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MEXICAN TARGET-DATE FUNDS BEHIND THE SCENES:
THE EFFECTS OF VALUE-AT-RISK ON RETIREMENT SAVINGS

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Mexican Target-Date Funds Behind the Scenes:

The effects of Value-at-Risk on retirement savings.

Francisco Javier Martínez Ramírez

Abstract

In this thesis, using data from the CONSAR, I simulated final pension savings that the workers would achieve with two different values of VaR, specifically using 97 and 99 levels of confidence. In order to perform the simulations I had to built one portfolio composed by all the 10 AFOREs, properly weighted with the managed accounts of each pension fund manager. After constructing the portfolio I used the Black-Scholes-Merton model to simulate 10 thousand returns of that constructed portfolio, which helped me to calculate the different levels of Value-at-Risk and its inherent returns. Next, by assuming some values regarding the wage, the contribution density, and the contributions made to the personal account, I constructed the final pension of a worker. Finally, with all the previous information I simulated the final pension savings of 10,000 workers, which are the main results of these thesis.

The results showed me what I suspected from the beginning: the increase in the significance level of the VaR allows the workers to achieve greater pension savings at the end of their working life. In addition, this conclusion is robust, provided that both proposed levels of VaR first-order stochastically dominate the current allowed level of VaR. Moreover, if allowing for a less conservative VaR increases the final wealth of the worker, combining these investment strategies with higher wages, higher contribution densities, and higher contributions to the AFOREs will lead the workers for a better and more dignified retirement.

Glossary

AFORE: Administradora de Fondos para el Retiro (Pension Fund Manager).

B-S-M: Black-Scholes-Merton.

CD: Contribution Density.

CONSAR: Comisión Nacional del Sistema de Ahorro para el Retiro (National Comission on the Retirement Savings System).

CVaR: Conditional Value at Risk.

DB: Defined Benefit.

DC: Defined Contribution.

DCVaR: Differential of Conditional Value at Risk.

IMSS: Instituto Mexicano del Seguro Social (Mexican Institute of Social Security).

MGP: Minimum Guaranteed Pension.

NR: Net Returns.

RR: Repalcement Rate.

SAR: Sistema de Ahorro para el Retiro (Retirement Savings System).

SIEFORE: Sociedad de Inversión Especializada en Fondos para el Retiro (Target-Date Fund).

TDF: Target-Date Fund.

VaR: Value at Risk

Contents

1 Introduction

Through years, as it was expected, the reformed Mexican pension system has become a paramount subject among social, political, and economical circles. Several reasons such as population ageing, fiscal pressures for the state, the informality sector, lower expected pensions, the level of fees charged by the AFOREs, and more, have turned the pensions debate into a controversial one; mostly because the number of personal accounts managed by the AFOREs is increasing year by year.

In 1997, the Mexican pension scheme experienced a major reform, which transformed it from a pay-as-you-go scheme into a private scheme of individual capitalization (defined contribution, DC, system), in which employees must save to fund their own retirement (Villagómez and Hernández, 2009). These personal contributions are complemented with those from employers and the government. Given this new scheme, the money contributed for the worker's retirement is administered by funds known as pension fund managers, AFOREs. These companies are authorized by the Minister of Finance and Public Credit to manage individual retirement accounts, where the employee's retirement savings are transferred. Therefore, the AFOREs', and the whole pension system, aim to accomplish two main goals: to protect workers from inflation and to maximize their replacement rate when they retire.

As the reform mandates, AFOREs must invest the worker's savings in SIEFOREs, which are investment funds with the only motivation of generating returns. Before 2019, individual retirement accounts were transferred, depending on the worker's age, from one SIEFORE to another; that is, the closer the individual was from retirement age, 65 years old, the account was transferred to a SIEFORE with more conservative portfolios (with more riskless assets). However, in the summer of 2019, the Mexican Congress approved an important reform to the SIEFOREs' operation [1]. This new reform transformed the SIEFOREs into Generational SIEFOREs, inspired by the findings of Robert Merton: to maximize the workers' savings (Fund Society, 2019). Now, instead

[1] DOF May 31st, 2019

of individual accounts going through several SIEFOREs according to the worker's age, the individual accounts would rest on one solely SIEFORE until all savings are retired when the target date arrives. Thus the portfolio's structure is adjusted depending on the employee's proximity to retirement, that is, decreasing the proportion of equity and risky assets, and increasing the share of fixed income, such as bonds and cash. The idea behind this new system is to "smooth the investment strategy, enhance long run investments, and to allow for better benefits to the workers" (Fund Society, 2019).

Since the 2019 reform, which transformed the SIEFOREs into target-date funds (TDFs), the AFOREs have further operational areas and can now, for instance, invest in foreign assets and riskier assets [2]. Therefore, this new operational system oblige us to look deeper into the construction of each AFORE glide path, which is the composition of the pension fund's portfolios through time: since the worker is registered in the Mexican Institution of Social Security, IMSS, until she retires.

It is commonly believed that portfolios become safer by increasing the proportion of government bonds and, at the same time, decreasing the share of equity. Nevertheless, this may not be completely true, as it is shown further in the analysis. Depending on the market structure at some time, a portfolio with a low proportion of equity may be riskier than if it had a greater share of those risky assets (Yoon, 2010).

Hence, there are several risk measures used by the AFOREs and the regulator, CONSAR, such as the value at risk (VaR), or conditional value at risk (CVaR), which measure the volatility of harmful unexpected events in the market to avoid catastrophic losses for investors (workers), which can deeply undermine their retirement wage, leading them, in the worst case scenario, to a severe poverty situation when old. For instance, in Mexico, 41 per cent of retired workers continue working due to lack of savings and the necessity of an income (SEGOB, et al, 2015).

The transformation of SIEFOREs to Generational SIEFOREs in 2019, as it was mentioned before, including the possibility for AFOREs to invest in international and

[2] See Notes about SAR no.3 (in Spanish).

riskier assets, increases the underlying risks of the pension funds' glide paths. However, the reformed regulation restricts these new portfolios to a low VaR and CVaR measures (less than 1% for the CVaR case), which could not entirely maximize the worker's replacement rate.

Therefore, the current strict rules regarding the VaR and CVaR leads us to this thesis objective: I will suggest a dynamic value-at-risk asset allocation approach regarding the investments of the Mexican TDFs. This new approach will consider two aspects: firstly, a looser parameter for both VaR and CVaR, and, secondly, that those risk quantifiers will change, restricted to the new criterion, not only according to the worker's age, but also according to market conditions.

Therefore, it is expected that changing the allowed risk for the AFOREs investments, through the VaR and the CVaR, while taking into consideration the market conditions (either favorable for the workers or not), when the target date arrives, the pension fund manager would have maximized the workers' replacement rate.

It is worth noting that the 2019 reform took effect on December, so there is not enough data to find reliable estimates using historical methods. Consequently, I will simulate AFOREs' returns. Besides, in the simulations, I will consider some of the latest changes to the pension system: the diminution of mandatory weeks affiliated to the IMSS, and the increase on the contributions made by the employers.

This dissertation is divided into six sections: the first one consists of the introduction. The second contains the theoretical framework, which is divided into two subsections: in the first one there is a literature review regarding optimal glide paths, portfolio risk analysis (focusing on VaR), and long-run asset allocation; while the second subsection contains some important characteristics about the Mexican labor market and the Mexican pension system, such as wages, IMSS affiliates. The third section consists of an analysis of the AFOREs' actual glide paths, their underlying risks, the number of accounts managed, and important regulation to keep in mind for the following section. The fourth section consists of the results, where I will find, through simulations, an optimal glide path and a VaR measure that minimizes risk to maximize the replacement ratio of the average worker in Mexico. The fifth section consists of the stochastic dominance of the proposed new approach over the current investment framework. Finally, the last section consists on the conclusion of the dissertation.

2 Theoretical Framework

2.1 Literature review on target-date funds

The main theoretical and empirical works on the field are suited for the United States context, regarding the 401(k) plan [3]. Therefore, this thesis uses the techniques proposed and used to the Mexican case. There are few Mexican studies regarding the possible or actual outcomes of the generational funds.

One of the main studies regarding TDFs, also known as life cycle investments, is the one made by Bodie, Merton and Samuelson (1992), where they consider the leisure-consumption choice on portfolio and consumption decisions through the individual's working age. An important assumption of labor supply flexibility, that is, the liberty workers have to manage their leisure-consumption problem. The importance of this assumption lies on the fact that labor supply flexibility can "play an important role in household asset allocation", and it is, according to their investigation, what generates the importance of glide paths (which refers to "a formula that defines the asset allocation mix of a target-date fund, based on the number of years to the target date"[Chen, 2019]).

A study that deserves important consideration is the one made by Poterba et al. (2006). Even though they do not only focus on a life-cycle model, they reach some conclusions worth considering not only for TDFs analysis but for the general construction of defined contribution systems. Their analysis simulates retirement wealth distributions and, by using historical asset returns, they determine how the distribution of retirement wealth and the expected utility of wealth at retirement are affected through different

[3] To understand 401(K) see: What Is a 401(k)? (n.d.)., from https://guides.wsj.com/personal-finance/retirement/what-is-a-401k/

asset allocation strategies. Provided the heterogeneity of households' earnings experiences, the expected utility associated with different 401(k) asset allocations is sensitive to three parameters: "the expected returns on corporate stock (due to correlation with labor income), the worker's relative risk aversion, and the non-401(k) wealth at retirement". The first conclusion consists of the similarity between the retirement wealth associated with a TDF and that associated with age-invariant asset allocation. Second, at modest levels of risk aversion, or having access to non-401(k) wealth at retirement, the expected utility of an all-stock portfolio is greater than others having a more conservative approach. Finally, life-cycle funds should be different regarding the marital status of households, given that "individuals have fewer opportunities to respond to an adverse economic shock than married couples, so their tolerance to equity market risk in their retirement accounts may be different from that for married couples". This last conclusion is important for the Mexican case due to the gender wage gap in the country, where in average, men are paid 14.6 per cent more than women (Estrella, 2020), and due to the fact that 28% of Mexican retired workers depend on someone else. For instance, 20% of those dependent workers rely on their children, while 6% rely on their couples (SEGOB, et al, 2015).

An important consideration regarding the findings of Bodie, Merton, and Samuelson (1992); and Poterba et al. (2006) is the fact that wages are treated as exogenous. Treating labor earnings exogenously is a constant in many studies, such as in Viceira (2001), who, by using risky labor income, concludes that savings level is "more responsive to changes in labor income risk than portfolio demand for risky assets when investors are moderately or highly risk-averse".

Cocco, Gomes, and Maenhout (2005) observe that using an empirically calibrated with (a small) probability of wage disaster, that is, losing all labor income, decreases, on average, demand for equities. The study of Gomes and Michaelides (2005) concludes that most households who present little risk aversion never invest in stocks, while more prudent ones do invest in equity, although not a fully all-stock investment. Gomes, Viceira, and Kotlikoff (2006) discuss the efficiency loss in investment decisions generated by the excess burden of government indecision and find that uncertainty generated by governments decreases welfare gains of investors.

Contrasting the previous studies, Gomes, Kotlikoff, and Viceira (2008) endogenize labor income in an optimal life-cycle investment model incorporating flexible labor supply. It is important to notice that they were not the first researchers to treat labor supply as flexible, see French (2005), Low (2005), Bodie, Merton, and Samuelson (1992), Bodie et al. (2004), and Chan and Viceira (2000) (as cited in Gomes, Kotlikoff, and Viceira, 2008). However, since wages are endogenous, their results seem to be closer to actual data than those assuming only adjustable labor supply. They (Gomes, Kotlikoff, and Viceira, 2008) find that if "an investor (an employee) follows a strategy of constantly rebalancing her portfolio to change with age along a deterministic path, mimics the strategy typically followed by life-cycle funds".

How come the studies exposed above are relevant to understanding the effects of value-at-risk on retirement savings? As it was explained above, assuming endogenous wages allows the life-cycle models to adjust to what the data says. Therefore, as I will be working with data from target date funds, an implicit assumption regarding the Mexican labor market is the flexibility of labor supply and the endogeneity of wages. Besides, these assumptions are reasonable given the high levels of informality in the Mexican system.

In addition to the preceding studies, it is the one presented by Campbell and Viceira (2001). In their study, they use a calibrated numerically solved life-cycle model to microeconomic US data. Like the rest of the authors, they obtain the same result regarding the high exposure to stock that young investors should face, due to their high horizon investment and the large human wealth they possess. Their second conclusion is also a well-known result in the defined contribution literature: as investors age, their attractiveness to risky assets diminishes provided their accumulation of financial wealth and the little human wealth they possess compared to their younger counterparts.

Furthermore, Poterba et al. (2006) discover heterogeneity across households' earnings processes and hence in their investment strategies. Moreover, they understand the importance of recognizing the difference between high risk-averse and impatient households provided that the former accumulate more financial wealth (given their willingness to save) and are more exposed to financial risks than the latter, which accumulate little savings and can invest more aggressively making financial risks unimportant to them. However, their model predicts that households should invest more aggressively than they do. To finish their work, they suggest that an important variable to consider, regarding the investment attitudes towards workers' retirement, should be their fixed commitments (mortgages, school tuition, etc.), which they think the model would recommend a more conservative approach to asset allocation.

After presenting the main theoretical works regarding life-cycle models, and given that it is considered a different asset allocation approach on TDFs, it is time to concentrate on literature regarding the analysis of glide paths construction by pension fund managers, the AFOREs in the Mexican case.

To begin, it is important to make clear the definition of a glide path, which refers to the long-term planning tool that defines the nature and objectives of the fund scheme to, progressively, reduce portfolio's risk (CONSAR, 2018) until the worker has accomplished at least 750 weeks listed for the IMSS and the worker is the retirement age, 60 to 65 years old, according to Mexican law. Given the alternative approach proposed in this thesis, which considers higher levels of value-at-risk allowed to increase workers' pensions savings, let me start considering studies that contemplate a risk perspective on glide path construction.

Lewis (2008) develops "a stochastic simulation model to capture the impact of different target date equity glide paths on retirement wealth creation". He questions the rule of thumb used to set equity allocation[4], since fund managers can apply several different asset allocation strategies apart from the rule. The strategies are based on how aggressive or conservative funds can be to maximize the long-term investors' wealth. To compare the results between multiple strategies, Lewis (2008) uses a combination of

[4]The percentage allocation of portfolios to stocks or risky assets should be 100 minus age in years.

bootstrapped historical data and Monte Carlo simulation to estimate real savings for each of the glide paths.

The estimations of Lewis (2008) show that the aggressive strategy (which allocates 100 percent to equities withing 35 years to retirement) has the same lower bound of the wealth distribution as the conservative scheme (which allocates 80 percent to equities withing 35 years to retirement); nonetheless, being aggressive returns a distribution upper bound 25 percent higher than the conservative, with a replacement ratio of 0.38 (compared to a 0.33 given by the conservative approach).

An interesting critic regarding the conventional asset allocation of TDFs is the one made by Basu, Byrne, and Drew (2011). They construct a dynamic life-cycle model conditional on the investors' wealth accumulation plan. Conversely to the conventional strategy, the dynamic life-cycle allocates fixed income and equity depending on the investors' accumulated wealth after some time, that is, the switch to fixed income from an all-equity portfolio only takes place if the worker accumulates wealth above the final objective. After the switch, whether the wealth accumulation falls below the target in any period, the portfolio's asset allocation should move towards stocks. After simulating wealth distribution for a hypothetical worker, they find that the dynamic strategy outperforms the conventional in terms of the mean, median, and lower- and upper-quartile outcomes. However, to them, that does not mean that the dynamic strategy is better than the traditional, thus, to prove this, they appeal to stochastic dominance (SD) of wealth distributions.

