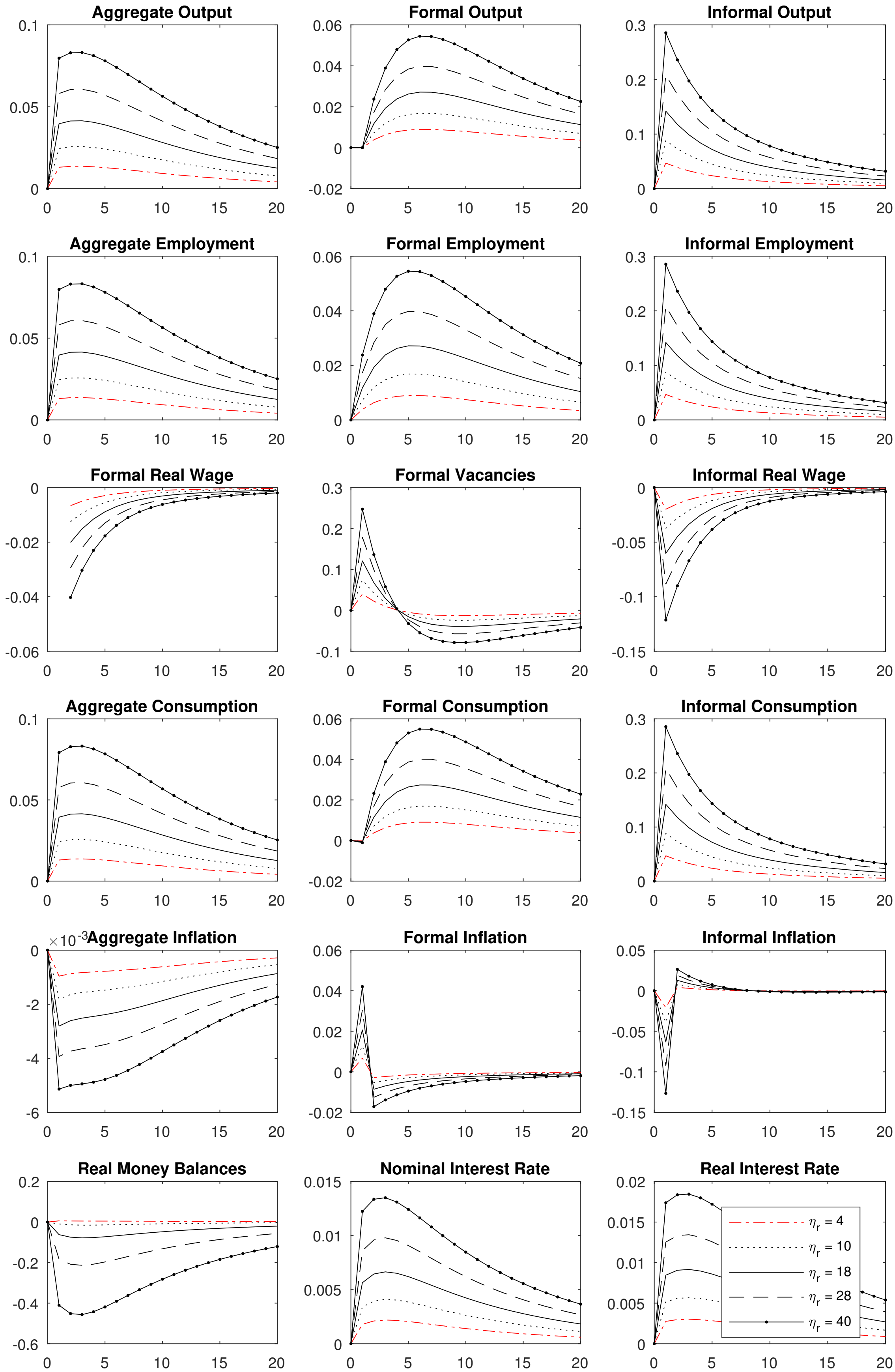

FIGURE B.4: Sensitivity of real balance effects before changes in η_r

Notes: In the figure, the difference between the IRFs with and without real balance effects (benchmark model) are plotted.

