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THE ROLE OF INFORMALITY AND NOMINAL WAGE RIGIDITIES ON MEXICAN BUSINESS CYCLES

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In your youth, you knew your place,
and kept time blissfully out of mind.
But time takes its toll, and
hardens all hearts with desire and ambition.
With all your hate and all your lies,
it will come as no surprise when you get lost.

You will have to prove you can work,
that you are deserving of the penance and the toil.
Then, you must move.
Invite yourself as conduit of alleged sages.
But reverence is temporary,
don't let gospel burnout keep your eyes closed.

The time will come
when, with desperation
you will send-off yourself
at your own door, in your own mirror
and each will sadden at the other's departure.
You will hate again the stranger that was your self.

To my mother and my sister.
To my friends: Eduardo and Carolina; Miguel, Paulina, Omar, and Francisco.
To my supervisor, Dr. McKnight, and my teachers.
To El Colegio de México.

Abstract

This thesis explores the influence of nominal wage rigidities on the cyclical behavior of informality in developing economies, specifically in Mexico. This research contributes in two key ways: firstly, it provides empirical estimates of business cycle fluctuations of real wages in formal and informal sectors, showing that both are more volatile in Mexico than in developed countries and both are procyclical. Secondly, it introduces a model featuring search and matching frictions in labor markets and nominal rigidities in the formal sector. This model successfully reproduces key stylized facts, including the countercyclicality of informal employment and unemployment, and the procyclicality of real wages. A plausible calibration finds that under this specification nominal wage rigidities are crucial to obtain the proper behavior of informal employment and unemployment. This finding suggests that policymakers should be wary of nominal adjustment frictions when designing policies to address informality during recessions.

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Chapter 1. Introduction

Large informal sectors are an important feature of developing countries. In the recent literature an interesting view on the role of informality has emerged: On the one hand, informality appears to lead to a long-run misallocation of resources which hurts economic growth. On the other hand, it acts as a substitute for unemployment insurance and a social safety net that developing countries cannot usually afford. Therefore, a dangerous trade-off arises where the short-run benefits of maintaining a sizeable informal sector are at odds with the long-run benefits of growth from reducing informality. Also, it has become evident from recent research that understanding the mechanisms which drive the business cycle behavior of informality are essential to determining appropriate policy responses in countries with high rates of informality. This thesis attempts to understand whether nominal wage rigidities can help explain the observed behavior of informality and wages over the business cycle in Mexico. The aims are to, first, understand the behavior of formal and informal wages during the business cycle and, second, to investigate whether a model with rigidities in formal wages can reproduce the stylized facts for Mexico.

This thesis is part of a recent strand of literature that is interested in modeling informality using search and matching frictions under a New Keynesian framework. The main articles to which this thesis is related are to De la O and McKnight (2019) and Leyva and Urrutia (2020). However, it is also related to other papers that try to reproduce the same stylized facts and investigate similar questions like Finkelstein (2017), Fernández and Meza (2015), Horvath (2018) and Restrepo-Echavarria (2014). The difference with this literature is that some of these models use competitive labor markets or are focused on explaining real business cycles, neither of which are of interest to this thesis.

This thesis makes two contributions to the literature: First, it undertakes an empirical estimation of the business cycle behavior of real wages in the formal and informal sectors. I find that real wages in both sectors are more volatile than output and procyclical in Mexico, which is very different from the behavior in developed countries, where wages are less volatile than output and mostly acyclical. Second, the thesis develops a framework that uses search and matching frictions in both labor markets and nominal rigidities in the formal sector, which is able to reproduce three important stylized facts: the countercyclicality of informal employment, the countercyclicality of

unemployment, and the procyclicality of real wages. In this sense, it proposes a novel mechanism to understand the observed behavior in the Mexican labor market.

This thesis is structured as follows: First, I review the related literature and compare and contrast the existing literature regarding their frameworks and features. Then, I present the stylized facts for the Mexican business cycle laying out the main aspects of employment and wage behavior in greater detail. In this chapter, I compare my results with those of the previous literature. Next, I describe the framework of the model and present its main equations. Subsequently, I present the results and sensitivity analysis of the model and a general discussion of its implications. Finally, the thesis concludes with a summary of its contributions to the literature and potential extensions to the modeling framework for future research.

Chapter 2. Literature Review

The modern macroeconomic literature on informality has two main characteristics. First, it has mostly abandoned the so-called "romantic view" of informality; informal firms are no longer seen as an efficient alternative to the tax-distorted formal firms (a brief discussion of this paradigm change is presented in Leal (2019). Empirical evidence has found that, controlling for aspects like firm size or business sector, informal firms are, on average, less productive than formal ones and suggests that the efficiency story may be backwards: informal firms may only be able to survive because they can escape taxation. Second, the current literature places particular importance on distortions that result in the misallocation of resources, whether they are introduced by informality or by the legal framework (mainly taxation).

The recent literature has mainly investigated the business cycle implications of an informal sector as a means to reconcile the stylized facts of developing countries with tenets of modern macroeconomic theory, for example, to resolve the contradiction between consumption smoothing and greater volatility of consumption than output. The primary tool of this literature is the use of Dynamic Stochastic General Equilibrium Models (DSGE) with two goods sectors. As this thesis is most related to the DSGE literature, in the remaining literature review I will compare and

contrast some of the papers in this literature and present a possible division of the articles depending on their assumptions and aims.

Most of the recent DSGE literature has focused on the self-employment definition of informality. This assumption is explicit in papers by De la O and McKnight (2019), and Fernández and Meza (2015), while Horvath (2018) and Restrepo-Echavarria (2014) permit and interpretation of the informal sector as small household firms. Nonetheless, even in the latter papers, the structure is essentially indistinguishable from self-employment. A general characteristic of informal production in these models is that it is normally equivalent to informal labor; that is, the existence of mixed firms or the legal status of informal firms is not essential.

This literature has generally focused on reproducing two stylized facts: greater consumption than output volatility and a countercyclical informal employment. Most of the authors comply with the former by allowing GDP to measure informal economy poorly, a hypothesis put forward by Restrepo-Echavarria (2014). However, more recent papers (Horvath, 2018; Leyva & Urrutia, 2020) use a novel mechanism that reproduces this behavior through the effect of interest rate shocks on the present value of job relations to firms. This structure also generates a countercyclical foreign interest rate which is another stylized fact of developing countries.

There is no consensus on how to reproduce the latter stylized fact, a countercyclical informality rate. Mainly on which type of shock or rigidity is appropriate to generate this behavior. A first approach is that of Horvath (2018) and, Leyva and Urrutia (2020) who propose that interest rate shocks are the primary driver; however, the mechanism through which the interest rate shock generates the countercyclical informality is quite different. Horvath (2018) assumes that formal firms have to borrow a fraction of their labor costs, so an increase in the interest rate naturally reduces formal labor demand; while in Leyva and Urrutia (2020) an increase in the interest rate decreases the present value of vacancies for formal firms given that they assume that labor relations are less flexible and have a longer duration. An important difference between the models in these papers is that Horvath (2018) assumes a competitive labor market, while Leyva and Urrutia (2020), who use search and matching frictions, do not.

A second approach is to use formal productivity shocks along with an incomplete pass-through to the informal sector to generate this behavior as in Fernández and Meza (2015). The incomplete

pass-through of the productivity shock affects the productivity ratio between the sectors reallocating resources to the sector that has become relatively more productive. A caveat of this approach is that it is difficult to contrast with data as we do not have reliable observations of informal productivity. A final approach relies on the structure of the labor market to reproduce this behavior which is the case of papers by De la O and McKnight (2019) and Finkelstein (2017). In both papers, the authors assume that formal labor is subject to greater rigidities (Search & Matching) than the informal sector. In De la O and McKnight (2019) there are no unemployed or out of the labor force states, so procyclical formal employment automatically generates a countercyclical informality rate. In contrast, there is unemployment in Finkelstein (2017), but given that the informal sector is less rigid it is possible to get the desired behavior.

It has become standard in the recent literature to include search and matching frictions in the labor market. This is the case for the papers included in the last approach, but also in the paper of Leyva and Urrutia (2020). However, its implementation is quite different among these three articles, which reflect mainly on the behavior of unemployment. In De la O and McKnight (2019) it is assumed that the counterpart of formal employment is informality and, therefore, there is no unemployment. In Leyva and Urrutia (2020) there is a fourth state as out-of-the-labor-force, and there are search costs so unemployment is procyclical, which is at odds with the data. Lastly, in Finkelstein (2017) there are rigidities in both sectors, and unemployment behaves countercyclically. It is worth noting that all of these implementations have drawbacks, either because they generate a cyclical behavior that contradicts the data or because they ignore an aspect or stock that is key in the labor market data.

Finally, there is another strand of research that has focused on the long-term implications of informality. There are some important differences between DSGE literature and the long-term literature which can be summarized by three aspects: stylized facts modeled, agent heterogeneity, and variables of interest. In the first aspect, Levy (2018) portrays a much richer image of informality using Mexico's economic census. This empirical evidence suggests there is a trench between informal labor and informal firms; that is, there are formal firms that hire informal labor and vice versa. In most of the DSGE literature informal employment can be hired only by informal firms. Second, agent-based literature (Bobba et al., 2019; Haanwinckel and Soares, 2016; Ulyssea, 2010) suggests that agent heterogeneity (mainly in human capital) can have significant

consequences on the economy's firm composition and long-run behavior. The distinction in agent heterogeneity has permitted this literature to analyze differences in cross-country informality evolution. Third, agent-based literature has focused on variables that generate distortions between formal and informal labor (mainly taxation) to explain the existence of informality, while DSGE literature usually assumes its existence and size.