More precisely, to the almost stochastic dominance (ASD) approach proposed by Leshno and Levy (2002) (as cited in Basu, Byrne, and Drew, 2011), provided the restrictive conditions of SD. After such an analysis, they prove that the dynamic strategy "invariably results in much higher wealth accumulation (...) compared to the conventional life-cycle strategy". Moreover, their study finds that even a balanced strategy (a portfolio with the same share of equity and fixed income during the investor's working life) outperforms the conventional strategy, where only age is solely reason to decline the stock exposure of the portfolio.

Yoon (2010) introduces “a new approach to define the glide path of target-date funds” by, firstly, determining the level of risk budget for each TDF “permitting only appropriate amounts of risks at the right time”. To do so, he mentions that it is necessary to avoid assuming constant risks and returns, since risk changes continually, impeding returns to be the same over time, making the assumption difficult to hold. However, as he proves with the findings of Campbell and Viceira (cited in Yoon, 2010), there is a mean-reverting behavior in stock returns in the long-run, that is, stocks through years experience a decrease in market volatility.

It is important to consider that, even though the stocks present mean-reverting processes in the long-run, as long as the term structure of risk is not flat, the strategy of just decreasing the share of stock does not necessarily make the portfolios safer, and it may cause serious shortfalls such as those in the United States during 2008 financial crisis. To prove that just declining the stock's exposure on the construction of glide paths may not be the best strategy to achieve the pension fund manager's objectives, Yoon (2010) finds that near-retirement workers face 30 percent more risk than their younger counterparts. He proves that the rule of thumb that is commonly used for constructing glide paths performs very poorly regarding risk allocation (volatility on Yoon's analysis).

Yoon (2010), besides considering the excessive risk taken by target-date funds, argues that the real problem regarding glide path construction is the inconsistent risk allocation that TDFs undergoes: provided that funds deal with several asset classes, they mistakenly assume time-invariant risk instead of focusing on risk tolerance as the basis of asset allocation. Other authors (Lewis and Okunev, 2009; Lewis, 2008) have started to consider different risk measures, for example, Value-at-Risk (VaR), Conditional Value-at-Risk (CVaR), volatility, and maximum drawdown as their basis for analyzing and proposing innovative asset allocation strategies to maximize worker's replacement rates.

Lewis and Okunev (2009) use a VaR approach to analyze and minimize risk on portfolios. They consider that using that approach is very useful, compared with others, because it does not require assumptions about the distribution of returns or the determination of the investor's utility function, such as in Bodie, Merton and Samuelson (1992); Gomes, Kotlikof and Viceira (2008); and in Poterba, et al., (2006).

To better understand the model used by Lewis and Okunev (2009), it is important to understand the concept of Value-at-Risk. VaR is "the loss in a portfolio's value from an adverse movement in market prices over a specific time horizon and with a given level of confidence" (Lewis and Okunev, 2009). It is a measure that gives an estimate of the largest losses a portfolio can experience during exceptional trading days (or during another time variable). Lewis and Okunev (2009) propose a dynamic VaR model to modify the investor's portfolio to the most attractive asset class, call it equity, fixed income, or a mix of both. The modifications or tilts are based on "current market conditions and constrained by a predefined VaR glide path, that becomes more risk-averse as the investor gets closer to retirement".

Lewis and Okunev (2009) argue that retirement fund managers may not be aware of the drastic changes the VaR can experience during a worker's working life. Therefore, it "determines the allocation to stocks and bonds as a function of prevailing market conditions conditional on the time to retirement". The authors consider the case of a life-cycle investment fund that can invest in the S&P 500 and long-term government bonds to compare the outcomes of the conventional life-cycle strategy and the dynamic VaR model at 99, 97, and 95 percent level of confidence. They see that at lower levels of confidence, the average return of the investment further outperforms the conventional equity glide path, provided that the model allows a higher share of equity allocation. Hence, for the authors, life-cycle investment funds "should be 'aged-based', 'risk-based', and take into account the current economic environment (...), and the VaR glide path elegantly achieves all of these objectives".

There are different studies which find contrasting results regarding the existence of Target-Date Funds in defined contribution systems by using life-cycle models. There are those that its results support the usage of TDFs (Poterba, et al, 2006; Bodie, Merton, and Samuelson, 1992; Viceira, 2006), which acknowledge the increase in benefits for the investors, mainly by considering flexible labor supply and endogenous wages. While others (Cocco, Gomes, and Maenhout, 2005; Gomes, and Michalides, 2005; Gomes, Viceira, and Kotlikoff, 2006) which do not consider that these type of mutual funds could present better results than others within defined contribution pension schemes. However, as I mentioned before, in this analysis I will use the endogeneity of wages and the flexibility of labor supply assumptions, which allow the model to better adjust to observed data. Therefore, following with the discussion about the operation and risk parameters of Target-Date funds, authors such as Yoon (2010), Lewis, and Okunev (2009), Basu, Byrne, and Drew (2011) test several values of value-at-risk to achieve higher pension savings for the workers. In the Mexican case, this pension scheme is clearly novel, hence, it is necessary to start auditing in order to ensure its correct functioning.

As it can be seen from all the information above, there are plenty of studies which analyse and evaluates either different pensions schemes or the institutions on those systems. However, the most relevant works for this thesis are those of Yoon (2010), Lewis, and Okunev (2009), and Basu, Byrne, and Drew (2011), because I will follow a similar methodology as them. I will focus on the different returns of using different VaR levels and prove their robustness with Stochastic Dominance.

2.2 The Mexican pension scheme and its characteristics

As it was mentioned before, Mexico passed from a Defined Benefit (DB) system to a Defined Contribution (DC) on the summer of 1997 due to a change in the Social Security Law. After the reform, there was only one SIEFORE, until 2005, for all workers regardless of their age, which had a very restrictive investment regime (it was only allowed to invest in fixed income securities issued by the Mexican government).

In the following years, it was clear the necessity for a new SIEFORE to manage the funds of people near retirement (from 56 years), so a new SIEFORE, the SB1, was allowed to operate in the market. Hence, each of the two SIEFOREs had different investment regimes: the SB1 could invest up to 15% of the portfolio on equity through structured notes, while the SB2 could only invest in Mexican Government Fixed Income securities (De la Torre Torres, et al, 2018). Thus, this is the point, according to the CONSAR, were the life-cycle approach began to be considered.

Three years later, the CONSAR decided to add three new SIEFOREs (the SB3, SB4, and SB5) into the system due to the new life cycle perspective pursued by the Mexican government. This new SIEFOREs would achieve better returns for the workers, provided that the CONSAR allowed to invest in foreign securities. In 2013, the SB5 SIEFORE merged with the SB4 [5].

In 2019, when the SIEFOREs operation system was reformed, the SAR allowed the incorporation of five more SIEFOREs, each one of them would invest worker's savings according to their age. That is, instead of workers changing from one SIEFORE to another as they age, they would remain for all of their working life in the same SIEFORE and, therefore, the AFORE will only adjust the portfolio's glide path according to the worker's age. Hence, Mexico joins countries such as the United States and New Zealand in the use of life-cycle systems, where the pension scheme focuses on maximizing the retirement wage rather than wealth accumulation (CONSAR, 2020) as proposed by Robert Merton.

Provided the new investing scheme, it can be seen in Table 1 the decreasing share of risky assets through the worker's age, for example, the maximum percentage of variable income allowed in the glide path of a worker will decrease from 60 percent, when the employee is less than 25 years, to 19 percent near the retirement or target date.

As it is noticed, this new scheme allows for a smoother transition to a more conservative investment strategy, because of the incorporation of the extra five SIEFOREs that allows for smaller age cohorts to maximize the worker's replacement rate. As the CONSAR (2020) informs, the regulations presented before in Table 1 consider a decrease in strategic risk, but not in cyclical risk, that is, the approach does not avoid the effects

[5] See Apuntes sobre el SAR no. 3 (n.d.) to know about the investment regimes of those SIEFOREs

Table 1: Investment Regime of Generational SIEFOREs.
(Investment society's maximum % of total assets allowed.)

	Basic Starter	Basic 90-94	Basic 85-89	Basic 80-84	Basic 75-79	Basic 70-74	Basic 65-69	Basic 60-64	Basic 55-59	Basic	Total[1]
Worker's age	<25	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64	≥65	
SIEFORE	New	New	SB4	New	SB3	New	New	SB2	SB1	SB0	
Government debt[2]	100	100	100	100	100	100	100	100	100	100	100
Foreign debt securities or national currency	51	51	51	51	51	51	51	51	51	51	51
FIBRAs	10	10	10	10	10	10	9	8	7	6	9
Goods	5	5	5	5	5	5	5	5	5	5	5
Structured instruments	20	20	20	20	20	20	17	14	11	10	18
Variable income[3]	60	60	58	56	53	49	43	34	19	15	48
Securities instruments[3]	40	40	38	36	33	30	27	23	21	20	31
International securities	20	20	20	20	20	20	20	20	20	20	20

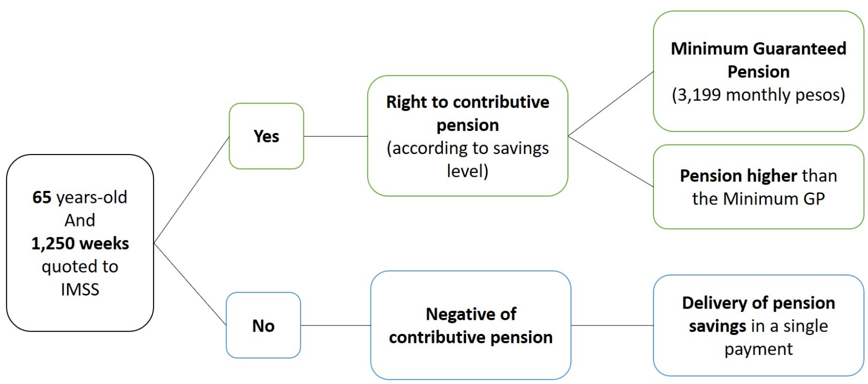
Data: CONSAR, 2020.
1.Weighted average by actives.
2.Composed of Assets subject to investment, obligations convertible into shares, and not convertible subordinated obligations.
3.Subject to the gradualness scheme determined by the Risk Analysis Committee (Fourth Transitory).

of the conditions of the financial markets. According to CONSAR (2020) calculations, the transition to Generational SIEFOREs will increase the average replacement rate up to 14 percent. Thus, as it was mentioned in the previous section, it is this assumption of time-invariant risks what might not allow for better returns, and therefore, higher pensions for the workers. As Yoon (2010) found, it may be that, at retirement, the portfolios could be riskier and present higher and unexpected losses.

To fully understand the whole panorama regarding the portfolio's asset allocation problem, it is important to consider some important variables regarding the Mexican pension system as a whole. I will consider a comprehensive panorama, considering variables that affect worker's contributions to AFOREs: workers' wages, the weeks listed to the Mexican Social Security Institute (IMSS), the number of workers affiliated to IMSS, transitions from the formal to informal labor markets (and vice versa), density contributions, replacement rates, among others. Considering these variables will help to better understand the Mexican pension system, and consequently, find new strategies that could maximize even more the pensions savings of all workers listed on the IMSS.

To possess the right of a pension, two main requirements must be fulfilled: being 60 or more years-old and listing 750 weeks in the IMSS system (approximately 15 years). If the second requirement is not accomplished, the government delivers a "Negative of Pension" document and the worker receives her accumulated savings in a single exhibition payment. However, if the worker accomplished both requirements, but the total of savings minus the *retirement, unemployment and old-age insurance* (RCV in Spanish) is not 30 percent higher than the Minimum Guaranteed Pension (GP) by the State, the worker receives the GP until her death. This procedure can be better understood with the help of Figure 1. In the present analysis, I will use as a benchmark the previous legislation regarding the mandatory weeks listed on the IMSS (1,250), so the reduction on mandatory weeks approved on July 2020 serves as a counterfactual to asses the impact of the new legislation on retirement savings.

Figure 1: Possibility of pension



Data: CONSAR, 2019.

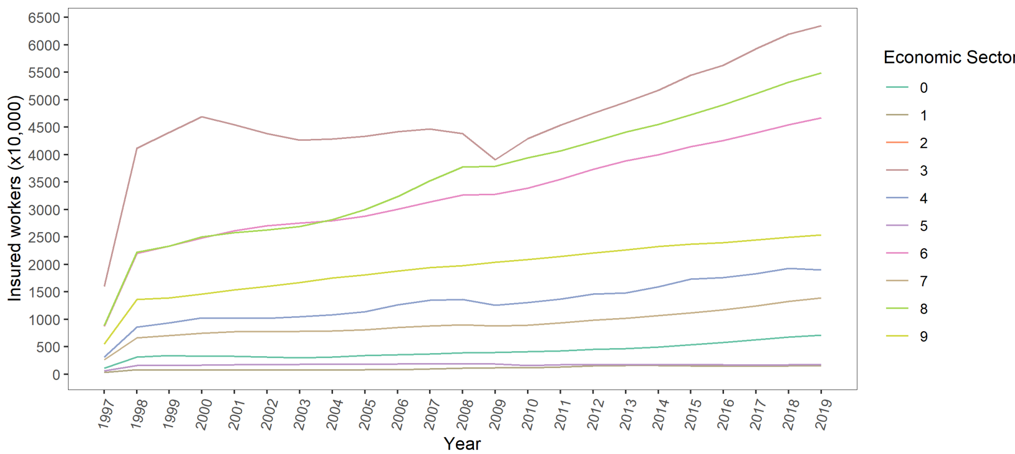
During the working life of the individuals, 6.5 percent of their contribution base salary is transferred to the worker's pension saving account in a three-party basis: 1.125 percent is contributed by the worker, 5.150 percent by the employer, and the rest, 0.255, by the government. Besides, the later complement some accounts with the Social Share with the only purpose to increase the savings of those workers with low income. According to CONSAR, in January 2019, the percentage of these complements made by the government to individual accounts vary from 5.604 to 0.000 percent (the latter for accounts whose investor's wage ranges from 16-25 times the value of UMA: 84.49 Mexican pesos in 2019). Therefore, the total contributions, as a percentage of the contribution base salary, and according to the worker's wage range, goes from 12.104% for individuals who are paid the minimum wage to 6.5% for those paid more than 15 times the UMA.

This issue is the first hurdle many workers are currently facing to achieve a poverty-free retirement. According to the CONSAR (2019), only 24 percent of workers (that were signed to the IMSS since July 1st, 1997: the AFORE Generation) will complete the listed time mandated by the IMSS law to get the right for a pension when turning 65, that is, 76 percent of the workers, when they are the retirement age, will receive a Negative of Pension. This low Mexican pension coverage represents a major issue for the population in the future, due to the lack of necessary resources for living a dignified life when old.

It is important to mention that only formal workers are listed on the IMSS, that is, only workers who have been formal for more 14 years will be considered to receive a pension. Hence, given the importance of the IMSS in the Mexican pension system, it is essential to start looking into the development of the number of insured workers through years, that is, the number of workers that are part of the IMSS conferred by their employers (which are forced by law) to reach towards issues that present significant importance in the Mexican pension scheme. Figures 2 and 3 exhibit the number of insured workers by the IMSS from 1998 to 2019. Figure 2 shows the insured workers by economic sector, while Figure 3, divided into three panels, shows insured workers by: wage, age, and gender.

