EL COLEGIO DE MÉXICO, A.C.
CENTRO DE ESTUDIOS ECONÓMICOS

STATIC AND DYNAMIC MORAL HAZARD WITH BARGAINING POWER

TESIS PRESENTADA POR

ITZA TLALOC QUETZALCOATL CURIEL CABRAL

PROMOCIÓN 2022-2025

CIUDAD DE MÉXICO AGOSTO DE 2025

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A mi familia y amigos.

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Contents

Introduction

The study of moral hazard and bargaining power in principal-agent relationships has been a central theme in economics, particularly in the fields of contract theory, corporate governance, and labor economics. The classical principal-agent problem, formulated by Holmström (1979) and Grossman and Hart (1983a), considers a scenario in which a principal—such as an employer or investor—offers a contract to an agent, such as an employee or manager, who takes an unobservable action that affects outcomes. Standard models often rely on take-it-or-leave-it contracts, where the principal unilaterally sets the terms. However, real-world negotiations frequently exhibit asymmetries in bargaining power, allowing both parties to influence contract outcomes (Laffont and Martimort, 2002).

This thesis builds upon the classical framework by incorporating bargaining power dynamics into both static and dynamic models of moral hazard. While much of the previous literature has focused on one-sided power structures, this research investigates more realistic negotiation environments where bargaining power affects contract efficiency, incentive compatibility, and welfare outcomes.

Bargaining power plays a critical role in various economic settings. In labor markets, firms and workers negotiate wages and performance incentives (Inderst, 2002; Bayindir-Upmann and Gerber, 2003). In corporate governance, executives bargain over compensation and control with shareholders and boards (Bebchuk and Fried, 2003). In financial markets, borrowers and lenders negotiate contract terms under asymmetric information (Pitchford, 1995).

This dissertation contributes to the literature by bridging static and dynamic models of moral hazard and by examining how bargaining power influences contract efficiency and equilibrium allocations over time. It combines analytical modeling with numerical simulations, offering a rigorous framework to address how power asymmetries shape contractual relationships. The methodological approach integrates optimization techniques to derive equilibrium solutions and uses multi-objective modeling to capture Pareto-efficient outcomes. To complement theoretical insights, numerical algorithms are implemented to approximate optimal contract structures under complex bargaining settings.

The core chapters of this dissertation address the relationship between bargaining power and incentive contract structures under moral hazard, progressively increasing in complexity. Chapter 1 develops a static agency model with continuous actions and limited liability, introducing compromise and Pareto-weights solutions, and demonstrating their theoretical equivalence through analytical and numerical results. Chapter 2 extends the framework to incorporate hidden actions, with a Board designing contracts under fixed bargaining weights and analyzing equilibrium contracts across different regions of the Pareto frontier based on reservation utilities. Chapter 3 introduces a dynamic model where bargaining power evolves endogenously according to firm performance, and examines how this bargaining drift shapes long-term compensation schemes through numerical simulations and econometric analysis.

This interdisciplinary approach allows for a nuanced exploration of static and dynamic settings, bridging theoretical insights with computational advancements. By combining these methodologies, the thesis offers a robust analysis of how bargaining power influences contract efficiency and evolves over time, contributing to the broader understanding of contract theory and organizational economics.

Chapter 1

Compromise Solutions of a Static Agency Model with Continuous Actions and Limited Liability

1.1 Introduction

In standard agency models, the principal typically designs take-it-or-leave-it contracts to offer to the agent (Holmström, 1979; Grossman and Hart, 1983b). However, these models overlook scenarios where bargaining power is shared between the principal and the agent. Real-world contracts often involve negotiations, such as firm-labor union agreements, bilateral monopolies, or direct principal-agent relationships (Yao, 2012). Despite its significance, the role of bargaining power in contract design remains underexplored in standard agency theory. This article addresses this gap by analyzing how bargaining power distribution affects optimal contract structures and economic outcomes.

This article contributes to contract theory by examining the effect of bargaining power distribution on the design of optimal contracts in a principal-agent model with continuous actions and limited liability. We characterize the Pareto frontier of feasible contracts and introduce two alternative approaches, the Pareto-weights solution and the compromise solution, demonstrating their theoretical equivalence. Our findings reveal a critical bargaining threshold, $\alpha = \frac{1}{2}$, at which the optimal contract undergoes a structural transition. Additionally, we explore a compromise solution that incorporates fairness considerations through distance minimization with respect to an endogenous ideal point. This alternative approach also leads to the same set of efficient outcomes as the standard model. In particular, we identify a critical value of the bargaining power parameter $p = 1$ at which the limited liability constraint ceases to bind, echoing the threshold result from the Pareto-weights formulation. This insight provides a refined framework for understanding incentive mechanisms, moral hazard, and economic surplus allocation, with direct applications in labor markets, corporate governance, and supply chain negotiations.

On the one hand, some form of bargaining power between manufacturers and retailers in a supply chain was introduced by Guo et al. (2018). In their work, the choice of bargaining power could be interpreted as the choice of partners. As Grout (1984) affirms, the allocation of bargaining power is likely to affect firms' investment decisions. Aust and Buscher (2014) also consider a bargaining problem when there is asymmetric information in the supply chain. They explore the dynamics of bargaining power between manufacturers and retailers.

On the other hand, Sheu and Gao (2014) investigate how the relative bargaining power of each actor affects negotiations between manufacturers and reverse logistics providers in a reverse supply chain. Finally, Feng and Lu (2013) contrast the contract outcome of a Stackelberg game, in which the manufacturer offers take-it-or-leave-it contracts to the retailers, with a bargaining game in which the firms bilaterally negotiate contract terms using an alternating offer. Thus, while the classic literature focuses on manufacturing investment and retailers' efforts, as these authors have pointed out, bargaining power also plays a significant role in supply chain operations and the development of optimal contracts.

Bargaining power also plays a central role in principal-agent models, as its distribution significantly affects contract design, the agent's effort level, and the overall efficiency of the relationship ((Demougin and Helm, 2006); Li et al. (2013); Li et al. (2015)). Traditional models assume that the principal has all the bargaining power and offers a take-it-or-leave-it contract to the agent (Demougin and Helm (2006); Dittrich and Städter (2014); Li et al. (2013); Li et al. (2015)). However, in reality, both parties often hold some degree of bargaining power (Demougin and Helm (2006); Li et al. (2013); Li et al. (2015)).

Several aspects of bargaining power within principal-agent models can be outlined:

- **Contract design:** When the agent has bargaining power, they can negotiate more favorable contractual terms, such as higher compensation or reduced risk exposure (Li et al., 2013; Chen et al., 2019). This may lead to contract structures different from those arising when the principal holds all the power (Pitchford, 1998). Particularly, when the agent has limited liability, bargaining power distribution significantly impacts contract terms and the agent's exerted effort (Pitchford, 1998).

- **Agent's effort level:** The agent's effort choice can be influenced by their bargaining power. Higher bargaining power might increase the agent's utility, potentially affecting their motivation and, consequently, their effort level (Chen et al., 2019). In some models, increased bargaining power can lead to the lower frequency of high-effort implementation but may intensify incentive contracts (Li et al., 2013, 2015).

- **Relationship efficiency:** The bargaining power allocation can have efficiency implications for the principal-agent relationship. If the agent holds all the bargaining power, the first-best solution may be reached, as they internalize and maximize the total surplus (Schmitz, 2005). However, in scenarios involving limited liability, the bargaining power distribution affects the joint surplus generated by the contract (Demougin and Helm, 2006).

- **Contract negotiation:** Some studies explicitly model the contract negotiation process between the principal and the agent, where each party's bargaining power influences the final contract terms (Li et al., 2013; Chen et al., 2019). These models depart from the traditional assumption that the principal has full bargaining power, offering a more realistic perspective on contract formation under moral hazard (Dittrich and Städter, 2014).

- **Determinants of bargaining power:** Several factors can influence the agent's bargaining power, such as labor market conditions, the presence of unions (Chen et al., 2019), the scarcity of the agent's skills (Edmans et al., 2017), or even specific characteristics of CEOs in executive compensation contexts (Choe et al., 2014; Pandher and Currie, 2013).

Closely linked to bargaining power, the reservation utility (often referred to as the outside option) plays a fundamental role in principal-agent models. It defines the minimum utility level that the agent must receive to accept the contract offered by the principal (Ábrahám et al., 2011; Bond and Gomes, 2009). If the agent's expected utility under the contract falls below their reservation utility, they will rationally reject the contract and seek an alternative (Acemoglu and Simsek, 2010).

The reservation utility plays a crucial role in contract design, as it sets the minimum level of utility that the agent must receive to participate in the contractual relationship. This constraint ensures that the agent's expected utility from accepting the contract is at least as good as their best available alternative. If the contract does not meet this threshold, the agent will rationally reject the offer in favor of their reservation utility. This fundamental principle shapes the design of incentive schemes, as the principal must structure the contract in a way that both satisfies this constraint and aligns the agent's incentives with the principal's objectives (Ábrahám et al., 2011; Bond and Gomes, 2009).

Beyond its role as a constraint, the reservation utility also serves as a reference point in bargaining situations. The stronger the agent's reservation utility, the greater their bargaining power, allowing them to negotiate more favorable contractual terms. Conversely, when the agent has a weak reservation utility, the principal gains more leverage in determining contract conditions. This dynamic is particularly relevant in settings where outside alternatives are scarce or highly valuable, influencing the overall distribution of surplus in the relationship (Aghion and Tirole, 1994; Demougin and Helm, 2006).

A characterization of all the possible contracts that arise from a static principal-agent model included in the associated Pareto frontier upon variation of the reservation utility would allow us to distinguish the different incentive mechanisms along this frontier. This paper provides an extension of Demougin and Helm's analysis of bargaining power in a moral hazard model with limited liability and a bonus contract (Demougin and Helm, 2006).

The rest of this chapter is organized as follows: In Section 1.2, we present the model in the standard way, along with a numerical example. Section 1.3 explains the Pareto-weights model and its solutions, and we also provide a numerical example. We analyze a compromise approach in Section 1.4. We also present the equivalence between the compromise solution and the standard Pareto frontier, highlighting the critical bargaining value $p = 1$ as a transition point for optimal contract structures. Finally, we provide the conclusion.

1.2 The Standard Model

In this section, we develop a principal-agent model with limited liability and a bonus contract that have been analyzed based on the moral hazard model of Demougin and Helm (2006); the purpose of this section is to provide the readers with all the necessary results that will be relevant for the following sections. We assume that there are two individuals: a principal and an agent, who are both risk-neutral. The variable a is the agent's effort choice made at the beginning, drawn from a compact set $A = [\underline{a}, \overline{a}]$, and it is unobservable to the principal. Suppose that there is a signal $S = \{0, 1\}$, where $S = 1$ is the favorable state and $S = 0$ is a nonfavorable state, thereby let $p(a)$ be the conditional probability that $S = 1$ given the level of effort a.

The principal-agent relationship described here is fundamentally shaped by bargaining power, which affects contract design and effort provision. As discussed by Chen et al. (2019), the distribution of bargaining power influences the incentive structures and can determine the level of effort exerted by the agent. This is particularly relevant when financial constraints limit the agent's outside options, reinforcing the principal's negotiating leverage (Aghion and Tirole, 1994).

The variable $w \geq 0$ is the agent's salary compensation at the end of the period, which is given by $w = F + bp(a)$; where F is the fixed payment and b is the bonus, and $bP(a)$ is the expected bonus in a favorable state. Let $U(w, a)$ be the agent's expected utility given by $U = F + bp(a) - c(a)$, where $c(a)$ is the cost associated when the agent makes an effort a. On the other hand, let $V(y, w)$

be the principal's expected profit given by $V = y(a) - w$ where $y(a)$ is the product's level associated with an effort, a, expressed in monetary terms.

Using the first-order approach to incentive-compatibility constraint (Rogerson (1985)), the incentive constraint is given by $bp'(a) - c'(a) = 0$. Therefore, from the bonus, $b = \frac{c'(a)}{p'(a)}$. Let $B(a) = \frac{p(a)c'(a)}{p'(a)}$ be the expected bonus function, which follows an increasing and convex pattern as a consequence of the model's structure. This increasing pattern is consistent with empirical findings in labor contracts, where agents with greater bargaining power negotiate higher bonus shares (Pitchford, 1998). Internalizing this result to the problem, the Karush-Kuhn-Tucker (KKT) program for this standard principal-agent model is:

$$V(\bar{u}) = \max_{\{F,a\}} \quad y(a) - [F + B(a)] \tag{1.1}$$

s.t.

$$F \geq 0, \tag{1.2}$$

$$F + B(a) - c(a) \geq \bar{u}; \tag{1.3}$$

where $\bar{u}$ is the agent's reservation utility. Given λ and μ as the multipliers associated with the limited liability and participation constraints, respectively.

The Lagrangian function is the following:

$$\mathcal{L} = y(a) - [F + B(a)] + \lambda F + \mu \left[F + B(a) - c(a) - \bar{u}\right].$$

These constraints illustrate the tension between limited liability and bargaining power, as seen in financial contracting models (Balkenborg, 2001). The principal's ability to impose incentive schemes depends on the agent's alternative options, a concept that is central to bargaining theory in contract negotiations.

The first-order conditions with respect to a and F are:

$$y'(a) - B'(a) + \mu\left[B'(a) - c'(a)\right] = 0, \tag{1.4}$$

$$-1 + \lambda + \mu = 0. \tag{1.5}$$

Only three of the four KKT cases are feasible; with these cases, we can describe the Pareto frontier upon variation of the reservation utility. [1] Next, we describe some of their results.

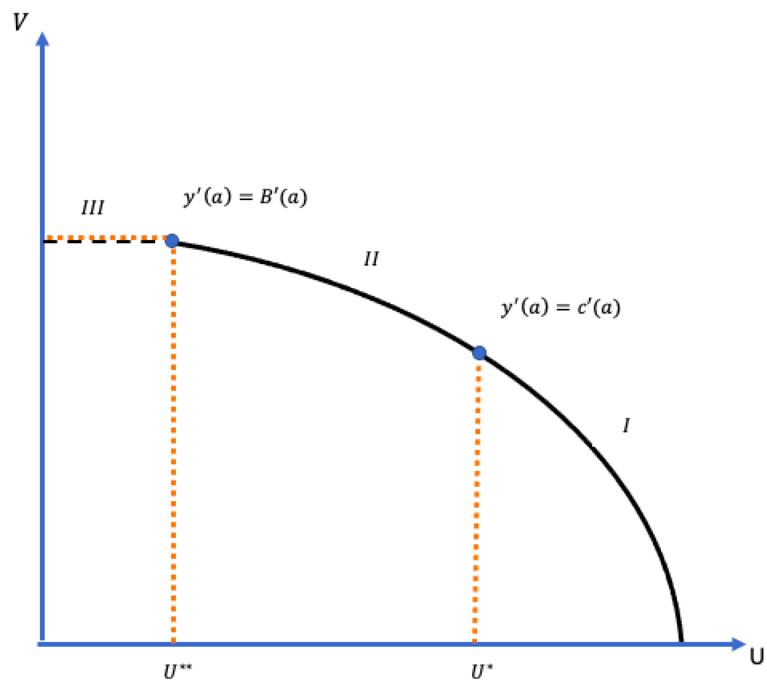

Figure 1.1: Pareto frontier

Let U^* be the agent's reservation utility when $y'(a) = c'(a)$. The region I of the constrained Pareto frontier, shown in figure 1.1, is the set of all the possible efficient utility pairs (V, U) where $U > U^*$; which are incentive-compatible contracts that satisfy the agent's financial constraint. Furthermore, since $\lambda = 0$ and $\mu > 0$ in this region, the fixed cost must be positive and $\mu^* = 1$. The optimality condition is $y'(a) = c'(a)$, from which it follows the corresponding effort levela^*. The fixed payment is a function of $\bar{u}$ that yields $F^*(\bar{u}) = \bar{u} - B(a^*) + c(a^*)$. Notice that only in

[1]For more details, see the proposition 1 of Demougin and Helm (2006).

the superior limit of the region the fixed payment is zero, and the agent's compensation is given by $w^* = F^*(\bar{u}) + B(a^*)$; that is, $w^* = \bar{u} + c(a^*)$.

The positive relationship between reservation utility and fixed payments supports findings by Choe et al. (2014), who highlight how agents with stronger negotiation power secure better compensation structures, reducing reliance on performance-based pay.

Let U^{**} be the agent's reservation utility when $y'(a) = B'(a)$. In region III, the set of possible utility pairs where $U < U^{**}$; or well, this region corresponds to the KKT case where $\lambda > 0$ and $\mu = 0$. In this case, the fixed payment is zero, $F = 0$, and $\lambda = 1$. Thus, the optimality condition $y'(a) = B'(a)$ determines the corresponding effort level a^{**}. Note that $V^{**} = y(a^{**}) - B(a^{**})$ is constant for all $\bar{u} < U^{**}$. The agent's compensation is $B(a^{**})$, *i.e.* the agent extracts a rent; that is, small variations in $\bar{u}$ keep a^{**}, V^{**} and U^{**} constant. Demougin and Helm (2006) affirm that region III does not belong to the Pareto frontier, since it cannot arise from a contract c.

This inefficiency aligns with the conclusions of Pitchford (1998), where rent extraction by agents leads to suboptimal contract outcomes. Furthermore, Aghion and Tirole (1994) argue that constrained bargaining settings often produce rigid contract terms that fail to maximize joint surplus.

In summary, the constrained Pareto frontier represents the set of possible utility pairs (V, U), which is decreasing and concave, with $U > U^{**}$. Additionally, there is a policy shift at U^* on the Pareto frontier, where the optimality condition changes from $y'(a) = c'(a)$ to $y'(a) = B'(a)$. Moreover, the compensation scheme is also affected by this shift.

These insights reinforce the importance of reservation utility in incentive alignment, as demonstrated by Guo et al. (2018), where stronger bargaining positions directly impact contract structure.

1.2.1 A Numerical Example Using the Standard Model

In this section, we introduce a numerical example that will be useful to illustrate the equivalence between the approximation described before (Demougin and Helm (2006)) and the ones we will develop next. Now consider, as a particular example, a product's level for the principal given by $y(a) = \sqrt{a}$, which is increasing and concave; an agent's effort cost $c(a) = a^2$, which is increasing and convex; a favorable signal's probability $p(a) = 1 - exp(-a)$, that is strictly increasing, strictly convex, with $p(0) = 0$ and $\lim_{a\to\infty} p(a) = 1$; finally, an expected bonus given by $B(a) = p(a)\frac{c'(a)}{p'(a)} = \frac{2a[1-exp(-a)]}{exp(-a)}$.

This numerical setup reflects common assumptions in contract theory, where the agent's effort cost function is convex, ensuring diminishing returns to effort, and the probability of success is increasing in effort, consistent with models of moral hazard (Chen et al., 2019). Furthermore, the concavity of $y(a)$ aligns with production functions where effort exhibits decreasing marginal returns, reinforcing its role in incentive-compatible contracts (Pitchford, 1998).

We start by checking that the expected bonus, $B(a)$, is strictly increasing. Since $B'(a) = \frac{2[1+a-exp(-a)]}{exp(-a)}$; besides $a > 0$ and $1 > exp(-a) \quad \forall a > 0$, then $1 + a > exp(-a) \quad \forall a > 0$. Finally $1 + a - exp(-a) > 0 \quad \forall a > 0$. In summary, the goal is achieved. Now, we have to check that $B(a)$ is strictly convex. Given that $B''(a) = \frac{2[2+a]}{exp(-a)} > 0$, the evaluation is complete.

The increasing and convex nature of $B(a)$ supports the idea that incentive schemes must account for risk-sharing considerations, as emphasized by Demougin and Helm (2006). When the agent's effort has increasing marginal costs, a well-designed contract must offer stronger incentives to maintain optimal effort levels, aligning with findings from wage bargaining literature (Choe et al., 2014).

The maximization program is given by:

$$V(\bar{u}) = \max_{\{a,F\}} \sqrt{a} - \left[F + \frac{2a[1-exp(-a)]}{exp(-a)}\right] \qquad s.t. \qquad F \geq 0, \;\; F + \frac{2a[1-exp(-a)]}{exp(-a)} - a^2 \geq \bar{u}$$

The Lagrangian function is the following:

$$\mathcal{L} = \sqrt{a} - \left[F + \frac{2a[1 - exp(-a)]}{exp(-a)}\right] + \lambda F + \mu\left[F + \frac{2a[1 - exp(-a)]}{exp(-a)} - a^2 - \bar{u}\right]$$

with F.O.C.s

$$\frac{1}{2\sqrt{a}} - \frac{2[1 + a - exp(-a)]}{exp(-a)} + \mu\left[\frac{2[1 + a - exp(-a)]}{exp(-a)} - 2a\right] = 0,$$

$$-1 + \lambda + \mu = 0.$$

If $\mu = 0$ and $\lambda > 0$ then $B(a) > \bar{u} + c(a)$ and $F = 0$. Therefore, $\lambda = 1$ and $\frac{1}{2\sqrt{a}} - \frac{2[1+a-exp(-a)]}{exp(-a)} = 0$; finally, by this last equation $a^{**} \approx 0.2232$, and evaluating this level of effort a^{**}, we obtain $U^{**} \approx 0.0618$ and $V^{**} \approx 0.3608$. Similarly, if $\lambda = 0$ and $\mu > 0$ thereupon $F > 0$ and $F + B(a) = \bar{u} + c(a)$. Given the above results and the F.O.C. we obtain $\mu = 1$ and $\frac{1}{2\sqrt{a}} - 2a = 0$; finally, $a^{*} \approx 0.3968$, and by evaluating with effort a^{*}, we obtain $U^{*} \approx 0.2291$, $V^{*} = 0.2433$ and $F^{*} \approx 0$. These results are expressed in figure 1.2.