Chapter 3. Stylized Facts

3.1 Business Cycles

Table 1. Business Cycle Dating for Canada, United States and Mexico (1993- 2019)

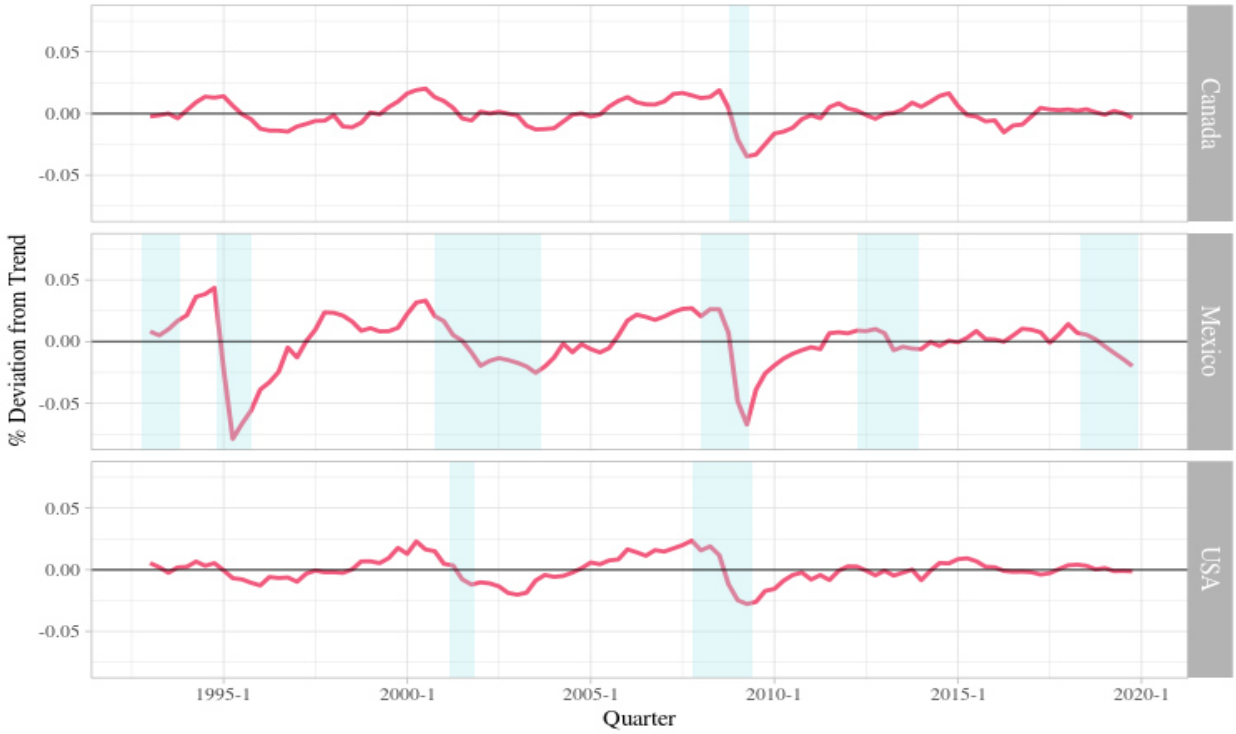
Country	Peak	Trough	Duration	Average
Canada	October 2008	May 2009	7	
				7 months
United States	March 2001	November 2001	8	
	December 2007	June 2009	18	
				13 months
Mexico	November 1994	October 1995	12	
	October 2000	September 2003	36	
	January 2008	May 2009	17	
	April 2012	December	20	
				21 months

Source: C.D. Howe Institute (Canada), NBER (United States), INEGI (Mexico).

National accounts data reveals that there are at least three critical differences between the behavior of business cycles in Mexico (and other developing economies) compared with their developed counterparts Aguiar and Gopinath (2004). First, the volatility of output in developing countries is on average double that of developed economies. For the 1993q1-2019q4 period, the volatility of

output [1] for Mexico is 2.16 and for Canada and the US is 1.04 and 1.02 respectively. This stylized fact is visually presented in Figure 1., where it can be seen that output's peaks and troughs are more extreme for Mexico than for the other two countries. Second, consumption is more volatile than output at business cycle frequencies. The main proposal to reconcile this fact with macroeconomic theory is that the measurement of informal output is quite imperfect and does not reflect the actual volatility of total output as in Restrepo-Echavarria (2014). This mismeasurement hypothesis may come at odds with current statements and procedures of national statistics agencies (INEGI, 2019, for example), however, both perspectives can be reconciled if it is noted that protocols to better include informal economy estimations are relatively recent and, hence, these changes are not reflected in longer time-series.

Figure 1: GDP Cyclical Component for Canada, Mexico and USA (1993-2019)



Note: Elaborated with HP-filtered deseasonalized constant-price time series from FRED (2020) and INEGI (2020); business cycle dates were taken from C.D. Howe Institute (Canada), NBER (USA), and INEGI (Mexico).

[1] Measured as the standard deviation of deseasonalized and HP-filtered (using a smoothing parameter of 1600) log-output.

There appears to be stark differences between the average duration of the business cycle and especially contractions between Mexico and developed countries like Canada and the U.S. As can be seen in Figure 1, business cycles in Mexico are shorter than those of both Canada and the United States; however, contractions appear to be longer. A caveat of this comparison is that in the selected time frame, as shown in Table 1., the three countries only share one contraction -that of the Great Recession- and in that case, the U.S. and Mexico had a much longer contraction than Canada. The former share one more recession at the beginning of the century; in this case, Mexico's contraction is more than four times as long as that of the U.S.. These facts suggest that Mexico spends more time in economic contraction than a comparable small open economy (Canada) and its major trade partner (U.S.).

3.2 Employment Aggregates

The main difference between Mexican employment dynamics (and that of other developing countries) and developed economies is the presence of a large informal share of total employment and its distinctive countercyclical behavior. These facts were mainly established by Fernández & Meza (2015) for five different measures of informality and have been confirmed by the literature reviewed in this text. Compared with its formal counterpart, informal employment is generally more volatile. In Table 2, I present business cycle statistics for the two informality measures most relevant to this text, self-employment, and employment in small firms (five or fewer employees). The two time-series were logged, seasonally adjusted, and HP-filtered with a smoothing parameter of 1600. The results are consistent with the literature in that the corresponding informal measure is more volatile than its formal counterpart (second column) and that formal [informal] employment is procyclical [countercyclical] (third column). Altogether, there appears to be more persistence in formal employment. Also, in Figure 2, I present a time series of the cyclical component of formal and informal measures. Leyva and Urrutia (2020) document that informal employment falls at the onset of recessions and recovers quickly afterwards. Unemployment in Mexico is low (on average 4.2% of total employment) and countercyclical (INEGI, 2020). The figure illustrates this behavior described for informality during downturns, its greater volatility, and its cyclicality.

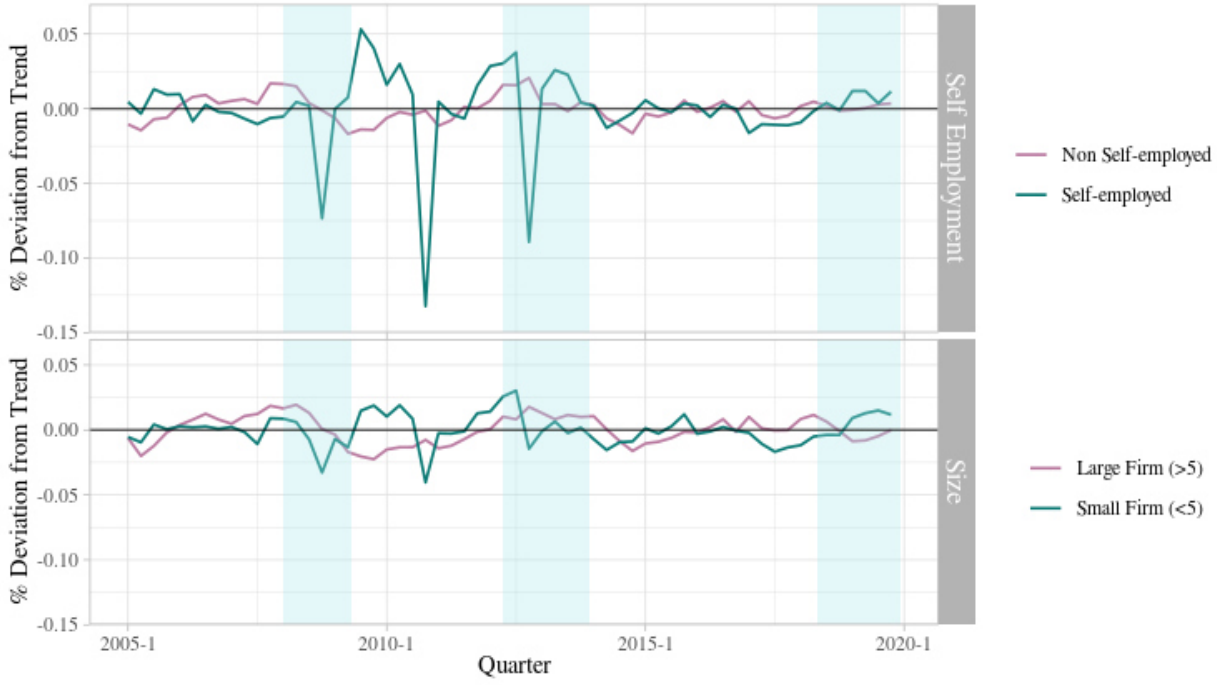
Recently, an analysis of gross employment flows by Leyva & Urrutia (2020) has challenged the view of informality as a buffer for formal workers during recessions and proposed instead that the

most crucial factor in explaining the observed behavior of formal employment is job creation from inactivity along with job reallocation to informality and job creation from unemployment. While the participation margin is important to explain employment dynamics, in the model developed in this thesis there is no endogenous participation as this would further complicate the model with minor benefits to the objectives of this work.

Table 2. Statistics for employment measures with respect to self-employment and firm size (2005-2019)

Variable	sd(·)	sd(·)/sd(Y)	corr(·, Y)	corr(·_{t}, ·_{t-1})
Non-Self Employment	0.84	0.49	0.66	0.71
Self-Employment	2.71	1.59	-0.21	0.04
Large Firm (>5)	1.07	0.63	0.69	0.83
Small Firm (<5)	1.22	0.72	-0.02	0.37

Figure 2: Cyclical Component of Employment Measures (2005-2019)



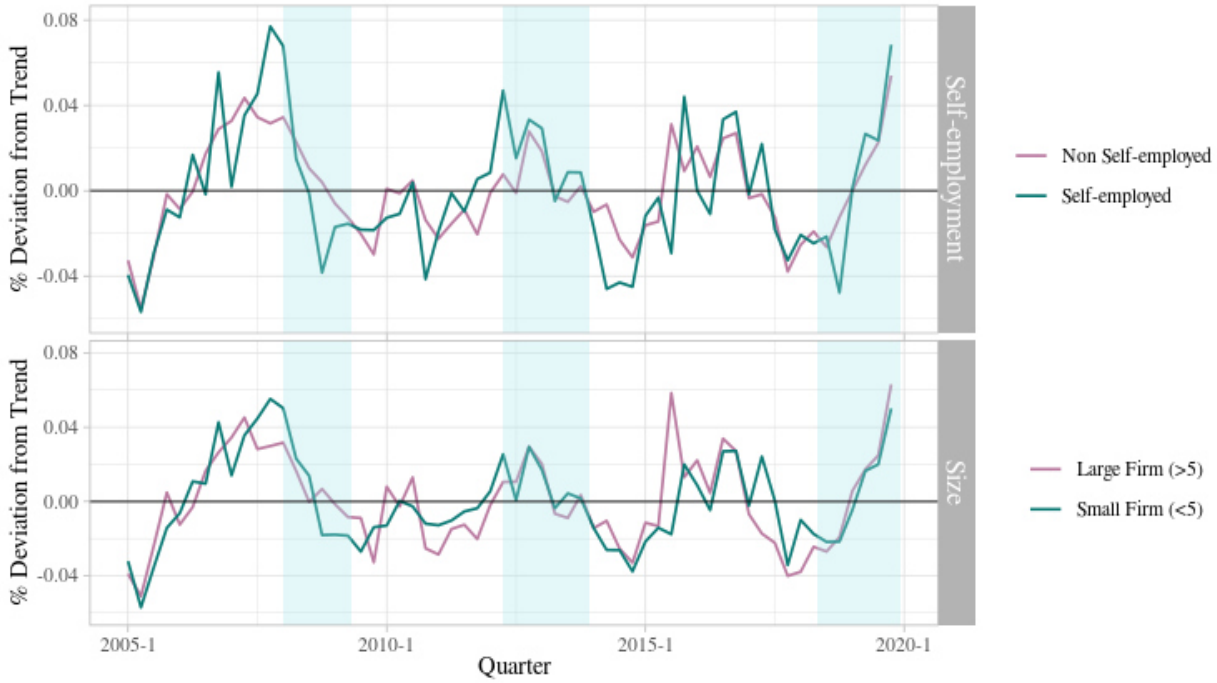
Note: Elaborated with HP-filtered deseasonalized time series constructed from ENOE. Business cycle dates were taken from INEGI

3.3 Formal and Informal Wages

Table 3. Statistics for real wages with respect to self-employment and firm size (2005-2019)

Variable	sd(·)	sd(·)/sd(Y)	corr(·, Y)	corr(·_{t}, ·_{t-1})
Non-Self Employment	2.23	1.31	0.40	0.69
Self-Employment	3.09	1.82	0.37	0.53
Large Firm (>5)	2.47	1.45	0.29	0.60
Small Firm (<5)	2.51	1.48	0.48	0.68

Figure 3: Cyclical Component of Real Wages (2005-2019)



Note: Elaborated with hourly mean real compensation time series constructed from ENOE. All series were transformed to constant prices, logged and HP-filtered. Business cycle dating was taken from ENOE.