At first view, in both Figures, there exists a positive trend regarding the numbers of protected workers by the IMSS. However, a closer examination to both shows us some important points. Figure 2 makes us notice that some sectors have shown a larger percentage increase compared with others. For instance, the social services sector almost quadrupled its number of insured workers from 1998 to 2019. Conversely, agriculture and manufacture the electric industry, collection and water distribution sectors presented a lower number of insured workers in 2019; the agriculture, hunting, fishing sectors presented almost no change over the whole period. Another interesting example

Figure 2: Insured workers by economic sector



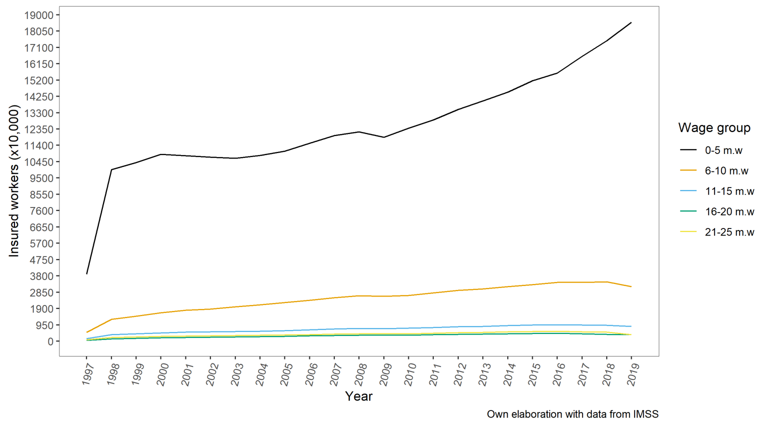
Own elaboration with data from IMSS.

Sector 1: Agriculture, hunting, fishing. Sector 2: Extracting industry.
Sector 3: Transformation industry. Sector 4: Construction.
Sector 5: Electric industry, collection and water distribution.
Sector 6: Commerce. Sector 7: Transportation and communication.
Sector 8: Services for firms, people and houses. Sector 9: Social services.

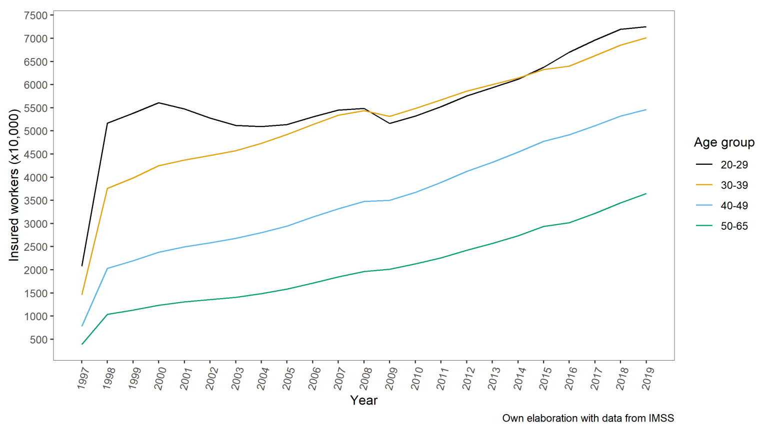
is the case for the services sector, which has seen an increasing number of workers . This could be due to the transition of the Mexican economy into a services economy. However, the transformation industry is the sector which employs most of the Mexican workers.

In panel 3a, of Figure 3, interesting phenomena emerge in the extremes of the wage distribution, for those who receive from one to five minimum wages per month and those who earn twenty-one and more minimum wages (m.w.) per month. On one hand, insured workers who made one to five m.w. have grown considerably, increasing from 40 million to 190 million in the 20-year span; noticing its higher increase at the end of the XX century. Since 2009, the number of insured workers appears to be increasing in a linear trend. On the other hand, the number of insured workers who make twenty-one or more mw did not increase since 1997. In fact, apart from those earning 1 to 5 minimum wages, the only group that has seen a modest increase in its number of insured workers is the group of people making 6 to 10 minimum wages. The rest of the groups have not changed at all. The importance of searching alternatives to increase the workers' pension savings (such as the one I propose), because structural problems like these seem far to be solved.

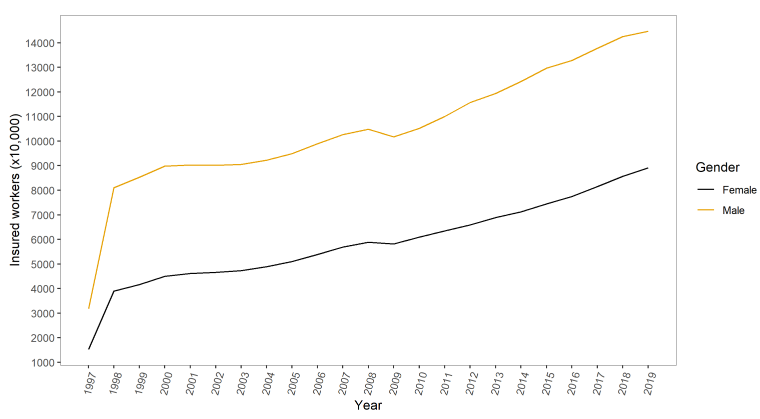
Figure 3: Insured workers in the IMSS from 1997 to 2019.



(a) Wage



(b) Age



(c) Gender

If we consider the development of insured workers according to their age group, panel 3b shows different trend behavior depending on the worker's age cohort. Although all the age groups present a positive trend, some tend to grow faster than others, for example, those far retirement, i.e., workers who are 20-29, almost tripled in 1997 to 1999, while older workers who are 50-65 years, through the twenty years, presented a modest increase in their number of insured at the IMSS, which could be that Mexican population, although ageing, it is still young (young workers have higher levels of insured that the oldest generation). Therefore, provided that population keeps on ageing, it is necessary to find better measures to maximize pension savings, because some day, all those young workers will retire and will have to live with its savings. It is worth remarking that in 2015, the insured workers of all age cohorts presented a sudden flat trend.

Regarding the worker's gender, in panel 3c, it can be seen that there is a wide gap between the percentage increase of insured male and female workers. By the end of the XX century, the gap seemed to contract, however, it has only become wider. As it can be inferred, this wide gap represents a serious problem for women to maximize their retirement wage, given that they may not list the mandated 750 weeks to the IMSS when being the retirement age.

After reviewing some basic aspects of the Mexican labor market, it is necessary to revise some important variables of the pension system that affect directly on maximizing the worker's retirement wage. To start, the concept of sufficiency, which refers to the magnitude of the benefits, and the replacement rate that will be achieved by workers, will be explored. According to the CONSAR, workers who accomplish the requirements to have a pension could save on average, when achieving the retirement age, 650,326 Mexican pesos with an average monthly pension of 3,843 MXN, attaining an average replacement rate (RR) of 49 percent. If we consider the fact that the OCDE estimates that a monthly contribution of 6.5 percent of the worker's salary could lead to a RR of 26 percent for a worker with an average wage, the average RR achieved in the Mexican case is high (CONSAR, 2019).

One of the main things that distinguish the definite contribution system of the Mexican pension scheme is the minimum guaranteed pension, which is sort of a “backup pension”, that is, if the worker accomplishes both of the requisites to have the right of a pension (750 weeks listed on the IMSS, and 60 years-old or more) but the amount she saved is not 30 percent higher that a benchmark (3,199 MXN by November 2020), the State will pay her a monthly pension of 3,199 MXN. According to an analysis made by the CONSAR (2019), the workers who will have the right of a pension (only the 24% of the AFORE generation), 71 percent will receive that government benefit, while the remaining 29 percent will have saved enough to access a bigger benefit, that is, only 7% of the AFORE generation will receive a higher monthly pension than the GP.

Therefore the RR estimated for those who will receive the GP is 59 percent, while the estimated RR for those who achieve higher benefits will be notoriously lower: 24 percent. As a consequence, the protection of the GP for workers with low wages may discourage those account holders to contribute more than the minimum required, given that most of the workers will receive almost the same benefits, no matter the differences in their level of pension savings (CONSAR, 2019).

According to CONSAR (2019), if the guaranteed pension would not exist, the replacement rate of workers with low wages (with no more than five times the level of the UMA) would range from 43% to 25% (for those who earn five times the UMA). In contrast, with the existence of the guaranteed pension, the R.R. of workers is considerably higher, especially for those earning the minimum wage. That is, the replacement rate ranges from 125% to 30% for workers who earn the minimum wage to workers with high wages, respectively. The above proves that the GP has important redistributive benefits among workers. Regarding the fiscal cost of the guaranteed pension, the CONSAR (2019) estimates that it will be low until 2050, due to the fact that by this time there will be a few individuals with a right to a pension and because the first payments will be issued using the workers’ resources.

The CONSAR’s calculations for the present value of the minimum guaranteed pension for the 2019-2100 period is calculated as 4.35% of the Mexican GDP, while the annual average inflows will represent 0.067% of the GDP. Finally, considering everything mentioned above, the contingent liability (expressed as present value) for the Federal government would imply a cost of 10.51% of the GDP for the 2019-2100 period (CONSAR, 2019).

Since January 2019, the Mexican government started a new program called pension for elders' well-being (PBAM, in Spanish), which gives 1,275 monthly MXN. Although the CONSAR estimates that the average replacement rate would increase from 49 percent to 67 percent, if we consider the PBAM as a non-contributive component of the pension system, the distortions generated by this new policy may increase those generated by the guaranteed pension, that is, it could further discourage workers to increase their savings and only contribute with the minimum required by law (CONSAR, 2019). However, this analysis goes beyond the purpose of this dissertation.

Another variable that directly affects the worker's retirement wage maximization is the contribution density, DC. This variable is defined as the ratio of the time listed in the retirement savings system (SAR) and the time that the worker has been in the labor market, multiplied by one hundred. The contribution rate is very important to the Mexican pension scheme due to the workers' constant changes from the formal to informal sectors of the economy, and vice versa (CONSAR, 2019). The time periods when the employee is on the informality are not counted for the SAR, therefore, those months or years worked are not considered for obtaining the right of a pension. The contribution density tells us that the higher it is, the higher the pension savings will be. Conversely, workers with a low contribution density will find several difficulties to achieve the required 750 weeks listed. It is important to mention that, regarding the importance of the density contribution, every time the worker withdraws money from her account (due to unemployment or marriage) there is a discount on its listed weeks proportional to the amount of resources withdrew.

A study made by Castañón and Ferreira (2017) regarding the contribution densities in Mexico showed that, on average, Mexican workers listed 42.9 percent of their working time for the SAR, which means that workers spend more than the half of its working life on the informality sector. In addition, they show that the distribution of the DC demonstrates that it is bimodal and asymmetric, condensed in small contribution densities. Thus, 27 percent of the workers present a contribution density from 0% to 10% in their working life. Conversely, 17 percent of them present a DC that ranges from 90% to 100%. Similar results are also find in several Latin American countries.

In Mexico, workers can select the AFORE they desire to manage their savings when they register to the IMSS, however, not all workers take the time to decide it, or do not know about this important step to obtain a pension. By law, all workers' savings must be managed by an AFORE. Therefore, those who do not sign a contract with a pension fund manager are automatically assigned to one of them under certain criteria; they are called assigned workers. While those who did sign a contract are known as registered workers. Thus, the contribution density of assigned and registered workers are completely different: while the DC for registered workers is, on average, 57 percent, the DC for assigned workers is 32.7% (Castañón and Ferreira, 2017).

The workers' sex also affect the contribution density according to Castañón and Ferreira (2017). Their results suggest that women spend more time on the informal sector or temporarily leave the labor market by several circumstances. For instance, 21 percent of male account holders have contribution rates between 90 and 100 percent, while 17 percent of women shareholders lied on the same contribution rate.

Alongside with the worker's sex, the wage level is also an important indicator of the contribution density. On average, workers with high wages remained on the formal sector 77 percent of the time between 1997 and 2006, according to Levy (2008), while low wage workers only remained, in the same time span, the 49 percent of the time in the formal sector. Therefore, low contribution densities are expected for workers with wages up to four minimum wages.

One of the central variables for pension research is the worker's wage. Hence, a study made by Castañón and Ibarra (2017) focused on the wage path of the Mexican workers. According to their findings, the wage path has an arc shape and reaches its highest point in a monthly income of about $9,500 MXN when the worker is 47 years-old. Besides, it is important to note that the growth rate of wages is not constant from 15 and 47 years. At the beginning of the worker's career, wages increase at growing rates. However, from 27 to 47 years-old, wages still grow, but at a lower pace each time. In addition, for the workers of SAR, there is a positive relation between wages and the years of potential experience. Therefore, wages reach a maximum when workers are 27 years of experience.

In the same paper, the authors studied the behaviour of the wage path if the sex of the worker was to be considered. They found that male and female workers between 21 and 29 years-old observed a wage increase of about 85 percent. However, after the end of that lapse, the wage growth rate for women, while still increasing, slowed down compared to the growth rate of men, that is, while women achieved a wage increase of 15 percent until being 47, men's wage was 33 percent higher than when they were 29 years-old. This means that the average highest wage for men is $12,303 MXN, while for women it is only $9,918 monthly MXN. Moreover, this wage gap becomes wider as the permanence in the labor market increases. Hence, the effect that sex has on wages is one of the multiple factors which determine that women's pensions are, in general, lower than those of their counterparts.

Besides the worker's sex, the wage path can also be analyzed on terms of the workers' registration status on the pension system (if they are self-registered or were automatically assigned to an AFORE) and on their contribution density. First, the workers who present higher wages are those who were registered by themselves to an AFORE and have a high contribution density. Hence, the wages of this type of worker grow at an increasing rate since the beginning of their career, until the time when they are 28 years-old. After that age, even though wages are still increasing, they grow at a slower rate until they achieve a maximum amount, on average, of $11,507 monthly pesos when the worker is 47 years-old; after that, wages tend to decrease. Conversely, workers that were automatically assigned to an AFORE and with low contribution rates present considerably lower wages than the other group, that is, the income of assigned and low DC workers stagnate at very low wage level at a very low age. Approximately at 27 years-old, they achieve their maximum average wage of $6,785 monthly pesos, and from then their salary is either flat or only tends to decrease (Castañón and Ibarra, 2017) Table 2 summarizes their findings.

Table 2: Influence of the wage path on the total pension savings*

Concept	Type of worker						
	Man	Woman	Registered	Assigned	Low DC	Medium DC	High DC
Pension savings (MXN)	$814,641	$715,028	$782,018	$510,217	$498,499	$637,766	$884,488
Monthly pension (MXN)	$4,210	$3,448	$4,042	$2,367	$2,576	$3,296	$4,571
Replacement Rate	42%	39%	42%	49%	42%	39%	40%

*Table 1 from Castañón and Ibarra (2017).

The Mexican pension scheme is a complex and complicated system which demands multiple measures in order to secure its correct functioning. As it was explained in the above paragraphs, there are structural problems, such as labor informality, low wages, low contribution rates, and low savings rates that cannot be solved easily. As a result, the problems aforementioned keep on affecting most of the Mexican workers, specially those in the lower deciles of the wage distribution. Therefore, as I brought up in the introduction of this thesis, one way to tackle the low pensions problem caused by the structural failures of the Mexican labor market is to modify the regulation, specially in the risk management field. We should allow the AFOREs to take further risks in order to achieve better returns for the workers. For the sake of finding an optimal risk parameter, in the preceding section, I show a deeper insight of the Mexican retirement funds managers, for we to better understand how the AFOREs invest and take risks.