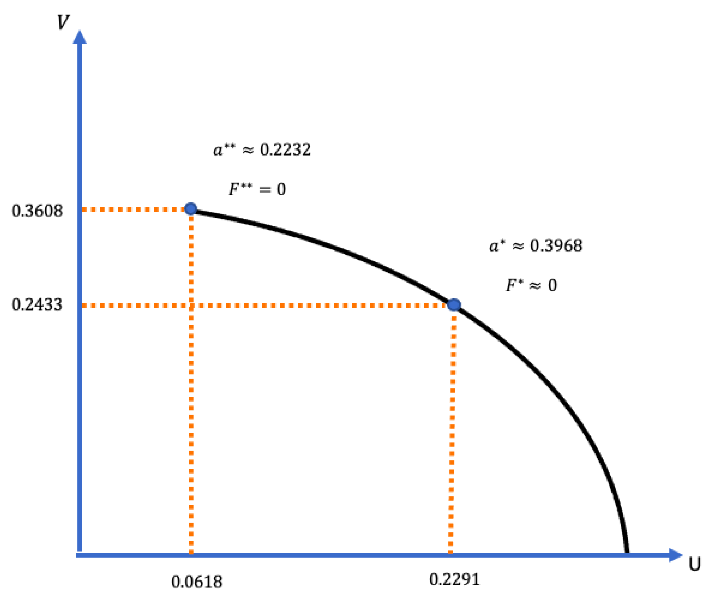

Figure 1.2: Pareto optimal contracts associated with numerical contract schemes

By examining Pareto optimal contracts associated with numerical contract schemes, one can determine the efficient allocation of utilities between a principal and an agent (see Figure 1.2). A higher outside option for the agent shifts the optimal contract along the Pareto Frontier from left to right. This shift reflects a change from a zero fixed payment, where incentives are provided through the expected bonus, to a variable fixed payment scheme.

The shift in the Pareto optimal contract due to changes in reservation utility aligns with empirical findings in contract negotiation, where agents with higher outside options negotiate contracts with reduced reliance on performance-based pay (Pandher and Currie, 2013). This highlights the importance of bargaining power in determining compensation structures.

Thus, understanding the principles of Pareto optimal contracts not only clarifies how outside options influence the choice of incentive schemes but also sets the foundation for analyzing alternative approaches to bargaining in principal-agent models. As highlighted by Guo et al. (2018), bargaining power asymmetries play a crucial role in defining contract efficiency, particularly in environments with information asymmetry. These insights motivate further exploration of bargaining solutions in contract theory.

This leads us to consider the Pareto-Weights (or utilitarian) solution, which provides a distinct perspective by focusing on maximizing the weighted sum of utilities without relying on a predefined disagreement point, which represents the utility each party would receive if no agreement is reached. The Pareto-Weights solution offers an alternative framework that contrasts with traditional bargaining approaches like the Nash bargaining solution.

By exploring alternative solutions such as the Pareto-Weights approach, we explicitly characterize the entire set of efficient bargaining outcomes along the Pareto frontier. As emphasized in cooperative game theory models (Dittrich and Städter, 2014), this perspective allows us to understand how specific bargaining solutions, such as the Nash outcome, arise from particular assumptions or weightings. This approach allows us to compare different contractual arrangements under varying levels of bargaining power, enriching the analysis of principal-agent interactions.

1.3 The Pareto-Weights Solution

However, one of the significant advantages of implementing a Pareto-weights solution (utilitarian solution) is that it does not require establishing a disagreement point. It is important to remember that Nash's solution selects a point on the constrained Pareto frontier where the product of utilities from the disagreement point is maximized (Nash (1950)).

The Pareto-Weights transformation provides a standard approach to solving optimization problems where bargaining power is explicitly incorporated into the objective function. Unlike Nash bargaining solutions, which rely on a disagreement point to determine the final allocation, Pareto-Weights approaches consider a direct convex combination of the principal's and the agent's utilities, allowing for a flexible negotiation framework (Demougin and Helm, 2006). This method is particularly useful in moral hazard settings where risk-sharing and incentive compatibility constraints interact with bargaining dynamics (Chen et al., 2019).

Therefore, a Pareto-Weights solution in the moral hazard model maximizes the convex combination of utilities, parametrized by α, where α represents the bargaining power of the agent, and $1 - \alpha$ represents the bargaining power of the principal.

This formulation aligns with prior work on bargaining power distributions in contractual settings. As noted by Guo et al. (2018), changes in bargaining power influence not only utility allocations but also the structure of incentive mechanisms. Thus, incorporating α as a parameter in the optimization framework extends standard moral hazard models by explicitly considering negotiation power asymmetries.

The associated optimization program is given by:

$$\max_{\{F,a\}} \ \alpha \left[F + B(a) - c(a)\right] \ + \ (1-\alpha)\left[y(a) - F - B(a)\right] \tag{1.6}$$

$$\text{s.t.} \quad F \geq 0, \tag{1.7}$$

where (1.6) denotes the convex combination of the utilities of both the agent and the principal and (1.7) represents the limited liability constraint.

We define the critical bargaining power as the threshold value of α at which the limited liability constraint ceases to bind, marking a shift in the structure of the optimal contract. This threshold determines when the contract transitions from having zero fixed payment to allowing a positive one.

Proposition 1 *Let the principal and agent be risk-neutral. In the context of a Pareto-Weights solution with limited liability and a bonus contract in a static moral hazard model, the critical bargaining power is given by* $\alpha = \frac{1}{2}$.

Proof. Revisiting the above optimization program, the Lagrangian associated with it is described by:

$$\mathcal{L} = \alpha \left[F + B(a) - c(a)\right] + (1 - \alpha) \left[y(a) - F - B(a)\right] + \lambda F,$$

where λ is the Lagrange multiplier associated with the limited liability constraint.

The first-order conditions for effort, a, and fixed payment, F, are given by:

$$\alpha \left[B'(a) - c'(a)\right] + (1 - \alpha) \left[y'(a) - B'(a)\right] = 0, \tag{1.8}$$

$$\alpha + (1 - \alpha)(-1) + \lambda = 0. \tag{1.9}$$

If the limited liability constraint is not binding, then $\lambda = 0$. Therefore, $F > 0$, which corresponds to Region I (see figure (1.1)). Hence, from equation (1.9), we obtain $\alpha = \frac{1}{2}$. ■

The result that $\alpha = \frac{1}{2}$ as a critical value confirms findings from Demougin and Helm (2006), where equal bargaining power leads to an optimal contract allocation balancing incentives and risk-sharing. This value represents a transition point between fixed payments and bonus-driven compensation, illustrating how bargaining power shapes the structure of optimal contracts (Choe et al., 2014).

When the agent and the principal have the same level of bargaining power, this represents the critical value for the optimal solution, where the payment scheme transitions between a fixed payment and a variable payment scheme.

Proposition 2 *Let the principal and agent be risk neutral. A Pareto-weights solution with limited liability and a bonus contract in a static moral hazard model is equivalent to a constrained Pareto frontier in a standard model.*

Proof. First, we suppose that $\lambda > 0$; that is, when the agent's financial constraint is binding, then $F = 0$. Accordingly, we are in Region II (see figure (1.1)). By differentiating equation (1.8) implicitly, $\frac{da}{d\alpha} > 0$, so a variation in α implies a variation in the effort and consequently in the contract. Thus, as the agent's bargaining power increases, the level of effort also increases. Consequently, the payment mechanism relies on the expected bonus, as discussed in Section 1.2. On the other hand, if $\lambda = 0$; the best solution is achieved for $\alpha \in \left[\frac{1}{2}, 1\right)$, Region I (see figure (1.1)). Specifically, when $\alpha = \frac{1}{2}$ and referring to equation (1.8), we obtain $y'(a) = c'(a)$, which is the same optimality condition obtained in the standard model, in Section 1.2, and the Nash-Bargaining solution (see Demougin and Helm (2006)). ■

This result aligns with empirical findings from labor contract theory, where stronger bargaining power for the agent results in contracts with higher effort levels and greater expected earnings (Pandher and Currie, 2013). Such an effect is particularly relevant in executive compensation models, where equity incentives play a central role in aligning interests.

This equivalence between the Pareto-Weights solution and the constrained Pareto frontier confirms that both approaches yield the same efficient outcomes. However, the Pareto-Weights method offers a computationally convenient alternative, as it does not rely on a predefined disagreement point. Instead, it allows the analyst to explore the entire frontier of efficient contracts by varying the bargaining weight parameter α, which provides a more tractable and flexible tool for analyzing negotiation settings under frictions like limited liability (Dittrich and Städter, 2014).

Indeed, this approach yields the same set of efficient outcomes as the constrained Pareto frontier. What makes the Pareto-Weights solution particularly useful is that the critical bargaining threshold is constant, $\alpha = \frac{1}{2}$, regardless of parameter values or functional form. In this sense, the method is robust: it provides a consistent and tractable way to characterize the entire Pareto frontier, including Regions I and II, as well as the transition point in the payment structure.

The generality and consistency of the Pareto-Weights approach reinforce its applicability in economic modeling, particularly in settings where bargaining power asymmetries shape contract outcomes. Its tractability makes it a valuable tool beyond moral hazard, extending to broader negotiation environments such as venture capital agreements and financial contracting (Balkenborg, 2001).

1.3.1 A Numerical Example Using Pareto-Weights Model

As in the previous section (1.2.1), we now consider specific functions as a particular example to analyse the convex combination of utilities, where $\alpha \in (0, 1)$ represents the bargaining power. This is explored through the following optimization program:

$$\max_{\{a,F\}} \quad \alpha \left[F + \frac{2a[1 - exp(-a)]}{exp(-a)} - a^2\right] + (1-\alpha)\left[\sqrt{a} - F - \frac{2a[1 - exp(-a)]}{exp(-a)}\right]$$

s.t. $\quad F \geq 0$ with the associated Lagrange function:

$$\mathcal{L} = \alpha \left[F + \frac{2a[1 - exp(-a)]}{exp(-a)} - a^2\right] + (1-\alpha)\left[\sqrt{a} - F - \frac{2a[1 - exp(-a)]}{exp(-a)}\right] + \lambda F$$

whose F.O.C. are given by, for effort a:

$$\alpha \left[\frac{2[1 + a - exp(-a)]}{exp(-a)} - 2a\right] + (1-\alpha)\left[\frac{1}{2\sqrt{a}} - \frac{2[1 + a - exp(-a)]}{exp(-a)}\right] = 0, \qquad (1.10)$$

and for fixed payment, F:

$$\alpha + (1 - \alpha) + \lambda = 0. \qquad (1.11)$$

This formulation builds upon the fundamental principles of bargaining power in contractual settings. As shown by Guo et al. (2018), differences in bargaining power influence contract outcomes significantly, impacting both incentive structures and surplus distribution. The use of α as a bargaining power parameter reflects real-world negotiations where asymmetric information and risk-sharing considerations dictate contract terms.

In the case where $\lambda = 0$, $F > 0$, and by (1.11) we get $\alpha = \frac{1}{2}$. Finally, by (1.10) and the above result for α, we obtain $a^* = \frac{1}{2\sqrt[3]{2}} \approx 0.3968$. These results are consistent with those of the standard model.

The critical value $\alpha = \frac{1}{2}$, where the principal and the agent hold equal bargaining power, is a common equilibrium point in contract theory. Similar results are observed in wage negotiations and supply chain agreements, where balanced bargaining positions often lead to stable contract structures (Choe et al., 2014; Pandher and Currie, 2013). The fact that this equilibrium arises naturally from the model highlights the robustness of Pareto-weighted solutions in economic analysis.

These results confirm the consistency with the standard model, where fixed parameters yield specific outcomes for bargaining power and utility distribution. However, this model, while providing a foundational understanding, limits the scope of analysis. As noted in Dittrich and Städter (2014), rigid contract structures often fail to capture dynamic bargaining processes, leading to suboptimal solutions in real-world negotiations. Particularly, in real-world settings, the standard model's fixed reserve utility levels do not fully capture the essence of bargaining power and compromise. This necessitates exploring alternative approaches that offer a more comprehensive perspective.

A key limitation of the standard approach is its assumption that bargaining power remains fixed. In reality, contract negotiations evolve dynamically, where external factors such as market conditions, institutional regulations, and individual bargaining histories shape final outcomes (Aghion and Tirole, 1994). This suggests the need for a more flexible framework that better captures these complexities.

While the Pareto-Weights and standard models are equivalent in terms of the efficient outcomes they generate, the Pareto-Weights framework offers a more flexible and tractable representation of bargaining. In particular, it allows for a continuous parametrization of bargaining power through α, enabling the analyst to trace the entire Pareto frontier without solving a new program for each utility configuration. This makes it especially suitable for studying environments with evolving external constraints or where comparative statics play a central role.

This perspective motivates the exploration of alternative bargaining frameworks that explicitly incorporate fairness or compromise considerations beyond pure efficiency.

One such approach is the compromise solution, which extends beyond the classical Nash bargaining framework to address these complexities. The compromise solution incorporates a more adaptable negotiation process, recognizing that optimal contracts may shift based on evolving constraints and trade-offs (Balkenborg, 2001). This broader perspective allows for a richer analysis of contract formation, particularly in cases where one party holds temporary bargaining advantages or faces liquidity constraints.

1.4 The Compromise Solution

At first, in this section, it is relevant to consider this motivation: "The primary argument for the canonical bargaining problem is that faced by management and labor in the division of a firm's profit" (Thomson (1994)).

This limitation arises because a fixed reserve utility level constrains the ability to analyze how bargaining power influences surplus allocation. Rather than treating the agent's reservation utility as an exogenous parameter, it should be considered an endogenous outcome of the negotiation process. This perspective aligns with empirical observations where bargaining power fluctuations impact contractual terms (Guo et al., 2018). When we obtain the complete characterization of the Pareto solution, it becomes essential to talk about social criterion or social equilibrium called compromise solutions.

The social solution criterion is a change of perspective with respect to other criteria since it focuses on asking what is the minimum distance with respect to a point at which we are all better off, unlike the Nash solution in which we do not ask what is the maximum distance from the disagreement point.

This distinction is crucial in bargaining theory, as it shifts the focus from maximizing individual gains to ensuring a more balanced allocation of utilities. Unlike Nash bargaining, which prioritizes efficiency by maximizing the product of utility gains, the compromise approach emphasizes proportional fairness, which has been widely explored in cooperative game theory (Balkenborg, 2001).

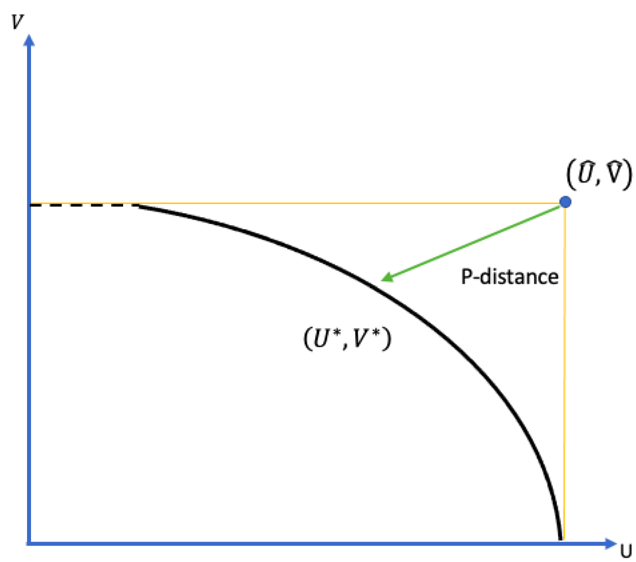

Figure 1.3: Pareto frontier and utopia point

Similarly to Nash bargaining solution as the bargaining process, the compromise approach requires a reference point, the ideal point; however, the ideal point is an endogenous point, unlike the disagreement point which is exogenous. Due to the disagreement point being exogenous, the drawback is that we must determine it in a subjective way. Let $(\hat{U}, \hat{\Pi})$ be the ideal point; where $\hat{U}$ $(\hat{\Pi})$ is the maximum utility that the agent (or the principal) can achieve on the Pareto frontier given a feasible contract arrangement. In particular, a contract (U^{*}, V^{*}) is the *p-Yu* solution if the distance between the ideal point and the Pareto frontier associated with this contract given a p-distance is minimal, see Figure (1.3). That is,

$$(U^*, V^*) = \arg\min_{\{U,V\}} \ [\ (\hat{U} - U)^p + (\hat{V} - V)^p\ ]^{1/p}.$$

This framework introduces a more flexible alternative to traditional bargaining models. As noted by Choe et al. (2014), the ideal point approach allows contract outcomes to be determined endogenously based on evolving negotiation dynamics rather than being restricted by predefined disagreement points.

In particular, the p parameter in the p-distance becomes the bargaining power.

Proposition 3 *Let the principal and the agent be risk neutral. A p-Yu solution with limited liability and bonus contract into moral hazard model is equivalent to the constrained Pareto frontier in a standard model, with critical bargaining power* $p = 1$.

Proof. The optimal program for the *p-Yu* solution is given by

$$\min_{a,F} \ \left\{\left[\hat{U} - [F + B(a) - c(a)]\right]^p + \left[\hat{V} - [y(a) - F - B(a)]\right]^p \ \right\}^{1/p}$$

s.t.

$$F \geq 0,$$

therefore the associated Lagrangian is

$$\mathcal{L} = \left\{ \ \left[\hat{U} - F - B(a) + c(a)\right]^p + \left[\hat{V} - y(a) + F + B(a)\right]^p \ \right\}^{1/p} + \ \lambda F$$

from which follow the first-order conditions, for a, we have:

$$\frac{1}{p}\left[\ \left(\hat{U} - F - B(a) + c(a)\right)^p + \left[\hat{V} - y(a) + F + B(a)\right]^p \ \right]^{1/p-1}$$

$$\left\{\ p\left[\hat{U} - F - B(a) + c(a)\right]^{p-1}[-B'(a) + c'(a)] + p\left[\hat{V} - y(a) + F + B(a)\right]^{p-1}[-y'(a) + B'(a)]\ \right\} = 0;$$

for F, we obtain:

$$\frac{1}{p}\left[\left(\hat{U}-F-B(a)+c(a)\right)^{p}+\left(\hat{V}-y(a)+F+B(a)\right)^{p}\ \right]^{1/p-1}$$
$$\left[p\left[\hat{U}-F-B(a)+c(a)\right]^{p-1}(-1)+p\left[\hat{V}-y(a)+F+B(a)\right]^{p-1}+\lambda\right]=0$$

which can be rewritten, with no loss of solutions, as:

$$p\left[\hat{U}-F-B(a)+c(a)\right]^{p-1}[-B'(a)+c'(a)]+p\left[\hat{V}-y(a)+F+B(a)\right]^{p-1}[-y'(a)+B'(a)]=0; \tag{1.12}$$

$$p\left[\hat{U}-F-B(a)+c(a)\right]^{p-1}(-1)+p\left[\hat{V}-y(a)+F+B(a)\right]^{p-1}+\lambda=0. \tag{1.13}$$

If $\lambda = 0$, limited liability is not binding, then $F > 0$ because of complementary slackness. We analyze the first-best solution. From (1.13), we obtain $\hat{U}-F-B(a)+c(a) = \hat{V}-y(a)+F+B(a)$; therefore, substituting into (1.12), we get $y'(a) = c'(a)$, the same optimal condition obtained in the previous section for the standard model.

This result confirms that when limited liability is not binding, the bargaining outcome coincides with the first-best solution, reinforcing the equivalence between the compromise and standard models under these conditions (Demougin and Helm, 2006).

On the other hand, since $\hat{U} - F - B(a) + c(a) = \hat{V} - y(a) + F + B(a)$, this can also be rewritten as:

$$d_1[\,\hat{U}\,, F+B(a^*)-c(a^*)\,] = d_1[\,\hat{V}\,,\ y(a^*)-F-B(a^*)\,]$$

where a^* comes from the optimal condition.[2]

Since the distances between the utopia point and the utility must be the same for both the agent and the principal, under this 1-metric (absolute metric), the optimal solution is equivalent to the regular *Yu* solution (see figure 1.4).

[2] d_1 is the absolute difference between two real numbers since the ideal point corresponds to the maximum utility values.