In general, Li (2011) finds that real wages [2] in developing economies are more volatile than output and procyclical; in contrast, for developed economies, wages are usually less volatile than output and acyclical. For Mexico, the ratio of wage to output volatilities is 1.91, and the unconditional

[2] In the paper by Li (2011) two different measures of hourly wage/earnings are used: wages across all economic activity and manufacturing wages. For Mexico, they use the latter.

correlation between real wages and output is .41 (Li, 2011, p. 692). These results are broadly consistent with those of Boz et al. (2015), who present a ratio of 2.22 and a correlation of .56.

In Table 3, I present business cycle statistics for self-employed workers, small firm employees [3] and their formal counterparts. The two time-series were logged, seasonally adjusted, and HP-filtered with a smoothing parameter of 1600. Formal and informal real wages are procyclical and more volatile than output. As in the case of employment, informal real wages are more volatile than formal real wages.

Regarding wage rigidity, Castellanos (2014) finds evidence for downward nominal wage rigidity in Mexican labor markets, which is stronger for formal than informal workers and similar to that in the United States for the former. In a more recent study, Juarez and Casarín (2016) find evidence of both nominal and real wage rigidities for Mexico. Their findings suggest that a greater fraction of workers was subject to real rather than nominal rigidities during the observed period (1996-2011); however, their results indicate that nominal rigidities were more prevalent during the Great Recession, probably due to an environment of low and stable inflation. My results are consistent with those in the literature in that real wages are more volatile than output and procyclical. My contribution to these facts is that informal real wages are more volatile than formal which supports the case for less wage rigidity for informal workers. Also, the correlation with output is high for informal measurements and higher for those who work in small firms with respect to their counterparts.

In Figure 3, I present the behavior of the cyclical component of real wages for these informality measurements across the 60 quarters from January 2005 to December 2019. Real wages are procyclical for informality measurements, and informal wages are slightly more volatile than formal ones. However, the behavior of real wages during the last contraction appears to be different. This behavior could be due to the significant hikes on the minimum wage, which many workers use as a reference (see Campos et al. (2015)), however further research would be needed, and this behavior may be subject to the continuation of the contraction further into 2020 and business cycle dating by INEGI.

[3] In Appendix A, I describe the process to get these results using ENOE survey data.

3.4 The Nature of Firms

The most comprehensive study of the nature of formal and informal firms was made by Levy (2018) using Mexico's economic censuses. The main advantage of this evidence is that most of the analysis is made at the establishment level; therefore, there is a more detailed description of firms. I will consider three aspects of the nature of firms: their size (number of employees), their share of the capital stock, and their productivity.

More than 90% of establishments in Mexico have five or fewer employees (Levy, 2018, p. 85), whether formal or informal. These small establishments absorb most of the workforce: each having on average 2.3 workers, representing around 40% of total employment. It is worth noting that probably this share is greater than census data shows, as most of the firms excluded are small non-established firms. In sum, most employment in Mexico occurs in firms with five or fewer employees (Levy, 2018, p. 87).

Regarding capital, the share between the formal and informal sectors is not as large as most of the DSGE literature assumes. In the 2013 economics census, informal establishments accounted for 43% of the capital stock; however, because of their greater number (more than 80% of all establishments are informal), we know that individually informal firms have a low capital stock. This is true for all small establishments but not for large ones. While large establishments represent a minimal share (0.27%) of the total (.75%), they account for over 60% of the capital stock. Especially, large, and fully formal firms appear to be more capital intensive than all others as they account for around 20% of the capital stock and 40% when considering a broader definition of formality. In summary, small formal and informal firms, where most employment occurs, are not capital intensive, and the general notion of formal firms being much more capital intensive than informal ones is a consequence of big formal firms being very capital intensive (Levy, 2018, pp. 87–89).

Regarding productivity, the scenario is very similar to that of capital. When controlling by size, formal firms have higher productivity than informal firms. The most productive firms are large formal ones, 80% more productive than small informal ones. However, in this respect, Levy (Levy, 2018) suggests that the critical margin is neither size nor formality but whether contracts are salaried. This is due to the structure of taxation and contributions, which is the principal mechanism this author studied. Nonetheless, given that non-salaried contracts are more prevalent

in informal firms than formal ones, regardless of size, it is a sufficient approximation to generally associate the characteristics of non-salaried contracts with informality.

The definition of informality that I will use to build the model will be that of the small firm. I have chosen to use this definition over the self-employment one because I think it is closer to the concept of a shadow economy parallel to the formal economy. That is, throughout the literature it is assumed that formal and informal firms are similar, and their output goods are substitutes, however, I think it is difficult to argue that the goods produced by self-employed workers are substitutes to that of firms with multiple employees, rather, it is probably more realistic to think of informal firms as smaller and less productive versions of the formal firms. The small firm definition also relates better to the findings in Leal (2014) and Levy (2018) where they argue that there is an institutional or cost barrier that prevents small informal firms from becoming large formal firms. This model choice has its drawbacks. First, the comparison between this model and others in the literature is less direct as different second moments are used to evaluate the model. Nonetheless, most of the second moments share their sign across the two definitions and allow a rough comparison between models. Second, it would probably be easier to match some second moments of self-employment, particularly its countercyclicality in contrast with the very low correlation of small firm employment, but this would come at the cost of it being more difficult to match other second moments, e.g. the much lower relative volatility of non-self-employment to self-employment, under the framework that will be proposed in the next chapter. Therefore, I think it is preferable to use the small firm definition and I will evaluate the model on its ability to match the second moments of this definition.

Chapter 4. Model

The economy in this model comprises a large number of infinitely lived households, formal and informal firms, and fiscal and monetary authorities. Households demand formal and informal consumption goods and provide an inelastic labor supply equal to one. Formal and informal firms have access to labor-intensive technology with constant returns to scale. The differences among firms in each sector are that formal firms have greater factor productivity, have monopoly power (and thus set prices), and face nominal wage rigidities. Both types of firms face search & matching

rigidities in the labor market, which allows unemployment to appear. Firms and workers negotiate wages, and it is assumed that this is resolved by Nash bargaining. The fiscal authority levies payroll taxes on formal labor, which it transfers to the unemployed as a kind of social insurance. Finally, the monetary authority sets a nominal interest rate following a Taylor rule.

4.1 Labor Dynamics

Individuals in this economy may be in one of three employment states: unemployed (U_t), formally employed (L_t^f) or informally employed (L_t^i). For simplicity, I normalize the total labor force to one, hence

$$U_t + L_t^f + L_t^i = 1 \qquad (4.1\text{–}1)$$

Following Ulyssea (2010) and Finkelstein (2017) flows among these states are governed by two sectorial matching functions:

$$\mathfrak{M}\left(V_t^k, U_{t-1}\right) = {V_t^k}^{1-\psi} U_{t-1}^{\psi} \qquad k = f, i \qquad (4.1\text{–}2)$$

where V_t^k are the vacancies posted by firms in sector k at the beginning of period t, U_t are the individuals who are searching for a job at the beginning of period t , and $\psi \in (0,1)$ is the matching elasticity.

It follows from 4.1–3 that sector k vacancy filling rate is given by $\lambda_t^k = \frac{\mathfrak{M}_t}{V_t^k} = {\theta_t^k}^{-\psi}$, where $\theta_t^k = \frac{V_t^k}{U_{t-1}}$ is a natural measure for each labor market tightness in both the formal and informal sectors. I will assume that there are job separations that happen with an exogenous rate $s^k \in (0,1)$. Accordingly, the laws of motion for formal and informal employment are given by:

$$L_t^k = (1 - s^k) L_{t-1}^k + \lambda_t^k V_t^k \qquad k = f, i \qquad (4.1\text{–}4)$$

Following Pissarides (2000), both formal and informal firms must pay a cost to post vacancies, but may not find a match instantaneously. However, I will assume that the vacancy cost is greater for

formal than informal firms, $\kappa^f > \kappa^i$. This is meant to model the relatively greater institutional barriers that formal firms face when hiring new workers.

4.2 Households

Households derive utility only from the consumption of formal and informal goods. Households do not derive utility from leisure because I will assume that they have an inelastic labor supply. Their preferences are described by the expected discounted lifetime utility:

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} \right] \qquad (4.2\text{–}1)$$

where $\mathbb{E}_t$ is the $\beta^t \in (0,1)$ is the discount factor, and $\frac{1}{\sigma}$ is the intertemporal elasticity of substitution.

The period budget constraint of the representative household is given by:

$$P_t C_t + Q_{t|t+1} B_t \leq B_{t-1} + W_t^f L_t^f + W_t^i L_t^i + b_t U_t + D_t \qquad (4.2\text{–}2)$$

where $P_t C_t$ is the aggregate expenditure on consumption goods, $W_t^f L_t^f$ is the nominal income from formal labor, $W_t^i L_t^i$ is the nominal income from informal labor, $b_t U_t$ are the benefits given by the government to the unemployed, D_t are the dividends from firms ownership, B_t are the holdings of one-period bonds and $Q_{t,t+1}$ is the price of the bonds at period t.

Household optimization of this problem yields the standard consumption Euler equation:

$$C_t^{-\sigma} = \frac{\beta}{Q_{t|t+1}} \mathbb{E}_t \left\{ C_{t+1}^{-\sigma} \frac{P_t}{P_{t+1}} \right\} \qquad (4.2\text{–}3)$$

The index C_t is an aggregate of formal and informal consumption goods according to the CES function:

$$C_t = \left[\omega^{\frac{1}{\varepsilon_c}}\left(c_t^f\right)^{\frac{\varepsilon_c-1}{\varepsilon_c}} + (1-\omega)^{\frac{1}{\varepsilon_c}}\left(c_t^i\right)^{\frac{\varepsilon_c-1}{\varepsilon_c}}\right]^{\frac{\varepsilon_c}{\varepsilon_c-1}} \quad (4.2\text{–}4)$$

where $\omega \in (0,1)$ represents the relative weight of the formal consumption goods on total consumption and $\varepsilon_c > 1$ is the elasticity of substitution between formal and informal goods.