3 Looking Deeper into the AFOREs

By mid 2020, according to the CONSAR, the total amount of pensions savings managed by the AFOREs has reached a level of around 4.2 billion MXN, which represents 17.2 percent of the Mexican GDP. Therefore, in order to make further reforms to the system, it is important to deeply learn and understand what is functioning, what is not, and what could be improved. Thus, this section of the text will serve two purposes: the first one will consist of a description of the AFOREs' current glide paths, that is, to comprehend how some of the biggest pension fund managers invest the workers' savings until they retire, focusing not only on the efficiency of the strategies used, but also on how optimal the investment regulation is, centered on risk measures, such as the VaR and the CVaR. The second purpose will be to compare with the results of the simulations made in the next section.

3.1 AFOREs' managed accounts, investment regimes, and glide paths.

Before starting the comparison between some of the AFOREs' glide path, I consider important to learn which pension funds have the largest number of managed personal accounts (registered or assigned) and the highest returns, plus the importance of learning the investment regime allowed by the regulator, CONSAR, regarding the value at risk of the portfolios. This information is important due to the fact that, as it was said in the previous section, most workers are automatically assigned to an AFORE, which, according to the research conducted by the CONSAR it has important effects on the total level of pension savings. Therefore, keeping in mind how the biggest AFOREs invest under their own criteria and under the regulation mandated by the CONSAR, could be determinant to show if the replacement rate can be further increased or not.

According to the CONSAR, by the end of May 2020, the AFOREs managed, in total, 66.8 million accounts, where 57 percent of them were registered and assigned in only three pension funds: Azteca, Citibanamex, and Coppel. As it is seen in Table 3, these three AFOREs account for more than half of the personal accounts, provided the high level of assigned workers to them; mainly for the cases of Citibanamex and Azteca. Even though the number of assigned workers is high, it accounts for less than 30 percent of the total of workers in the retirement savings system, SAR. However, in the same table, it can be noticed that the AFOREs with the highest number of managed accounts do not have the highest returns, in fact, Azteca and Citibanamex belong to the group of the worst performers. This can be a sign of little knowledge from workers regarding the pension system in the country.

Table 3: AFOREs at a glance[1]

AFORE	Managed Accounts			Managed Resources	Fees	Returns[2]
	Registered	Assigned	Total	Millions of MXN	Annual %	Annual %
Azteca	5,109,008	9,025,254	14,134,262	232,318.2	0.98	6.76
Citibanamex	9,182,614	2,965,148	12,147,762	1,030,059	0.88	6.43
Coppel	11,051,549	685,055	11,736,604	395,707.70	0.98	7.08
XXI Banorte	8,080,762	492,903	8,573,665	1,198,941.10	0.88	7.25
Sura	4,353,212	3,255,534	7,608,746	829,742.60	0.92	6.99
Profuturo	3,156,795	1,250,102.00	4,406,897	831,738.30	0.92	7.81
Principal	2,458,445	499,807	2,958,252	366,003.70	0.97	6.82
PensionISSTE	1,707,843	441,481	2,149,324	491,690	0.79	7.66
Invercap	1,791,869	261,246	2,053,115	256,337.60	0.98	5.68
Inbursa	1,029,694	11,825	1,041,519	186,093.40	0.92	7.72
Total	47,921,791	18,888,355	66,810,146	5,818,631	0.92[3]	7.02[4]

1. Data from CONSAR (May 2020).
2. Simple average of the 10 SIEFOREs' returns (until last 36 moths, stock prices).
3, 4. Simple average of the 10 AFOREs

According to the CONSAR, in 2019, 8.2 billions of MXN were redistributed to the AFOREs with the highest earnings, in order to boost competitiveness among the pension funds. The AFOREs where the money was reallocated were Citibanamex, Coppel, Inbursa, PensionISSTE, Principal, Profuturo, and Sura. However, among these AFOREs, Citibanamex, Principal and Sura are the worst performers if we consider the information on table 3. Although, by the time the reassignment was performed, the last three AFOREs had the highest returns they are not performing as before. Moreover, those AFOREs are charging higher fees compared to others, specially for the case of Principal, which has the second highest in fee in the pension funds market.

One further thing to notice regarding the information on Table 3 is that we could argue little knowledge from workers, whether they are assigned or registered, about the existing pension funds and their performance in the retirement savings system, SAR. After inspecting the managed resources and the annual returns of the AFOREs, it can be noticed that some pension fund managers manage a very low level of resources, even though they have a high percentage of annual returns, such is the case of PensionISSTE, which is not only the AFORE with the lowest level of managed accounts, but the third one with the highest annual returns. It is also the case for Inbursa, which is the second

AFORE with the best performance but the second one with the lowest level of managed resources after PensionISSTE. Therefore, this quick glance of the AFOREs' composition reveals us some inefficiencies in one of the main aspects of the pension system, regarding the workers understanding of it and the improvement of their future retirement wage.

Another interesting point regarding the structure of the pension system exposed in Table 3 is that some AFOREs with a relative low number of managed accounts have a high level of managed resources; that is the case, for instance, of Profuturo, which is the fifth AFORE with less managed accounts, but the second with the highest level of managed resources. To the same extent, it is the case of Invercap compared with one of the AFOREs that manages most of the accounts: Azteca. It can be noticed that both pension funds manage approximately the same level of resources, however, Azteca has almost seven times more managed accounts than Invercap. These disparities may be due to the fact that what makes some of the AFOREs to manage a large number of accounts is that lots of workers are assigned to them. Therefore, AFOREs with a large number of assigned workers actually do not manage a large number of resources, due to the fact that the accounts of assigned workers tend to be relatively small compared to those of registered workers.

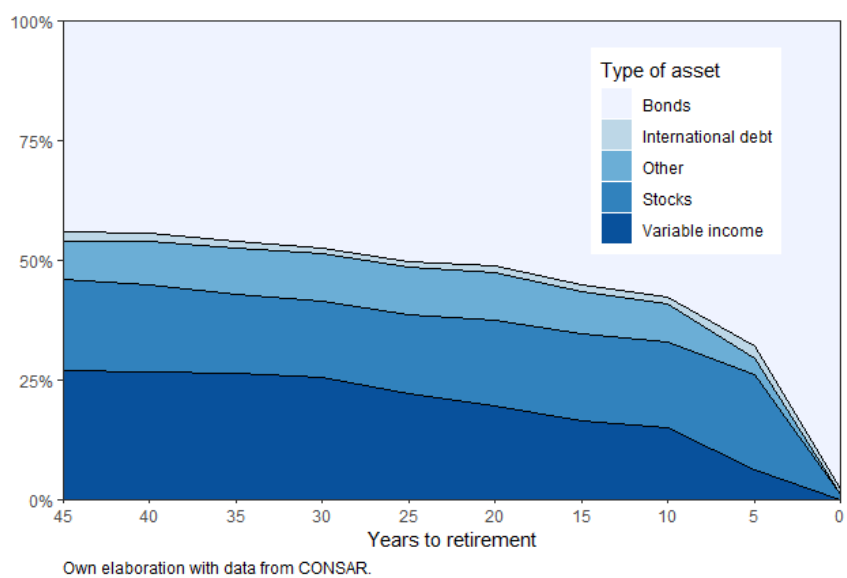
To understand how the AFOREs manage the worker's resources, it is important to know how are their glide paths constructed. Recall from Section 2 the definition of glide path, which refers to the long-term planning tool that defines the nature and objectives of the fund scheme to, progressively, reduce portfolio's risk (CONSAR, 2018).

Provided that there are ten AFOREs, I will only show five glide paths which I consider serve the purpose to expand our knowledge on the new generational pension system: a general glide path constructed with the weighted averages of all the pension funds in the system, the glide paths regarding the three pension funds with the largest number of managed accounts, and the glide path of the AFORE with the best performance in the financial markets.

The first glide path is depicted in Figure 4 and it shows the one of the whole system, that is, it is constructed with the weighted average of all ten glide paths of the ten AFOREs (weighted by the number of managed accounts). The glide path is constructed with five categories: variable income (which includes both national and international, without further specifications), stocks, international debt, bonds, and others (FIBRAs, structured instruments, and commodities).

The general glide path tells us that, on average, the AFOREs will have a relative constant investment regime, i.e., the proportions will rarely change until ten years before the retirement date, where the proportion of bonds increase fast until representing 100 percent of the worker's portfolio. In the set of risky assets, it is important to notice that, on average, variable income and stocks are invested in almost the same proportions and both represent around 40 percent of the portfolio, almost all the working life of the worker. However, variable income decreases in a faster pace. Furthermore, it is worth noticing the small percentage invested in international debt, due to the regulation allowed by the CONSAR. [6]

Figure 4: General Glide Path (June, 2020)



In figure 5, I show the glide paths of the three AFOREs that manage most of the personal accounts in the Mexican pension system. The pensions funds with the

[6]See Appendix to learn how I constructed the Glide Paths

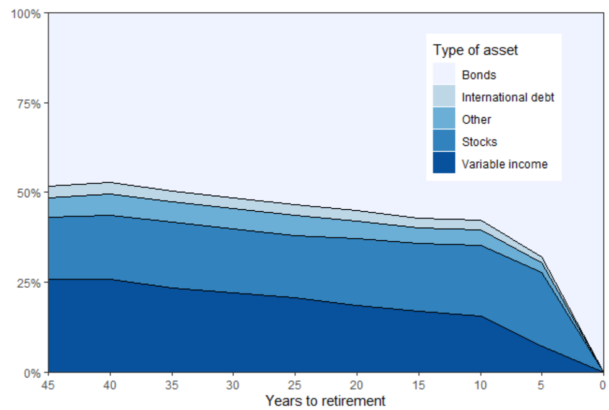
most managed account, as it was said before with the information presented in table 3, are Azteca, Citibanamex, and Coppel. Their glide paths, as we can notice, depict some similarities among them, for example, on the proportion invested on variable income. The three AFOREs start investing around 25 percent of the worker's money on variable income; while Coppel and Azteca have a relative smooth path (until decreasing the proportion of variable income from ten years to retirement), Citibanamex starts decreasing the level of money invested on this category twenty years before retirement.

Regarding the rest of the portfolios composition, it is very clear that after bonds and variable income, stocks is the category in which the money of the workers is invested the most, though there are some differences among the AFOREs' strategies. For instance, Coppel is the AFORE that invest the most in stocks, almost 25 percent of the portfolio, while Azteca and Citibanamex prefer a more conservative strategy, investing 20 and 15 percent on this category respectively. However, if we consider the investments on other instruments, it is noticed that Citibanamex is the pension fund which invests the most in this category, approximately 15 percent of the portfolio the majority of time; the other two pension funds invest almost the same low proportion.

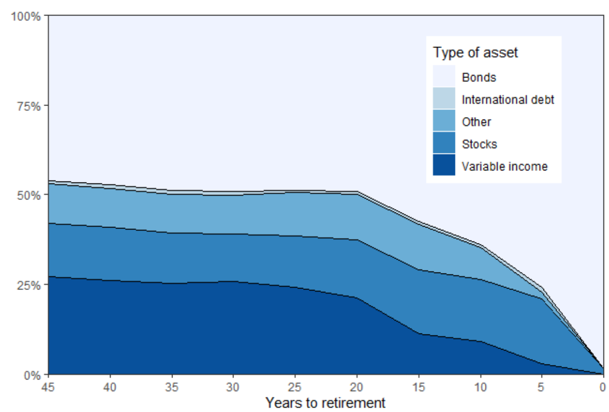
Notwithstanding the previous differences, it is important to mention how Coppel is the less conservative pension fund among those with the highest number of accounts managed. As it is depicted in figure 5, Coppel invests 30 percent of the pension savings in bonds for almost all the working life of the investor. It is 10 years to retirement when the proportion of bonds increase dramatically until reaching the 100 percent when the target date arrives. Conversely, for the cases of Azteca and Citibanamex, we observe more conservative portfolios, where the risky assets (everything but bonds) never represent more than 55 percent of the portfolio's composition. In fact, the proportion invested in bonds start increasing when there are 40 years to retirement, compared to Coppel, where the proportion of bonds actually decrease. However, how do these different investment strategies differ or look like to the technique used by the pension fund with the highest returns?

To answer the previous question, in figure 6 is depicted the glide path of Profuturo,

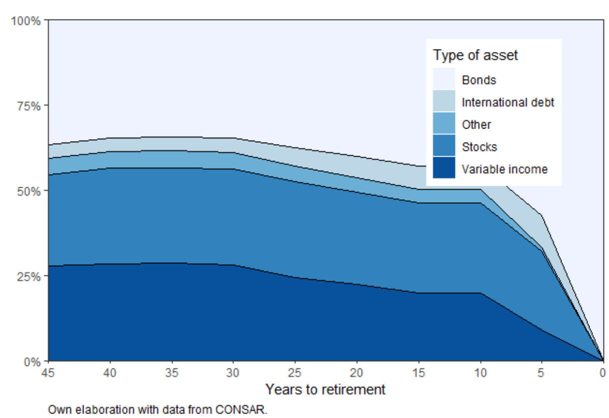
Figure 5: Glide paths of the AFOREs with the highest level of managed accounts (June, 2020)



(a) Azteca



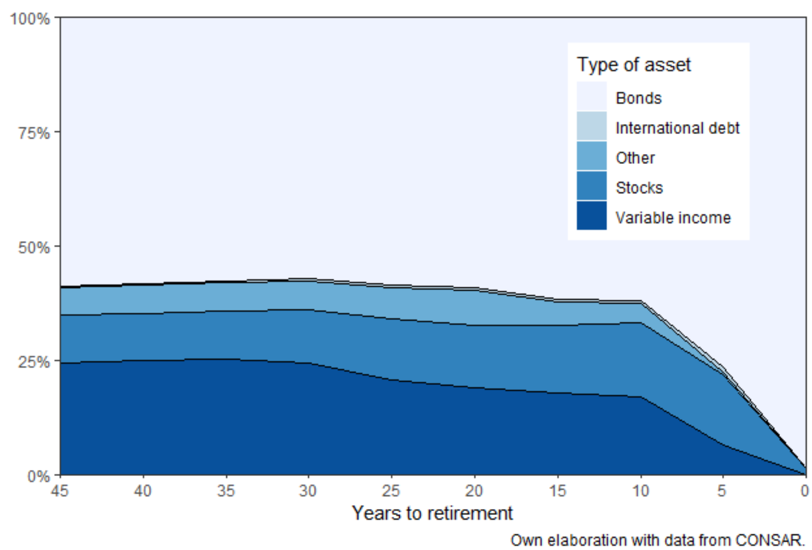
(b) Citibanamex



(c) Coppel

the AFORE that had, on average, the highest returns by mid-2020. Comparing these glide path to the previous three, the Profuturo's investment strategy tells us a completely different story. First, the percentage of risky assets never surpasses 45 percent of the portfolio's composition while being quite constant until ten years to retirement. Second, and as a consequence of the first difference, the proportion invested in stocks is very small compared to the other three pension funds. In fact, for Profuturo, the sum of the proportions of stocks, international debt and other instruments is less than the proportion invested on variable income, which in the case for Azteca, Citibanamex, and Coppel, the same sum is equal to or bigger than the proportion of variable income. The only similarity found among the four glide paths is that the proportion invested in bonds rapidly increases from ten years to retirement. This could be entirely due to regulation (see Table 4, where investments limits are shown).