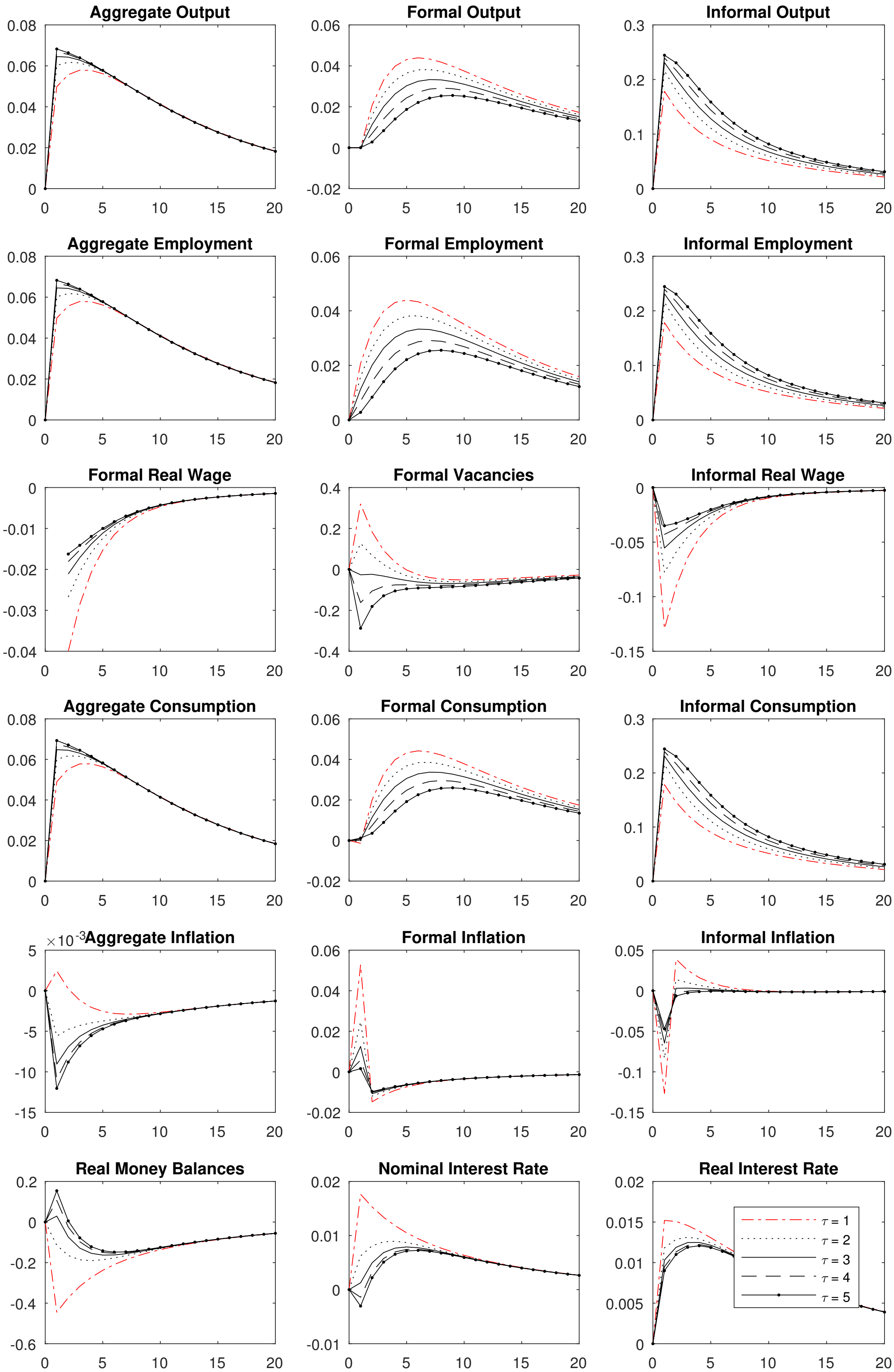

FIGURE B.5: Sensitivity of real balance effects before changes in τ

Notes: In the figure, the difference between the IRFs with and without real balance effects (benchmark model) are plotted.