Given this index, the household must allocate optimally its consumption expenditure among formal and informal goods ($P_t^f c_t^f + P_t^i c_t^i$). The optimality conditions of this maximization problem imply that:

$$c_t^f = \omega\left(\frac{P_t^f}{P_t}\right)^{-\varepsilon_c} C_t \quad (4.2\text{–}5)$$

$$c_t^i = (1-\omega)\left(\frac{P_t^i}{P_t}\right)^{-\varepsilon_c} C_t \quad (4.2\text{–}6)$$

$$P_t = \left[\omega {P_t^f}^{1-\varepsilon_c} + (1-\omega){P_t^i}^{1-\varepsilon_c}\right]^{\frac{1}{1-\varepsilon_c}} \quad (4.2\text{–}7)$$

In the same manner, households must optimize their consumption among the different formal goods j, which yields a similar condition:

$$c_t^f(j) = \left(\frac{P_t^f(j)}{P_t^f}\right)^{-\varepsilon_f} c_t^f \quad (4.2\text{–}8)$$

An analogous condition is not needed for informal firms because, as the informal goods market is assumed to be competitive, their goods are homogeneous, and aggregation is straightforward.

4.3 Formal Firms

Formal firms are assumed to have monopoly power, access to the same labor-intensive technology, pay payroll taxes, face price rigidities and encounter search and matching frictions in the labor

market. I will assume there is a continuum of formal firms in the interval $[0,1]$ indexed by j. The formal production function has constant returns to scale:

$$Y_t^f(j) = A_t^f L_t^f(j) \qquad (4.3\text{–}1)$$

where $Y_t^f(j)$ is the output of formal firm j, A_t^f is the technology common to all formal firms and $L_t^f(j)$ is the labor input in formal firm j's production. As is standard in the literature, I will model price rigidities à la Calvo: there is a constant and independent probability $\theta_p \in [0,1]$ that a firm will be unable to update its prices. This implies that the formal price can be described by:

$$P_t^f = \left[\theta_p {P_{t-1}^f}^{(1-\epsilon_p)} + (1-\theta_p){P_t^{f*}}^{(1-\epsilon_p)}\right]^{\frac{1}{1-\epsilon_p}} \qquad (4.3\text{–}2)$$

It is optimal for formal firms to minimize their costs which are constrained by the formal production function and labor mobility conditions, then formal firms solve:

$$\begin{aligned} \min_{\{L_t^f(j), V_t^f(j)\}_{m=0}^{\infty}} \quad & (1+\tau)W_t^f + P_t \kappa^f V_t^f \\ s.t. \quad & Y_t^f(j) = A_t^f L_t^f(j) \\ & L_t^f = (1-s^f)L_{t-1}^f + \lambda_t^f V_t^f \end{aligned}$$

which yields the following expression for formal firm j's marginal cost:

$$MC_t(j) = \frac{1}{A_t^f}\left(W_t^f(1+\tau) + \frac{P_t \kappa^f}{\lambda_t^f}\right) \qquad (4.3\text{–}3)$$

Given the market power and nominal price rigidities assumptions over formal firms, they can set prices but must consider that they may be unable to change prices for an uncertain number of periods. Therefore, the problem that a firm j that chooses its price P_t^{f*} at t is the following:

$$\max_{\{P^{f*}_{t|t+m}(j)\}^{\infty}_{m=0}} \quad \mathbb{E}_t \sum_{m=0}^{\infty} \theta_p^m \, Q_{t|t+m}[P_t^{f*}(j) Y^f_{t+m}(j) - MC_{t+m}]$$

$$s.t.$$

$$Y^f_{t+m}(j) = \left(\frac{P^f_{t+m}(j)}{P^f_{t+m}}\right)^{-\epsilon_f} Y^f_{t+m}$$

where $MC_t(j)$ is the nominal marginal cost of formal firm j and is derived from the cost minimization of the firm subject to the formal production function and labor mobility conditions.

From the first order condition of this problem an equation for the formal price is derived:

$$P_t^{f*} = \frac{\epsilon_f}{\epsilon_f - 1} \mathbb{E}_t \frac{\sum_{m=0}^{\infty} \theta_p^m \, Q_{t|t+m} P^{\epsilon_f}_{t+m} Y^f_{t+m} MC_{t+m}}{\sum_{m=0}^{\infty} \theta_p^m \, Q_{t|t+m} P^{\epsilon_f}_{t+m} Y^f_{t+m}} \qquad (4.3\text{–}4)$$

where $\frac{\epsilon_f}{\epsilon_f - 1} > 1$ is the desired markup of formal firms in the absence of nominal price rigidities. The intuition behind equation (4.3–5) is that formal firms will choose a price that corresponds to the desired markup over a weighted average of their nominal marginal costs in present and future periods.

4.4 Informal Firms

Informal firms are competitive, produce homogeneous goods, and have access to the same labor intensive production technology:

$$Y_t^i(n) = A_t^i L_t^i(n) \qquad (4.4\text{–}1)$$

where A_t^i is the labor productivity common to all informal firms. The informal firm chooses contingency plans for vacancies posted. Then, informal firms choose such quantities as to solve:

$$\max_{\{V^i_{t+m}\}^{\infty}_{m=0}} \quad \mathbb{E}_t \sum_{m=0}^{\infty} Q_{t,t+m}[P^i_{t+m} Y^i_{t+m} - W^i_{t+m} L^i_{t+m}(n) - P_t \kappa^i V^i_{t+m}(n)]$$

$$s.t.$$

$$Y^i_{t+m}(k) = A^i_{t+m} L^i_{t+m}(k)$$

$$L^i_{t+m} = \left(1 - s^i\right) L^i_{t-1+m} + \lambda^i_{t+m} V^i_{t+m}$$

The first order condition of this problem yields a standard vacancy posting condition:

$$\left(A_{t+m}^{i}P_{t+m}^{i} - W_{t+m}^{i}\right)\lambda_{t+m}^{i} = P_t\kappa^{i} \qquad (4.4\text{–}2)$$

4.5 Wage Dynamics

As is standard in the literature that uses search and matching frictions, I will assume that wages are determined through Nash bargaining in both sectors. In this framework, the total surplus from a match is split between the worker and the firm following an effective bargaining power parameter ($\chi^k, k = f, i$). The surplus of a match for a worker at period t ($V_t^k, k = f, i$) depends on the match's value (with either type of firm) minus the value of unemployment. The value of a match depends on current wages plus the expectation of future benefits of remaining employed or the value of being unemployed. Therefore, the value of the match for formal and informal workers takes the following form:

$$\Xi_t^f = W_t^f + \mathbb{E}_t Q_t\left\{(1 - s^f)\Xi_{t+1}^f + s^f\Xi_{t+1}^U\right\} \qquad (4.5\text{–}1)$$

$$\Xi_t^i = W_t^i + \mathbb{E}_t Q_t\left\{\left(1 - s^i\right)\Xi_{t+1}^f + s^i\Xi_{t+1}^U\right\} \qquad (4.5\text{–}2)$$

The value of unemployment for a worker at t (Ξ_t^U) depends on the current benefits given by the government and the expected benefits of getting a match at period $t + 1$ with either a formal or informal firm or remaining unemployed:

$$\Xi_t^U = b_t + \mathbb{E}_t Q_{t|t+1}\left\{(1 - s^f)\lambda_t^f\Xi_{t+1}^f + \left(1 - s^i\right)\lambda_t^i\Xi_{t+1}^i + \left[1 - (1 - s^f)\lambda_t^f - \left(1 - s^i\right)\lambda_t^i\right]\Xi_{t+1}^U\right\} \qquad (4.5\text{–}3)$$

The surplus of a match for a firm at t ($Y_t^k\ k = i, f$) is composed of the additional net revenue that the match produces plus future benefits of maintaining that match. Then, the value of the match for formal and informal firms takes the form:

$$\Upsilon_t^f = P_t^f A_t^f - W_t^{f*}(1+\tau) + \mathbb{E}_t Q_t\{(1-s^f)\Upsilon_{t+1}^f\} \quad (4.5\text{–}4)$$

$$\Upsilon_t^i = P_t^i A_t^i - W_t^i + \mathbb{E}_t Q_t\{(1-s^i)\Upsilon_{t+1}^i\} \quad (4.5\text{–}5)$$

Consequently, firms and workers who bargain over the real wages solve the following problems:

$$W_t^{f*} \equiv \arg\max\left\{\left(\Xi_t^f - \Xi_t^U\right)^{\chi^f}\left(\Upsilon_t^f\right)^{1-\chi^f}\right\}$$
$$W_t^i \equiv \arg\max\left\{\left(\Xi_t^i - \Xi_t^U\right)^{\chi^i}\left(\Upsilon_t^i\right)^{1-\chi^i}\right\}$$

The solution to these problems yields the two following conditions for the nominal wages of both sectors:

$$W_t^{f*} = \chi^f\left[P_t^f A_t^f + P_t \kappa^f \theta_t^f\right] + (1-\chi^f)\left[\frac{\chi^i}{1-\chi^i} P_t \kappa^i \theta_t^i\right] \quad (4.5\text{–}6)$$

$$W_t^i = \chi^i\left[P_t^i A^i + P_t \kappa^i \theta_t^i\right] + \left(1-\chi^i\right)\left[\frac{\chi^f}{1-\chi^f} P_t \kappa^f \theta_t^f\right] \quad (4.5\text{–}7)$$

Following Hall (2005), I will assume that formal wages are sticky and follow a backwards-looking process. I will model this rigidity à la Calvo [4] where only a fraction $\theta_w \in [0,1]$ of matches can update their wages to the optimal W_t^{f*}. This is expressed by the following equation defining the average formal wage:

$$W_t^f = \left[\theta_w {W_{t-1}^f}^{1-\epsilon_w} + (1-\theta_w){W_t^{f*}}^{1-\epsilon_w}\right]^{\frac{1}{1-\epsilon_w}} \quad (4.5\text{–}8)$$

[4] While there are other microfounded methods of embedding wage rigidity into the wage negotiations, for example, Bodart et al. (2006), they yield the same log-linearized expression. For the sake of simplicity, I have decided to use this more straightforward method.

4.6 Monetary Policy Rule

Finally, I will assume that the monetary authority in this economy sets the interest rate reacting to price inflation, and the deviations of output from its steady state.

$$i_t = \left(\widehat{\Pi_t^P}\right)^{\phi_P} \left(\widehat{Y_t}\right)^{\phi_Y} \qquad (4.6\text{–}1)$$

where, i_t is the nominal interest rate, $\widehat{\Pi_t^P}$ is deviations of inflation from its steady state level, $\widehat{Y_t}$ is deviations of output from its steady state level, and ϕ_k for $k \in \{P, Y\}$ is the strength with which the monetary authority responds to fluctuations in the respective variables. I will assume that the official statistics capture only formal output and so the interest rate reacts only to developments in that sector.