Figure 6: Profuturo's Glide Path (June, 2020)



After inspecting different investment strategies: the glide path of the system as a whole, the glide path of the AFOREs with the highest number of managed accounts, and the glide path of Profuturo, the pension fund with the highest average returns, it comes to the discussion the following question: how does these glide paths are reflected on the returns of the AFOREs? Figure 7 helps us answer the question. In graph 7a, we notice that there are plenty of differences among returns regarding young people (people that is 45 years-to-retire). One can observe, for the aforementioned time period, that Citibanamex is the worst performer with an annual return less than 5.5 percent, while Profuturo is the best with an almost 7.5 percent of annual return. The rest of the AFOREs are located in the range of returns of 6.25 percent and 6.60 percent. However, as the workers approximates retirement age, the returns of most of the AFOREs gather around 6.75 percent of annual returns, approximately, when the retirement age arrives; except from Azteca, which by the target date it has an annual return of less than 6.50 percent.

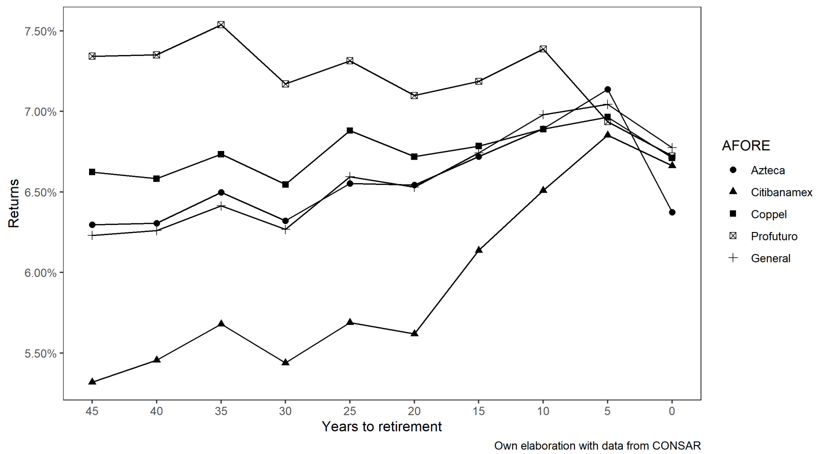
What it is interesting about graph 7a is the fact that the AFORE with the most conservative glide path has the highest annual returns for most of the time, and even on average, according to figure 7b [7]. However, one should be careful of thinking that being conservative increases returns, because that may not be completely the case. For instance, Coppel, the AFORE with the riskiest glide path is the second best performer for almost all the working life of the investor, and, thus, it is the second best performer in terms of annual average return (6.74 percent) according to graph 7b. A further interesting fact that is depicted in graph 7a is the fact that for almost all AFOREs, their returns decrease (for some more than others) when the investor is ten years from retirement, which coincides when the proportion of bonds in the AFOREs' glide paths increases dramatically.

Therefore, to completely understand the differences of annual returns among AFOREs, in spite of their glide paths —either conservative or not—, it is necessary to look deeper into other variables that not only affect returns, but the glide paths themselves. A set of these type of variables are known as risk-measure variables, which include, among others, value at risk, volatility, and liquidity coefficient. However, as it was said before, in this thesis I will only focus on value at risk, given the fact that it is a variable that

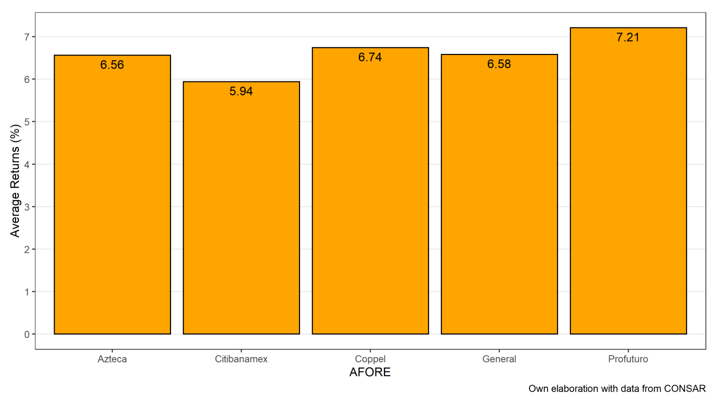
[7] The values of this Figure are calculated averaging the ten returns of each of the depicted five AFOREs

has gained a lot of popularity among pension systems regulators around the globe since the financial crisis of 2008.

Figure 7: Returns of the AFOREs (June, 2020)



(a) Returns through the investor's working life



(b) Average annual returns

As expressed in Figure 7, the glide path is not the only factor that affects the returns of the AFOREs. It is therefore convenient to analyze other variables in the financial system in order to further comprehend the behavior of AFOREs' returns. Thus, the following paragraphs will focus on the value at risk (VaR) used in the Mexican pension system. Recall from section 2 the definition of VaR: "the loss in the market value over a given time period, such as one day or two weeks, that it is exceeded with a small probability" (Duffie and Pan, 1997).

At the time of the introduction of the generational SIEFOREs in December 2019, the

CONSAR established new risk regulations regarding the investments of the AFOREs, provided that five new SIEFOREs were introduced. In these new regulation, the value at risk was considered in two forms: conditional value at risk (CVaR), and the differential conditional value at risk (DCVaR), which is the difference between CVaR of the portfolio and the CVaR of the same portfolio but without derivatives. However, between both measures, the DCVaR is the risk unit in which the CONSAR focuses the most. Therefore, the analysis must not only focus on the VaR —as it was originally planned—, but also on the DCVaR.

Table 4 shows the limits established by the CONSAR on the DCVaR per basic SIEFORE. It is seen that that the investment regime is relatively loose for the basic SIEFOREs from the basic initial to the basic 75-79, where the DCVaR ranges from 1.00 percent to 0.90 percent. However, the story is different for the rest of the SIEFOREs, where the regulation is very strict, decreasing rapidly from 0.78 percent to 0.25 percent. This could be one explanation on why the returns of the AFORE decrease from ten years to retirement.

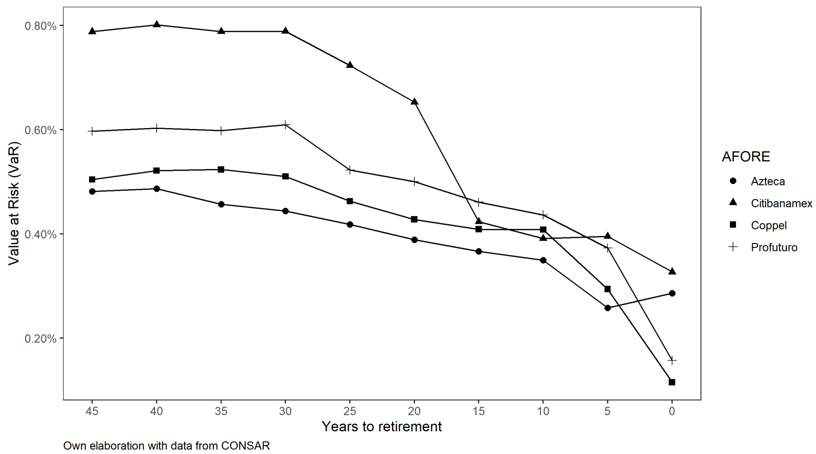
Table 4: Limits per Basic SIEFORE

Risk measure	Basic SIEFORE									
	Initial	90-94	85-89	80-84	75-79	70-74	65-69	60-64	55-59	Pensions
DCVaR	1.00%	1.00%	0.98%	0.95%	0.90%	0.78%	0.59%	0.39%	0.28%	0.25%

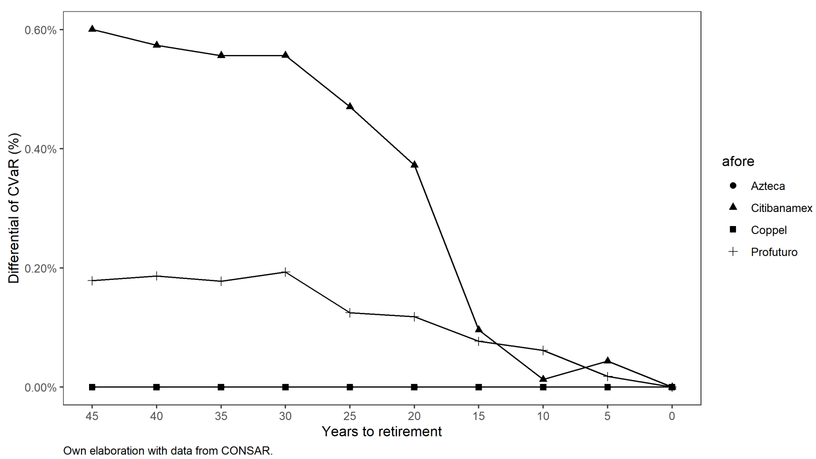
Data from CONSAR.

Provided the information in table 4, it may look like that the pension funds, in order to obtain higher returns, would set their VaR in such a way that the DCVaR is on the limit allowed by the CONSAR. However, this might not be case. Figure 8 depicts the reported VaR and DCVaR of the AFOREs analyzed previously, that is, the pension funds with the highest level of managed accounts (Azteca, Citibanamex, and Coppel) and the AFORE with the highest percentage of average annual returns (Profuturo). Thus, graph 8b shows us that all the previously analyzed AFOREs are far from reporting a DCVaR that lies on the limit allowed by the regulator. This is an interesting fact worth trying to understand, however, that exercise goes beyond the scope of this thesis.

Figure 8: VaR and DCVaR of the AFOREs (June, 2019)



(a) Reported VaR



(b) Reported DCVaR

Looking deeper into both graphs in Figure 8, we can find some interesting things. For instance, in Figure 8a it is observed that the AFORE that has the highest VaR does not has the highest returns, as depicted in Figure 7a, where, in fact, Citibanamex is the worst performer for almost all the working life of the investor. However, what it is worth observing is the fact that, while Profuturo is the best performer in terms of returns, it has the second highest VaR reported. How is that? What may cause that Profuturo performs much better than Citibanamex? It could be that the former decreases its VaR to almost zero, while the latter decreases it to a value of around 0.35 percent, when the retirement age arrives. For the case of Coppel, the best second performer in terms of annual average returns —according to graph 7b—, it follows practically the same strategy of Profuturo, but with a lower value of VaR for all the working lifespan of the employee.

If we observe Figure 8b, it is noticeable the existing difference among the AFOREs' risk strategy. Both, Azteca and Coppel, report a differential of CVaR of zero for the whole period. This could be due to two main reasons: because of reporting mistakes or because of the CVaRs of both pensions funds are the same with or without dividends. Citibanamex, as said above, has a very large DCVaR at the beginning of the employee's working lifespan, which slowly decreases until there are left 25 years for retirement. Nonetheless, since the worker is 24 years-to-retire, the risk measure decreases faster as times go by, until reaching a zero value when the target date arrives. Conversely, for the case of Profuturo, the strategy for allocating DCVaR is to maintain it quite smooth. The differential stays in the same value until there are 30 years to retirements; after that, the DCVaR slowly decreases until reaching zero at retirement.

In this section I reviewed the current state of some features of the pension system focused on the investment strategies of the AFOREs and the regulation that limits them. The cornerstone of this third section was the presentation and comparison of the glide paths of four Generational SIEFOREs (selected by the number of managed accounts and returns), their average annual returns, and their reported VaR and DCVaR. This section also serves as an introduction and as a benchmark for comparing the results obtained after the simulations on the next section.

4 The Model

4.1 The data.

Using data from the CONSAR I will simulate, using the Monte Carlo method, what would be the final pension savings of a worker considering different confidence levels of the Value-at-Risk, specifically 95%, 97%, and 99%. Besides, I will also consider in the simulation some structural parameters such as the contribution densities and wages. The inclusion of these parameters will allow to have a clearer panorama regarding the Mexican pension system.

The data that I use for the estimation is the reported net returns of the AFOREs to the CONSAR. The net returns, NR, are simply the return minus the fee charged by the AFORE. It is important to mention that these returns are annual returns and not monthly as it may be thought. The data covers from December 2019 (when the Generational SIEFOREs started to operate) to October 2020. Fortunately, the CONSAR reports the net returns for each AFORE in each of the ten Generational SIEFOREs.

One of the caveats of the data is that not all the required information is available for the public, therefore, I have to tackle the problem of the simulation using an alternative approach, which I will be explaining through the section. In order to perform a traditional approach for analysing the VaR of each AFORE I would need the returns of each of the investments made by the AFOREs, that is, the returns gained from investing in variable income, government bonds, and more. However, provided that I only have the aggregate returns of each AFORE in each SIEFORE I will construct only one portfolio per SIEFORE and made the simulations through it.

4.2 The model.

As I mentioned before, I will construct only one portfolio, instead of 10 (one per AFORE), per Generational SIEFORE. To start building the portfolio and running the simulations, I have to use a key assumption for the model to be identified: there is only one agent in the economy and she invests her money in only one portfolio managed by a Target-Date Fund; she has 40 years left to retire. The portfolio is composed by 10 assets correlated among them: each of them are the 10 AFOREs. The weights of the assets in the portfolio are not the same, they represent the proportion of managed accounts by each AFORE. In table 5, I show the composition of the portfolio, displaying the net returns (net return of the AFORE) and the weights of the assets that compose the portfolio.

Table 5: Portfolio of the investor. 40 years to retire.

Asset	Net Return	Weight
Azteca	5.07%	18.92%
Citibanamex	4.78%	19.12%
Coppel	5.37%	50.04%
Inbursa	4.87%	0.41%
Invercap	3.63%	0.77%
PensionISSSTE	4.90%	0.31%
Principal	4.90%	0.50%
Profuturo	6.58%	2.85%
SURA	5.35%	4.91%
XXI-Banorte	4.92%	2.18%

Data from CONSAR.

To carry the simulations on, there are also some other assumptions to take into considerations. One of them is the value of the portfolio. Provided that the assets of the portfolio are the AFOREs, it is easy to know the value of the portfolio with the information reported by the CONSAR. In this analysis I will consider the sum of the net assets of each of the AFOREs as the value of the portfolio. This let me know how the different values of the VaR change with the value of the portfolio, and, therefore, the final pension savings of the worker. Another assumption I must consider is the risk-free rate of return. Provided that the investment horizon of the target-date funds is large, it is safe to consider the risk-free rate as 0.00% (Christofferson, 2012). Finally, given that the CONSAR considers the 1-day VaR, it is important to consider it when working with the net returns of the portfolio, provided that they are annual returns.

Once I have established all the assumptions needed for the model, I begin with the calculations to perform the simulations. The first step is to consider the expected volatility of the portfolio. Because the assets are correlated among them, I calculated the variance-covariance of the portfolio as in equation (1) to obtain the value of the expected variance of the portfolio.

$$\sigma_{t+1}^2 = w_t^{'}\Sigma_{t+1}w_t = \begin{bmatrix} w_{1,t} & \cdots & w_{10,t} \end{bmatrix} \begin{bmatrix} \sigma_{1,1;t+1}^2 & \cdots & \sigma_{1,10;t+1} \\ \vdots & \ddots & \vdots \\ \sigma_{10,1;t+1} & \cdots & \sigma_{10,10;t+1}^2 \end{bmatrix} \begin{bmatrix} w_{1,t} \\ \vdots \\ w_{10,t} \end{bmatrix} \quad (1)$$

Where w_t is the 10x1 matrix of weights, Σ_{t+1} is the variance-covariance matrix of the portfolio's returns, and $w_t^{'}$ is the transposed matrix of w_t. In the matrix, the values of the diagonal are the variances of each asset, while the values off the diagonal are the covariances among them. After calculating the expected variance of the portfolio, in order to proceed to the simulation, we need to obtain the expected volatility of the portfolio, which is the standard deviation of the portfolio, which is the square root of σ_{t+1}^2. The scalar resulting after the previous calculations not only shows us the volatility of the portfolio, but the volatility of the whole Generational SIEFORE.[8]. In table 6 I show the implied volatilities of each of the Generational SIEFOREs [9]. although I will only use the volatility of the Basic Initial SIEFORE in the rest of the thesis.