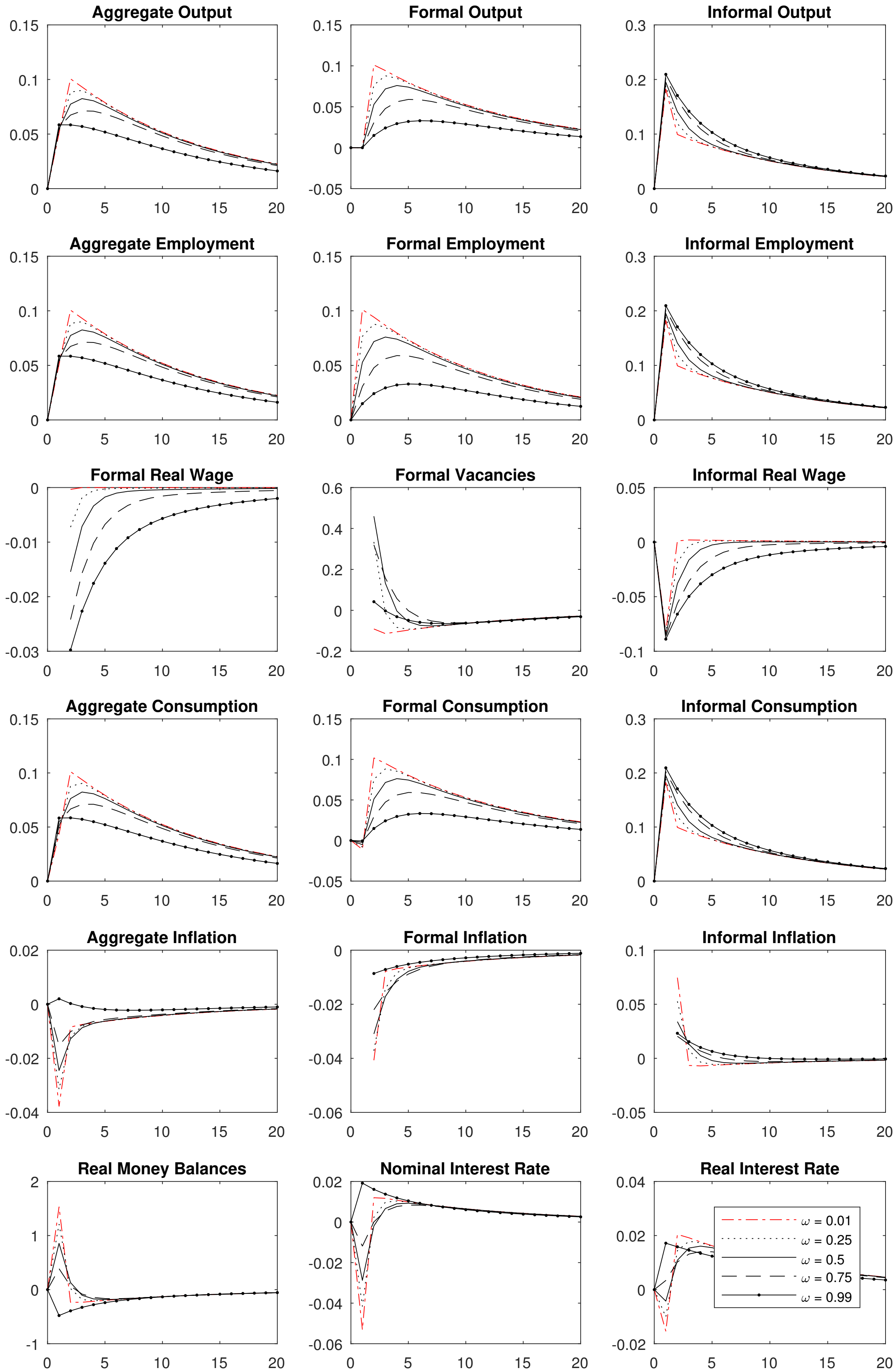

FIGURE B.6: Sensitivity of real balance effects before changes in ω

Notes: In the figure, the difference between the IRFs with and without real balance effects (benchmark model) are plotted.