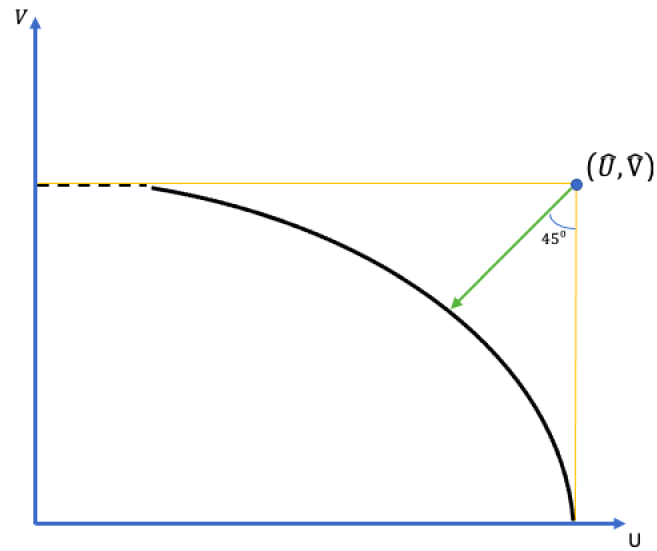

Figure 1.4: Pareto frontier and utopia point

In the case where limited liability is binding, i.e., $\lambda > 0$, then $F = 0$. Therefore, the first-order conditions become:

$$p\left[\hat{U} - B(a) + c(a)\right]^{p-1}\left[-B'(a) + c'(a)\right] + p\left[\hat{V} - y(a) + B(a)\right]^{p-1}\left[-y'(a) + B'(a)\right] = 0; \tag{1.14}$$

$$\left[p(\hat{U} - B(a) + c(a))^{p-1}(-1) + p(\hat{V} - y(a) + B(a))^{p-1} + \lambda\right] = 0. \tag{1.15}$$

These equations (1.14) and (1.15) describe a scenario where limited liability restricts the feasible contract set, shifting the bargaining outcome away from the first-best solution. As emphasized by Pitchford (1998), rent extraction by the agent in constrained settings alters the optimal contract structure, necessitating adjustments in incentive mechanisms. From (1.14) and applying implicit differentiation, we get:

$$\frac{da}{dp} > 0.$$

This result indicates that as bargaining power increases, so does the agent's effort level. This aligns with findings from contract theory, where stronger negotiation positions lead to higher effort provisions and improved contractual efficiency (Guo et al., 2018). ■

1.4.1 A Numerical Example Using the *P-Yu* Solution as Compromise Solution

As in the previous sections, consider the same specific functions whose minimization program is given by:

$$\min_{\{a,F\}} \left[\left(\hat{U} - \left[F + \frac{2a[1-exp(-a)]}{exp(-a)} - a^2 \right] \right)^p + \left(\hat{V} - \left[\sqrt{a} - F - \frac{2a[1-exp(-a)]}{exp(-a)} \right] \right)^p \right]^{1/p}$$

$$\text{s.t.} \qquad F \geq 0.$$

This optimization problem seeks to minimize the p-distance between the ideal utility point and the feasible allocation on the Pareto frontier, adjusting bargaining outcomes accordingly. By incorporating a flexible reference point, this formulation generalizes standard moral hazard models and allows for a richer characterization of bargaining dynamics (Balkenborg, 2001).

The Lagrange function is given by:

$$\mathcal{L} = \left[\left(\hat{U} - \left[F + \frac{2a[1-exp(-a)]}{exp(-a)} - a^2 \right] \right)^p + \left(\hat{V} - \left[\sqrt{a} - F - \frac{2a[1-exp(-a)]}{exp(-a)} \right] \right)^p \right]^{1/p} + \lambda F$$

with the simplified first-order conditions (F.O.C.):

$$\left(\hat{U} - \left[F + \frac{2a[1-exp(-a)]}{exp(-a)} - a^2 \right] \right)^{p-1} \left(-\frac{2[1+a-exp(-a)]}{exp(-a)} + 2a \right)$$
$$+ \left(\hat{V} - \left[\sqrt{a} - F - \frac{2a[1-exp(-a)]}{exp(-a)} \right] \right)^{p-1} \left(-\frac{1}{2\sqrt{a}} + \frac{2[1+a-exp(-a)]}{exp(-a)} \right) = 0$$

$$\left[\left(\hat{U} - \left[F + \frac{2a[1-exp(-a)]}{exp(-a)} - a^2 \right] \right)^p + \left(\hat{V} - \left[\sqrt{a} - F - \frac{2a[1-exp(-a)]}{exp(-a)} \right] \right)^p \right]^{1/p-1} *$$
$$\left[\left(\hat{U} - \left[F + \frac{2a[1-exp(-a)]}{exp(-a)} - a^2 \right] \right)^{p-1} + \left(\hat{V} - \left[\sqrt{a} - F - \frac{2a[1-exp(-a)]}{exp(-a)} \right] \right)^{p-1} \right] + \lambda = 0$$

These conditions characterize the optimal contract under the compromise solution approach. Unlike standard principal-agent models, where contract parameters are typically derived from exogenously set disagreement points, this framework adapts the allocation dynamically in response to bargaining power variations (Guo et al., 2018).

When $\lambda = 0$, then $F > 0$ (first-best segment). Therefore, from the F.O.C. with respect to F, we get:

$$\hat{U} - \left[F + \frac{2a[1 - exp(-a)]}{exp(-a)} - a^2\right] = \hat{V} - \left[\sqrt{a} - F - \frac{2a[1 - exp(-a)]}{exp(-a)}\right]$$

This equality implies that the optimal contract allocation balances the deviation from the ideal point between both players. This property makes the p-distance minimization approach particularly useful for studying negotiation frameworks that seek fairness while maintaining incentive compatibility (Dittrich and Städter, 2014).

Substituting this last expression into the F.O.C. with respect to a and simplifying, we obtain:

$$\left(\hat{U} - \left[F + \frac{2a[1 - exp(-a)]}{exp(-a)} - a^2\right]\right)\left[2a - \frac{1}{2\sqrt{a}}\right] = 0$$

This result indicates that, under equal bargaining weights, the effort level is adjusted to equate the marginal productivity of effort to the marginal disutility of exertion. This mirrors the first-best effort level found in classical agency models but within a framework that accounts for endogenous bargaining power (Demougin and Helm, 2006).

1.5 Conclusions

This chapter extends the standard principal-agent framework by incorporating bargaining power in a moral hazard setting with limited liability and a bonus contract. Unlike traditional models, which assume that the principal unilaterally dictates contract terms, our approach explicitly accounts for the distribution of bargaining power between the principal and the agent. This refinement allows for a more realistic representation of contractual relationships, where both parties may influence the design of incentives.

A key contribution of this study is the characterization of the *constrained Pareto frontier* under different bargaining power distributions. We analyze two distinct approaches, the *Pareto-Weights solution* and the *compromise solution*, and show that, despite their conceptual differences, they yield equivalent allocations along the Pareto frontier. In particular, we show that the compromise solution is equivalent to the standard model's Pareto frontier and captures the same optimal contracts under a fairness criterion based on distance minimization. This equivalence highlights the generality and interpretive strength of these methodologies in identifying optimal contracts that balance incentive compatibility and surplus distribution.

Our results highlight a *critical threshold of bargaining power,* $\alpha = \frac{1}{2}$, where the contract structure undergoes a fundamental shift. This threshold is independent of parameter values or specific functional forms, indicating a universal property of the model. When bargaining power is evenly split, the compensation scheme transitions from a *bonus-based* structure to a *fixed-payment* regime, aligning incentives in a manner consistent with an egalitarian distribution of surplus. Similarly, in the compromise solution, we identify a critical point at $p = 1$ where the limited liability constraint ceases to bind, and the optimal contract reaches the first-best. This result strengthens the robustness of the equivalence across frameworks.

Beyond its theoretical implications, this study provides a foundation for understanding how bargaining power influences contract efficiency in various economic contexts, including *labor markets, corporate governance, and supply chain negotiations*. The findings suggest that incorporating bargaining power asymmetries into contract theory not only refines existing models but also offers practical insights into the design of incentive schemes in real-world settings.

Chapter 2

Pareto Weights with Hidden Actions Model

2.1 Introduction

Traditional agency models often assume that the principal unilaterally designs take-it-or-leave-it contracts for the agent, as seen in (Holmström, 1979) and (Grossman and Hart, 1983b). However, real-world situations are rarely so straightforward, and a more realistic approach considers varying degrees of bargaining power between the principal and the agent. The principal does not always hold all the bargaining power. For example, negotiations between firms and labor unions over contract terms, bilateral monopoly situations, or one-on-one principal-agent relationships often reflect scenarios where both parties have some degree of bargaining power (Yao, 2012).

Several studies depart from the full-control assumption and instead analyze how bargaining power shapes contract outcomes. For instance, (Innes, 1990) introduces a debt model where the agent operates under limited liability, while (Pitchford, 1995) extends this framework to examine the indirect liability of a lender, positioning the borrower as the agent. In both cases, the agent holds full bargaining power during contract negotiations, mirroring the dynamics of a competitive capital market. In contrast, (Aghion and Tirole, 1994) explores extreme distributions of bargaining power in research and development settings, showing how such asymmetries affect ownership structures. The role of bargaining power in incentive structures, particularly when the agent is wealth-constrained, has become a central theme in growth and development economics (Dam and Ruiz Pérez, 2012).

This chapter builds on the analysis of bargaining power and outside options (reservation utilities) in incentive contracts. We emphasize how the distribution of bargaining power between the manager and the owner determines the characteristics of the optimal incentive contract. Unlike traditional models that primarily consider the agent's reservation utility, we incorporate the owner's reservation utility as well, leading to contracts that are more adaptable to different economic environments.

Bargaining power in principal-agent relationships plays a crucial role in shaping economic interactions beyond theoretical modeling. Power asymmetries influence corporate governance, labor markets, and strategic decision-making. Empirical evidence suggests that unequal negotiation leverage often leads to suboptimal contracts that favor stronger parties at the expense of overall welfare. The following examples illustrate how bargaining power shapes economic outcomes:

- **CEO Compensation:** The CEO Pay Slice (CPS), which measures the percentage of total executive compensation received by the CEO, reflects bargaining power (Choe et al., 2014). A higher CPS corresponds to greater influence over remuneration, often at the expense of shareholder interests.

- **Influence on the Board of Directors:** CEOs with significant bargaining power can shape board appointments, affecting governance structures and the objectivity of compensation negotiations (Bebchuk et al., 2002).

- **Strategic Decisions:** CEOs with extensive bargaining power can implement controversial yet profitable strategies, such as offshoring production, despite resistance from stakeholders (Pandher and Currie, 2013). Their professional networks further enhance their influence (Saidu, 2018).

- **Labor Negotiations:** Unions negotiate wages and benefits based on their bargaining power, often leading to industry-wide agreements that shape broader economic structures (Aust and Buscher, 2014).

- **Supply Chain Bargaining:** The negotiation leverage between manufacturers and retailers influences contract terms, profit-sharing agreements, and market efficiency (Feng and Lu, 2013).

Bargaining power also influences contract dynamics across various fields, including insurance markets (Kihlstrom and Roth, 1982), wage bargaining (Laroque and Salanié, 2004), and moral hazard situations (Bental and Demougin, 2010; Demougin and Helm, 2011). Multiple models demonstrate that contract distortions are primarily driven by imbalances in bargaining power, as illustrated by frameworks based on Nash bargaining, Kalai-Smorodinsky solutions, and generalized Nash approaches (Bayindir-Upmann and Gerber, 2003; Edmans et al., 2017).

One of the core contributions of this paper is the full characterization of all feasible contracts along the Pareto frontier of a static principal-agent model, varying with reservation utility. Extending the analysis by (Demougin and Helm, 2006), we show that the optimal contract depends not only on bargaining power but also on outside options, significantly influencing payoffs and incentive schemes. See also (Curiel-Cabral et al., 2022) for a related discussion on the role of risk aversion, reservation utility, and bargaining power in incentive contract design.

Along the Pareto frontier, efficient bargaining contracts can be defined using various approaches, such as Nash bargaining and utilitarian solutions. We highlight this early because these solution concepts provide a benchmark to understand the structure and diversity of contracts along the frontier. The disagreement point plays a central role, serving as an external reference for contract negotiation and design. In contexts where both the outside option and bargaining power are relevant (such as in the case of star athletes, tenured professors, or top executives) contracts are typically shaped by a combination of bargaining position and alternative offers.

We introduce a Pareto weights model with hidden actions, incorporating the figure of the Board as the entity responsible for contract design. The Board's utility function is a convex combination of the manager's and owner's utilities, weighted by their respective bargaining power. The Board acts as a neutral party with no particular interest in supporting either side. The Pareto weights programme is constrained by the manager's incentive compatibility and the individual rationality conditions for both the manager and the owner.

This modeling approach fills a gap in the literature by fully characterizing contracts along the Pareto frontier in a principal-agent framework with hidden actions and bargaining power asymmetries. Unlike traditional models that focus on specific bargaining solutions such as Nash or Kalai-Smorodinsky, our analysis provides a more flexible structure that explicitly accounts for variations in bargaining power and outside options. This allows us to capture a broader range of contractual distortions and strategic interactions, offering insights into how incentive schemes evolve under different negotiation dynamics.

A central finding of our analysis is that the optimal contract exhibits distinct features depending not only on the level of bargaining power but also on the specific combination of the agent's and the principal's outside options. Based on this interaction, we fully characterize the set of feasible contracts along the Pareto frontier in a principal-agent model with hidden actions. This characterization allows us to identify three distinct regions: when the owner has both greater bargaining power and a better outside option, the contract minimizes base compensation and relies primarily on performance-based incentives; when the manager holds more power and a superior outside option, the contract structure shifts toward higher fixed salaries and reduced reliance on variable pay. Finally, we identify an intermediate region in which neither party dominates. In this region, contracts tend to balance fixed and variable components more equitably, resulting in more stable and efficient incentive schemes.

Moreover, our analysis reveals that the effects of risk aversion and output uncertainty on incentive structures are not always straightforward. In particular, these effects vary depending on the bargaining regime and which individual rationality constraints are binding.

To gain further analytical insight, we later examine a particular environment with linear output, quadratic effort cost, and constant risk. This special case allows us to obtain closed-form solutions that clarify how bargaining power and outside options shape contract design across the different regions of the Pareto frontier.

The rest of the chapter is organized as follows: Section 2.2 outlines the model. Section 2.3 discusses the characterization of the solution to the game, while Section 2.4 focuses on the implications within a specific contractual environment. Finally, Section 2.5 concludes the paper.

2.2 The Model

We consider a static game that integrates agency theory, managerial power theory, and Nash bargaining, accounting for hidden actions. Bargaining power between the manager and the owner plays a central role in the design of the compensation structure. The firm consists of a risk-neutral owner with a lifetime expected utility denoted by $\bar{V}$, and a risk-averse manager with a lifetime expected utility denoted by $\bar{U}$. The manager's preferences are characterized by a Constant Absolute Risk Aversion (CARA) utility function, given by $U(w, e) = -\exp[-r(w - c(e))]$ where r is the coefficient of constant absolute risk aversion, w denotes the monetary payoff, and $c(e)$ represents the strictly convex monetary cost of exerting effort, with $c'(e) > 0$ and $c''(e) > 0$. The convexity of $c(e)$ is justified by two key factors (Martinez-Gorricho and Sanchez Villalba, 2021). First, the manager experiences *i*ncreasing disutility of effort, meaning that sustaining higher levels of effort becomes progressively more costly, which aligns with standard models of effort aversion. Second, the production process often exhibits *d*iminishing marginal returns to effort, implying that additional effort contributes less and less to output, making it increasingly expensive to achieve marginal gains in performance. The owner, observing only the firm's stock price but not the manager's effort, must design an optimal contract under these constraints.

The output is represented as $y(e) = \bar{y}(e) + \varepsilon$, where $\bar{y}(e)$ denotes the mean output for a given effort level e, and ε represents an error term with $\varepsilon \sim N(0, \sigma(e))$ (Murphy, 1999). Note that managerial effort affects the variance of the error term, reflecting the stochastic relationship between the firm's stock price and the manager's effort. We assume that $\bar{y}'(e) > 0$ and $\bar{y}''(e) \leq 0$, meaning that the mean output increases as effort increases, but at a decreasing or constant rate. Furthermore, we assume that $\sigma'(e) \leq 0$ and $\sigma''(e) \geq 0$, indicating that effort reduces uncertainty, though with diminishing returns to this reduction (Choe et al., 2014). Together, these conditions describe a scenario where increased effort enhances productivity and reduces risk, but with progressively smaller gains in risk reduction. These assumptions are crucial for ensuring the existence of an optimal effort level and an overall optimal solution.

In this model, we employ linear compensation schemes for the manager, as suggested by (Holmstrom and Milgrom, 1987), who notes that linear contracts are commonly observed in real-world scenarios. The manager's compensation is given by $w(y) = ay + b$, where $a \in [0, 1]$ represents the share of output allocated to managerial compensation, and b is the base compensation. Assuming normality, the manager's expected utility can be expressed as $\mathbb{E}[U(w(y), e)] = a\bar{y}(e) + b - c(e) - \frac{1}{2}ra^2\sigma(e)$, while the owner's expected utility is $\mathbb{E}[V(y, w(y))] = (1-a)\bar{y}(e) - b$.

The third player in the game is the Board. Its payoff is defined as a convex combination of the manager's and owner's expected utilities, consistent with its role as a neutral contract designer. Specifically, the Board maximizes $PW = \rho\mathbb{E}[U(w(y), e)] + (1-\rho)\mathbb{E}[V(y, w(y))]$, where $\rho \in [0, 1]$ denotes the bargaining power of the manager. This structure reflects the Board's balancing role between both parties' interests and is consistent with real-world observations where Boards are expected to mediate between executive demands and shareholder expectations (Choe et al., 2014; Bebchuk et al., 2002).

There are three players in this game: the Board, the manager (M), and the owner (O). We begin by describing the sequential game model, represented in Figure 2.1. The Board determines the compensation scheme, which depends on the output or realization level, and recommends the level of effort to be exerted during the contract. Essentially, the Board is responsible for creating the contract scheme and acts as a neutral party with no particular interest in supporting either side. Subsequently, both the manager and the owner must separately decide whether to sign the contract. If either party decides not to sign, the game ends, and the payoffs are $(0, \bar{U}, \bar{V})$. If both parties decide to sign, the manager then selects the level of effort to exert based on the agreed compensation scheme. Finally, nature determines the stock price.

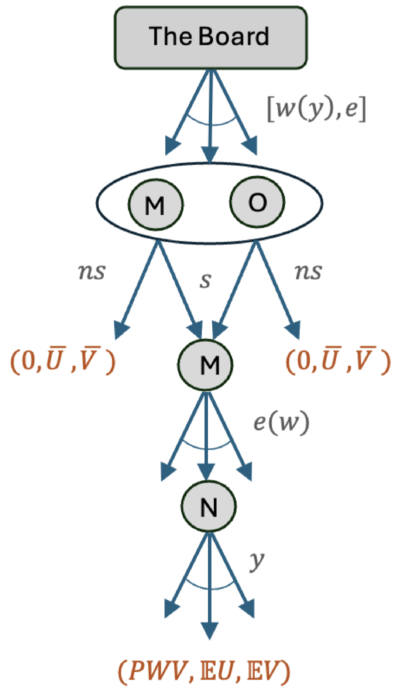

Figure 2.1: Timing of the game

At the final stage of the game, the payoffs are represented as $(PWV, \mathbb{E}[U], \mathbb{E}[V])$, where PWV denotes the Pareto weights value, which we define below, given $\rho \in [0, 1]$ and the expected lifetime utilities. Here, ρ represents the exogenous bargaining power of the manager. We adopt a Pareto weights structure because it is equivalent to models based on variations in reservation utilities, such as the principal-agent model or Managerial model, while also allowing for the introduction of a bargaining parameter. Additionally, $\mathbb{E}[U]$ denotes the manager's expected utility, and $\mathbb{E}[V]$ denotes the owner's expected utility. Consequently, through backward induction, a Pareto weights problem with hidden actions arises, formulated as:

$$\max_{\{a,b,e\}} \quad \rho\, \mathbb{E}U(a, b, e) + (1 - \rho)\, \mathbb{E}V(y(e), a, b) \quad \text{s.t.}$$

$$e = \arg\max \mathbb{E}U(a, b, e), \tag{2.1}$$

$$\mathbb{E}U(a, b, e) \geq \bar{U}, \tag{2.2}$$

$$\mathbb{E}V(y(e), a, b) \geq \bar{V}, \tag{2.3}$$

$$\{a, b\} \geq 0, \tag{2.4}$$

where (2.1) describes the managerial incentive compatibility constraint, (2.2) and (2.3) represent the individual rationality constraints for the manager and the owner, and (2.4) establishes the limited liability constraint (Macho-Stadler and Pérez-Castrillo, 2001).