Once the implied volatility is calculated, I can now simulate the price of the portfolio. To achieve this I will use the Black-Scholes-Merton model (B-S-M). As mentioned before, I will use Monte Carlo simulation to generate 10 thousand random portfolio prices. These random prices will let me calculate the Value-at-Risk of the Generational SIEFORE. The price of the portfolio is calculated with the following stochastic differential equation:

$$dS_t = rS_tdt + \sigma S_tdZ_t \quad (2)$$

[8]Due to the assumptions made, the obtained implied volatility is of the Basic Initial SIEFORE (for people born after 1994).

[9]The variance-covariance matrices are displayed in the Appendix.

Table 6: Implied volatilities by Generational SIEFORE

SIEFORE	Volatility
SB Pensions	1.474%
SB 55-59	1.656%
SB 60-64	1.282%
SB 65-69	1.185%
SB 70-74	1.066%
SB 75-79	1.032%
SB 80-84	0.864%
SB 85-89	0.913%
SB 90-94	0.824%
SB Initial	0.780%

Own generated with data from CONSAR.

Where S_t is the value of the portfolio at time t, r is the risk-free rate, σ is the implied volatility of the portfolio, and Z_t is a Brownian motion. Therefore, to simulate equation (2) it is necessary to solve it first, and then perform the simulation. I applied the Euler Discretization Scheme (Haugh, 2017) to solve for equation (2). The solution is given by the expression:

$$\widehat{S_t} = S_t * exp\left[\left(r - \frac{1}{2}\sigma^2\right)\Delta t + \sigma\sqrt{\Delta t}Z_t\right], \quad \text{where} \quad Z_t \sim N(0,1) \tag{3}$$

Where $\widehat{S_t}$ is the simulated value of the portfolio. Provided that we already made some assumptions about the risk free rate, and the time of the portfolios. Equation (3) can be expressed as:

$$\widehat{S_t} = S_t * exp\left[\left(-\frac{1}{2}\sigma^2\right) + \frac{21}{252} + \sigma\sqrt{\frac{21}{252}}Z_t\right], \quad \text{where} \quad Z_t \sim N(0,1) \tag{4}$$

where $\frac{21}{252}$ is the trading days in a month over the trading days in a year. The previous expression is necessary due to we want to calculate the 1-day VaR.

After the 10 thousand prices have been simulated, I use them to calculate the 1-day Value-at-Risk with different levels of confidence. In this thesis I will calculate the 1-day VaR with 99%, 97%, and 95% levels of confidence. To calculate the VaR of the portfolio, I first calculate the price changes of the portfolio to obtain its simulated returns. Finally, I use the distribution of those simulated returns to calculate its 1st, 3rd, and 5th percentiles. Therefore the returns and the VaR are calculated as follow in equations (5) and (6):

$$\widehat{R_t} = \ln\left(\widehat{S_{t+1}}\right) - \ln\left(\widehat{S}_t\right) = \ln\left(\frac{\widehat{S_{t+1}}}{\widehat{S}_t}\right) \tag{5}$$

$$VaR^{\alpha}_{t+1:t+K} = \left|1 - Percentile\left\{\left[\widehat{R_{i,t+1:t+K}}\right]_{i=1}^{10{,}000},\ 1-\alpha\right\}\right| \tag{6}$$

Where from equation (6) α is the significance level at which we want to calculate the 1-day VaR, and $\left[\widehat{R_{i,t+1:t+K}}\right]_{i=1}^{10{,}000}$ is the set of all returns, which were calculated from the price simulation using the Black-Scholes-Merton model. In the next section I present the results of the VaR simulations on the final pension savings of the workers.

5 Estimations and Results

5.1 Simulated VaR and Final Pension Savings

After simulating the VaR with the procedure described on section 4, I constructed what would be the final average pension savings of a worker. The construction considered, as mentioned in subsection 4.1, the wage, the contribution density, the percentage of wage transferred to the worker's AFORE, the annual average net return, and the standard deviation of the net returns. Before moving on to the results, I describe how I simulated the final pension savings of a worker:

1. I generated 40 random numbers between 0 and 1. Each random number is interpreted as a probability, and will serve to construct the net return of all the years that the agent has left to retire. In this case is 40, because of initial assumption.

2. Then, using the random numbers, the standard deviation of the net returns, and the simulated returns with different VaRs, I calculated the net returns of each year

using the inverse of the normal cumulative distribution function: $\phi^{-1}(P, \mu, \sigma)$. Where the probability, P, are the random umber between 0 and 1, μ is the simulated return (with different VaR levels), and σ is the standard deviation of the net returns.

3. After simulating the returns, I multiplied the first contribution to the AFORE, that is the 6.5% or 12.5% of the monthly wage times one plus the generated return. Then I added to it the annual contribution to obtain the pension savings of the first year.

4. I did the same for the rest of the years, but instead o adding the annual contribution I added the last year's pensions savings, in order to calculate the cumulative pension savings.

5. At the end of step four I had already calculated the accumulated pension in the year 40, that is, the pensions savings at retirement. After that, I multiplied this number with 10 thousand numbers normally distributed with zero mean and a standard deviation of one. This allowed me to simulate many different scenarios: each scenario represents the pensions savings of a worker, that is, I simulated the final pensions savings of 10 thousand workers.

6. Finally, I calculated the summary statistics of the 10 thousand portfolios, which I report below.

The main results of the analysis are shown in tables 7 and 8. To make the analysis in table 7, I assumed a constant monthly wage of $10,000 MXN, with a contribution density of 100%, that is, the worker is always listed in the IMSS. In the table it is displayed the different final pensions achieved when using different confidence levels of VaR and contributions to the worker's AFORE. The column named benchmark depicts the results when no change is made to the system, that is, what a worker faces today. The next two columns show us how would the final average pensions be if there were a change in the allowed levels of VaR. As it can be observed, in panels A and B, changing the allowed VaR confidence level from 99% to 97% results in a dramatic increase on the final average pension and in the replacement rate, which goes from 45% to 69.5% of the final wage. Passing to a 95% VaR also increases both, the final pension and the replacement rate, however, the increase is not as big as before.

Table 7: Pensions Savings with different levels of VaR.

Metric	Benchmark	97% VaR	95% VaR
Panel A. 6.5% of wage as a contribution to the AFORE (before 2020 reform).			
Mean	$ 1,080,442.54	$ 1,668,696.02	$ 1,731,400.70
Median	$ 1,069,567.72	$ 1,651,900.34	$ 1,713,973.89
St. Dev	$ 1,073,444.06	$ 1,657,887.17	$ 1,720,185.69
0.05	-$ 687,487.55	-$ 1,061,794.31	-$ 1,101,693.41
0.25	$ 346,349.29	$ 534,921.25	$ 555,022.02
0.75	$ 1,827,470.82	$ 2,822,448.37	$ 2,928,507.66
0.9	$ 2,463,288.35	$ 3,804,440.60	$ 3,947,400.27
R. Rate[1]	45.05%	69.53%	72.14%
Panel B.12.5% of wage as a contribution to the AFORE (after 2020 reform).			
Mean	$ 2,079,104.84	$ 3,209,030.81	$ 3,329,616.73
Median	$ 2,058,178.33	$ 3,176,731.42	$ 3,296,103.63
St. Dev	$ 2,065,637.60	$ 3,188,244.57	$ 3,308,049.41
0.05	-$ 1,322,938.19	-$ 2,041,912.13	-$ 2,118,641.18
0.25	$ 666,482.90	$ 1,028,694.72	$ 1,067,350.04
0.75	$ 3,516,617.76	$ 5,427,785.32	$ 5,631,745.50
0.9	$ 4,740,126.88	$ 7,316,231.92	$ 7,591,154.36
R. Rate	86.63%	133.71%	138.73%

1. Replacement Rate.
2. Assumptions: Monthly wage of $10,000.00 (Mexican Pesos) and a contribution density of 100%.

Table 7 also shows the final pension savings when increasing the percentage of the wage that is retained in the AFORE account of the worker. As I mentioned in section 3, in mid-2020 there was a major reform which increased the contributions from 6.5 to 12.5% of the wage. Considering this change in the analysis allows me to predict for how much the final pensions, on average, will increase. As the results show, the final pensions (independently of the level of VaR) almost doubles. These are excellent news, because as it is shown in table 8, the replacement rate doubles for all, even if the worker has a low contribution density.

Table 8: Pensions with Different Contribution Densities

CD	Benchmark			97% VaR			95% VaR		
	Final Pension	Monthly	R.R.[1]	Final Pension	Monthly	R.R.	Final Pension	Monthly	R.R.
Panel A. 6.5% of wage as a contribution to the AFORE (before 2020 reform).									
Low[2] (20%)	$ 216,226.90	$ 900.95	9.01%	$ 333,739.20	$ 1,390.58	13.91%	$ 346,280.14	$ 1,442.83	14.43%
Medium (50%)	$ 540,567.26	$ 2,252.36	22.52%	$ 834,348.01	$ 3,476.45	34.76%	$ 865,700.35	$ 3,607.08	36.07%
High (80%)	$ 864,907.61	$ 3,603.78	36.04%	$ 1,334,956.82	$ 5,562.32	55.62%	$ 1,385,120.56	$ 5,771.34	57.71%
Panel B.12.5% of wage as a contribution to the AFORE (after 2020 reform).									
Low (20%)	$ 415,820.97	$ 1,732.59	17.33%	$ 641,806.16	$ 2,674.19	26.74%	$ 665,923.35	$ 2,774.68	27.75%
Medium (50%)	$ 1,039,552.42	$ 4,331.47	43.31%	$ 1,604,515.40	$ 6,685.48	66.85%	$ 1,664,808.37	$ 6,936.70	69.37%
High (80%)	$ 1,663,283.87	$ 6,930.35	69.30%	$ 2,567,224.65	$10,696.77	106.97%	$ 2,663,693.39	$11,098.72	110.99%

1. Replacement Rate.
2. Using the measures of Castañón e Ibarra (2017).
3. The same as in table 7 plus a life expectancy of 20 years after retirement.

In table 8, I consider the same levels of Value-at-Risk and contribution rates, but now I perform the simulations with different levels of contribution densities(20%, 50%, and 80%). Compared to table 7, this chart tries to approach to the Mexican reality. Besides the assumptions I made before, I include an extra assumption: the worker lives 20 years (240 moths) after her retirement date. This assumption allows me to know the monthly pension that the worker will receive.

As depicted in the table, one can observe how low the replacement rates are when considering a wage contribution of 6.5% and an unchanged level of VaR. However, increasing both of them allows the final pension to increase considerably. What I find important about table 8 is the huge difference between the replacement rates between the workers with low and highest contribution densities. Without changing the levels of VaR, the replacement rate of a worker with a contribution density of 20% is four times lower than the replacement rate of a worker with a high contribution density. The distinction between low and high contribution densities comes from the work of Castañón e Ibarra (2017).

Furthermore, in table 8, the increase in the level Value-at-Risk and the contributions to the worker's AFORE, let the replacement rate of all workers to increase exceptionally. With a contribution to AFORE of 12.5% of the wage, and a 95% VaR, the replacement rate of a worker with a contribution density of 80% is 11 percent greater than the last wage received by the worker. Considering a 97% VaR, the replacement rate is 7 percent greater. Although not as impressive as in the case of a contribution rate of 80%, listing in the IMSS 50% percent of the working life, and with levels of 97 and 95 percent of VaR, the replacement rates increases to an excellent rate of 66 and 67 percent respectively. If it were not enough, the workers who have spend only 20 percent of their working lives in the formal sector, reach a better percentage of replacement rate, which increases their probability a pension higher than the minimum guaranteed pension by the State.

5.2 Stochastic Dominance of the simulated Pension Savings Distributions

In the previous subsection I presented the results of simulating total pension savings by considering different confidence levels of the 1-day Value at Risk. The results are drawn from performing 10 thousand Monte Carlo iterations of the model derived in section 4.2. The outcomes in section 5.1 show that increasing the significance level of the 1-day VaR, the worker would, on average, reach greater pension savings, and, consequently, greater replacement rates. They show that the adjustment outperforms the current strategy in terms of the mean, median, and the interquartile range outcomes. Still, we cannot conclude that increasing the significance levels of the allowed VaR is a superior investment strategy for the workers' retirement plans (Basu, Byrne, and Drew, 2011). It is necessary to compare the entire range of outcomes (final pension savings) to fully understand if the proposed strategy is dominant.

For the previous reason, the purpose of this section is to test whether the strategy proposed through this thesis is dominant to the current VaR scheme or not. Provided that I have simulated a whole distribution of final pensions, the ideal approach to perform the test is through stochastic dominance. This method offers minimal restrictions regarding the assumptions required for the model to be identified, for instance, the data need not to be normally distributed, and the investors' utility of wealth must be non-decreasing and convex ($u^{'}(W) \geq 0$ and $u^{''}(W) \leq 0$).

If F and G represent the pension distributions with a 3% and 5% significance levels of VaR respectively, and if Z represents the pension distributions with the benchmark level of VaR, each of the former dominate the latter under the First-Order Stochastic Dominance rule if, and only if,

$$F(W) \leq Z(W), \quad \forall W \quad \text{for a 3 percent 1-day VaR}$$

$$G(W) \leq Z(W), \quad \forall W \quad \text{for a 5 percent 1-day VaR}$$

In other words, the latter means that for the pension distribution of either the 3% or 5% VaR to dominate the current pensions distribution, they have to always remain below the cumulative pension savings distribution of the benchmark level of VaR. Formally, given $u^{'}(W) \geq 0$ and $u^{''}(W) \leq 0$, F or G distributions are preferred to Z if, and only if,

$$\int u(W)\,dF \geq \int u(W)\,dZ \quad \text{for a 3 percent 1-day VaR}$$

$$\int u(W)\,dG \geq \int u(W)\,dZ \quad \text{for a 5 percent 1-day VaR}$$

In figures 9, 10, and 11 I show the cumulative distribution functions of three different simulations. Figure (9) is the distribution of the final pension savings considering a 6.6% contribution rate of the worker's wage the her AFORE. As it can be appreciated, both cumulative distributions proposed with levels of VaR first-order stochastically dominate the distribution with the benchmark VaR. It is clear how using different levels of VaR in the asset allocation process increases, in all the distribution, the final wealth of the workers. It is important to mention that using a 95% VaR is a dominant investment strategy, stochastically speaking, to the rest of levels used, however, the difference between it and using a 97 percent level is not as huge as it is with the benchmark VaR.

In figure 10 we can appreciate almost the same story as in 9. In one hand, the only difference with the latter is that the simulation was performed considering that the worker contributed 12.5% of her wage to its personal AFORE account. We can appreciate that, again, the investment strategy of using a 95% level dominates the rest of them. As a well as in the previous example, the total difference between the 95% and 97% curves is minimal On the other hand, the only substantial change in the analysis is the magnitude of the final pension, that is, the highest final pension achieved almost doubles, from 4 million pesos to 7.5 million pesos in total.