4.7 Market Clearing and Government's Budget Constraint

The goods market clearing requires that each sector's output equals the consumption goods produced and the total cost of vacancies posted:

$$Y_t^f = C_t^f + \kappa^f V_t^f \qquad (4.7\text{–}1)$$

$$Y_t^i = C_t^i + \kappa^i V_t^i \qquad (4.7\text{–}2)$$

$$\mathrm{Y_t} = Y_t^f + Y_t^i \qquad (4.7\text{–}3)$$

I will assume that the government balances its budget every period, therefore, the benefits given to the unemployed must equal the payroll taxes levied:

$$b_t U_t = \tau W_t^f L_t^f \qquad (4.7\text{–}4)$$

4.8 Productivity Processes

Finally, I will assume that the level of technology for each sector evolves according to the AR(1) process:

$$A_t^k = \rho_a A_{t-1}^k + \epsilon_a \qquad k = f, i \qquad (4.8\text{–}1)$$

where $\rho_a \in [0,1]$ is interpreted as the persistence in the productivity process and ϵ_a is and i.i.d shock distributed $(0, \sigma_a^2)$ with $\sigma_a^2 > 0$. In the baseline model, I will assume that ϵ_a is common to both sectors.

4.9 Equilibrium

A rational-expectations equilibrium for this economy consists of an initial set of condition $\{B_0, L_0^f, L_0^i\}$, an exogenous process for the productivity shock a_t, a sequence of prices $\{P_t, P_t^i, P_t^f, W_t^i, W_t^f, i_t\}$, and a sequence of allocations $\{U_t, L_t^i, L_t^f, C_t, C_t^i, C_t^f, Y_t, Y_t^i, Y_t^f, \theta_t^f, \theta_t^i, V_t^i, V_t^f, b_t\}$ for all $t \geq$ such that the following hold: the optimality conditions for the household (4.2-4, 4.2-5, 4.2-6, 4.2-7) and the transversality condition, the optimality conditions for the formal and informal firms (4.3-4, 4.4-2), the conditions for the labor markets (4.1-3, 4.1-2 for f and i), the monetary policy rule (4.6-1), the equations for the wages (4.5-6, 4.5-7), the government budget constraint (4.7-4), the production functions (4.3-1, 4.4-1), and the conditions for labor (4.1-1) and goods markets clearing (4.7-1, 4.7-2, 4.7-3).

Chapter 5. Model Solution and Calibration

To solve the model, I use log-linearization of the model around the deterministic zero-inflation steady state. The steady-state equations are presented in Appendix B, and a brief description of the log-linearization of the model and the full log-linearized model are presented in Appendix C. Table 4 summarizes the values used for calibration in three groups depending on their origin or target. As is standard in the literature, I used a risk aversion coefficient (σ) of 1 and a discount factor (β) of .99. Following Pissarides (2000) and as is usually done in the literature, the elasticity of the matching function (ψ) is set to a value of .5. For the formal (s_f) and informal (s_i) separation rates,

I have followed the empirical values from Bosch and Esteban-Pretel (2012). For the payroll tax (τ), I use the effective tax on labor calculated by the OECD (2019). For the empirical estimation of the productivity ratio, I use the empirical estimation obtained in Busso et al. (2013). For the elasticity of substitution of sector goods I follow the value used by Restrepo-Echavarria (2014) and Fernández and Meza, (2015). Finally, I follow Leyva and Urrutia (2020) who set the persistence of aggregate technology equal to the persistence of GDP, which is equal to .92.

The formal (χ_f) and informal (χ_i) bargaining powers, the bias for formal consumption (ω), and the elasticity of substitution of formal goods (ε_f) were jointly calibrated to match the following four stylized facts for Mexico: (1) a wage ratio of 1.23 ($\frac{W^f}{W^i}$, consistent with Esteban-Pretel and Kitao, 2021), (2) an unemployment rate of 4.2% (INEGI, 2020, average from 2005q1 to 2019q4), (3) a labor informality rate of 53% (INEGI, 2019), and (4) a sector output ratio ($\frac{Y^i}{Y^f}$) of .64 (INEGI, 2020). The calibration of these parameters implies that informal workers have more bargaining power than formal ones. This is reasonable under this scenario because informal firms are supposed to model small, family-based firms in which family members may have greater bargaining power than mostly non-unionized formal workers. Nevertheless, alternative models with different levels of bargaining power for formal workers is presented in the sensitivity analysis section. A second implication is that there is a bias towards formal goods, which aligns with the literature reviewed. Finally, it is also implied through the elasticity of substitution of formal goods that firms in that sector get a markup of roughly 10% which is similar to the other literature with this kind of production framework.

Finally, the formal (κ_f) and informal (κ_i) vacancy costs, the indices for the price (θ_p) and wage (θ_w) rigidities, and productivity shock (ϵ_A) are set to match the volatility of GDP. This calibration complies with the assumption that formal vacancy costs should be higher than informal ones. Taking these into account with separation rates complies with the stylized fact that, in general, the informal labor market is more flexible than the formal one.

Table 4. Parameters for the Baseline Model

	Symbol	Value	Source
From outside the model			
Risk Aversion coefficient	σ	1.00	Standard
Discount Factor	β	0.99	Standard
Elasticity of Matching Function	ψ	0.50	Pissarides (2000)
Formal Separation Rate	s_f	.032	Bosch and Maloney (2008)
Informal Separation Rate	s_i	.062	Bosch and Maloney (2008)
Payroll Tax	τ	0.12	OECD (2019)
Elasticity of Substitution of Sector Goods	ε_c	8.00	Fernandez & Meza (2015)
Productivity Ratio	$\frac{A_f}{A_i}$	2.19	Busso et al. (2013)
Persistence of aggregate productivity shock	ρ_a	0.92	Leyva and Urrutia (2020)
Calibrated to steady state			
Formal Bargaining Power	χ_f	0.20	
Informal Bargaining Power	χ_i	0.40	
Bias for Formal Consumption	ω	0.73	
Elasticity of Substitution of Formal Goods	ε_f	11.00	
Other Parameters			
Formal Vacancy Cost	κ_f	0.06	
Informal Vacancy Cost	κ_i	0.02	
Price Stickiness	θ_p	0.75	
Wage Stickiness	θ_w	0.60	
Productivity Shock	ϵ_A	1.00%	

Chapter 6. Results

Table 5 presents the model's second moments and compares them with those observed in the data. The model produces the appropriate cyclicality for all the variables of interest, that is, both wages are procyclical, informal employment and unemployment are countercyclical and, total and formal employment are procyclical. However, the model predicts a higher correlation than is present in the data for most variables except in the unemployment rate which is matched closely. The main discrepancy of the model with respect to the data is that informal employment is roughly acyclical (or slightly countercyclical), while the model suggests a strong counter-cyclicality of informality which mirrors the procyclicality of formal employment. For this variable, the model prediction is closer to the observed correlation for other informality definitions as can be seen in the stylized facts presented in Chapter 3.

Table 5. Baseline Model Results

	Data	Baseline	Flexible Wages	Flexible Prices
$\sigma(Y^f)$	1.73	1.73	1.35	1.80
$\sigma(C^f)/\sigma(Y^f)$	0.99	0.99	1.00	0.99
$\sigma(L)/\sigma(Y^f)$	0.42	0.05	0.00	0.05
$\sigma(U)/\sigma(Y^f)$	0.44	0.76	0.12	0.82
$\sigma(L^f)/\sigma(L^i)$	0.87	0.81	0.52	0.67
$\sigma(W^f)/\sigma(W^i)$	0.98	0.43	0.85	0.45
$Corr(Y^i, Y^f)$	-	0.92	1.00	0.90
$Corr(C^f, Y^f)$	0.93	1.00	1.00	1.00
$Corr(L, Y^f)$	0.76	0.34	-0.97	0.34
$Corr(L^f, Y^f)$	0.69	0.94	-0.60	0.97
$Corr(L^i, Y^f)$	-0.02	-0.89	0.28	-0.89
$Corr(U, Y^f)$	-0.43	-0.44	0.97	-0.34
$Corr(W^f, Y^f)$	0.29	0.91	1.00	0.91
$Corr(W^i, Y^f)$	0.48	0.93	0.99	0.94

In terms of the relative volatilities, the model performs relatively well for most variables. The relative volatilities of formal labor and formal wages to their informal counterparts display the

appropriate ratio as the informal variables are more volatile that the formal ones. The labor ratio is especially close to the data; however, the wages ratio is lower than in the data. Nonetheless, the wage volatilities ratio also implies an important feature. Given the greater volatility of informal wages with respect to formal ones in the model, the wage gap between formal and informal workers is countercyclical as seen in the empirical literature. There are two relative volatilities for which the model does not perform well: the relative volatility of labor to output is too low and the relative volatility of unemployment to output is too high. This result is probably a consequence of the assumption of inelastic labor supply of the model whereby the unemployment rate becomes a channel for adjustment, hence, it is more volatile.

The assumption of sticky wages -which is the main feature of the model- is key for the cyclicality of the unemployment rate and informality. The third column of Table 5 presents the model results with flexible wages. For that model specification informality and unemployment are procyclical, and formality and total labor are countercyclical. Therefore, the novel mechanism present in this model is that it is possible to generate a countercyclical informality rate through an asymmetry in the rigidity of wages. In response to a negative aggregate productivity shock, as formal firms are left with temporarily greater wages than they would have under flexible conditions, they face a greater marginal cost which ultimately, through formal prices, translates to lesser demand for formal goods relative to informal ones. This in turn implies a lesser demand for labor from formal firms which, given the greater relative demand for informal goods and the flexible informal wages, results in greater demand for informal labor. The intuition for these results can be seen in Figure 4 which presents the Impulse Response Functions for a negative productivity shock.