Let $\varsigma(\rho, \bar{U}, \bar{V}) = \{a^*(\rho, \bar{U}, \bar{V}), b^*(\rho, \bar{U}, \bar{V}), e^*(\rho, \bar{U}, \bar{V})\}$ represent the optimal contract scheme that solves the Pareto weights program with hidden actions, as faced by the Board when the manager's bargaining power is ρ. Then, $PWV(\rho, \bar{U}, \bar{V})$ is the value function of the Board's program, defined as:

$$PWV(\rho, \bar{U}, \bar{V}) = \rho \, \mathbb{E}U(a^*, b^*, e^*) + (1 - \rho) \, \mathbb{E}V(y(e^*), a^*, b^*)$$

where PWV denotes the Pareto weights value for a fixed level of the manager's bargaining power ρ, and $1 - \rho$ reflects the owner's bargaining power. It is important to note that when $\rho = 0$, the optimal contract mirrors the solution of the principal-agent model with hidden actions and limited liability. In this case, the manager's individual rationality constraint is active, as the owner holds all the power in the contract negotiation. Conversely, when $\rho = 1$, the solution follows the managerial theory. Here, the owner's individual rationality constraint is active, as the manager controls the contract terms.

Understanding the structure of the optimal contract under different bargaining power and outside options regimes is crucial for determining how incentives and compensation evolve in response to shifts in the economic environment. Contracts that allocate decision-making power differently can lead to significant variations in managerial incentives, risk-sharing mechanisms, and overall firm performance. By characterizing the equilibrium along the Pareto frontier, we can distinguish between contractual arrangements that emphasize managerial incentives, where the manager has significant bargaining power, and those that reinforce owner control, where compensation structures favor risk minimization and lower agency costs.

2.3 Characterization of the Solution of the Game

In this section, we provide a detailed characterization of the equilibrium solution of the game. The solution is derived from the optimization of the contract scheme,

$$\varsigma(\rho, \bar{U}, \bar{V}) = \left\{a^*(\rho, \bar{U}, \bar{V}), b^*(\rho, \bar{U}, \bar{V}), e^*(\rho, \bar{U}, \bar{V})\right\},$$

which maximizes the Pareto weights problem under conditions of hidden actions.

Proposition 4 . *If the manager's expected utility is given by*

$$\mathbb{E}[U(w(y), e)] = a\bar{y}(e) + b - c(e) - \frac{1}{2}ra^2\sigma(e),$$

then the first-order condition for managerial incentive compatibility (Rogerson, 1985) is

$$a\bar{y}'_e(e) - c'_e(e) - \frac{1}{2}ra^2\sigma'_e(e) = 0. \tag{2.5}$$

Furthermore, the optimal effort, e^*, which is derived through Rogerson's condition, increases with respect to the share of the output, a.

Proof: See Appendix A.1.

This condition reflects the classic trade-off between providing incentives and ensuring risk-sharing in the contract. The term $\frac{1}{2}ra^2\sigma'(e)$ captures how the agent's risk aversion interacts with the variability of output, illustrating the tension between incentivizing effort and limiting risk exposure (see (Holmström, 1979; Grossman and Hart, 1983a; Rogerson, 1985)). In particular, when output becomes riskier, the agent demands stronger incentives to maintain effort, making the optimal contract highly sensitive to both the agent's risk aversion and the nature of output uncertainty.

Additionally, this first-order condition reveals how the optimal effort level depends on the contract's structure. The left-hand side represents the marginal benefit of effort in terms of increased expected output, while the right-hand side captures the marginal cost of exerting effort, adjusted for the risk-sharing constraint. The presence of the term $a^2\sigma'_e(e)$ shows that higher risk or stronger risk aversion diminishes incentives, reinforcing the necessity of an appropriate balance between fixed and variable compensation. This insight is consistent with the findings of (Holmström, 1979), who demonstrated that optimal contracts must trade off risk-sharing against incentive compatibility.

We apply the first-order approach (Rogerson, 1985) because it facilitates analysis by identifying and simplifying relationships between variables, thus optimizing decision-making through a more accessible mathematical framework. This approach is widely used in agency models with moral hazard, particularly in settings where the principal cannot directly observe effort but must rely on stochastic performance indicators (see (Holmström, 1979; Grossman and Hart, 1983a)).

According to Proposition 4, this approach allows us to substitute the managerial incentive compatibility constraint (2.1) with its first-order condition (2.5), enabling us to define the optimal effort as a function of the share of output, $e^*(a)$, by solving equation (2.5). Since we only consider the limited liability constraint for the base compensation, the program can be reformulated as follows:

$$\max_{\{a,b\}} \left[\rho \mathbb{E}U(a, b, e^*(a)) + (1-\rho)\mathbb{E}V(y(e^*(a)), a, b)\right] \tag{2.6}$$

subject to:

$$\mathbb{E}U(a, b, e^*(a)) \geq \bar{U},$$

$$\mathbb{E}V(y(e^*(a)), a, b) \geq \bar{V},$$

$$b \geq 0.$$

Since there are two cases, when the limited liability constraint is binding and when it is not, we can characterize two regions of the Pareto frontier corresponding to the Pareto weights (see Figure 2.2). We do not consider the individual rationality constraints, as our aim is to achieve a complete characterization of the Pareto frontier.

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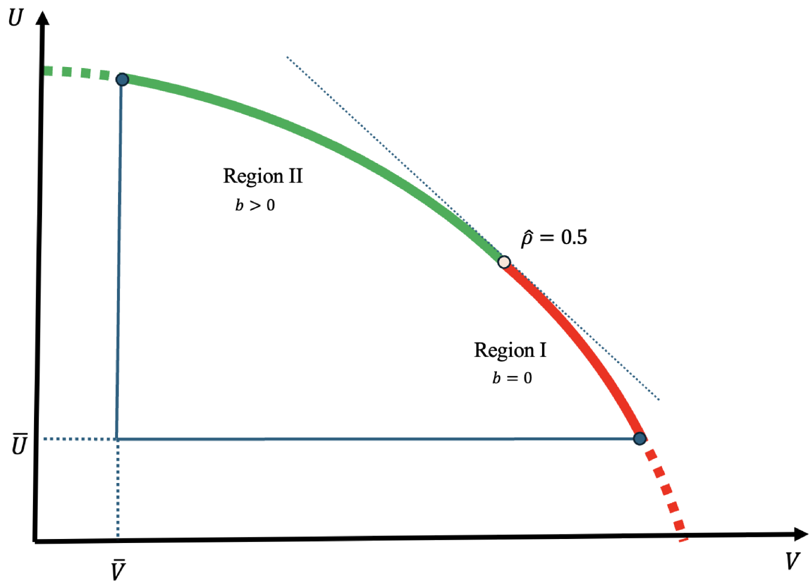


Figure 2.2: The Pareto frontier

2.2). We do not consider the individual rationality constraints, as our aim is to achieve a complete characterization of the Pareto frontier.

Region I, the red arc in Figure (2.2), corresponds to scenarios where the manager possesses a low bargaining power level, $\rho < \hat{\rho}$, for a fixed value $\hat{\rho} \in [0, 1]$, indicating that the limited liability constraint is binding. Conversely, Region II, the green arc in Figure (2.2), pertains to situations where the manager exhibits a high bargaining power level, $\rho \geq \hat{\rho}$, signifying that the limited liability constraint is not binding. In Proposition 5, we establish the hinge value of bargaining power.

Proposition 5 . *Given the Pareto weights problem with hidden actions, the hinge value of manager's bargaining power is* $\hat{\rho} = \frac{1}{2}$.

Proof: See Appendix A.2.

As indicated in Proposition 5, the critical value of bargaining power does not depend on any specific parameter. This result is straightforward to implement and can be explained as follows: if the manager has less bargaining power than the owner, $\rho < \hat{\rho}$, the contract scheme sets the base compensation to zero. However, when the manager's bargaining power exceeds that of the owner,

$\rho > \hat{\rho}$, the base compensation must be positive to ensure participation.

The shift from Region I to Region II is driven by the relaxation of the limited liability constraint. In Region I, the manager's outside option is low, leading to an equilibrium where the base salary is zero. The principal, having stronger bargaining power, sets a contract that maximizes the owner's surplus while ensuring the manager's participation at the lowest possible cost. This result aligns with classic principal-agent models where the agent receives only their reservation utility (Holmström, 1979; Grossman and Hart, 1983a).

Once bargaining power surpasses the critical threshold $\hat{\rho}$, the base salary becomes positive to sustain participation, modifying the incentive structure in a way that resembles high-stakes managerial compensation models (Bebchuk et al., 2002; Choe et al., 2014). In this scenario, stronger bargaining power allows the manager to negotiate more favorable contracts, including guaranteed fixed compensation, reducing reliance on performance-based pay. This result is consistent with empirical findings in executive compensation, where CEOs with greater influence over the Board tend to receive higher base salaries and lower variable incentives.

Additionally, the shift in contract structure reflects a broader trend in the literature on bargaining power and effort provision. (Chen et al., 2019) argue that the relationship between bargaining power and effort is non-monotonic. Initially, increased bargaining power enhances effort provision, but beyond a certain threshold, further increases in bargaining power can lead to declining effort as managers secure high fixed compensation, reducing their reliance on performance-based incentives. This suggests that the transition from Region I to Region II can have significant implications for effort and firm performance, depending on how bargaining power is distributed.

However, the characterization of the solution depends not only on the bargaining power ρ, but also on the lifetime expected (reservation) utilities of the manager and the owner, denoted by $(\bar{U}, \bar{V})$. To fully characterize the solution, we begin by focusing on Region I (see the red area in Figure 2.3), which is defined by the conditions $\bar{U} < \hat{U}$ and $\bar{V} > \hat{V}$. Note that the pair $(\hat{U}, \hat{V})$ is used solely as a reference point and is indeed intended to indicate the case when $\rho = \frac{1}{2}$. In Region

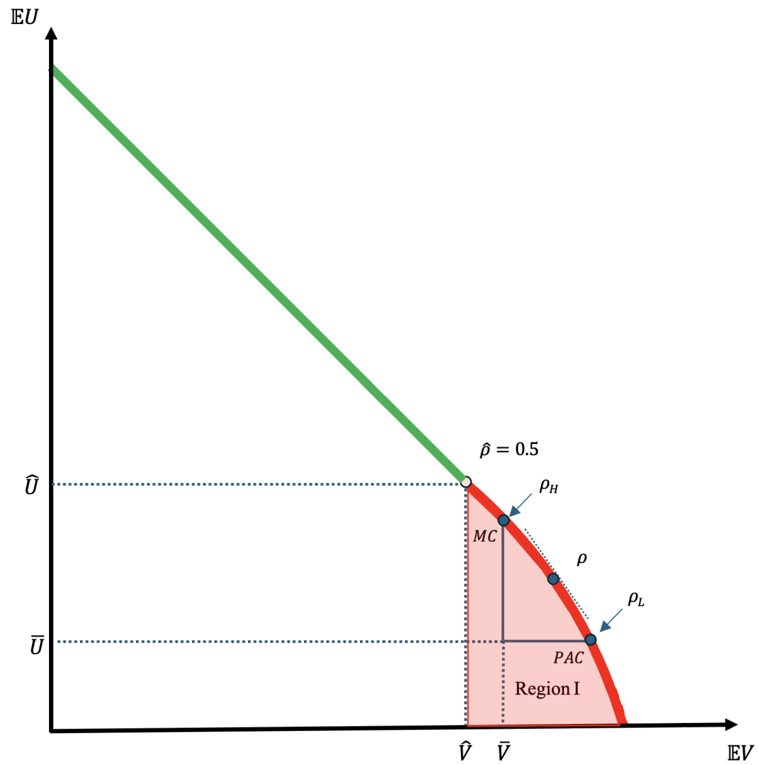


Figure 2.3: Region I

I, the owner's reservation utility is relatively high while the manager's is comparatively low relative to these reference values. In Proposition 6, we describe how the share of output allocated to the manager is determined when both parties' outside options lie within this region, as the bargaining power ρ varies over the interval $[0, 1]$.

In this context, ρ_H represents the bargaining power level associated with the contract where the managerial participation constraint is binding, and the owner's participation constraint is not. This is referred to as the Managerial Contract (MC) (Bebchuk et al., 2002; Choe et al., 2014), where the manager holds all the power and must meet the owner's reservation utility. Conversely, ρ_L denotes the bargaining power level for the contract where the owner's participation constraint is binding, and the managerial constraint is not. This is known as the principal-agent Contract (PAC) (Holmström, 1979; Grossman and Hart, 1983a), where the owner has all the power and must ensure the manager's reservation utility is satisfied.

Proposition 6 . *In Region I, where $\bar{U} < \hat{U}$ and $\bar{V} > \hat{V}$, given the Karush-Khun-Tuker program* *when the participation constraints are considered and the limited liability constraint is binding, the optimal share of output to the manager's compensation, denoted by a^*, is the solution of the following equations that depend on ρ:*

i) $(1-a)\bar{y}(e^*(a)) = \bar{V}$ *if* $\rho_H \leq \rho$*;*

ii) $\rho[\bar{y}(e^*(a)) - ra\sigma(e^*(a))] + (1-\rho)[(1-a)\bar{y}'_e(e^*(a)){e^*}'_a(a) - \bar{y}(e^*(a))] = 0$ *if* $\rho_L < \rho < \rho_H$*;*

iii) $a\bar{y}(e^*(a)) - c(e^*(a)) - \frac{1}{2}ra^2\sigma(e^*(a)) = \bar{U}$ *if* $\rho \leq \rho_L$*.*

Proof: See Appendix A.3.

Finally, since we are working in Region I, where the base compensation is zero, we only need to describe the share of output allocated to the manager's compensation. The effort level is determined by equation (2.5) in Proposition 4. Notably, the optimal choice of the output share is described across three sections of ρ, as outlined in Proposition 6. When $\rho \leq \rho_L$, we are in a corner solution, and the optimal share choice matches that of the principal-agent Contract (PAC) solution. Conversely, if $\rho_H \leq \rho$, we encounter another corner solution, corresponding to the Managerial Contract (MC) perspective. In both of these cases, the optimal choice of the output share does not depend on the level of bargaining power, but rather on the reservation utilities. In the intermediate section, when $\rho_L < \rho < \rho_H$, we obtain an interior solution. Here, the optimal choice depends on the bargaining power, the coefficient of constant absolute risk aversion, and the risk associated with the error. Unlike the corner solutions, this interior solution does not depend on the reservation utilities.

Similarly, in order to fully characterize the solution, we will focus on Region II, where $\bar{U} > \hat{U}$ and $\bar{V} < \hat{V}$ (see the green area in Figure (2.4)). In this region, the owner has a lower reservation utility and the manager a higher reservation utility, again relative to $\hat{U}$ and $\hat{V}$, respectively. In Proposition (7), we will describe the contract $\varsigma(\rho, \bar{U}, \bar{V})$, where the bargaining power ρ ranges over $[0, 1]$. In this region, $\lambda^* = 0$; therefore, the optimal contract structure is given by a^*, $b^* > 0$, and $e^* = e(a^*)$.

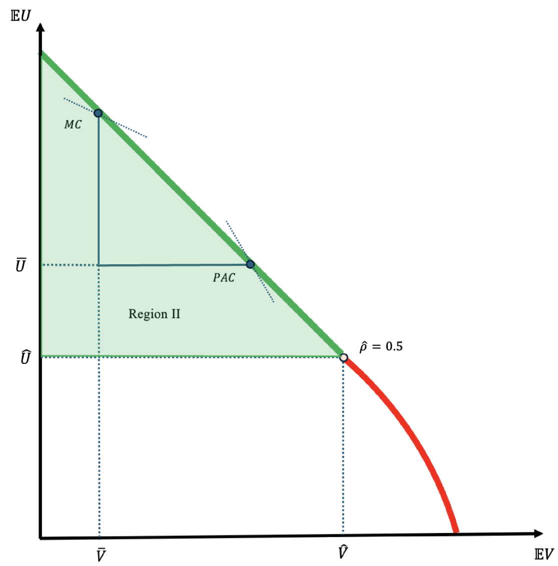


Figure 2.4: Region II

Proposition 7 . *In Region II, where $\bar{U} > \hat{U}$ and $\bar{V} < \hat{V}$, when the limited liability constraint is not binding and the participation constraints are taken into account, the optimal share of output allocated to the manager's compensation, denoted by a^*, is the solution of the following equations*

$$(1-a)\bar{y}'_e(e^*(a)){e^*}'_a(a) - ra\sigma(e^*(a)) = 0. \tag{2.7}$$

Additionally, a^* does not depend on ρ, therefore e^* doesn't either. Finally, in Region II, managers are incentivized through the base compensation, which are given by:

$$b^* = \begin{cases} (1-a)\bar{y}(e^*(a)) - \bar{V} & \text{if} \quad \frac{1}{2} < \rho, \\ \bar{U} - a\bar{y}(e^*(a)) + c(e^*(a)) + \frac{1}{2}ra^2\sigma(e^*(a)) & \text{if} \quad \rho < \frac{1}{2}; \end{cases}$$

when $\rho = \frac{1}{2}$, the base compensation takes a value within the convex set:

$$\left[\bar{U} - a\bar{y}(e^*(a)) + c(e^*(a)) + \frac{1}{2}ra^2\sigma(e^*(a)), (1-a)\bar{y}(e^*(a)) - \bar{V}\right].$$

Proof: See Appendix A.4.

In the above Proposition (7), we observe that, in the optimal contract, the share of output allocated to managerial compensation is invariant to bargaining power. Equation (2.7) is a familiar expression in the literature, as noted, for example, by (Choe et al., 2014) and (Murphy, 1999). Instead, the variation in the share of output depends on the coefficient of constant absolute risk aversion and the risk associated with the error. Since effort is a function of the share of output, as shown in equation (2.5), it is also independent of bargaining power. Consequently, the contract is primarily characterized by the level of base compensation. It is important to note that, in Region II, incentives are provided through the base compensation, which is determined by the level of bargaining power.

In the first case, when the manager has more bargaining power than the owner, we encounter a corner solution (the MC solution), where the base compensation is equal to the owner's share of the output minus the owner's reservation utility, ensuring that the owner's individual rationality constraint is satisfied. In the opposite case, where the manager has less bargaining power than the owner, we again have a corner solution (the PCA solution), where the base compensation equals the manager's reservation utility minus the expected utility of the manager with zero base compensation. In this scenario, the bonus ensures that the manager's individual rationality constraint is satisfied. Finally, when both the manager and the owner have the same bargaining power, $\rho = 1/2$, the optimal base compensation can be any quantity between the MC solution and the PCA solution.

In summary, in Region II, the base compensation b^* is a linear transfer between the owner and the manager. Therefore, the Region II is represented as a straight line with a slope of -1 in the plane of the owner's expected utility and the manager's expected utility, as shown in Figure (2.4). However, in the plane of the owner's utility and the manager's utility, it appears as a concave line, as illustrated in Figure (2.2).

In order to complete the full characterization of the solution given the values of the lifetime expected utilities, it is necessary to analyze the case where $\bar{U} < \hat{U}$ and $\bar{V} < \hat{V}$; that is, when both the manager and the owner have lower reservation utility than $\hat{U}$ and $\hat{V}$, respectively. This region is illustrated in Figure 2.5 (the blue area). In Proposition 8, we will express the optimal contract

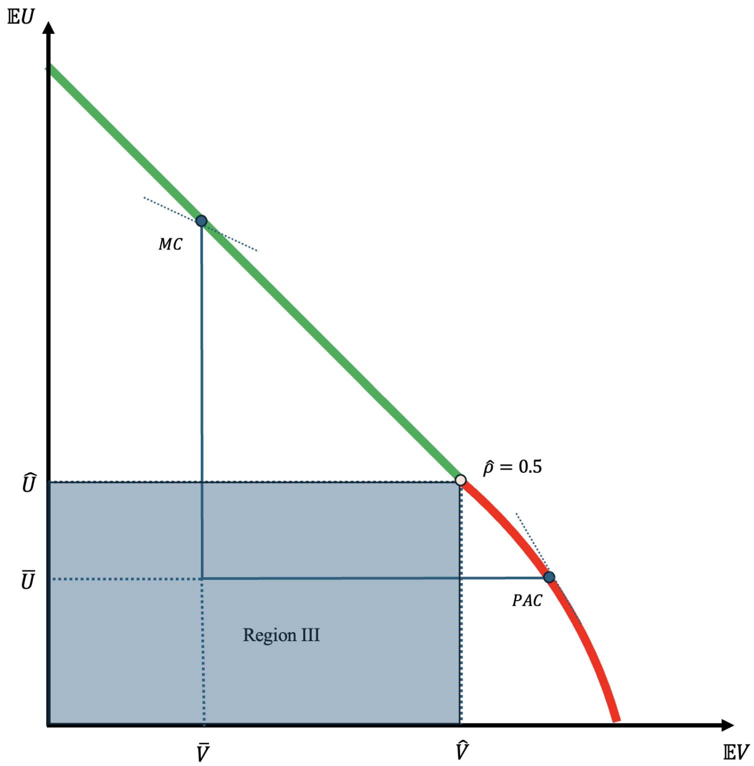


Figure 2.5: Region III

$\varsigma(\rho, \bar{U}, \bar{V})$ using the characterization for each level of bargaining power. As the above analysis has been made for Region I and Region II, the optimal effort will be determined by the equation (2.5), which only depends on the share of output allocated to the manager.