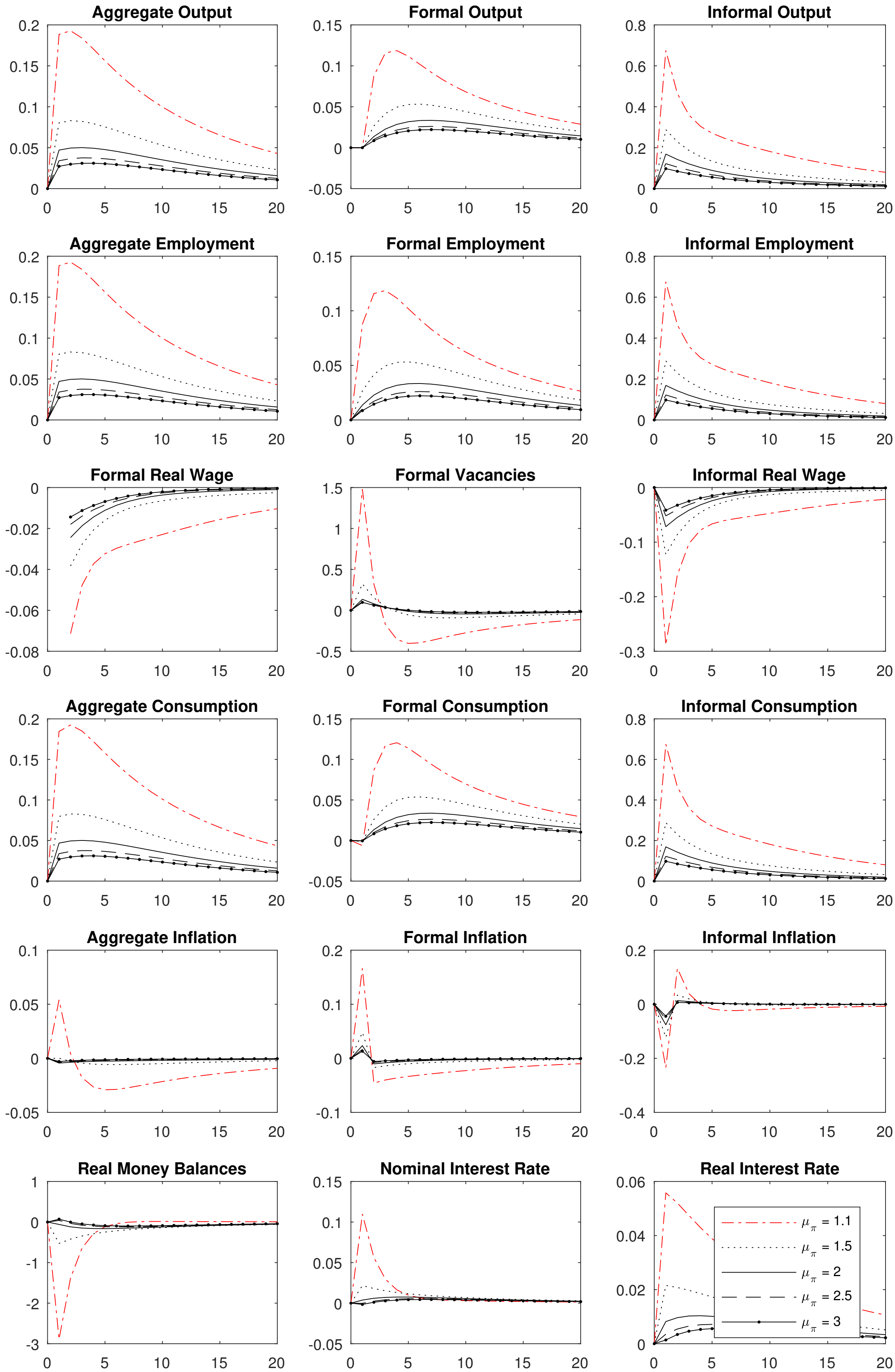

FIGURE B.7: Sensitivity of real balance effects before changes in μ_π

Notes: In the figure, the difference between the IRFs with and without real balance effects (benchmark model) are plotted.