Figure 4: Impulse Response Functions to a Negative Productivity Shock

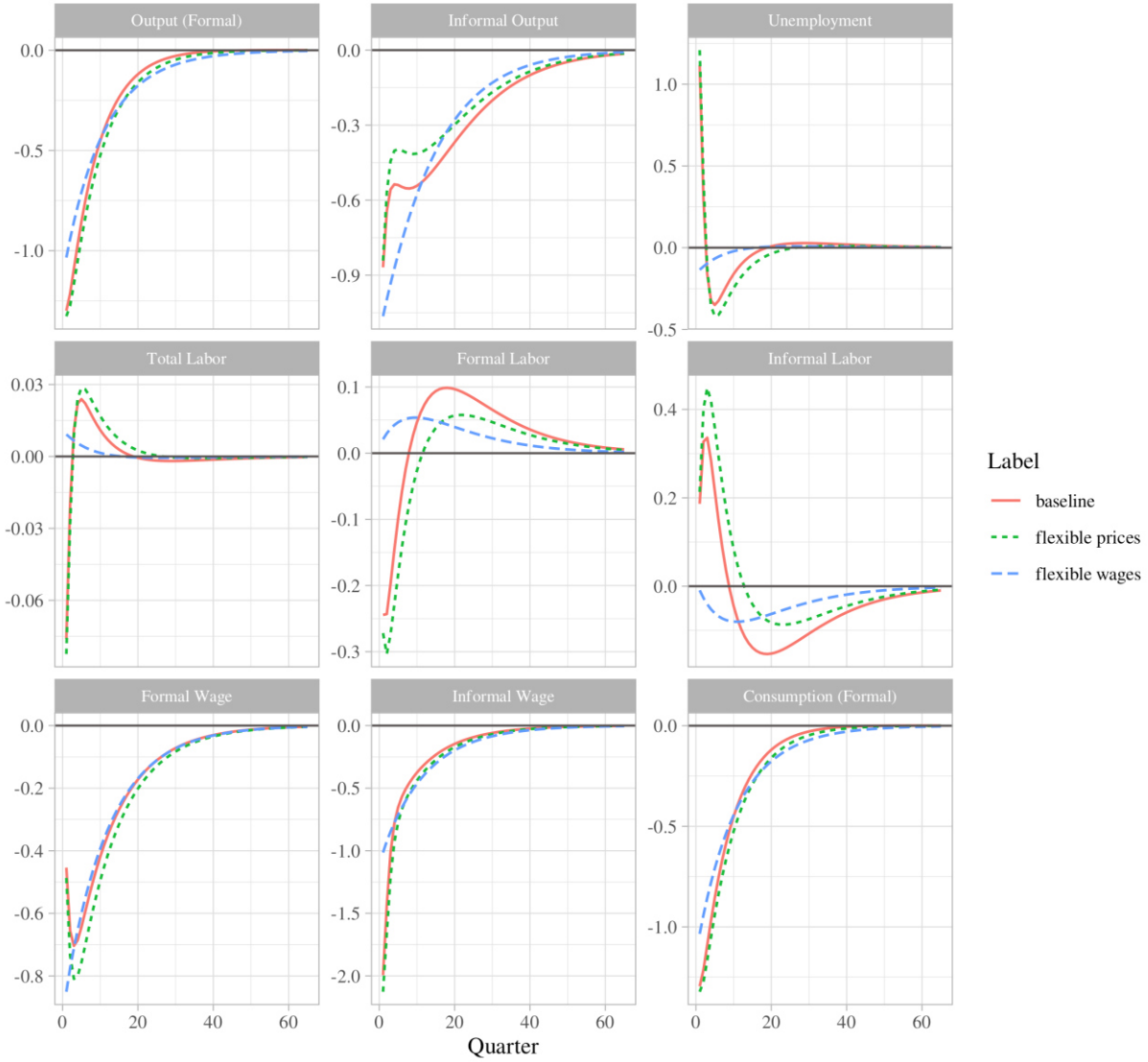


Figure 4: *Impulse Response Functions to a Negative Productivity Shock*

While the model includes sticky prices, this is not a necessary model assumption to obtain the main results. This is mainly a consequence of the absence of other production inputs (i.e., capital) besides labor in this New Keynesian framework. As the price is a markup over expected marginal costs, rigidities in wages imply rigidities in prices as well. This can be seen in the results of the model with flexible prices in the fourth column of Table 5. Most of the second moments of that version of the model are in the neighborhood of the baseline model, although with greater volatility. Therefore, it would be possible to build a version of the model with flexible prices without harming the performance of the model.

Chapter 7. Sensitivity Analysis

In this section I explore the robustness of the results to two assumptions over non-observable parameters -the pass-through of the productivity shock to the informal sector and the bargaining power of labor in the formal sector- and show that they are not important for the results.

The Level of Pass-Through of the Productivity Shock

As was discussed in the Literature Review, an asymmetry in the pass-through of a shock from the formal to the informal sector is one of the ways in which the countercyclicality of informality is reproduced in the literature, mainly in the models of Fernández and Meza (2015). In the baseline model I have assumed a common productivity shock between sectors. Table 6 compares the second moments of the model with alternative versions with incomplete and zero pass-through.

Table 6. Business Cycle Statistics: Alternative Pass-Through Levels

	Data	Baseline	No Pass Through	0.5 Pass Through
$\sigma(Y^f)$	1.73	1.73	1.84	1.78
$\sigma(C^f)/\sigma(Y^f)$	0.99	0.99	1.00	.99
$\sigma(L)/\sigma(Y^f)$	0.42	0.05	0.05	.05
$\sigma(U)/\sigma(Y^f)$	0.44	0.76	0.76	0.75
$\sigma(L^f)/\sigma(L^i)$	0.87	0.81	0.61	0.65
$\sigma(W^f)/\sigma(W^i)$	0.98	0.43	0.55	0.48
$Corr(Y^i, Y^f)$	-	0.92	-0.89	0.90
$Corr(C^f, Y^f)$	0.93	1.00	1.00	1.00
$Corr(L, Y^f)$	0.76	0.34	0.14	0.25
$Corr(L^f, Y^f)$	0.69	0.94	0.98	0.97
$Corr(L^i, Y^f)$	-0.02	-0.89	-0.89	-0.89
$Corr(U, Y^f)$	-0.43	-0.44	-0.14	-0.34
$Corr(W^f, Y^f)$	0.29	0.91	0.94	0.91
$Corr(W^i, Y^f)$	0.48	0.93	0.86	0.94

The level of pass-through affects mainly three second moments, the correlation of formal and informal outputs, the correlation of unemployment to formal output and correlation of total labor and formal output, however, it does not affect the countercyclicality of unemployment and

informal labor nor the procyclicality of wages which are the main aims of this model. In this sense, the main results of the model are robust to changes in the shock's pass-through.

Figure 5: Impulse Response Functions with Different Pass-through Levels

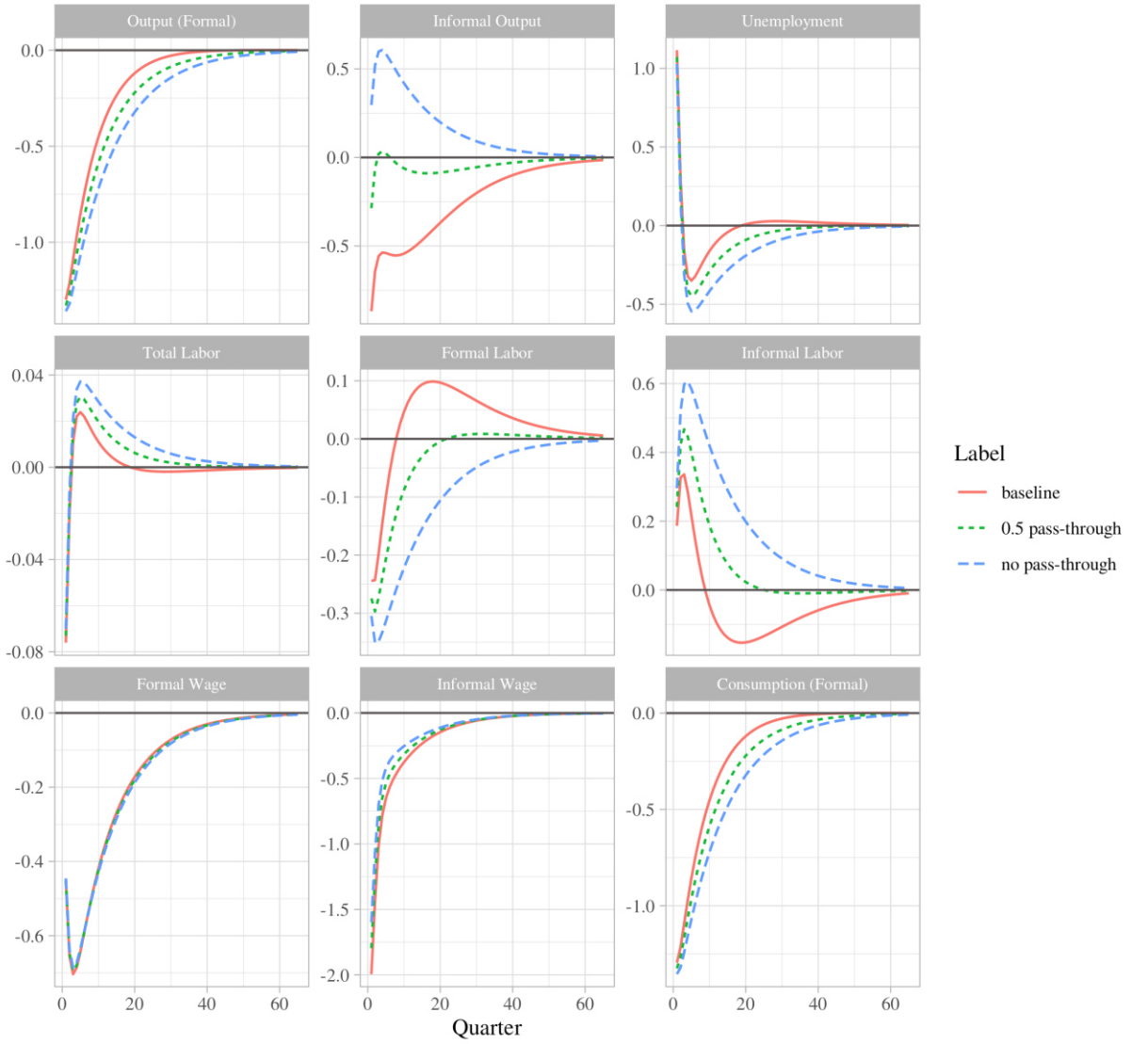


Figure 5: *Impulse Response Functions with Different Pass-through Levels*

For the correlations of unemployment and total labor to formal output, a lesser pass-through reduces the level of correlation pulling them away from the observed second moments, although they retain the appropriate sign. Regarding the correlation between the formal output and informal output, the model with no pass-through displays a countercyclical relation between them whereas the baseline model displays the contrary. Nonetheless, given the shadow-like nature of informality discussed earlier, I have assumed that there is not an observable counterpart of this second moment.

Specifically, there is no estimation for the share in GDP from the definition of informality that I have used in the model[5]. As can be seen in Figure 5, it is also to informal output that the level of pass-through has a greater effect on the shape of the impulse response function. While in the baseline model informal output falls at the offset of the shock, when there is no pass-through it increases in the first time-periods and then converges towards the steady-state.

The main argument I advocate for the complete pass-through and the predicted procyclicality of informal output from the baseline model is the nature of the firms that hire most labor in Mexico, that is, as has been documented by Levy (2018), most formal and informal firms are small, labor intensive and perform relatively similar activities; the capital intensive and productive firms are a small subset in comparison. Therefore, I think that no pass-through may be a less credible assumption, even so, it is possible to deviate from complete pass-through without much loss in performance as can be seen in the third column of Table 6.

The Bargaining Power of Labor

The bargaining power parameters were chosen to match the steady-state levels mentioned in Chapter 5. An increase in the bargaining power of formal worker increases the wage ratio, sector output ratio and the informality rate. However, its value is not important for generating the key transmission mechanism of the model.