Unlike Regions I and II, where reservation utility is either low or high for one party, Region III emerges as an intermediate case where both the manager and the owner have low reservation utilities. Consequently, neither party possesses enough negotiating leverage to entirely dictate contract terms, resulting in an equilibrium in which both participation constraints are operative. This leads to a hybrid incentive structure that balances fixed and variable compensation.

Proposition 8 . *In Region III, where $\bar{U} < \hat{U}$ and $\bar{V} < \hat{V}$, the optimal share of output allocated to the manager's compensation, denoted by a^*, is the solution of the following equations depending on ρ:*

i) $(1-a)\bar{y}_e'(e^*(a))e^{*\prime}(a) - ra\sigma(e^*(a)) = 0 \quad$ *if* $\quad \frac{1}{2} \leq \rho,$

ii) $\rho[\bar{y}(e^*(a)) - ra\sigma(e^*(a))] + (1-\rho)[(1-a)\bar{y}_e'(e^*(a))e^{*\prime}(a) - \bar{y}(e^*(a))] = 0 \quad$ *if* $\quad \rho_L < \rho < \frac{1}{2},$

iii) $a\bar{y}(e^*(a)) - c(e^*(a)) - \frac{1}{2}ra^2\sigma(e^*(a)) = \bar{U}$, *if* $\rho \leq \rho_L$.

The managers are incentivized through the base compensation, which is given by:

$$b^* = \begin{cases} (1-a)\bar{y}(e^*(a)) - \bar{V} & \text{if} \quad \frac{1}{2} < \rho, \\ 0 & \text{if} \quad \rho < \frac{1}{2}; \end{cases}$$

For $\rho = \frac{1}{2}$*, the base compensation is determined as a value within the convex set:*

$$\left[0, (1-a)\bar{y}(e^*(a)) - \bar{V}\right].$$

Proof: See Appendix A.5.

The emergence of Region III suggests that contract structures are not strictly divided between principal-dominant and manager-dominant regimes, but instead can exhibit intermediate characteristics. This aligns with (Demougin and Helm, 2006) and (Chen et al., 2019), who highlight that real-world contracts often balance incentives and base compensation in response to labor market conditions.

This situation commonly arises in settings where external labor market conditions prevent either party from holding a dominant bargaining position. For instance, in industries with high managerial mobility, outside options for managers remain low, while competitive pressures constrain the bargaining power of firm owners. In such environments, contracts tend to be more balanced, distributing risk and incentives across both parties.

The existence of Region III adds depth to the characterization of contracts, illustrating how bargaining power and reservation utilities shape compensation structures. As noted in (Dittrich and Städter, 2014), distortions in bargaining power can introduce efficiency trade-offs:

- **Region I**: Owner-dominant contracts, characterized by zero base compensation and full reliance on performance-based incentives.

- **Region II**: Manager-dominant contracts, where base compensation is positive, reducing reliance on variable pay.
- **Region III**: Hybrid contracts where bargaining power is evenly distributed, leading to a mix of fixed and variable compensation.

In terms of implications for corporate governance, the existence of Region III has significant implications for labor market efficiency and corporate governance. When bargaining power is evenly distributed, extreme distortions caused by one party's dominance are mitigated, leading to more stable incentive structures. This balance is crucial in executive compensation design, where governance mechanisms must align managerial incentives with shareholder interests.

In environments where firm owners exert excessive control, managers may receive contracts with low or zero base compensation, relying entirely on performance-based incentives. This can increase managerial risk aversion, potentially discouraging effort and innovation (Holmström, 1979). Conversely, when managerial power is high, contracts often feature substantial fixed salaries, reducing the manager's exposure to performance fluctuations but weakening incentives (Choe et al., 2014).

Region III represents an intermediate governance setting, where neither party fully dictates contract terms, leading to a balance between risk-sharing and incentive provision. This aligns with empirical evidence showing that firms with stronger governance structures tend to adopt compensation schemes that incorporate both fixed salaries and performance-related pay (Li et al., 2013).

Moreover, the role of external market conditions in shaping governance structures is particularly relevant in Region III. In industries with competitive labor markets and high managerial mobility, governance mechanisms must account for external job opportunities and reservation utilities, ensuring that contracts remain attractive to both managers and firm owners (Chen et al., 2019).

Ultimately, the existence of Region III highlights the importance of designing incentive structures that are neither overly restrictive nor excessively flexible. By maintaining an appropriate trade-off between ownership control and managerial discretion, firms can create governance frameworks that enhance firm performance while ensuring long-term stability.

In summary, contracts in Region III exhibit a more balanced structure, where neither party has complete control over contract terms. Here, "balanced structure" refers to agreements that equitably distribute decision-making authority between the owner and the manager, ensuring that neither side has an overwhelming advantage in shaping the contract terms. The following section formalizes these findings by exploring closed-form solutions that quantify contract characteristics under different bargaining power regimes.

2.4 A Particular Environment

To derive closed-form solutions for each region described in the previous section—including the manager's output share, base compensation, and the effort level recommended by the Board; we now consider a particular environment. We assume the mean output as $\bar{y}(e) = e$, which satisfies the standard assumptions; a quadratic cost of effort $c(e) = \frac{c}{2}e^2$, where $c > 0$ is the cost parameter; and a constant output risk $\sigma(e) = \overline{\sigma}$ with $\overline{\sigma} > 0$.

To ensure the solution is meaningful, we assume that $0 < 1 - rc\overline{\sigma} < 2\frac{1-\rho}{\rho}$ and $0 \leq 1 - 4c\bar{V} \leq 1$. Given these assumptions, the first-order condition in Equation (2.5) implies that the optimal effort is $e^*(a) = \frac{a}{c}$. Hence, in the optimal contract, the effort recommended by the Board is proportional to the manager's share of output and inversely related to the cost parameter. The threshold values ρ_H and ρ_L required for characterizing the solution in Region I are computed and presented in Appendix A.6.

In Region I, the characterization of the solution is that the base compensation is zero, $b^* = 0$, and the share of the output for the manager compensation, a^*, is determined by the following equations, which depend on ρ and the reserve utility levels:

$$a^* = \begin{cases} \frac{1-\sqrt{1-4c\bar{V}}}{2} & \text{if} \quad \rho_H \leq \rho; \\ \frac{1-\rho}{2(1-\rho)-\rho(1-rc\overline{\sigma})} & \text{if} \quad \rho_L < \rho < \rho_H; \\ \sqrt{\frac{2c\bar{U}}{1-rc\overline{\sigma}}} & \text{if} \quad \rho \leq \rho_L. \end{cases}$$

The comparative statics analysis of the share of output is shown in Table (2.1)[1]. In the case where ρ is at least ρ_H, we present a corner solution, the managerial solution, that satisfies the individual rationality constraint for the owner. We observe that as either the cost parameter or the owner's reservation utility increases, the share of the output allocated to managerial compensation also increases.

This occurs because, when the manager dominates the negotiation, the owner's participation constraint is binding. An increase in $\bar{V}$ requires the contract to generate higher total output so that the owner's utility is met. This is achieved by inducing more effort from the manager, which is in turn stimulated through a higher a^*. Similarly, an increase in the cost of effort c implies that the manager requires stronger incentives to supply the same level of effort. In both cases, a^* rises.

On the other hand, in the alternative corner solution, where the principal-agent solution applies when $\rho \leq \rho_L$, the individual rationality constraint for the manager must hold with equality. In this scenario, the manager's share of output increases with respect to the cost parameter, the manager's risk aversion, the output variance, and the manager's reservation utility. These results are consistent with the theoretical predictions and have been studied in the corresponding literature.

Under this regime, compensation is purely performance-based. As r, $\overline{\sigma}$, or c increase, the manager faces higher risk or effort disutility. Therefore, the only way to preserve incentive compatibility and satisfy the participation constraint is to increase a^*. The same logic applies if $\bar{U}$ increases: a larger share must be granted to reach the reservation utility level.

[1]The sign of the comparative statics analysis is shown in Appendix A.7.

	$\rho_H \leq \rho$	$\rho_L < \rho < \rho_H$	$\rho \leq \rho_L$
$\bar{V}$	$+$		
ρ		$+$	
r		$-$	$+$
c	$+$	$-$	$+$
$\overline{\sigma}$		$-$	$+$
$\bar{U}$			$+$

Table 2.1: Comparative statics analysis of the optimal share of output, a^*, in Region I

The effect of the manager's risk aversion parameter r on the optimal incentive intensity a^* is not monotonic across bargaining regimes. In the interior region, where no individual rationality constraint binds, a higher r leads to a lower a^*, as the Board internalizes the cost of risk exposure. In contrast, when the owner dominates (that is, when $\rho \leq \rho_L$), the manager's individual rationality constraint binds, and the contract must compensate for higher risk aversion with stronger incentives. This results in a positive relationship between r and a^*. This shift in the sign of the derivative illustrates how the binding constraint determines the direction of adjustment in the contract: when participation is not binding, the Board can prioritize efficiency; when it is, the contract must meet utility thresholds regardless of incentive costs.

Interestingly, our comparative statics reveal that the effect of output uncertainty on incentives is not uniform across bargaining regimes. In the interior region, an increase in the variance of output ($\overline{\sigma}$) actually reduces the optimal share of output allocated to the manager. This contrasts with the boundary regions, where higher risk leads to stronger incentives. This difference arises because, when participation constraints do not bind, increasing $\overline{\sigma}$ makes incentive provision less efficient, leading the Board to reduce a^*. However, in corner solutions, the contract cannot reduce incentives without violating participation; hence, a^* must rise with risk. This non-monotonic behavior highlights the complex interaction between risk, bargaining power, and contract design.

Finally, we consider the interior solution, which arises when $\rho_L < \rho < \rho_H$. This is the only solution that depends on the bargaining power, as well as the cost parameter, the risk of error, and the manager's risk aversion. In this case, the share of the output decreases with respect to all parameters except bargaining power, for which it increases.

This outcome reflects the fact that, in the interior case, the Board chooses a^* to balance incentives and risk-sharing, since neither party's reservation utility binds. As r, c, or $\overline{\sigma}$ rise, strong incentives become less efficient, and thus a^* decreases. Conversely, when the manager gains more bargaining power, a larger share is allocated.

There is a notable change in the sign of the comparative statics between the corner solutions: the Managerial Contract and the principal-agent Contract and the interior solution. At extreme values of managerial power, the sign of the comparative statics aligns with theoretical predictions from the literature when the individual rationality constraints of either the manager or the owner are binding, assuming that the base compensation is zero. Conversely, in the interior solution, where the individual rationality constraints for both the manager and the owner are not binding, both parties receive more than their respective expected utilities. Therefore, the structure of the optimal share resembles that of models with base compensation, where marginal incentive costs are weighed against total surplus gains. This case will be analyzed in the next Region II.

In Region I, based on the optimality condition, the contract prescribes the optimal level of effort for the Board to exert as follows:

$$e^* = \begin{cases} \frac{1-\sqrt{1-4c\bar{V}}}{2c} & \text{if} \quad \rho_H \leq \rho; \\ \frac{1-\rho}{2c(1-\rho)-c\rho(1-rc\overline{\sigma})} & \text{if} \quad \rho_L < \rho < \rho_H; \\ \sqrt{\frac{2\bar{U}}{c-rc^2\overline{\sigma}}}\overline{\sigma} & \text{if} \quad \rho \leq \rho_L. \end{cases}$$

In Region II, when the manager has a high outside option, the characterization of the solution for the share of output allocated to managerial compensation, a^*, is determined by the following equation: $(1-a)\frac{1}{c} - ra\overline{\sigma} = 0$. Thus, the optimal share of output offered by the Board in the contract is given by: $a^* = \frac{1}{1+rc\overline{\sigma}}$, which corresponds to a well-known expression (see (Choe et al., 2014) and (Murphy, 1999)). Notice that a^* decreases when we increase r, c or $\overline{\sigma}$.[2] Based on the

[2]Details are provided in Appendix A.8.

optimal condition, the effort suggested in Region II is: $e^* = \frac{1}{c(1+rc\overline{\sigma})}$. It is important to note that a^* does not depend on bargaining power, ρ, and consequently, the suggested optimal effort, e^*, is also independent of ρ.

Finally, in Region II, the manager is incentivized primarily through the base compensation, as their outside option or reservation utility is higher than the hinge value associated with the bargaining power. In summary, the base compensation in this region is given by:

$$b^* = \begin{cases} \frac{rc\overline{\sigma}}{c(1+rc\overline{\sigma})^2} - \bar{V} & \text{if} \quad \frac{1}{2} < \rho; \\ \bar{U} - \frac{1-rc\overline{\sigma}}{2c(1+rc\overline{\sigma})^2} & \text{if} \quad \rho < \frac{1}{2}. \end{cases}$$

The optimal base compensation values are inherently positive by construction, as they result from corner solutions corresponding to the values of $\bar{U}$ and $\bar{V}$ at the boundary of the feasible region.

When $\rho = \frac{1}{2}$, the base compensation lies within the convex set: $\left[\bar{U} - \frac{1-rc\overline{\sigma}}{2c(1+rc\overline{\sigma})^2}, \frac{rc\overline{\sigma}}{c(1+rc\overline{\sigma})^2} - \bar{V}\right]$. Table 2.2 presents the comparative statics analysis of the optimal base compensation for the manager.[3] In this case, when $\rho > \frac{1}{2}$, the base compensation decreases only with increases in the effort cost and the owner's outside option. Conversely, when $\rho < \frac{1}{2}$, the base compensation increases with all parameters.

	$1/2 < \rho$	$\rho < 1/2$
$\bar{V}$	-1	
r	$+$	$+$
c	$-$	$+$
$\overline{\sigma}$	$+$	$+$
$\bar{U}$		1

Table 2.2: Comparative statics analysis of the base compensation, b^*, in Region II

In the Region III, the characterization of the solution for the share of output is expressed by:

$$a^* = \begin{cases} \frac{1}{1+rc} & \text{if} \quad \frac{1}{2} \leq \rho; \\ \frac{1-\rho}{2(1-\rho)-\rho(1-rc\overline{\sigma})} & \text{if} \quad \rho_L < \rho < \frac{1}{2}; \\ \sqrt{\frac{2c\bar{U}}{1-rc\overline{\sigma}}} & \text{if} \quad \rho \leq \rho_L. \end{cases}$$

[3]The calculation of the comparative statics is in the Appendix A.9.

The optimal effort suggested by the Board in this contract scheme is structured as follows:

$$e_2^* = \begin{cases} \frac{1}{c(rc\overline{\sigma}+1)} & \text{if} \quad \frac{1}{2} \leq \rho; \\ \frac{1-\rho}{2c(1-\rho)-c\rho(1-rc\overline{\sigma})} & \text{if} \quad \rho_L < \rho < \frac{1}{2}; \\ \sqrt{\frac{2\bar{U}}{c-rc^2\overline{\sigma}}} & \text{if} \quad \rho \leq \rho_L. \end{cases}$$

Regarding the base compensation, we face a mixed situation. When the bargaining power of the manager is greater than the owner's bargaining power, the base compensation is non-zero. However, when the manager's bargaining power is less than the owner's, the base compensation is zero. The scheme that the Board offers in the contract with respect to base compensation in this region is:

$$b^* = \begin{cases} \frac{r\overline{\sigma}}{(cr\overline{\sigma}+1)^2} - \bar{V} & \text{if} \quad \frac{1}{2} < \rho; \\ 0 & \text{if} \quad \rho < \frac{1}{2}; \end{cases}$$

when $\rho = \frac{1}{2}$, the base compensation can take a value in the convex set $\left[0, \frac{r\overline{\sigma}}{(cr\overline{\sigma}+1)^2} - \bar{V}\right]$.

In this section, we examined the explicit solutions for the optimal contract conditions across Regions I, II, and the Region III, focusing on the share of output allocated to managerial compensation, the base compensation, and the effort level proposed by the Board in an efficient contract. Assuming a linear mean output function, a quadratic cost of effort, and constant risk, we characterized the solutions under varying bargaining power levels (ρ). In Region I, the base compensation is zero, and the share of output is determined by the manager's and owner's reservation utilities and bargaining power, with distinct comparative static patterns emerging based on whether ρ lies within, above, or below the critical thresholds, ρ_H and ρ_L. Notably, this article introduces a novel approach where outcomes are directly influenced by bargaining power when it lies within the critical thresholds. In Region II, when the manager's outside option is high, the share of output decreases with all parameters, and base compensation becomes crucial, particularly when $\rho > \frac{1}{2}$. In all cases, the optimal effort is proportional to the share of output and inversely related to the cost parameter, consistent with theoretical predictions and established literature

This work provides a novel contribution by fully characterizing contracts along the Pareto frontier in a principal-agent model with hidden actions and Pareto weights. Rather than restricting the analysis to specific bargaining solutions such as Nash or Kalai-Smorodinsky, our framework extends existing approaches by describing how contracts vary depending on bargaining power and outside options.

A key aspect of our contribution is the interaction between fixed pay and output-based compensation, and how this balance affects managerial incentives. In the traditional literature, optimal contracts tend to focus on maximizing incentives without explicitly characterizing how fixed and variable pay respond to changes in bargaining conditions. Our work shows that the compensation structure depends not only on the manager's risk aversion and output variability but also on the distribution of bargaining power and participation constraints.

Additionally, it is important to explicitly clarify how our approach builds upon and generalizes models such as Nash or Kalai-Smorodinsky. While our framework captures a broader range of contract distortions, highlighting the key mechanisms that differentiate our analysis from traditional solutions can further illustrate its contributions.

2.5 Conclusions

This paper has analyzed how bargaining power and outside options shape the design of incentive contracts in a principal-agent model with hidden actions. We introduced the Board as the central coordinating entity, responsible for designing take-it-or-leave-it contracts between the firm's owner and manager. The Board's objective function is modeled as a convex combination of the owner's and manager's utilities, weighted by their respective bargaining power. This structure reflects how negotiation dynamics are embedded into contract design. The resulting Pareto weights framework is subject to the manager's incentive compatibility constraint and the individual rationality conditions of both parties.

Our approach extends the analysis of standard principal-agent models by incorporating bargaining power as an explicit determinant of contract equilibrium. Rather than focusing on a specific bargaining solution such as Nash or Kalai-Smorodinsky, our model generalizes the analysis by characterizing the entire Pareto frontier of efficient contracts, demonstrating how changes in bargaining power and reservation utilities shape incentive structures.

One key insight from our results is that optimal contracts exhibit distinct characteristics across different bargaining power regimes. When the owner holds more bargaining power, contracts tend to minimize fixed compensation, relying heavily on performance-based incentives. Conversely, when the manager's bargaining power increases, contracts shift toward higher fixed salaries, reducing reliance on variable pay. The existence of an intermediate region (Region III), where both parties have relatively low outside options, highlights that incentive structures are not strictly divided between principal-dominant and manager-dominant regimes. Instead, contracts in this region exhibit a more balanced mix of fixed and variable compensation, aligning managerial incentives with firm objectives while ensuring a stable governance framework.

Our findings also emphasize the critical role of outside options in shaping equilibrium outcomes. Unlike models that primarily focus on the agent's reservation utility, we show that the owner's outside option significantly influences the contract design. The interplay between bargaining power and outside options creates a richer contract space, where incentive schemes dynamically adjust to shifts in economic conditions and governance structures.

From a corporate governance perspective, these results offer valuable implications. Contracts that allocate excessive control to one party may lead to distortions, such as excessive risk exposure for the manager or inefficient compensation schemes that fail to align incentives. By characterizing optimal contracts along the entire Pareto frontier, our approach provides a more flexible framework for understanding executive compensation, firm governance, and strategic decision-making under asymmetric power dynamics.

Future research could explore the extension of this framework to dynamic settings, where bargaining power evolves over time due to firm performance or external market conditions. Additionally, empirical validation of our model using executive compensation data could provide further insights into how bargaining power and outside options interact in real-world corporate environments.

Our comparative statics highlight that risk-related parameters do not have uniform effects across bargaining regimes. For example, the effect of the manager's risk aversion parameter r on the optimal incentive share a^* is non-monotonic. In the interior region, where no individual rationality constraint binds, an increase in r leads to lower incentives. However, when the manager's constraint is binding, as in the owner-dominant regime, the contract must compensate for higher risk aversion, resulting in stronger incentives.

A similar non-monotonic pattern emerges with output uncertainty. In the interior region, greater output variance $\overline{\sigma}$ reduces the manager's incentive share, while in the boundary regions, higher uncertainty leads to more performance-based compensation. These findings underscore the complex interaction between risk, bargaining power, and contract design in shaping optimal incentive structures.

Overall, our results contribute to the literature on contract theory by providing a comprehensive characterization of optimal incentive schemes in principal-agent relationships with hidden actions. By explicitly modeling bargaining power and outside options, we offer a novel perspective on how contract design can be tailored to different negotiation settings, enhancing both firm performance and managerial incentives.