Provided that the difference between the 95% and 97% curves is minimal, I would recommend using the 97% VaR investment strategy, because the final wealth would be

Figure 9: First-order stochastic dominance I

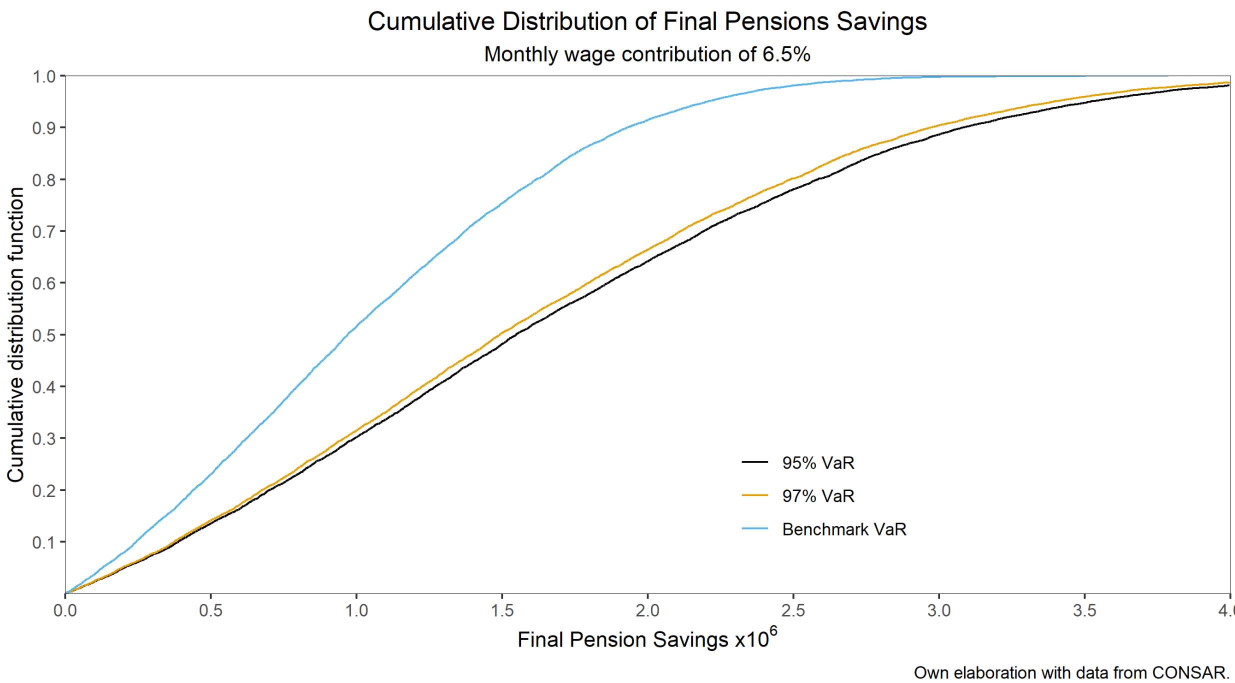
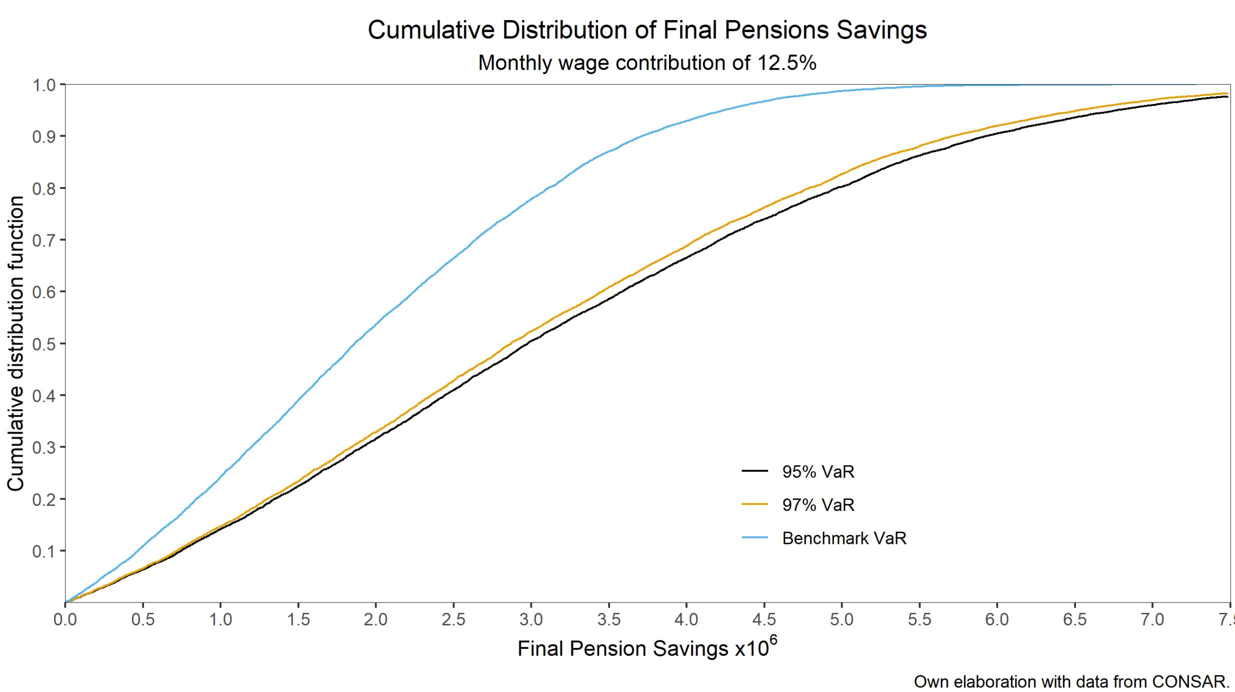
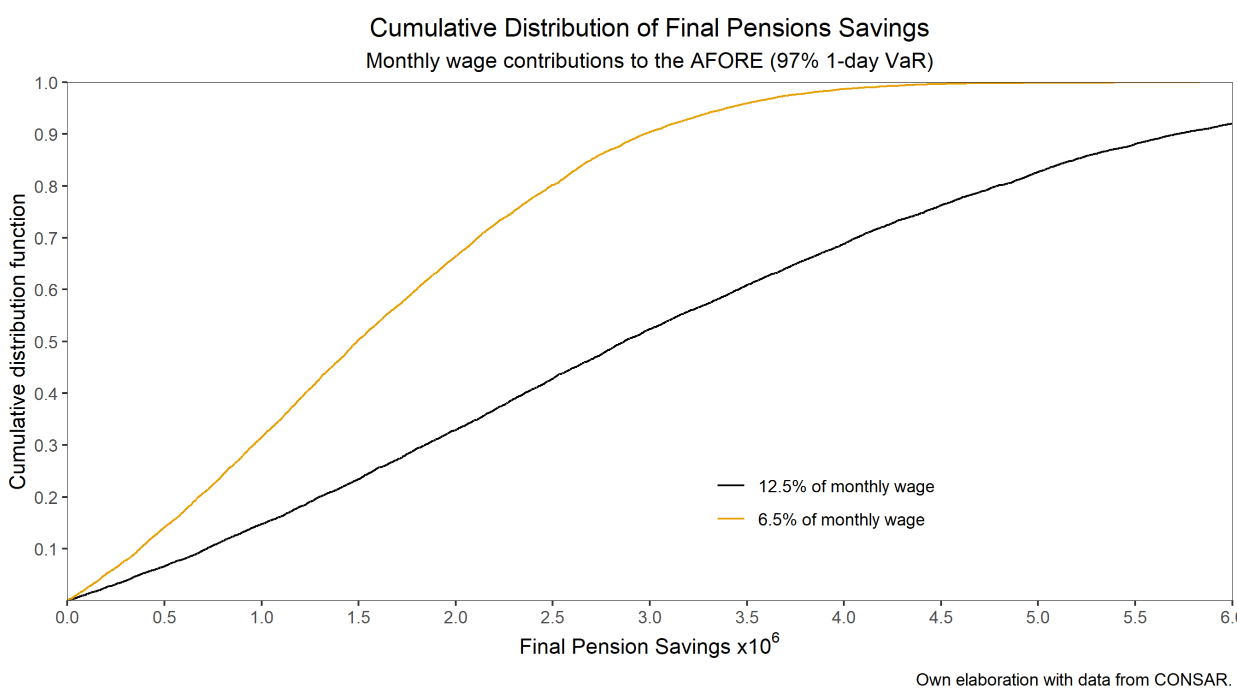


Figure 10: First-order stochastic dominance II



practically the same as with using the 95% VaR (due to the similarity of the distributions), but with the advantage of incurring in less risk. Figure 11 shows how dominant for the worker is to contribute 12.5% of her wage to her personal AFORE account, compared to only transferring the 6.5% of her monthly wage. As a matter of fact, the simulation of the distributions is made considering only a 97% 1-day VaR. It is observed how drastically the final pension savings increase. Contributing the double to the AFORE translates into a doubling of the pension achieved. For instance, with a contribution of 6.5%, half the workers achieved a final pension of 1.5 million pesos, while contributing the double made the same half of workers to achieve a final pension of 3 million pesos

Figure 11: First-order stochastic dominance III



6 Conclusion

In the previous sections I described and explained how the Target-Date Funds work, what have been some of the studies that have proposed improvements to them or that tried to simply evaluate them. Recalling from section 2.1, there are mixed opinions about the benefits of Target-Date Funds, since it depends on the assumptions made and how the pension system in each country works. It is clear that having pay-as-you-go systems is no longer sustainable, the fiscal pressures for the government increases over time, while ageing population keeps on growing. Therefore, transitioning to a defined contribution system may be an answer to the problem. Target-Date Funds have been emerging as an easy alternative for the workers to grow their money in an automatic way, that is why, studying and improving them is necessary if we want the workers retire with dignity.

In this thesis, using data from the CONSAR, I simulated final pension savings that the workers would achieve with two different values of VaR, specifically using 95 and 97 levels of confidence. In order to perform the simulations I had to built one portfolio composed by all the 10 AFOREs, properly weighted with the managed accounts of each pension fund manager. After constructing the portfolio I used the Black-Scholes-Merton model to simulate 10 thousand returns of that constructed portfolio, which helped me to calculate the different levels of Value-at-Risk and its inherent returns. Next, by assuming some values regarding the wage, the contribution density, and the contributions made to the personal account, I constructed the final pension of a worker. Finally, with all the previous information I simulated the final pension savings of 10,000 workers, which are the main results of these thesis.

The results showed me what I suspected from the beginning: the increase in the significance level of the VaR allows the workers to achieve greater pension savings at the end of their working life. In addition, this conclusion is robust, provided that both proposed levels of VaR first-order stochastically dominate the current allowed level of VaR. Moreover, if, allowing for a less conservative VaR increases the final wealth of the worker, combining these investment strategies with higher wages, higher contribution densities, and higher contributions to the AFOREs will lead the workers for a better and more dignified retirement.

This thesis shows that there a interesting effects of VaR on final pension savings. As discussed above, the final pensions of the workers would be higher if the AFOREs could invest with less restrictive allowed levels of VaR. Hence, the results obtained here justify a policy change regarding the investment regimes of the AFOREs. I provide enough information to Mexican policymakers, so they could modify the actual risk management policy of the AFOREs and let workers achieve greater final pensions savings without incurring on unnecessary risks.

Provided that Mexican Target-Date Funds are relatively new, there is little research about their functioning and benefits. Hence, this thesis is a first attempt for evaluating and understanding them, in order to make them function right and maximise worker's pensions savings. Therefore, considering the results obtained in this dissertation, the next steps would be to analyse why the implied volatilities (displayed in Table 6) are higher when the worker is nearer to its retirement date, and how this affects their final pension savings. Understanding why that happens would help the AFOREs, and the CONSAR itself, to implement a more dynamic risk management policies for the sake of the Mexican workers.

One of the main limitations of the analysis is the fact that I could not obtain the returns of each investment of the AFORE, that is, the returns of each of the categories in which the funds are invested (variable income, international debt, stocks, bonds, among others). Having this information would allow me to construct a portfolio per AFORE and not only one by considering each AFORE as an asset. I consider that having those returns would allow me to obtain more precise measures of VaR

I strongly believe that until there are no important structural changes in the Mexican economy, the workers will not be able to retire with dignity and out of the poverty. The results found in this dissertation is just part of the solution to the actual problem. They show a temporal solution to cope with some of the issues in the system while the really big problems are truly faced: low wages, minimal or non existent savings, and informality.

Appendices

A Basic SIEFOREs' Variance-Covariance Matrices

Table A1: SB of Pensions variance-covariance matrix

	Azteca	**Citibanamex**	**Coppel**	**Inbursa**	**Invercap**	**PensionISSSTE**	**Principal**	**Profuturo GNP**	**SURA**	**XXI Banorte**
Azteca	0.00001483	0.00001827	0.00001343	0.00002098	0.00001198	0.00001212	0.00001503	0.00001132	0.00002187	0.00001629
Citibanamex	0.00001827	0.00002261	0.00001656	0.00002608	0.00001472	0.00001489	0.00001854	0.00001392	0.00002705	0.00002011
Coppel	0.00001343	0.00001656	0.00001230	0.00001940	0.00001102	0.00001120	0.00001373	0.00001044	0.00001968	0.00001478
Inbursa	0.00002098	0.00002608	0.00001940	0.00003126	0.00001735	0.00001769	0.00002162	0.00001648	0.00003083	0.00002320
Invercap	0.00001198	0.00001472	0.00001102	0.00001735	0.00000991	0.00001011	0.00001228	0.00000941	0.00001743	0.00001316
PensionISSSTE	0.00001212	0.00001489	0.00001120	0.00001769	0.00001011	0.00001034	0.00001248	0.00000961	0.00001758	0.00001332
Principal	0.00001503	0.00001854	0.00001373	0.00002162	0.00001228	0.00001248	0.00001534	0.00001163	0.00002206	0.00001653
Profuturo GNP	0.00001132	0.00001392	0.00001044	0.00001648	0.00000941	0.00000961	0.00001163	0.00000894	0.00001645	0.00001244
SURA	0.00002187	0.00002705	0.00001968	0.00003083	0.00001743	0.00001758	0.00002206	0.00001645	0.00003250	0.00002405
XXI Banorte	0.00001629	0.00002011	0.00001478	0.00002320	0.00001316	0.00001332	0.00001653	0.00001244	0.00002405	0.00001791

Table A2: SB 55-59 variance-covariance matrix

	Azteca	**Citibanamex**	**Coppel**	**Inbursa**	**Invercap**	**PensionISSSTE**	**Principal**	**Profuturo GNP**	**SURA**	**XXI Banorte**
Azteca	0.00001913	0.00002402	0.00001907	0.00002713	0.00001785	0.00001799	0.00002040	0.00001908	0.00001858	0.00002252
Citibanamex	0.00002402	0.00003017	0.00002393	0.00003402	0.00002237	0.00002255	0.00002561	0.00002395	0.00002333	0.00002828
Coppel	0.00001907	0.00002393	0.00001905	0.00002700	0.00001784	0.00001787	0.00002034	0.00001911	0.00001861	0.00002245
Inbursa	0.00002713	0.00003402	0.00002700	0.00003871	0.00002538	0.00002576	0.00002892	0.00002689	0.00002612	0.00003189
Invercap	0.00001785	0.00002237	0.00001784	0.00002538	0.00001677	0.00001682	0.00001903	0.00001786	0.00001736	0.00002099
PensionISSSTE	0.00001799	0.00002255	0.00001787	0.00002576	0.00001682	0.00001722	0.00001919	0.00001771	0.00001718	0.00002114
Principal	0.00002040	0.00002561	0.00002034	0.00002892	0.00001903	0.00001919	0.00002176	0.00002035	0.00001982	0.00002402
Profuturo GNP	0.00001908	0.00002395	0.00001911	0.00002689	0.00001786	0.00001771	0.00002035	0.00001929	0.00001881	0.00002248
SURA	0.00001858	0.00002333	0.00001861	0.00002612	0.00001736	0.00001718	0.00001982	0.00001881	0.00001836	0.00002190
XXI Banorte	0.00002252	0.00002828	0.00002245	0.00003189	0.00002099	0.00002114	0.00002402	0.00002248	0.00002190	0.00002651