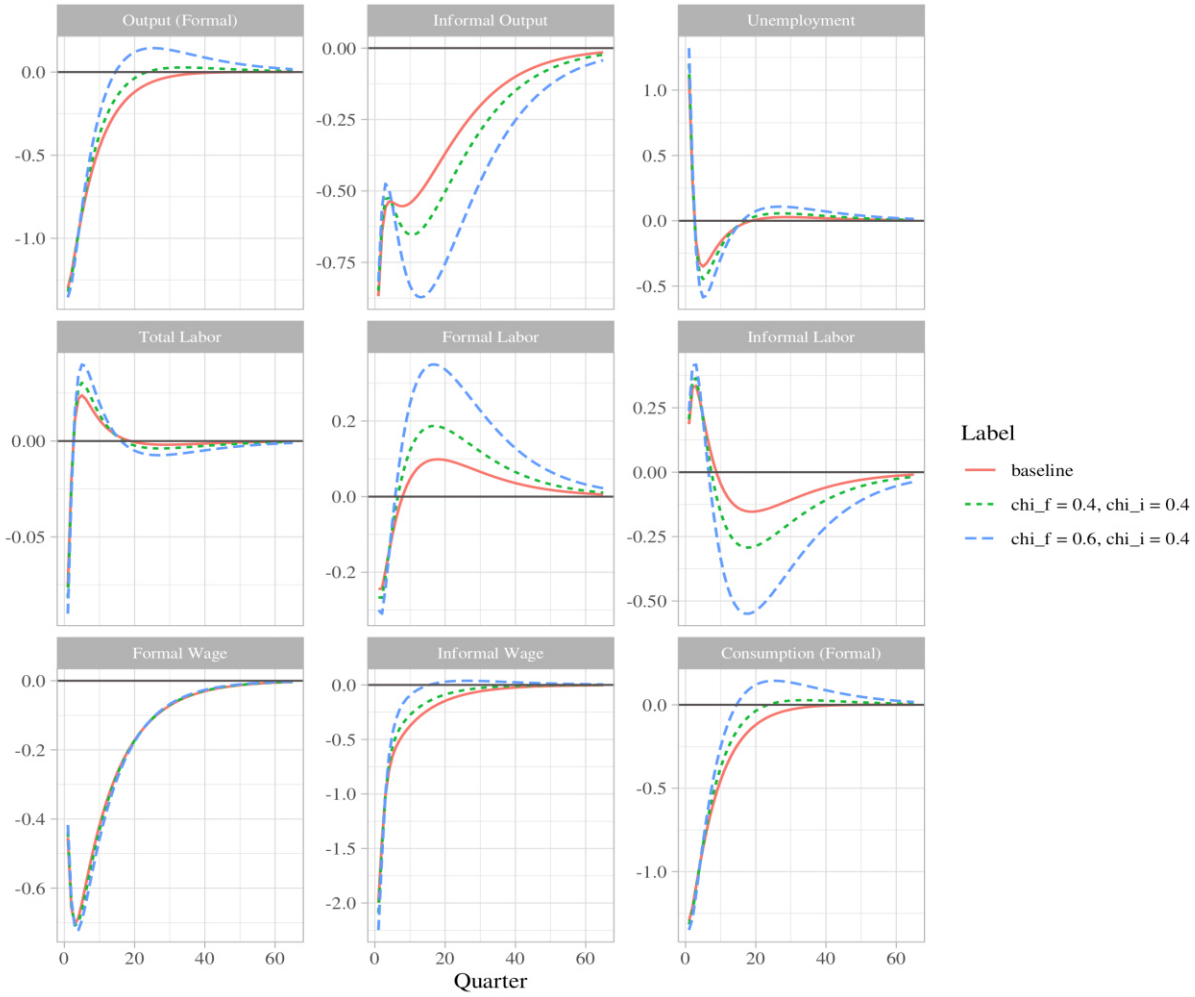
[5] INEGI publishes an estimation of the informal sector share in GDP, however, they use a broader definition of both informal sector and labor than I do.

Table 7. Business Cycle Statistics: Alternative Bargaining Power Levels

	Data	Baseline	$\chi_f = 0.4$ $\chi_i = 0.4$	$\chi_f = 0.6$ $\chi_f = 0.4$
$\sigma(Y^f)$	1.73	1.73	1.79	1.91
$\sigma(C^f)/\sigma(Y^f)$	0.99	0.99	1.00	1.00
$\sigma(L)/\sigma(Y^f)$	0.42	0.05	0.06	0.06
$\sigma(U)/\sigma(Y^f)$	0.44	0.76	0.83	0.92
$\sigma(L^f)/\sigma(L^i)$	0.87	0.81	0.69	0.68
$\sigma(W^f)/\sigma(W^i)$	0.98	0.43	0.41	0.39
$Corr(Y^i, Y^f)$	-	0.92	0.83	0.59
$Corr(C^f, Y^f)$	0.93	1.00	1.00	1.00
$Corr(L, Y^f)$	0.76	0.34	0.31	0.28
$Corr(L^f, Y^f)$	0.69	0.94	0.91	0.90
$Corr(L^i, Y^f)$	-0.02	-0.89	-0.87	-0.87
$Corr(U, Y^f)$	-0.43	-0.44	-0.31	-0.27
$Corr(W^f, Y^f)$	0.29	0.91	0.89	0.85
$Corr(W^i, Y^f)$	0.48	0.93	0.91	0.89

As can be seen in Table 7, the model retains the countercyclicality of informal employment and unemployment and the procyclicality of wages. Nonetheless, the model does improve in the level of correlation between wages and formal output. While the baseline model predicts too high of a correlation between them, an increase in the bargaining of formal workers reduces those correlation bringing these second moments closer to the observed correlations. An important aspect to notice, is that the three specifications shown in Table 7, show that the correlation between formal wages and formal output should be lower than that of informal wages, which also appears in the data. This is a consequence of the assumption of sticky wages, which reduces the procyclicality of formal wages.

Figure 6: Impulse Response Functions with Different Bargaining Levels



As can be seen in Figure 6, the increase in the formal bargaining power also increases the volatility of unemployment, formal employment, and informal employment. This is probably related to the way in which the formal bargaining affects the determination of optimal formal wages. Greater formal bargaining power decreases the contribution of the outside option to formal wages, which increases the formal wage, but it also affects the level of informal wages through the outside option term in the informal wage. As formal wages are sticky, the effect is mainly reflected on informal wages which decrease further in the alternative models; in turn, this produces a greater shedding of labor from formal firms increasing unemployment. As unemployment increases, the lower wages and labor tightness of the informal sector increase the absorption of labor by informal firms, hence the greater labor reallocation seen in the impulse response functions.

Given that there is no empirical estimate for bargaining power of workers, I think that the greater bargaining power of informal workers is a sensible assumption as the definition of informality used implies that informal firms tend to be based around family ties.

Chapter 8. Conclusion

In this thesis I have proposed a business cycle model for a closed economy in which labor market frictions, an informal sector and nominal rigidities play a key role to reproduce the stylized facts for the Mexican economy. The model features a novel mechanism in the literature: the inclusion of nominal wage rigidities in the formal labor market. I show that under this framework it is possible to generate a transmission mechanism of productivity shocks that matches the data, mainly the countercyclicality of the informal employment and unemployment rates and the procyclicality of wages.

A second contribution of this thesis has been to include data for formal and informal wages for two measures of informality. Through these statistics, we have learned that both formal and informal wages are procyclical and that informal wages are more volatile than formal ones, which supports the inclusion of nominal rigidities. While this is an important margin to understand the behavior of informality in Mexico, it is relatively absent from the developing countries business cycle literature and usually is not regarded as a behavior to match when evaluating a model of informality.

This model has the potential to be extended with features that are already common in the literature. For example, the inclusion of capital in the formal sector or the inclusion of differentiated capital in both sectors. Also, a possible extension would be to make it the basis for a small open economy model, mainly to investigate if the model is able to reproduce the countercyclical interest rates observed in the data. Lastly, interest rate and cost-push shocks have been shown in the literature as other possible sources to generate a countercyclical informality rate. Given that these kinds of shocks are a feature of the Mexican economy, their inclusion could also shed light on the role of nominal wage rigidities on the behavior of informality.

Finally, while this thesis belongs primarily to the literature of macro models, I think it should also be regarded as an initial contribution to the literature of institutions that generate informality. However, I do not think that this thesis necessarily implies that labor should be deregulated in pursuit of flexible wages in the formal sector, rather, that there should be more research on the possible effects of a universal unemployment insurance and labor rights enforcement that could reduce the asymmetry that these models assume to reproduce the observed behavior of informality.

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Appendix A

The data used in chapter 3.1 for Business Cycle Dating comes from three sources depending on the country. For the United States and Canada, the cycle dating comes from the National Bureau of Economic Research (NBER) and the C.D. Howe Institute respectively. In the case of these countries, the business cycle dating is official and there are special councils that determine peaks and troughs dates based on several indices including GDP. In the case of Mexico there is no official dating council, however, INEGI publishes reports monthly on the *Índice Global de Actividad Económica* (IGAE) [6] where a business cycle dating is presented and hence the dates were taken from it. The data for the three countries was extracted as was published in October of 2020, further revisions to the dates may have been done by the respective national agency.

The data used in chapters 3.2 Employment Aggregates and 3.3 Wages come from INEGI's Encuesta Nacional de Ocupación y Empleo (ENOE), specifically, from Tabla de Datos Sociodemográficos (SDEMT) [7]. The data ranges from the first quarter of 2005 to the last quarter of 2019, all encompassed in what is referred to as "new ENOE" in the previous literature. As this is a survey, the steps followed to get the aggregate time series is described next.

The first step to process the data from the tables was to filter for all individuals in the labor force which can be done by following the document Reconstrucción de Variables. [8]

Then, to obtain the self-employed workers and their counterpart one must use corresponding variable (POS_OCU) and to get the workers of small firms one must follow a similar procedure using the variable for firm's size (EMPLE7C). To get the nominal hourly wages one must keep the variable for hourly income (ING_X_HRS). To get the quarterly employment totals and

[6] The IGAE is supposed to be a short run (mothly) proxy for real economic activity in contrast with GDP which INEGI reports quarterly. Therefore, I have taken these dates to be reasonably equivalent to those reported on Canada and the United States.

[7] As of April 2021, available at ENOE

[8] Available at Reconstrucción de Variables

nominal mean wages it is necessary to use the survey's expansion factor (FAC). [9]. To make nominal wages real I used the Consumer Price Index series from INEGI [10]. To deseasonalize all the time series I used the X-13ARIMA-SEATS Seasonal Adjustment Program of the US Census Bureau.

Finally, to get the cyclical behavior of all these variables I filtered the time series with the Hodrick-Prescott filter using the standard λ value of 1600 for quarterly data.