Chapter 3

The Dynamics of Bargaining Power in a Principal-Agent Model

3.1 Introduction

The dynamics of bargaining power and its implications for executive compensation have long been central to both academic research and corporate governance. Traditional principal-agent models, such as those of Holmström (1979) and Grossman and Hart (1983a), provided a foundational framework for analyzing incentive-based relationships under asymmetric information. These models were later extended by Spear and Srivastava (1987), who introduced dynamic frameworks to explore how incentives evolve over time. Over the past few decades, such models have become essential for understanding long-term contractual relationships, particularly in settings characterized by moral hazard and repeated interactions. Edmans and Gabaix (2016) offer a comprehensive review of executive compensation, highlighting the importance of bargaining power in shaping pay structures and emphasizing the need for models that account for its evolution over time. However, existing frameworks often treat bargaining power as static, failing to capture its gradual development within the principal-agent relationship and its interaction with firm performance and executive compensation.

This chapter introduces a dynamic principal-agent model where bargaining power evolves over time through a mechanism we name the bargaining drift coefficient. This approach, developed as part of a joint research with Sonia Di Giannatale and Genaro Basulto, departs from traditional dynamic bargaining models by allowing negotiation leverage to adjust continuously in response to firm performance, capturing the feedback loops between power, incentives, and executive pay. By incorporating a law of motion for bargaining power, we extend standard models, such as those of Spear and Srivastava (1987) and Sannikov (2008), to examine the long-term relationship between managerial influence and compensation design. Our framework aligns with the managerial power theory, which argues that executives, particularly CEOs, actively leverage their influence to shape pay structures (Bebchuk and Fried (2003); Choe et al. (2014)). Although the proposed bargaining drift mechanism is not necessarily optimal, it provides an intuitive and tractable first approximation for understanding how bargaining power accumulates, shifts, and responds to performance over time. This model offers a lens through which to examine the gradual adaptation of bargaining power, emphasizing how initial conditions and firm performance shape executive compensation trajectories.

This study makes several key contributions. First, it extends dynamic principal-agent models by introducing bargaining power as an evolving state variable, offering a richer framework to analyze long-term strategic interactions in principal-agent relationships. Second, it bridges theoretical modeling with empirical relevance by deriving an equation that directly links CEO compensation, bargaining drift, and firm performance, allowing for real-world validation. Third, it employs numerical simulations to explore the impact of key parameters, such as bargaining drift sensitivity, revealing how initial conditions influence compensation and managerial power trajectories. Fourth, it integrates insights from the managerial power theory, illustrating how bargaining power evolves endogenously, influencing compensation structures and governance mechanisms. Finally, the study develops a computational algorithm for solving multiobjective optimization problems, enhancing its practical applicability in executive compensation and governance design.

This research also contributes to the broader debate on agency models and their ability to explain CEO compensation. Critics have argued that traditional agency models struggle to fully capture real-world executive pay dynamics (Jensen and Murphy (1990); Edmans et al. (2017); Edmans and Gabaix (2016)). By explicitly linking bargaining power evolution to compensation and firm performance, our framework addresses this critique, providing a dynamic perspective on the CEO-shareholder relationship. The infinite-horizon approach allows for the study of long-term incentive structures, emphasizing how bargaining drift accumulates over time, shaping contractual relationships, managerial influence, and governance structures. This model not only captures the immediate effects of compensation policies but also their cumulative impact on long-term organizational behavior and performance.

The remainder of this chapter is structured as follows. Section 3.2 introduces the dynamic principal-agent model and the bargaining drift mechanism, including a discussion of its role as a first approximation for understanding bargaining power dynamics. Section 3.3 establishes the equivalence between our framework and traditional dynamic models, highlighting key innovations. Section 3.4 details the computational strategy and parameter assumptions, while Section 3.5 presents numerical results and simulations. Section 3.6 explores empirical implications and discusses potential avenues for future validation. Finally, Section 3.7 concludes by summarizing the theoretical and practical contributions of this study.

3.2 Model

In this section, we develop a dynamic principal-agent model based on the standard repeated moral-hazard model of Spear and Srivastava (1987). We assume that time is discrete and that it goes on until infinity: $t = 0, 1, 2, \ldots$. There are two individuals: a risk neutral principal and a risk averse agent, both of whom maximize their expected discounted utility with a common discount rate $\beta \in (0, 1)$.

Suppose that the agent has a continuous utility function represented by: $v(w_t, a_t)$, which is assumed to be bounded, strictly increasing, and strictly concave with respect to w_t; and strictly decreasing and convex with respect to a_t. Here, $w_t \geq 0$ denotes the agent's salary or present compensation at the end of every period, while a_t represents the agent's effort choice made at the beginning of every period, drawn from a compact set $A = [\underline{a}, \overline{a}]$ and unobservable to the principal.

Furthermore, we assume that v is additively separable in its arguments w_t and a_t, and is also twice continuously differentiable.

In every period $t \geq 1$, the principal and the agent observe a realization of the output y_t, drawn from the compact set Y. The stochastic relationship between the output realization and the agent's effort choice is described by the time-invariant distribution $F(y_t \mid a_t) > 0$ for all $y_t \in Y$ and for all $a_t \in A$. We also assume that this distribution has a density f and that the distribution of the outputs is *i.i.d.* from period to period for a given action.

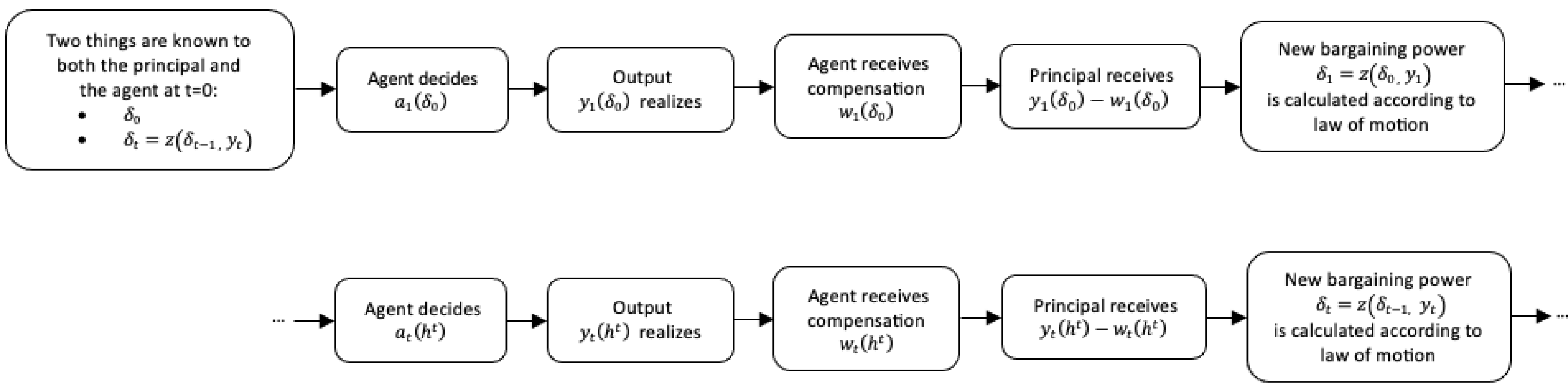
Now we introduce the agent's bargaining power into our framework. At the beginning of the principal-agent relationship, at $t = 0$, the agent has an exogenously determined bargaining power level, $\delta_0 \in [0, 1]$, while the principal's bargaining power is $(1 - \delta_0)$. The agent's bargaining power reflects their ability to extract a portion of the surplus generated by their productive activities. At the outset, the principal and the agent agree on the evolution of the agent's bargaining power, which is governed by a predetermined law of motion. Thus, unlike the bargaining process described by (Binmore et al. (1986)), where the bargaining power is renegotiated in each period, our model assumes that both the agent's initial bargaining power and its evolution are established at $t = 0$. Additionally, unlike Liu and Xuan (2020), our model does not incorporate supervision; instead, incentives are provided through the agent's compensation and the law of motion governing the agent's bargaining power.

We assume that the agent's bargaining power evolves according to the function $z(\delta_{t-1}, y_t)$, which we define as the bargaining drift mechanism. This function maps $[0, 1]$ into itself and is bounded, continuous, and increasing in both arguments, capturing the gradual shifts in bargaining power over time in response to firm performance. Rather than abrupt changes, this bargaining drift mechanism models the accumulation or erosion of negotiation leverage in a path-dependent manner, reinforcing the agent's influence in periods of strong performance and diminishing it when results are weak.

Although assuming no further bargaining between the principal and the agent after the initial contract at $t = 0$ is simplistic, it can be justified by considering the mobility costs associated with changing jobs, as discussed by Baily (1974). However, it is well documented that CEOs often face significant turnover rates after poor industry performance and, to a lesser extent, poor market performance (Jenter and Kanaan (2015)). While this is a limitation of the present model, it allows us to introduce a framework to study how the CEO's ability to extract surplus from the relationship changes over time. Our modeling strategy, where the state variable has continuation values, is partially inspired by Sannikov (2008) although unlike Sannikov, these continuation values do not emerge endogenously in our model. This limitation notwithstanding, our approach provides a useful vehicle for exploring the dynamics of bargaining power in an environment characterized by moral hazard.

The contract that defines the infinite relationship of the principal and the agent follows this timeline: At $t = 1$, given an initial bargaining power $\delta_0 \in [0, 1]$, the agent chooses an effort level $a_1(\delta_0) \in A$. The output $y_1(\delta_0) = y_1(a_1(\delta_0))$ is then drawn from the distribution $F(y_1 \mid a_1(\delta_0))$, and the agent receives compensation $w_1(y_1(\delta_0))$. As previously mentioned, the principal observes $y_1(\delta_0)$ but not $a_1(\delta_0)$, so $w_1(y_1(\delta_0))$ depends solely on $y_1(\delta_0)$. Consequently, the agent's compensation at $t = 1$ is $w_1(y_1(\delta_0))$. The principal receives $y_1(\delta_0) - w_1(y_1(\delta_0))$, and the agent's bargaining power for $t = 2$, $\delta_1 = z(\delta_0, y_1(\delta_0)) \in [0, 1]$ is determined by the exogenous law of motion. The first line of Figure 3.1 provides a graphical representation of the principal-agent relationship at $t = 0$ and $t = 1$.

Figure 3.1: The timeline of the principal and agent relationship



Now assume that the principal and the agent employ history-dependent pure strategies. At $t = 2$, given δ_1, the agent chooses $a_2(\delta_1(\delta_0, y_1(\delta_0))$. The output $y_2(\delta_1(\delta_0, y_1(\delta_0)))$ is then drawn from the distribution $F(y_2 \mid a_2(\delta_1(\delta_0, y_1(\delta_0))))$. The agent receives compensation $w_2(y_2(\delta_1(\delta_0, y_1(\delta_0))))$, and the principal receives $y_2(\delta_1(\delta_0, y_1(\delta_0))) - w_2(y_2(\delta_1(\delta_0, y_1(\delta_0))))$. The agent's new bargaining power is then determined by the law of motion, resulting in $\delta_2 = z(\delta_1(\delta_0, y_1(\delta_0)), y_2(\delta_1(\delta_0, y_1(\delta_0)))) \in [0, 1]$. This process repeats from $t = 3, \ldots$ See the second line of Figure 3.1 for the timeline of the contract at time t.

Therefore, at any time t, there is a history of agent's bargaining powers and output realizations $h^t = \{(\delta_s, y_{s+1}(\delta_s))\}_{s=0}^{t-1}$; with $h^0 = \delta_0$, such that $y_{s+1} \in Y$ and $\delta_s \in [0, 1]$ for all $s = 1, 2, \ldots, t-1$. The principal's decision is $w_t(h^t)$, and the agent's decision is $a_t(h^t)$, as the effort decision must be made before $y_t(h^t)$ is realized and given the value of $\delta_{t-1} = z(\delta_{t-2}, y_{t-1})$ at the beginning of the period. Let $\pi(h^{t+\tau} \mid h^t, a_t)$ be the probability distribution of $h^{t+\tau}$ conditional on h^t and a_t. This distribution is recursively expressed as follows:

$$d\pi(h^{t+\tau} \mid h^t, a_t) = f(y_{t+\tau} \mid a(h^{t+\tau-1}))d\pi(h^{t+\tau-1} \mid h^t, a_t)$$

with

$$d\pi(h^{t+1} \mid h^t, a_t) = f(y_{t+1} \mid a(h^t)).$$

The value functions derived by the principal and the agent, respectively, from the sub-game starting from h^t, are as follows:

$$\begin{aligned} U(h^t, w_t, a_t) &= \sum_{\tau=0}^{\infty} \beta^\tau \int_Y \left[y_{t+\tau}(h^{t+\tau}) - w_{t+\tau}(h^{t+\tau})\right] d\pi(h^{t+\tau} \mid h^t, a_t), \\ V(h^t, w_t, a_t) &= \sum_{\tau=0}^{\infty} \beta^\tau \int_Y v(w_{t+\tau}(h^{t+\tau}), a_{t+\tau}(h^{t+\tau}))d\pi(h^{t+\tau} \mid h^t, a_t). \end{aligned}$$

Given sequences $\delta_t = \{z(\delta_{t-1}, y_t)\}$ and $w_t = \{w_t(h^t)\}$, the sequence $a_t = \{a_t(h^t)\}$ is incentive compatible at h^t if:

$$V(h^t, w_t, a_t) \geq V(h^t, w_t, \overline{a}_t) = \sum_{\tau=0}^{\infty} \beta^{\tau} \int_{Y} v\left(w_{t+\tau}(h^{t+\tau}), \overline{a}_{t+\tau}(h^{t+\tau})\right) d\overline{\pi}(h^{t+\tau}; h^t, \overline{a}_t),$$

for any other sequence $\overline{a}_t = \{\overline{a}_t(h^t)\}$, and $\overline{\pi}$ is the distribution in the future histories induced by δ_{t-1}, y_t, w_t and $\overline{a}_t$.

A contract σ^{δ_0} is defined by a history-dependent agent's effort recommendation $\{a_s(h^s)\}_{s=1}^{t}$, and a history-dependent agent's compensation plan $\{w_s(h^s)\}_{s=1}^{t}$. The agent's history-dependent bargaining power values $\{\delta_s = z(\delta_{s-1}, y_s)\}_{s=0}^{t-1}$ are determined by the agreed-upon law of movement. Thus, a contract is characterized by:

$$(\sigma^{\delta_0} \mid h^t) = \{a_s(h^s), w_s(h^s)\}_{s=1}^{t}.$$

We say that a contract $(\sigma^{\delta_0} \mid h^t)$ is feasible if:

$$a_t(h^t) \in A; \qquad \forall h^t \in ([0,1] \times Y)^t \quad \forall t \geq 1, \tag{3.1}$$

$$0 \leq w_t(h^t) \leq y_t(h^t); \qquad \forall h^t \in ([0,1] \times Y)^t \quad \forall t \geq 1, \tag{3.2}$$

and also the agreed-upon law of motion of the agent's bargaining power must hold:

$$\delta_t = z(\delta_{t-1}, y_t) \in [0,1]; \qquad \forall (\delta_{t-1}, y_t) \in ([0,1] \times Y) \quad \forall t \geq 0. \tag{3.3}$$

Condition (3.1) ensures that the agent's efforts fall within the permissible range of effort values. Condition (3.2) mandates that the agent's salary remains non-negative and does not exceed the current output. Condition (3.3) dictates that the agent's bargaining power must lie within the interval $[0,1]$.

In this setup, for any given δ_t, two conflicting objective functions are simultaneously maximized: the *ex-ante* principal's discounted expected utility, and the *ex-ante* agent's discounted expected utility, subject to incentive compatibility and feasibility. The resulting solution is not a unique contract, but rather a unique series of contracts that satisfies the Pareto optimality criterion.

A contract $(\sigma^{\delta_0} \mid h^t)$ is deemed Pareto optimal if there exists no other feasible and incentive-compatible contract $(\varphi^{\delta_0} \mid h^t)$ such that $(U(h^t, \varphi^{\delta_0}), V(h^t, \varphi^{\delta_0})) \succeq (U(h^t, \sigma^{\delta_0}), V(h^t, \sigma^{\delta_0}))$, for all h^t. Each Pareto optimal contract $(\sigma^{\delta_0} \mid h^t)$ maximizes both $U(h^t, \sigma^{\delta_0})$ and $V(h^t, \sigma^{\delta_0})$ subject to feasibility, and incentive compatibility: $V(h^t, w_t, a_t) \geq V(h^t, w_t, \overline{a}_t)$, for all h^t and for all $\overline{a}_t$.

As previously mentioned, we formulate the dynamic relationship between the principal and the agent as a multiobjective optimization problem, where both the discounted expected utility of the principal and the agent are simultaneously optimized subject to feasibility and incentive constraints. It's noteworthy that we omit the participation constraint since the agent's reservation utility is not included in this model. This modeling decision could prove advantageous as it avoids potential time inconsistency problems that might arise from having a forward-looking constraint (the participation constraint in principal-agent environments), as analyzed by Marcet and Marimon (2019). Therefore, we can proceed to transform this problem into a static variational one as in Spear and Srivastava (1987).

The continuation profile from time $t+1$ onwards for contract σ^{δ_0} at any t, where δ_0 is the initial bargaining power of the agent, given h^t, is determined by $(\sigma_t^{\delta_0} \mid h^t)$. This implies a continuation value from time $t+1$ onwards of $U(\sigma^{\delta_0} \mid h^t)$ for the principal, and of $V(\sigma^{\delta_0} \mid h^t)$ for the agent.

A contract $(\sigma^{\delta_0} \mid h^t)$ is temporary incentive compatible if, for all t and for all h^t:

$$a_t(h^t) \in \arg\max_{a \in A} \int_Y [v(w_t(h^t), a) + \beta V(\sigma^{\delta_0} \mid h^t)] f(y_t \mid a) dy_t. \tag{3.4}$$

This constraint guarantees that there will be no deviations in the optimal path of the agent's effort decisions for any δ_t. Furthermore, to ensure the validity of the first-order approach to this incentive compatibility constraint, we assume that the Monotone Likelihood Ratio Property and the Convexity of the Conditional Distribution Condition are satisfied, following Rogerson (1985). It's worth noting that our definitions of feasibility and incentive compatibility imply that the optimal contract is Markovian - the relevant history up to period t can be summarized by the agent's bargaining power δ_t.

Let D denote the set of all values of the agent's initial bargaining power δ_0 that are associated with feasible and incentive-compatible contracts. This set is defined as:

$$D = \{\delta_0 \in [0,1] \mid \exists \ \sigma^{\delta_0} \ \ s.t. \ (3.1), (3.2), (3.3), \ \ and \ \ (3.4)\}. \tag{3.5}$$

For every $\delta_0 \in D$, let $\mathcal{W}(\delta_0)$ denote the set of discounted expected utility values for both the principal and the agent, generated by contracts that are feasible, incentive-compatible, and characterized by the agent's initial bargaining power δ_0 and the agreed-upon law of motion z. This set can be formally defined as:

$$\mathcal{W}(\delta_0) = \{(U(\sigma^{\delta_0} \mid h^0), V(\sigma^{\delta_0} \mid h^0)) \mid \ \ \delta_0 \in D\}. \tag{3.6}$$

Proposition 9 *$\mathcal{W}(\delta)$ is compact for all δ.*

Proof: See Appendix B.1.

Proposition 9 guarantees the existence of a unique series of Pareto optimal contracts, from which we define $(U^*(\delta), V^*(\delta))$ as the Pareto optimal values of the principal's and the agent's discounted expected utilities, respectively, belonging to $\mathcal{W}(\delta)$. Let Γ be an operator that maps from the space of the Cartesian product of two spaces of continuous and bounded functions—one for the principal and one for the agent—into itself with the $\sup^*$ norm, defined as $\sup^* = \sup(\sup, \sup)$[1]. Since the functions $(U(\delta), V(\delta))$ defined on a compact set are bounded and continuous, we can express the operator $\sup^*$ as $\sup^* = \max(\sup, \sup)$. The function $U(\delta) : \mathcal{W}(\delta) \to \mathbb{R}$ is bounded because the principal's rewards are bounded, and the function $V(\delta) : \mathcal{W}(\delta) \to \mathbb{R}$ is also bounded

[1]Let A and B be two non-empty subsets of $\mathbb{R}$ that are bounded above (i.e., they have a supremum). Then, we define: $\sup\{\sup A, \sup B\} = \max\{\sup A, \sup B\} = \begin{cases} \sup A, & \text{if } \sup A \geq \sup B, \\ \sup B, & \text{if } \sup B > \sup A. \end{cases}$ Here, the supremum (sup) of a set is the least upper bound of that set (Bartle and Sherbert (2011)).

because the agent is risk-averse and their compensations are bounded. The problem Γ should be understood as a multiobjective optimization problem, and its solutions, in the non-negative orthant, are Pareto optimal or non-dominated (Sawaragi et al. (1985)). Therefore, for all $(U(\delta), V(\delta)) \in \mathcal{W}(\delta)$:

$$\Gamma(U, V)(\delta) = \max_{w(\delta,y),\overline{V}(\delta,y),\overline{U}(\delta,y)} \{U(\delta), V(\delta)\}$$

where:

$$U(\delta) = \int_Y [y - w(\delta, y) + \beta\overline{U}(\delta, y)] f(y \mid a^*(\delta)) dy,$$

$$V(\delta) = \int_Y [v(w(\delta, y), a^*(\delta)) + \beta\overline{V}(\delta, y)] f(y \mid a^*(\delta)) dy;$$

subject to

$$a^*(\delta) \in \arg\max_{a(\delta)\in A} \int_Y [v(w(\delta, y), a(\delta)) + \beta\overline{V}(\delta, y)] f(y \mid a(\delta)) dy, \tag{3.7}$$

$$0 \leq w(\delta, y) \leq y \quad \forall\ y \in Y, \tag{3.8}$$

$$\delta' = z(\delta, y) \in [0, 1], \tag{3.9}$$

$$(\overline{U}(\delta, y), \overline{V}(\delta, y)) \in \mathcal{W}(\delta') \quad \forall\ y \in Y, \tag{3.10}$$

where $\overline{V}(\delta, y)$ and $\overline{U}(\delta, y)$ represent the utility in the next period for the agent and the principal, respectively. Equation (3.7) represents the incentive compatibility constraint, while (3.8) indicates the agent's temporary inability to borrow. Additionally, (3.9) guarantees that the future value of the agent's bargaining power lies within the interval $[0, 1]$, and (3.10) ensures that the principal's and the agent's future utility plans are feasible. Finally, we establish that $(U^*(\delta), V^*(\delta))$ is a fixed point of Γ.