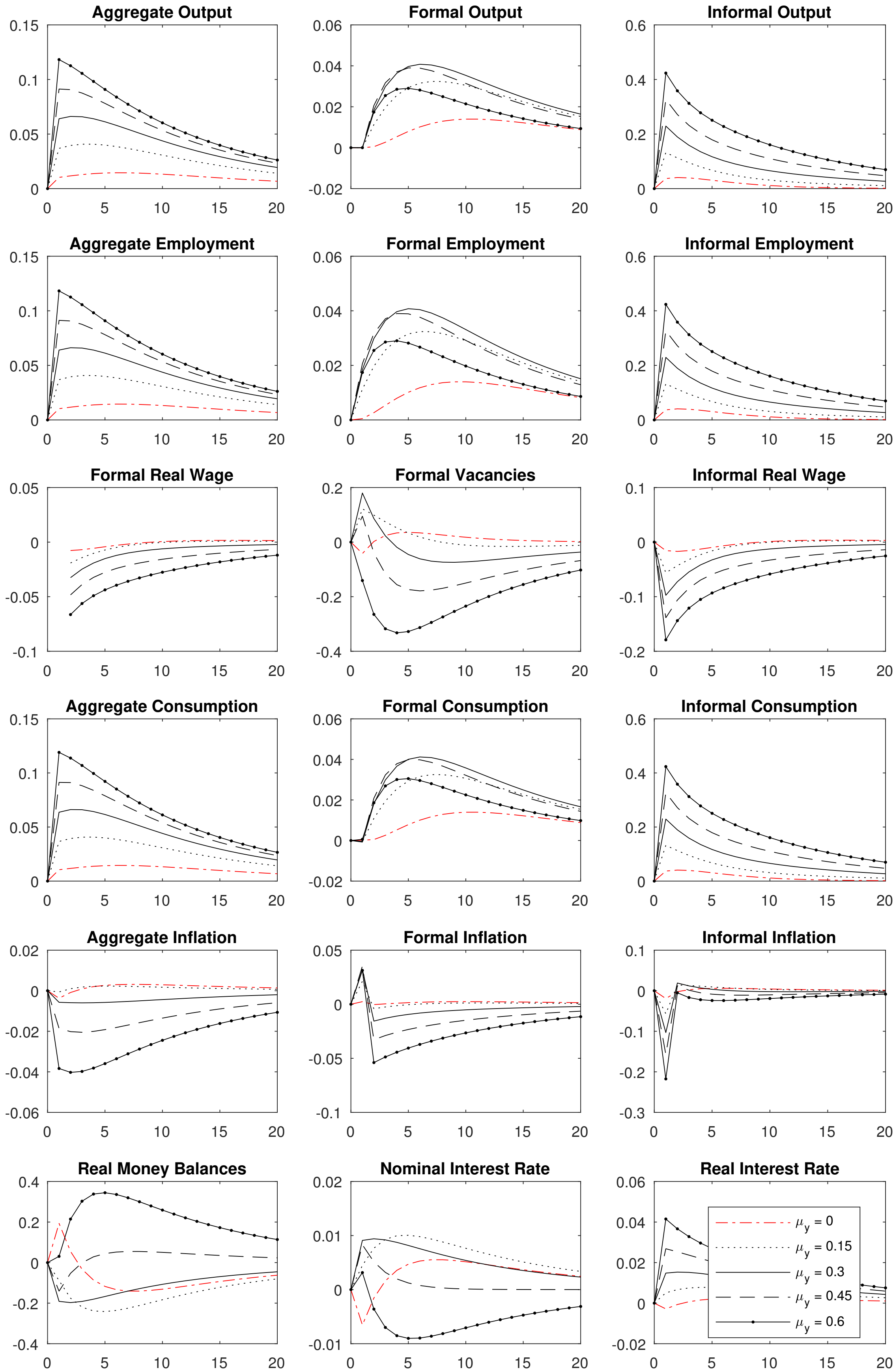

FIGURE B.8: Sensitivity of real balance effects before changes in μ_y

Notes: In the figure, the difference between the IRFs with and without real balance effects (benchmark model) are plotted.