Table A3: SB 60-64 variance-covariance matrix

	Azteca	**Citibanamex**	**Coppel**	**Inbursa**	**Invercap**	**PensionISSSTE**	**Principal**	**Profuturo GNP**	**SURA**	**XXI Banorte**
Azteca	0.00001196	0.00001209	0.00001466	0.00001761	0.00001656	0.00001451	0.00001460	0.00001092	0.00000972	0.00001388
Citibanamex	0.00001209	0.00001222	0.00001481	0.00001776	0.00001673	0.00001469	0.00001476	0.00001102	0.00000984	0.00001404
Coppel	0.00001466	0.00001481	0.00001803	0.00002141	0.00002025	0.00001755	0.00001788	0.00001353	0.00001215	0.00001702
Inbursa	0.00001761	0.00001776	0.00002141	0.00002650	0.00002451	0.00002182	0.00002152	0.00001588	0.00001368	0.00002036
Invercap	0.00001656	0.00001673	0.00002025	0.00002451	0.00002297	0.00002023	0.00002023	0.00001505	0.00001331	0.00001921
PensionISSSTE	0.00001451	0.00001469	0.00001755	0.00002182	0.00002023	0.00001849	0.00001777	0.00001266	0.00001099	0.00001685
Principal	0.00001460	0.00001476	0.00001788	0.00002152	0.00002023	0.00001777	0.00001783	0.00001330	0.00001183	0.00001695
Profuturo GNP	0.00001092	0.00001102	0.00001353	0.00001588	0.00001505	0.00001266	0.00001330	0.00001043	0.00000938	0.00001266
SURA	0.00000972	0.00000984	0.00001215	0.00001368	0.00001331	0.00001099	0.00001183	0.00000938	0.00000881	0.00001134
XXI Banorte	0.00001388	0.00001404	0.00001702	0.00002036	0.00001921	0.00001685	0.00001695	0.00001266	0.00001134	0.00001614

Table A4: SB 65-69 variance-covariance matrix

	Azteca	Citibanamex	Coppel	Inbursa	Invercap	PensionISSSTE	Principal	Profuturo GNP	SURA	XXI Banorte
Azteca	0.00001026	0.00000944	0.00001299	0.00001489	0.00001400	0.00001317	0.00001412	0.00000897	0.00000832	0.00001234
Citibanamex	0.00000944	0.00000897	0.00001202	0.00001328	0.00001273	0.00001164	0.00001302	0.00000850	0.00000811	0.00001145
Coppel	0.00001299	0.00001202	0.00001648	0.00001876	0.00001770	0.00001657	0.00001789	0.00001141	0.00001064	0.00001565
Inbursa	0.00001489	0.00001328	0.00001876	0.00002225	0.00002056	0.00001976	0.00002043	0.00001270	0.00001143	0.00001774
Invercap	0.00001400	0.00001273	0.00001770	0.00002056	0.00001923	0.00001832	0.00001926	0.00001204	0.00001108	0.00001680
PensionISSSTE	0.00001317	0.00001164	0.00001657	0.00001976	0.00001832	0.00001809	0.00001812	0.00001076	0.00000980	0.00001575
Principal	0.00001412	0.00001302	0.00001789	0.00002043	0.00001926	0.00001812	0.00001944	0.00001233	0.00001149	0.00001700
Profuturo GNP	0.00000897	0.00000850	0.00001141	0.00001270	0.00001204	0.00001076	0.00001233	0.00000835	0.00000779	0.00001081
SURA	0.00000832	0.00000811	0.00001064	0.00001143	0.00001108	0.00000980	0.00001149	0.00000779	0.00000753	0.00001015
XXI Banorte	0.00001234	0.00001145	0.00001565	0.00001774	0.00001680	0.00001575	0.00001700	0.00001081	0.00001015	0.00001489

Table A5: SB 70-74 variance-covariance matrix

	Azteca	Citibanamex	Coppel	Inbursa	Invercap	PensionISSSTE	Principal	Profuturo GNP	SURA	XXI Banorte
Azteca	0.00000869	0.00000674	0.00001132	0.00001284	0.00001252	0.00001088	0.00001153	0.00000737	0.00000653	0.00001057
Citibanamex	0.00000674	0.00000719	0.00000889	0.00000898	0.00000952	0.00000702	0.00000920	0.00000668	0.00000667	0.00000859
Coppel	0.00001132	0.00000889	0.00001476	0.00001666	0.00001631	0.00001412	0.00001504	0.00000962	0.00000859	0.00001381
Inbursa	0.00001284	0.00000898	0.00001666	0.00001952	0.00001858	0.00001671	0.00001689	0.00001050	0.00000888	0.00001540
Invercap	0.00001252	0.00000952	0.00001631	0.00001858	0.00001809	0.00001590	0.00001661	0.00001046	0.00000924	0.00001523
PensionISSSTE	0.00001088	0.00000702	0.00001412	0.00001671	0.00001590	0.00001493	0.00001432	0.00000829	0.00000698	0.00001305
Principal	0.00001153	0.00000920	0.00001504	0.00001689	0.00001661	0.00001432	0.00001535	0.00000984	0.00000886	0.00001410
Profuturo GNP	0.00000737	0.00000668	0.00000962	0.00001050	0.00001046	0.00000829	0.00000984	0.00000696	0.00000638	0.00000907
SURA	0.00000653	0.00000667	0.00000859	0.00000888	0.00000924	0.00000698	0.00000886	0.00000638	0.00000623	0.00000825
XXI Banorte	0.00001057	0.00000859	0.00001381	0.00001540	0.00001523	0.00001305	0.00001410	0.00000907	0.00000825	0.00001298

Table A6: SB 75-79 variance-covariance matrix

	Azteca	Citibanamex	Coppel	Inbursa	Invercap	PensionISSSTE	Principal	Profuturo GNP	SURA	XXI Banorte
Azteca	0.00000770	0.00000599	0.00001074	0.00001153	0.00000886	0.00000965	0.00001113	0.00000744	0.00000513	0.00000970
Citibanamex	0.00000599	0.00000662	0.00000843	0.00000814	0.00000728	0.00000609	0.00000906	0.00000673	0.00000577	0.00000803
Coppel	0.00001074	0.00000843	0.00001501	0.00001603	0.00001237	0.00001347	0.00001557	0.00001037	0.00000721	0.00001357
Inbursa	0.00001153	0.00000814	0.00001603	0.00001767	0.00001309	0.00001495	0.00001646	0.00001085	0.00000696	0.00001427
Invercap	0.00000886	0.00000728	0.00001237	0.00001309	0.00001027	0.00001084	0.00001289	0.00000874	0.00000627	0.00001126
PensionISSSTE	0.00000965	0.00000609	0.00001347	0.00001495	0.00001084	0.00001342	0.00001374	0.00000841	0.00000510	0.00001190
Principal	0.00001113	0.00000906	0.00001557	0.00001646	0.00001289	0.00001374	0.00001621	0.00001088	0.00000777	0.00001415
Profuturo GNP	0.00000744	0.00000673	0.00001037	0.00001085	0.00000874	0.00000841	0.00001088	0.00000788	0.00000587	0.00000952
SURA	0.00000513	0.00000577	0.00000721	0.00000696	0.00000627	0.00000510	0.00000777	0.00000587	0.00000506	0.00000689
XXI Banorte	0.00000970	0.00000803	0.00001357	0.00001427	0.00001126	0.00001190	0.00001415	0.00000952	0.00000689	0.00001236

Table A7: SB 80-84 variance-covariance matrix

	Azteca	Citibanamex	Coppel	Inbursa	Invercap	PensionISSSTE	Principal	Profuturo GNP	SURA	XXI Banorte
Azteca	0.00000533	0.00000380	0.00000758	0.00000844	0.00000809	0.00000750	0.00000720	0.00000551	0.00000191	0.00000692
Citibanamex	0.00000380	0.00000526	0.00000541	0.00000494	0.00000592	0.00000354	0.00000568	0.00000549	0.00000434	0.00000584
Coppel	0.00000758	0.00000541	0.00001080	0.00001196	0.00001156	0.00001074	0.00001026	0.00000776	0.00000268	0.00000988
Inbursa	0.00000844	0.00000494	0.00001196	0.00001385	0.00001270	0.00001252	0.00001113	0.00000815	0.00000183	0.00001055
Invercap	0.00000809	0.00000592	0.00001156	0.00001270	0.00001241	0.00001148	0.00001102	0.00000829	0.00000298	0.00001063
PensionISSSTE	0.00000750	0.00000354	0.00001074	0.00001252	0.00001148	0.00001222	0.00000982	0.00000631	0.00000041	0.00000921
Principal	0.00000720	0.00000568	0.00001026	0.00001113	0.00001102	0.00000982	0.00000987	0.00000771	0.00000318	0.00000958
Profuturo GNP	0.00000551	0.00000549	0.00000776	0.00000815	0.00000829	0.00000631	0.00000771	0.00000694	0.00000395	0.00000762
SURA	0.00000191	0.00000434	0.00000268	0.00000183	0.00000298	0.00000041	0.00000318	0.00000395	0.00000425	0.00000348
XXI Banorte	0.00000692	0.00000584	0.00000988	0.00001055	0.00001063	0.00000921	0.00000958	0.00000762	0.00000348	0.00000936

Table A8: SB 85-89 variance-covariance matrix

	Azteca	Citibanamex	Coppel	Inbursa	Invercap	PensionISSSTE	Principal	Profuturo GNP	SURA	XXI Banorte
Azteca	0.00000518	0.00000395	0.00000784	0.00000788	0.00000655	0.00000771	0.00000798	0.00000603	0.00000278	0.00000696
Citibanamex	0.00000395	0.00000534	0.00000601	0.00000519	0.00000534	0.00000417	0.00000681	0.00000595	0.00000497	0.00000623
Coppel	0.00000784	0.00000601	0.00001192	0.00001190	0.00000996	0.00001176	0.00001214	0.00000907	0.00000420	0.00001058
Inbursa	0.00000788	0.00000519	0.00001190	0.00001240	0.00000979	0.00001219	0.00001187	0.00000886	0.00000330	0.00001023
Invercap	0.00000655	0.00000534	0.00000996	0.00000979	0.00000839	0.00000966	0.00001022	0.00000769	0.00000387	0.00000896
PensionISSSTE	0.00000771	0.00000417	0.00001176	0.00001219	0.00000966	0.00001320	0.00001143	0.00000761	0.00000185	0.00000977
Principal	0.00000798	0.00000681	0.00001214	0.00001187	0.00001022	0.00001143	0.00001257	0.00000967	0.00000514	0.00001103
Profuturo GNP	0.00000603	0.00000595	0.00000907	0.00000886	0.00000769	0.00000761	0.00000967	0.00000814	0.00000507	0.00000857
SURA	0.00000278	0.00000497	0.00000420	0.00000330	0.00000387	0.00000185	0.00000514	0.00000507	0.00000509	0.00000483
XXI Banorte	0.00000696	0.00000623	0.00001058	0.00001023	0.00000896	0.00000977	0.00001103	0.00000857	0.00000483	0.00000973

Table A9: SB 90-94 variance-covariance matrix

	Azteca	**Citibanamex**	**Coppel**	**Inbursa**	**Invercap**	**PensionISSSTE**	**Principal**	**Profuturo GNP**	**SURA**	**XXI Banorte**
Azteca	0.00000377	0.00000251	0.00000622	0.00000600	0.00000694	0.00000625	0.00000639	0.00000471	0.00000212	0.00000561
Citibanamex	0.00000251	0.00000420	0.00000387	0.00000299	0.00000421	0.00000214	0.00000446	0.00000450	0.00000449	0.00000442
Coppel	0.00000622	0.00000387	0.00001036	0.00000995	0.00001164	0.00001070	0.00001058	0.00000751	0.00000310	0.00000925
Inbursa	0.00000600	0.00000299	0.00000995	0.00001001	0.00001111	0.00001061	0.00001003	0.00000705	0.00000220	0.00000861
Invercap	0.00000694	0.00000421	0.00001164	0.00001111	0.00001316	0.00001225	0.00001187	0.00000818	0.00000324	0.00001036
PensionISSSTE	0.00000625	0.00000214	0.00001070	0.00001061	0.00001225	0.00001254	0.00001059	0.00000634	0.00000085	0.00000893
Principal	0.00000639	0.00000446	0.00001058	0.00001003	0.00001187	0.00001059	0.00001091	0.00000798	0.00000378	0.00000963
Profuturo GNP	0.00000471	0.00000450	0.00000751	0.00000705	0.00000818	0.00000634	0.00000798	0.00000687	0.00000444	0.00000727
SURA	0.00000212	0.00000449	0.00000310	0.00000220	0.00000324	0.00000085	0.00000378	0.00000444	0.00000501	0.00000393
XXI Banorte	0.00000561	0.00000442	0.00000925	0.00000861	0.00001036	0.00000893	0.00000963	0.00000727	0.00000393	0.00000861

Table A10: SB Initial variance-covariance matrix

	Azteca	**Citibanamex**	**Coppel**	**Inbursa**	**Invercap**	**PensionISSSTE**	**Principal**	**Profuturo GNP**	**SURA**	**XXI Banorte**
Azteca	0.00000313	0.00000090	0.00000555	0.00000530	0.00000476	0.00000589	0.00000520	0.00000426	0.00000148	0.00000419
Citibanamex	0.00000090	0.00000346	0.00000112	0.00000061	0.00000103	-0.00000083	0.00000178	0.00000283	0.00000408	0.00000197
Coppel	0.00000555	0.00000112	0.00001002	0.00000947	0.00000863	0.00001104	0.00000926	0.00000722	0.00000203	0.00000737
Inbursa	0.00000530	0.00000061	0.00000947	0.00000931	0.00000806	0.00001053	0.00000865	0.00000685	0.00000150	0.00000683
Invercap	0.00000476	0.00000103	0.00000863	0.00000806	0.00000750	0.00000959	0.00000799	0.00000614	0.00000178	0.00000637
PensionISSSTE	0.00000589	-0.00000083	0.00001104	0.00001053	0.00000959	0.00001368	0.00000974	0.00000645	-0.00000023	0.00000741
Principal	0.00000520	0.00000178	0.00000926	0.00000865	0.00000799	0.00000974	0.00000873	0.00000714	0.00000275	0.00000707
Profuturo GNP	0.00000426	0.00000283	0.00000722	0.00000685	0.00000614	0.00000645	0.00000714	0.00000678	0.00000391	0.00000602
SURA	0.00000148	0.00000408	0.00000203	0.00000150	0.00000178	-0.00000023	0.00000275	0.00000391	0.00000491	0.00000283
XXI Banorte	0.00000419	0.00000197	0.00000737	0.00000683	0.00000637	0.00000741	0.00000707	0.00000602	0.00000283	0.00000582

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