Proposition 10 $(U^*(\delta), V^*(\delta)) = \Gamma(U^*, V^*)(\delta), \qquad \forall\delta \in [0, 1].$

Proof: See Appendix B.2.

The operator Γ satisfies Blackwell's sufficient conditions for a contraction, and the contraction mapping theorem ensures that the fixed point $(U^*(\delta), V^*(\delta))$ is unique for all $(U, V) \in \mathcal{W}(\delta)$. This implies that along the resulting Pareto Frontier, PF^*, there exists only one pair of maximal values of the principal's and the agent's discounted expected utilities, given a value of δ, for all

$(U, V) \in \mathcal{W}(\delta)$; and vice versa. Moreover, PF^* must be non-increasing because otherwise either the principal or the agent can achieve a higher level of discounted expected utility and the other individual would be better off (Spear and Srivastava (1987)).

Now, we use Abreu et al. (1990)'s notion of self-generation to propose a converging algorithm aimed at computing the set D defined by (3.5), as in Wang (1997) and Clementi et al. (2010). Let us define the operator B in the following manner:

$$B(M) = \{\delta \in [0,1] \mid \exists\, a^*(\delta), w(\delta, y), \overline{V}(\delta, y), \overline{U}(\delta, y) \;\; s.t. \;\; (3.7), (3.8), (3.9), \;\; (3.10), \text{ and } \delta' \in M \; \forall\, y\}$$

where $M \subseteq [0, 1]$ is an arbitrary set.

The following result establishes that the set D is the largest fixed point of the operator B, and that it can be approximated through a recursive procedure starting from the full interval $[0, 1]$:

Proposition 11 *(i) $D = B(D)$; (ii) Let $M_0 = [0, 1]$. Let $M_{n+1} = B(M_n)$ for $n = 0, 1, 2, \ldots$. Then $\lim_{n\to\infty} M_n = D$.*

Proof: See Appendix B.3.

By virtue of Proposition 11, we can initiate the process with any arbitrary set $M_0 \subseteq [0, 1]$ and iterate on itself to determine the set D, which constitutes the fixed point of the operator B. Thus, we have established an algorithm that enables us to compute the space of feasible and incentive-compatible values of the state variable in our model, δ. In the subsequent section, we will delve into exploring the equivalence between the dynamic multiobjective principal-agent problem delineated in this section and the dynamic principal-agent problem articulated by Spear and Srivastava (1987). This comparison will facilitate a direct assessment of the optimal contracts derived from both problem formulations.

3.3 Equivalence Results

In this section, our focus is on establishing the equivalence between the dynamic multiobjective principal-agent problem outlined in the preceding section and the dynamic standard principal-agent problem articulated by Spear and Srivastava (1987). Our approach involves two key steps: firstly, we will demonstrate the equivalence of the two problems and validate the Pareto Weights representation as an appropriate representation of the multiobjective formulation. Subsequently, we will delve into a comparative analysis of the structures of the optimal contracts arising from each formulation. This comparative examination will provide valuable insights into the similarities and differences between the contractual arrangements derived from the two problem formulations.

Proposition 12 *Let* $\{w^*(\delta, y), a^*(\delta)\}$ *be a solution to the stationary problem defined above. Then, the contract* $\{w^*(\delta, y), a^*(\delta)\}$ *also solves the standard dynamic principal-agent problem for some value of agent's reservation utility* $\hat{V}$.

Proof: See Appendix B.4.

This outcome enables us to conclude that each value of the state variable δ corresponds to a specific value of the agent's reservation utility on the Pareto Frontier resulting from the resolution of these problems. Moreover, following the principles outlined in Hernández-Lerma and Romera (2004), our multiobjective dynamic optimization problem can be represented using Pareto Weights, with δ serving as the state variable:

$$\max_{w(\delta,y)} [\ \delta\, V(\delta) + (1-\delta)\, U(\delta)\ ] \tag{3.11}$$

subject to constraints (3.7), (3.8), (3.9), and (3.10). Each objective function in the multiobjective problem has a designated level of priority. Specifically, δ represents the priority assigned to the agent's *ex-ante* discounted expected utility, while $1-\delta$ denotes the priority assigned to the principal's *ex-ante* discounted expected utility. Based on Proposition 12 and the validity of the Pareto Weights representation, we can assert that for every value of the state variable in the multiobjective problem—the agent's initial bargaining power—there exists a corresponding value of the state variable in the problem posed by Spear and Srivastava (1987), namely the agent's reservation utility, within the resulting Pareto Frontier.

In the remainder of this section, we will study the conditions that characterize the optimal contract that solves problem (3.11), and then we proceed to compare this contract with that that solves the model of Spear and Srivastava (1987).

Proposition 13 *Any optimal contract that solves (3.11) satisfies the following necessary and sufficient conditions:*

$$\frac{1}{v_w(w(\delta, y), a^*(\delta))} = \frac{\delta}{1-\delta} - \frac{\mu(\delta)}{1-\delta}\frac{f_a(y \mid a^*(\delta))}{f(y \mid a^*(\delta))}, \tag{3.12}$$

and

$$(1-\delta)\left[\int_Y [y - w(\delta, y) + \beta\overline{U}(\delta, y)] f_a(y \mid a^*(\delta)) dy\right] +$$

$$\mu(\delta)\left[\int_Y [v(w(\delta, y), a^*(\delta)) + \beta\overline{V}(\delta, y)] f_{aa}(y \mid a^*(\delta)) dy - v_{aa}(w(\delta, y), a^*(\delta))\right] = 0. \tag{3.13}$$

where $\mu(\delta)$ is the Lagrange multiplier associated with the incentive constraint of the multiobjective formulation.

The conditions are obtained by differentiating the Lagrangian multiplier of this optimization problem. The proof that these conditions are necessary and sufficient for an optimal contract is given in Appendix B.5.

Proposition 14 *By the equivalence stated in Proposition 11 and the risk sharing result given by (3.12), the following relationships are obtained:*

$$\lambda = \frac{\delta}{1-\delta} \tag{3.14}$$

and

$$\mu = \frac{\mu(\delta)}{1-\delta} \tag{3.15}$$

where λ and μ are the Lagrange multiplier associated with the participation constraint and incentive constraint, respectively, in the dynamic agency model of Spear and Srivastava (1987).

The equivalence between the Lagrange multiplier linked to the participation constraint in the standard problem and the ratio defined in (3.14) signifies that the marginal impact of increasing the agent's reservation utility corresponds to the ratio of the agent's bargaining power to that of the principal. Put differently, in the standard problem, elevating the agent's reservation utility prompts proportional adjustments in the bargaining powers of both the agent and the principal within the Pareto Weights framework. Similarly, the equivalence between the Lagrange multiplier tied to the incentive constraint in the standard formulation and the ratio expressed in (3.15) can be interpreted in line with Mele (2014)'s definition. This suggests that the cost, in terms of resources allocated, of implementing the optimal effort reduces as the principal's bargaining power, $(1 - \delta)$, increases.

3.4 Computational Strategy

In this section, we outline the computational methodology employed to approximate the optimal solutions of a parameterized version of the multiobjective dynamic principal-agent model introduced earlier. Subsequently, we conduct a comparative analysis by juxtaposing the numerical outcomes with those of the equivalent standard model. We commence by delineating the algorithms that underpin our computational approach, followed by describing the functional forms and parameter configurations utilized in our computational framework.

3.4.1 The Computational Algorithm

In this subsection, we present two algorithms outlining the computational program designed to obtain a numerical solution for the model proposed in this chapter. Algorithm 1 deals with identifying the admissible values of the state variable δ, as per Proposition 11. This involves determining the minimum admissible value, δ_{min}, and the maximum admissible value, δ_{max}, followed by discretizing the space of admissible values for δ. In the second part of our computational strategy, we recursively seek the stationary solution of the Bellman equation for the Pareto Weights representation of our multiobjective dynamic principal-agent model. This involves iterating the Bellman Equation until convergence is achieved, as outlined in Algorithm 2.

Algorithm 1 Admissible Values of State Variable δ

Require: $\delta_{min} > 0$ ▷ Find δ_{min}

$\mathbf{w} \leftarrow (w_H, w_L)$

$EV_i(\mathbf{w}) \leftarrow \left[f(y_H; a_i) \left(\frac{w_H^{1-h}}{1-h} - a_i^2 \right) + f(y_L; a_i) \left(\frac{w_L^{1-h}}{1-h} - a_i^2 \right) \right], \quad i = H, L$

$EU_i(\mathbf{w}) \leftarrow [f(y_H; a_i)(y_H - w_H) + f(y_L; a_i)(y_L - w_L)], \quad i = H, L$

$\delta_0 \leftarrow 0$

$EV_i^*(0) \leftarrow \max\limits_{0 \le \mathbf{w} \le y} \{\delta_0 EV_i(\mathbf{w}) + (1 - \delta_0) EU_i(\mathbf{w})\}, \quad i = H, L$ ▷ s.t. (3.7) and (3.8)

$EV^* \leftarrow \max\{EV_i^*(0)\}, \quad i = H, L$

$EV^{**} \leftarrow EV^*$

$t \leftarrow 0$

while $EV^* = EV^{**}$ **do**

  $t \leftarrow t + 1$

  $\delta_t \leftarrow \delta_{t-1} + \Delta$ for $t = 1, 2, ...;$ where $\Delta > 0$ is arbitrarily small

  $EV_i^*(t) \leftarrow \max\limits_{0 \le \mathbf{w} \le y} \{\delta_t EV_i(\mathbf{w}) + (1 - \delta_t) EU_i(\mathbf{w})\}, \quad i = H, L$ ▷ s.t. (3.7) and (3.8)

  $EV^{**} \leftarrow \max\{EV_i^*(t)\}, \quad i = H, L$

end while

$\delta_{min} \leftarrow \delta_t$

Require: $0 < \delta_{max} < 1$ ▷ Find δ_{max}

$\delta_0 \leftarrow 1$

$EV_i^*(0) \leftarrow \max\limits_{0 \le \mathbf{w} \le y} \{\delta_0 EV_i(\mathbf{w}) + (1 - \delta_0) EU_i(\mathbf{w})\}, \quad i = H, L$ ▷ s.t. (3.7) and (3.8)

$EV^* \leftarrow \max\{EV_i^*(0)\}, \quad i = H, L$

$EV^{**} \leftarrow EV^*$

$t \leftarrow 0$

while $EV^* = EV^{**}$ **do**

  $t \leftarrow t + 1$

  $\delta_t \leftarrow \delta_{t-1} + \Delta$ for $t = 1, 2, ...;$ where $\Delta > 0$ is arbitrarily small

  $EV_i^*(t) \leftarrow \max\limits_{0 \le \mathbf{w} \le y} \{\delta_t EV_i(\mathbf{w}) + (1 - \delta_t) EU_i(\mathbf{w})\}, \quad i = H, L$ ▷ s.t. (3.7) and (3.8)

  $EV^{**} \leftarrow \max\{EV_i^*(t)\}, \quad i = H, L$

end while

$\delta_{max} \leftarrow \delta_t$

$N \leftarrow \frac{(\delta_{max} - \delta_{min}) \times 2}{\varepsilon + 1};$ where $\varepsilon > 0$ is arbitrarily small ▷ Discretize δ

$K \leftarrow 0$

while $K \le N$ **do**

  $K \leftarrow K + 1$

  $D(K) \leftarrow \delta_{min} + \frac{K-1}{N-1}[\delta_{max} - \delta_{min}]$

end while

Algorithm 2 Stationary solution of Bellman Equation

$\mathbf{w} \leftarrow (w_H, w_L)$
$K \leftarrow 0$
while $K \leq N$ **do**
  $K \leftarrow K + 1$
  $S_0(K) \leftarrow 0$ ▷ Initialize value function at zero
  $U_0(K) \leftarrow 0$ ▷ Initialize principal's future utility at zero
  $V_0(K) \leftarrow 0$ ▷ Initialize agent's future utility at zero
end while
$t \leftarrow 0$
while $S_t \neq S_{t-1}$ **do**
  $K \leftarrow 0$
  $P \leftarrow 1$
  $Q \leftarrow 1$
  while $K \leq N$ **do**
    $K \leftarrow K + 1$
    $P \leftarrow \min(K + 2, N)$ ▷ Index of δ' at y_H
    $Q \leftarrow \max(K - 1, 0)$ ▷ Index of δ' at y_L
    $EV_t^i(K; \mathbf{w}) \leftarrow \left[f(y_H; a_i)\left(\frac{w_H^{1-h}}{1-h} - a_i^2 + \beta V_{t-1}(P)\right) + f(y_L; a_i)\left(\frac{w_L^{1-h}}{1-h} - a_i^2 + \beta V_{t-1}(Q)\right)\right]$, $i = H, L$
    $EU_t^i(K; \mathbf{w}) \leftarrow [f(y_H; a_i)(y_H - w_H + \beta U_{t-1}(P)) + f(y_L; a_i)(y_L - w_L + \beta U_{t-1}(Q))]$, $i = H, L$
    $S_t^i(K) \leftarrow \max_{0 \leq \mathbf{w} \leq y} \{D(K)EV_t^i(K; \mathbf{w}) + (1 - D(K))EU_t^i(K; \mathbf{w})\}, i = H, L$ ▷ s.t. (3.7) and (3.8)
    $S_t(K) \leftarrow \max\{S_t^i(K); i = H, L\}$
  end while
  $t \leftarrow t + 1$
end while

3.4.2 Functional Forms and Parameter Values

In this subsection, we outline a series of assumptions concerning functional forms and parameter values necessary for implementing the computational strategy outlined in the previous subsection.

Firstly, the principal's temporary utility function is defined as $u(y, w(y, \delta)) = y - w(y, \delta)$. The agent's temporary utility function takes the form $v(w(y, \delta), a(\delta)) = \frac{w^{1-h}(y,\delta)}{1-h} - a^2(\delta)$, where $0 < h < 1$. It's worth noting that the agent's temporary utility function is separable and follows a Constant Relative Risk Aversion (CRRA) type structure with respect to current compensation, with the coefficient of relative risk aversion denoted by h. Higher values of h correspond to higher degrees of relative risk aversion. The agent's feasible effort choices are discrete and belong to the set $A = \{a_L, a_H\}$, where a_L represents the low effort choice and a_H represents the high effort choice.

Furthermore, there are two levels of output: low (L) and high (H), characterized by the set $Y = y_L, y_H$. The stochastic relationship between effort and output is described by the following probability function:

$$f(y_L \mid a_L) = f(y_H \mid a_H) = \frac{2}{3},$$
$$f(y_H \mid a_L) = f(y_L \mid a_H) = \frac{1}{3},$$

and these probabilities reflect the notion that higher effort levels chosen by the agent increase the likelihood of achieving the high output level.

Regarding the law of motion for the agent's bargaining power, we propose the following:

$$\delta' = z(\delta, y) = \begin{cases} \min\{1, \delta + \varepsilon \cdot \frac{y}{y_H}\} & if \quad y = y_H, \\ \max\{0, \delta - \varepsilon \cdot \frac{y}{y_H}\} & if \quad y = y_L. \end{cases}$$

In the proposed law of motion, the parameter ε, which we refer to as the bargaining drift coefficient, plays a crucial role in governing how bargaining power evolves over time in response to performance outcomes. As a small positive number, it determines the rate at which the agent's bargaining power adjusts based on firm performance. A higher bargaining drift coefficient (ε) leads to more responsive adaptations in bargaining power, while a lower value results in greater stability despite fluctuations in output. Specifically, when a high output y_H is observed, the agent's bargaining power increases in the next period, whereas a low output y_L leads to a decline. This mechanism reinforces incentives by rewarding favorable outcomes and penalizing unfavorable ones, aligning with our assumptions about the function $z(\delta, y)$.

The interpretation of ε is particularly significant as it captures the extent to which changes in the CEO's bargaining power reflect performance-based rewards or penalties. A higher ε amplifies the link between current performance and future bargaining power, intensifying the agent's sensitivity to firm outcomes. Additionally, we assume that the principal's response is asymmetrical, favoring rewards over punishments. This asymmetry is reflected in the adjustment process: increases in the agent's bargaining power are equal to ε, while reductions are moderated—potentially less than ε. Such uneven adjustments mirror empirical findings on CEO compensation, where pay tends to be more responsive to positive rather than negative performance, as documented in studies such as Gopalan et al. (2010) and Bell et al. (2021).

Our set of parameters are the following:
$h = \frac{1}{2}$; $\beta = 0.96$; $Y = \{y_L = 0.4, y_H = 0.8\}$; $A = \{a_L = 0.1, a_H = 0.2\}$; $\varepsilon = 0.001$.

Indeed, it's crucial to clarify that our numerical exercise should not be interpreted as a calibration exercise. While we utilize specific functional forms and parameter values to demonstrate the computational approach and analyse comparative results, these choices are not intended to precisely match real-world data. Instead, they serve to illustrate the model's behaviour under certain assumptions and to provide insights into the dynamics of the principal-agent relationship.

3.5 Numerical Results

In this section, we present some of the stationary and simulation results obtained from our numerical implementation.[2] Both sets of results offer distinct, but complementary perspectives on the model proposed in this chapter.

3.5.1 Stationary Results

As the name suggests, this subsection presents numerical results derived from the model under stationary conditions, focusing on how bargaining power influences the agent's compensation structure and expected utility while comparing different model specifications. We have organized the presentation of these results into three modules: Firstly, we provide results concerning the comparison of the model we propose with related models, considered as benchmark or reference models: the standard dynamic principal-agent model (SDPA), the multiobjective static principal-agent model (MOSPA2), the multiobjective dynamic principal-agent model with no dynamics for the agent's bargaining power (MODPA1), and the full-fledged multiobjective dynamic principal-agent model with dynamics for the agent's bargaining power (MODPA2). [3] Then, we present results obtained from varying the values of relevant parameters. Finally, we showcase results obtained from considering various utility functions for the agent.

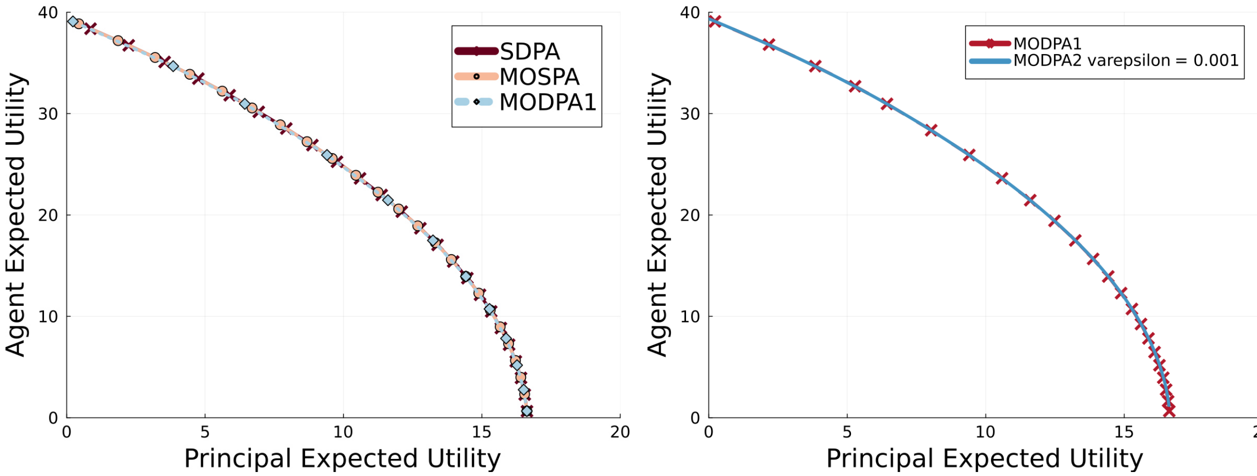
Model Comparison

Figure 3.2 illustrates the Pareto frontiers for different models. The left panel shows that SDPA, MOSPA2, and MODPA1 generate identical outcomes, confirming that when bargaining power remains static, the standard results hold. However, introducing bargaining power dynamics in MODPA2 leads to a noticeable, albeit small, shift in the frontier for low values of bargaining drift ($\varepsilon = 0.001$). This suggests that while the dynamic evolution of bargaining power introduces new elements into the model, its short-term impact remains moderate, preserving the efficiency of standard optimal contracts.