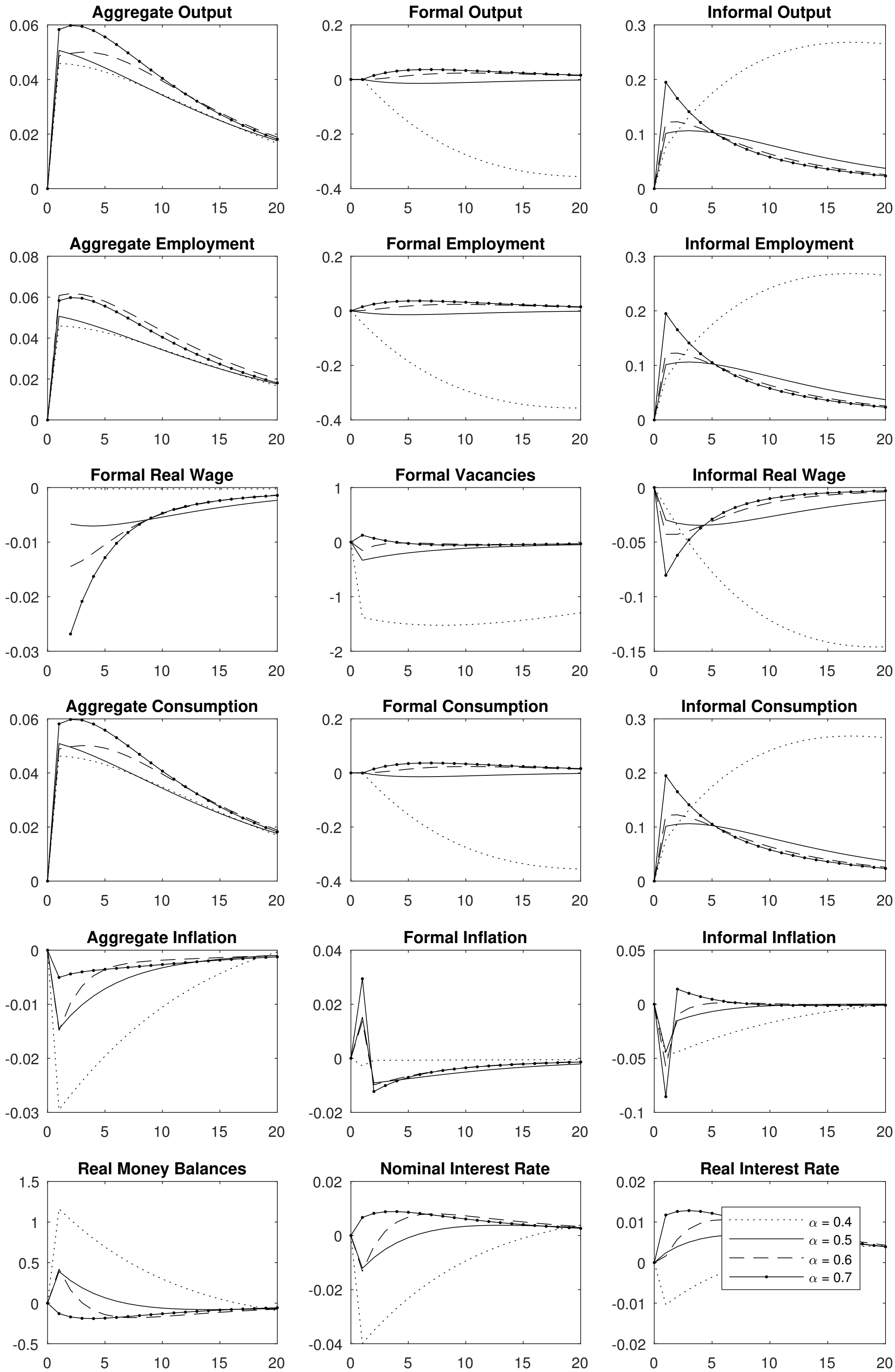

FIGURE B.9: Sensitivity of real balance effects before changes in α

Notes: In the figure, the difference between the IRFs with and without real balance effects (benchmark model) are plotted.

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