[9] The description of all ENOE variables is available at Descriptor de Archivos

[10] Available at INEGI's Banco de Información Económica

Appendix B

A unique zero-inflation steady-state equilibrium exists for this economy. Variables not indexed by time denote their steady state values:

$$\theta^f = \frac{V^f}{U}$$
$$\theta^i = \frac{V^i}{U}$$
$$s^f L^f = \lambda^f V^f$$
$$s^i L^i = \lambda^i V^i$$
$$U + L^f + L^i = 1$$
$$A^i \frac{P^i}{P} - \frac{W^i}{P} = \kappa^i \theta^{i\psi}$$
$$Q = \beta$$
$$c^f = \omega \left\{\frac{P^f}{P}\right\}^{\varepsilon_c} C$$
$$c^i = (1-\omega) \left\{\frac{P^i}{P}\right\}^{\varepsilon_c} C$$
$$P = \left[\omega {P^f}^{1-\varepsilon_c} + (1-\omega) {P^i}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon_c}}$$
$$Y^f = A^f L^f$$
$$Y^i = A^i L^i$$
$$P^f = \frac{\epsilon_f}{\epsilon_f - 1} MC$$
$$W^f = \chi^f [P^f A^f + P\kappa^f \theta^f] + (1-\chi^f) \left[\frac{\chi^i}{1-\chi^i} P\kappa^i \theta^i\right]$$
$$W^i = \chi^i \left[P^i A^i + P\kappa^i \theta^i\right] + \left(1-\chi^i\right) \left[\frac{\chi^f}{1-\chi^f} P\kappa^f \theta^f\right]$$
$$Y^f = C^f + P\kappa^f V^f$$
$$Y^i = C^i + P\kappa^i V^i$$

Where we set the general price P as the numerator and solve the system of equation for: $\{Q, U, L^i, L^f, W^i, W^f, P^f, P^i, C, C^i, C^f, Y^f, Y^i, \theta^f, \theta^i, V^i, V^f\}$

Appendix C

As our interest is on the impulse deviations from the steady-state path, it is convenient to reduce the complexity of the model by transforming the model to a log-linear version. Following the usual notation:

$$\widehat{x_t} = \frac{x_t - x}{x}$$

where x represents the steady state value of variable x_t and $\widehat{x_t}$ is the percent deviation of x_t from its steady state.

We then proceed to obtain the log-linearized versions of the equilibrium conditions.

Goods Market Clearing

From the log-linearization of the formal market clearing equation 4.7-1 follows that:

$$\widehat{Y_t^f} = \left(1 - \frac{\kappa^f V^f}{Y^f}\right)\widehat{c_t^f} + \frac{\kappa^f V^f}{Y^f}\left[\widehat{\theta_t^f} + \widehat{U_t}\right]$$

To log-linearize the informal goods market we follow the same procedure and use 4.1-2 to substitute informal vacancies:

$$\widehat{Y_t^i} = \left(1 - \frac{\kappa^i V^i}{Y^i}\right)\widehat{c_t^i} + \frac{\kappa^i V^i}{Y^i}\left[\widehat{\theta_t^i} + \widehat{U_t}\right]$$

Labor Market

Using the laws of motion for each sector 4.1-3 and using 4.1-2 we get:

$$\widehat{L_t^f} = (1 - s^f)\widehat{L_{t-1}^f} + s^f\left(\widehat{U_{t-1}} + (1 - \psi)\widehat{\theta_t^f}\right)$$
$$\widehat{L_t^i} = \left(1 - s^i\right)\widehat{L_{t-1}^i} + s^i\left(\widehat{U_{t-1}} + (1 - \psi)\widehat{\theta_t^i}\right)$$

The log-linearization of the labor market clearing equation yields:

$$L^f\widehat{L_t^f} + L^i\widehat{L_t^i} + U\widehat{U_t} = 0$$

Households

The households Euler equation becomes:

$$\widehat{C_t} = \mathbb{E}_t\widehat{C_{t+1}} - \frac{1}{\sigma}[\widehat{\imath_t} - \mathbb{E}_t\widehat{\pi_{t+1}}]$$

where $\pi_t = \log\left(\frac{P_{t+1}}{P_t}\right)$ is the inflation rate for this economy. Demand equations 4.2-5 and 4.2-6 are log-linearized as:

$$\begin{aligned}\widehat{c_t^f} &= \varepsilon_c\left(\widehat{P_t} - \widehat{P_t^f}\right) + \widehat{C_t}\\ \widehat{c_t^\iota} &= \varepsilon_c\left(\widehat{P_t} - \widehat{P_t^\iota}\right) + \widehat{C_t}\end{aligned}$$

The aggregate price index 4.2-7 can be represented in terms of inflation:

$$\widehat{\pi_t} = \omega\left(\frac{P^f}{P}\right)^{1-\varepsilon_c}\widehat{\pi_t^f} + \left(1 - \omega\left(\frac{P^f}{P}\right)^{1-\varepsilon_c}\right)\widehat{\pi_t^\iota}$$

Firms

The log-linearization of the production functions of informal 4.4-1 and formal 4.3-1 firms yield:

$$\widehat{Y_t^\iota} = \widehat{L_t^\iota}$$

$$\widehat{Y_t^f} = \widehat{L_t^f}$$

From the optimal price decision 4.3-4 of the formal firm we get a condition for formal inflation:

$$\widehat{\pi_t^f} = \beta\hat{\pi}_{t+1}^f + \frac{(1-\theta_p)(1-\theta_p\beta)}{\theta_p}\left(\widehat{W_t^f}(1+\tau) + \kappa^f\psi\widehat{\theta^f} - \hat{a}_t\right)$$

The log-linearzation of the informal firm's optimality condition yields:

$$\widehat{\theta_t^\iota} = \frac{1}{\psi}\left\{\frac{P^i}{\theta^{i\psi}}\widehat{P_t^\iota} - \frac{W^i}{\theta^{i\psi}}\widehat{W_t^\iota}\right\}$$

Wages

From the solutions to the Nash bargaining problems 4.5-6 and 4.5-7 we obtain:

$$\widehat{W_t^\iota} = \chi^i\left(\frac{P^i}{W^i}\left(\widehat{P_t^\iota} + \hat{a}_t\right) + \frac{\kappa^i\theta^i}{W^i}\widehat{\theta^\iota}\right) + \left(1-\chi^i\right)\left(\frac{b}{W^i}\widehat{b_t} + \frac{\chi^f}{1-\chi^f}\frac{\kappa^f\theta^f}{W^i}\widehat{\theta^f}\right)$$

$$\widehat{W_t^f} = \theta_w \widehat{W_{t-1}^f} + (1-\theta_w)\left\{\chi^f\left(\frac{P^f}{W^f}\left(\widehat{P^f} + \widehat{a_t}\right)\right) + (1-\chi^f)\left(\frac{b}{W^f}\widehat{b_t} + \frac{\chi^i}{1-\chi^i}\frac{\kappa^i\theta^i}{W^f}\widehat{\theta_t^i}\right)\right\}$$

Monetary Authority

The monetary policy 4.6-1 yields:

$$\widehat{\iota_t} = \rho\widehat{\iota_{t-1}} + (1-\rho)\left(\phi_p\widehat{\Pi_t}\right) + \phi_Y\left(\widehat{Y_t}\right)$$

Government Budget

Finally, the log-linearized version of the balanced government budget is:

$$\widehat{b_t} = \tau\left(\widehat{W_t^f} + \widehat{L_t^f}\right) - \widehat{U_t}$$

In summary, the log-linearized version of the model consists of the next equations which define the profile of variables: $\{\widehat{Y_t^f}, \widehat{Y_t^\iota}, \widehat{C_t^f}, \widehat{C_t^\iota}, \widehat{C_t}, \widehat{L_t^f}, \widehat{L_t^\iota}, \widehat{U_t}, \widehat{\theta_t^f}, \widehat{\theta_t^\iota}, \widehat{W_t^f}, \widehat{W_t^\iota}, \widehat{b_t}\, \hat{\pi}, \widehat{\pi^f}, \widehat{P_t^\iota}, \widehat{\iota_t}\}$.

$$\widehat{Y_t^f} = \left(1 - \frac{\kappa^f V^f}{Y^f}\right)\widehat{c_t^f} + \frac{\kappa^f V^f}{Y^f}\left[\widehat{\theta_t^f} + \widehat{U_t}\right]$$

$$\widehat{Y_t^\iota} = \left(1 - \frac{\kappa^i V^i}{Y^i}\right)\widehat{c_t^\iota} + \frac{\kappa^i V^i}{Y^i}\left[\widehat{\theta_t^\iota} + \widehat{U_t}\right]$$

$$\widehat{L_t^f} = (1 - s^f)\widehat{L_{t-1}^f} + s^f\left(\widehat{U_{t-1}} + (1-\psi)\widehat{\theta_t^f}\right)$$

$$\widehat{L_t^\iota} = \left(1 - s^i\right)\widehat{L_{t-1}^\iota} + s^i\left(\widehat{U_{t-1}} + (1-\psi)\widehat{\theta_t^\iota}\right)$$

$$L^f\widehat{L_t^f} + L^i\widehat{L_t^\iota} + U\widehat{U_t} = 0$$

$$\widehat{C_t} = \mathbb{E}_t\widehat{C_{t+1}} - \frac{1}{\sigma}[\widehat{\iota_t} - \mathbb{E}_t\widehat{\pi_{t+1}}]$$

$$\widehat{c_t^f} = \varepsilon_c\left(\widehat{P_t} - \widehat{P_t^f}\right) + \widehat{C_t}$$

$$\widehat{c_t^\iota} = \varepsilon_c\left(\widehat{P_t} - \widehat{P_t^\iota}\right) + \widehat{C_t}$$

$$\widehat{\pi_t} = \omega\left(\frac{P^f}{P}\right)^{1-\varepsilon_c}\widehat{\pi_t^f} + \left(1 - \omega\left(\frac{P^f}{P}\right)^{1-\varepsilon_c}\right)\widehat{\pi_t^\iota}$$

$$\widehat{Y_t^\iota} = \widehat{L_t^\iota}$$

$$\widehat{Y_t^f} = \widehat{L_t^f}$$

$$\pi_t^f = \beta\widehat{\pi_{t+1}^f} + \frac{(1-\theta_p)(1-\theta_p\beta)}{\theta_p}\left(\widehat{W_t^f}(1+\tau) + \widehat{\kappa^f}\psi\theta^f - \widehat{a_t}\right)$$

$$\widehat{\theta_t^\iota} = \frac{1}{\psi}\left\{\frac{P^i}{\theta^{i\psi}}\widehat{P_t^\iota} - \frac{W^i}{\theta^{i\psi}}\widehat{W_t^\iota}\right\}$$

$$\widehat{W_t^\iota} = \chi^i\left(\frac{P^i}{W^i}\left(\widehat{P_t^\iota} + \widehat{a_t}\right) + \frac{\kappa^i\theta^i}{W^i}\widehat{\theta^\iota}\right) + \left(1 - \chi^i\right)\left(\frac{b}{W^i}\widehat{b_t} + \frac{\chi^f}{1-\chi^f}\frac{\kappa^f\theta^f}{W^i}\widehat{\theta_t^f}\right)$$

$$\widehat{W_t^f} = \theta_w\widehat{W_{t-1}^f} + (1-\theta_w)\left\{\chi^f\left(\frac{P^f}{W^f}\left(\widehat{P^f} + \widehat{a_t}\right)\right) + \left(1 - \chi^f\right)\left(\frac{b}{W^f}\widehat{b_t} + \frac{\chi^i}{1-\chi^i}\frac{\kappa^i\theta^i}{W^f}\widehat{\theta_t^\iota}\right)\right\}$$

$$\widehat{\iota_t} = \rho\widehat{\iota_{t-1}} + (1-\rho)\left(\phi_p\widehat{\Pi_t}\right) + \phi_Y\left(\widehat{Y_t}\right)$$

$$\widehat{b_t} = \tau\left(\widehat{W_t^f} + \widehat{L_t^f}\right) - \widehat{U_t}$$