[2]Further information is available here: https://github.com/genarobasulto/Project-Dynamics-of-Bargaining-Power.

[3]See Appendix B.7 for more details about these models.

Figure 3.2: Pareto Frontier: Model comparison

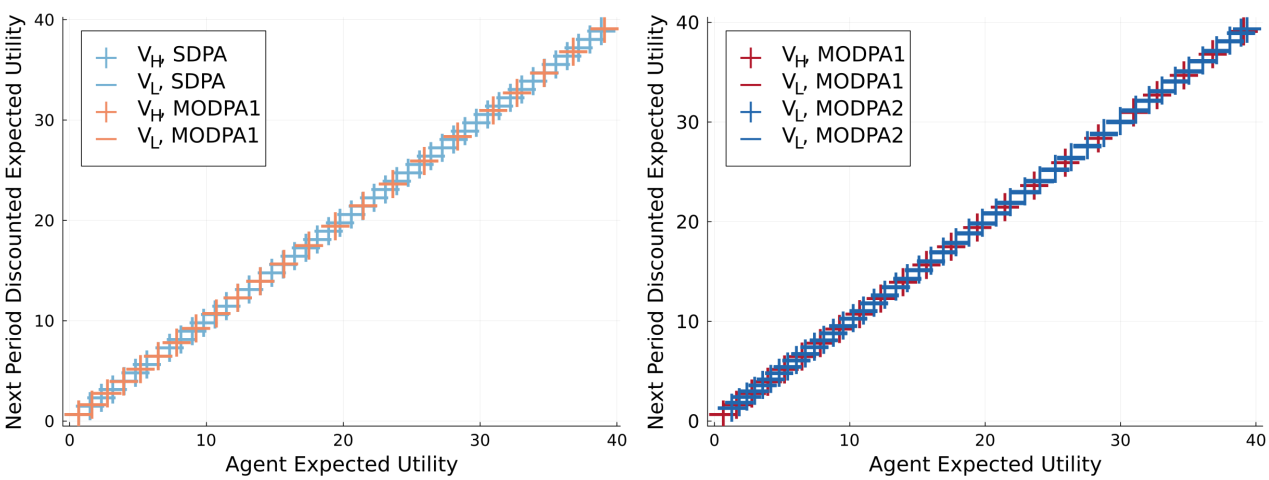


(a) Single objective and multiobjective approach

(b) Bargaining dynamics and no dynamics

Figure 3.3 further explores the agent's promised discounted expected utility across models, showing a linear relationship between the agent's current utility and future expected utility. This implies that bargaining power grows predictably over time. Because future compensation does not respond aggressively to past performance, it suggests that bargaining power acts as an alternative incentive mechanism, reducing the reliance on direct financial compensation.

Figure 3.3: Agent's promised discounted expected utilities: Model comparison



(a) Single objective and multiobjective approach

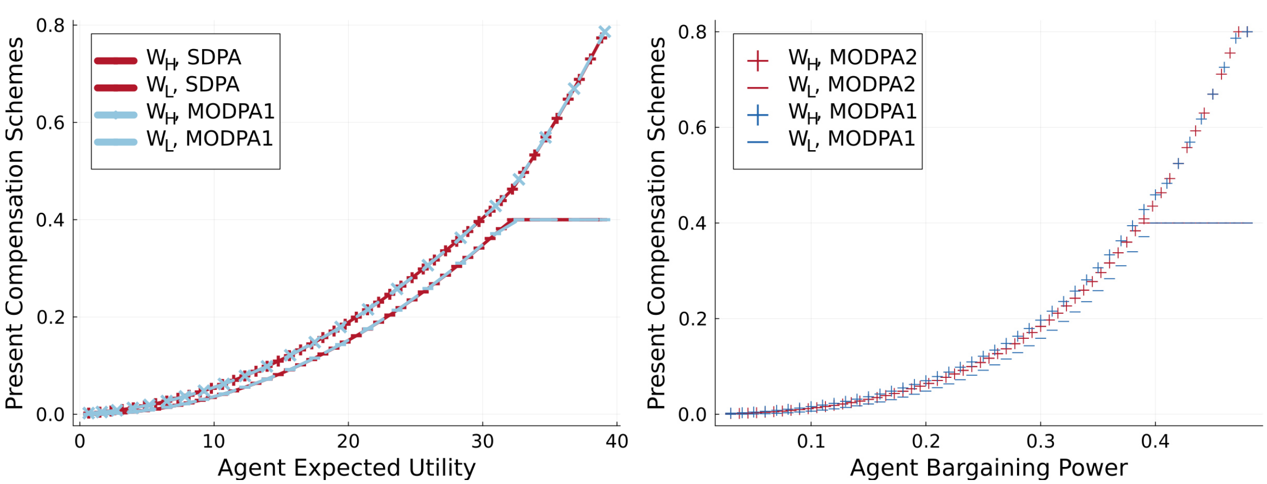
(b) Bargaining dynamics and no dynamics

Figure 3.4 compares present compensation across different models. Panel (a) shows compensation as a function of the agent's expected utility under single and multiobjective formulations. Panel (b) displays compensation as a function of bargaining power, contrasting models with and without endogenous dynamics.

Under MODPA2, compensation grows more gradually with bargaining power. This reflects the principal's anticipation of future shifts in negotiation leverage: the agent is rewarded less through immediate performance-based pay and more through the accumulation of future influence. Compared to models where bargaining power is static, this dynamic specification softens the link between current output and pay.

This mechanism highlights a substitution effect. Accumulated bargaining power can act as an alternative to direct incentive pay. Executives who expect to gain negotiation strength over time may receive higher compensation in anticipation of that influence, even when current performance is not exceptionally strong. Consequently, Figura 3.4 helps illustrate how dynamic considerations of bargaining power can reshape the structure of optimal incentive contracts.

Figure 3.4: Current compensation: Model comparison



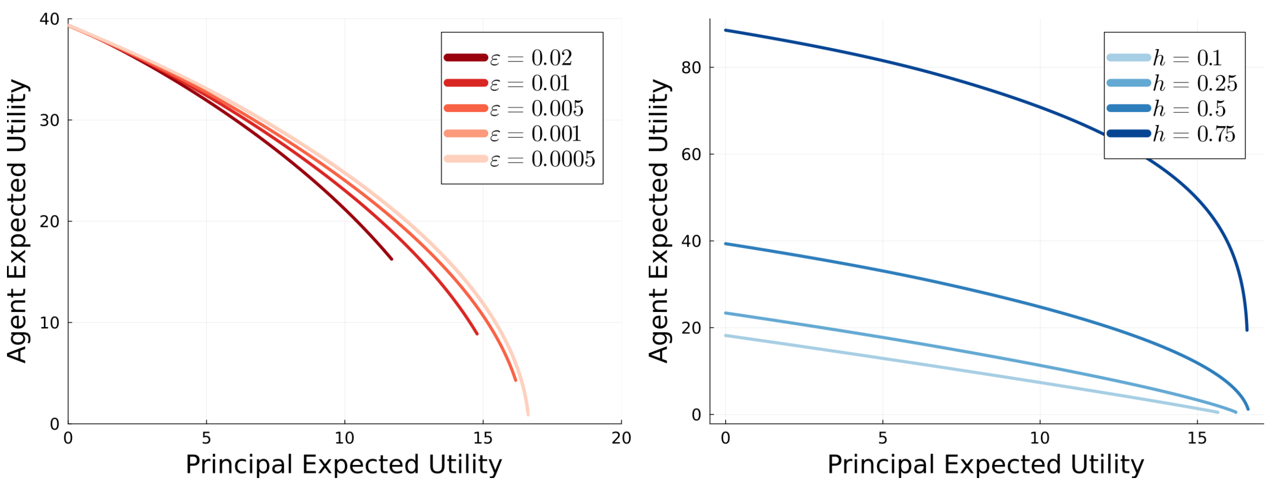
(a) Single objective and multiobjective approach

(b) Bargaining dynamics and no dynamics

Parameter Values

Figure 3.5 demonstrates that increasing the bargaining drift coefficient (ε) amplifies the responsiveness of bargaining power to firm performance, leading to higher volatility in negotiation leverage over time. This heightened drift effect causes greater uncertainty in long-term compensation structures, ultimately shifting the Pareto frontier inward and reducing both the agent's and the principal's expected utility. Notably, the loss in efficiency suggests that while increased bargaining drift can enhance short-term responsiveness, it also introduces higher contract instability, potentially discouraging long-term cooperation between the agent and the principal.

Figure 3.5: Pareto Frontier: Parameter comparison

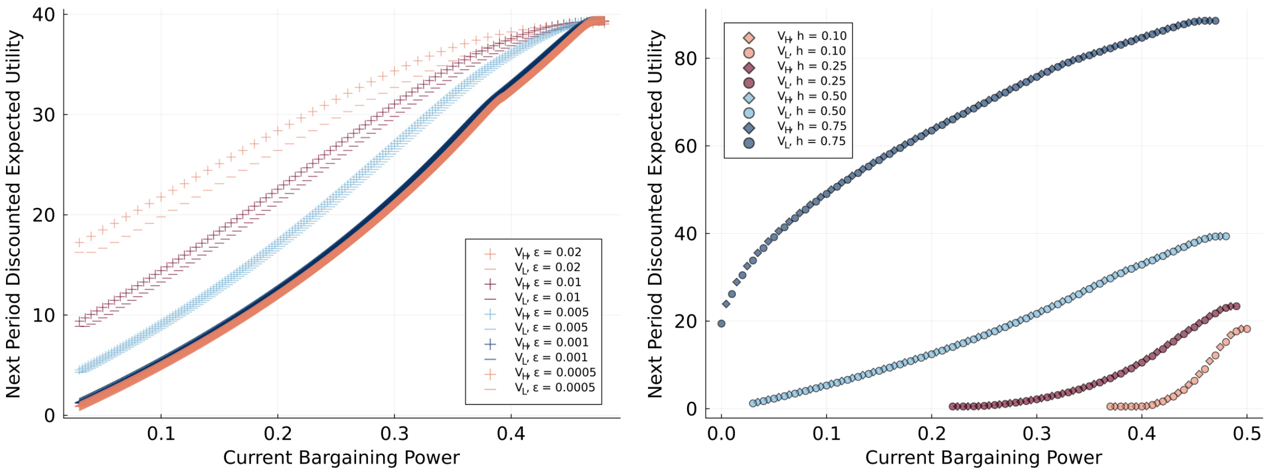


(a) Different Values for ε (b) Different Values for h

Figure 3.6 shows that the agent's expected utility as a function of bargaining power follows a bell-shaped curve, indicating diminishing returns. When bargaining power is too high, additional gains do not translate into proportional increases in expected utility, suggesting that agents may not always seek to maximize bargaining power.

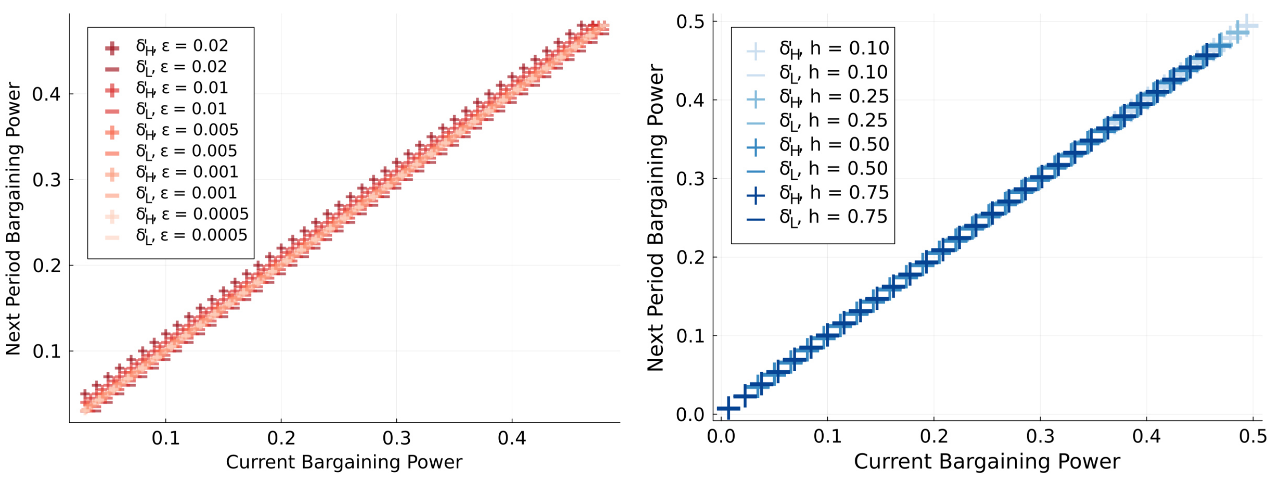
Figure 3.7 highlights that higher values of ε increase fluctuations in bargaining power, reducing the agent's insurance against risk. However, the agent's risk aversion does not significantly impact the difference in compensation between high and low output scenarios.

Figure 3.6: Agent's promised expected utility on current bargaining power: Parameter comparison



(a) Different Values for ε (b) Different Values for h

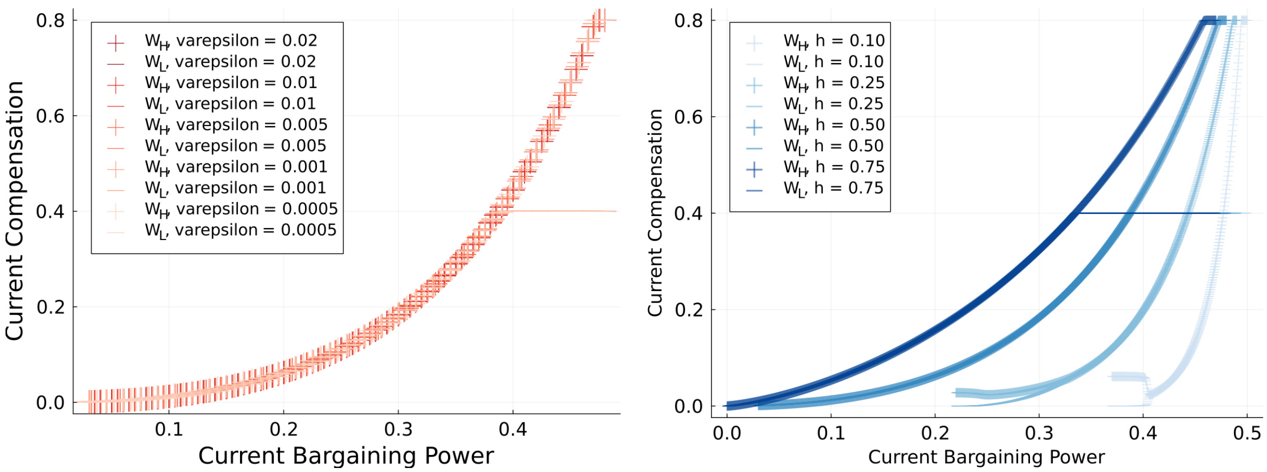
Figure 3.7: Agent's promised bargaining power: Parameter comparison



(a) Different Values for ε (b) Different Values for h

Figure 3.8 confirms that adjusting ε has minimal effects on salaries, while higher risk aversion leads to increased compensation. This suggests that risk-averse agents require higher salaries to offset uncertainty, but bargaining power remains a primary driver of utility.

Figure 3.8: Current compensation: Parameter comparison



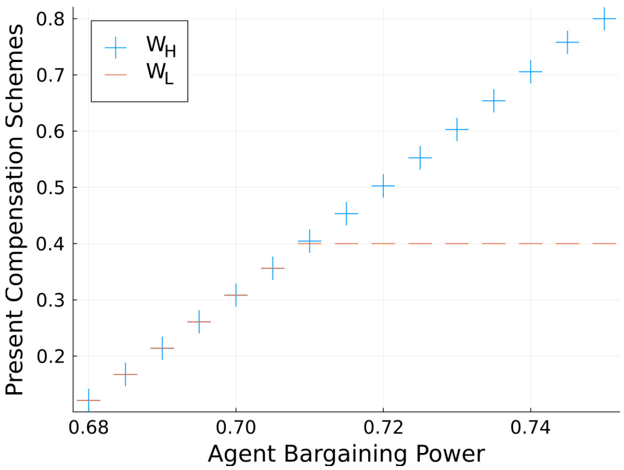
(a) Different Values for ε (b) Different Values for h

Agent's Utility Functions

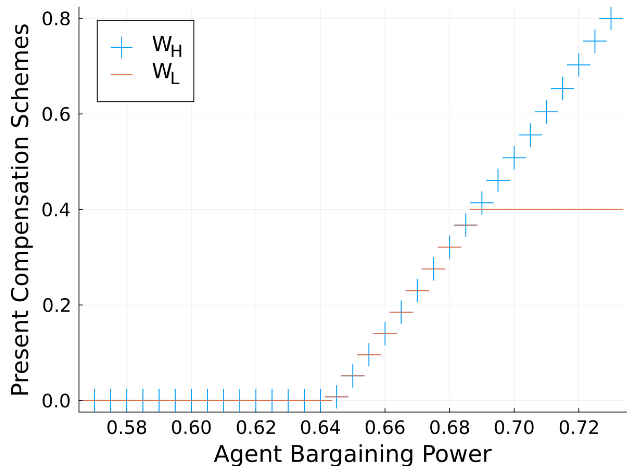
As a robustness check, we solve the model using alternative specifications of the agent's utility function to assess the stability of our results. Figure 3.9 shows that the relationship between reservation utility and salary remains consistently concave across different utility formulations, reinforcing the predictive reliability of the model. This consistency suggests that the model's key insights hold regardless of the specific functional form used to represent agent preferences.

The stationary analysis highlights three main conclusions. First, bargaining power effectively replaces traditional financial incentives, meaning that once it becomes a dynamic element, compensation becomes independent of performance. Second, higher values of ε increase contract volatility, lowering overall efficiency and introducing instability in compensation. Third, risk aversion influences salary levels but does not change bargaining power dynamics significantly.

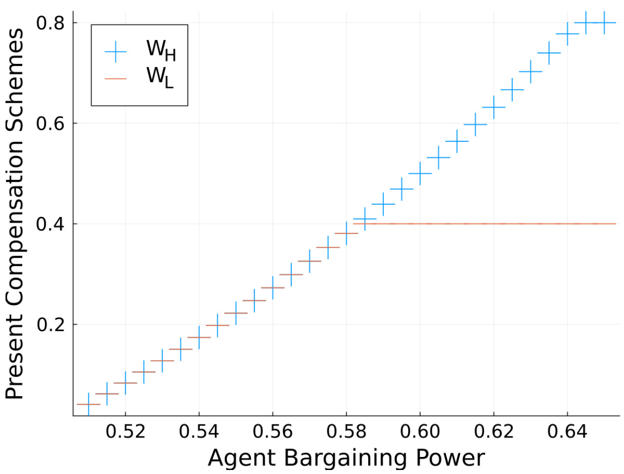
Figure 3.9: Robustness check: Agent's compensation.



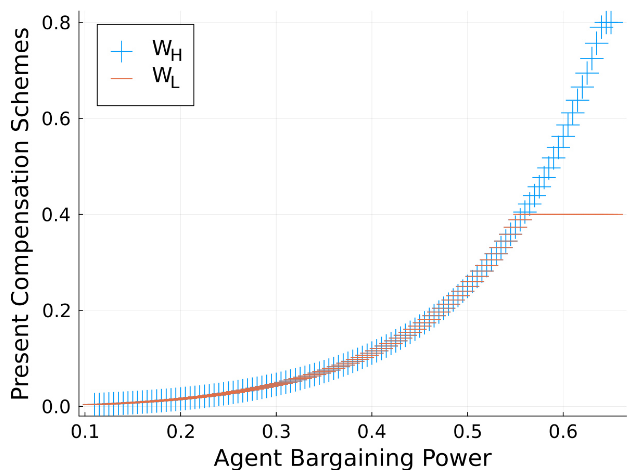
(a) $v(w,a) = -exp(-0.5w) - a^2$



(b) $v(w,a) = -exp[a - \gamma w]$



(c) $v(w,a) = \log(w+1) - a^2$



(d) $v(w,a) = \sqrt{w} - a^2$

3.5.2 Simulation Results

To complement the stationary analysis, the model is simulated over 100 contracting periods to observe how bargaining power and compensation evolve over time. The results provide insight into how initial bargaining power conditions shape long-term outcomes.

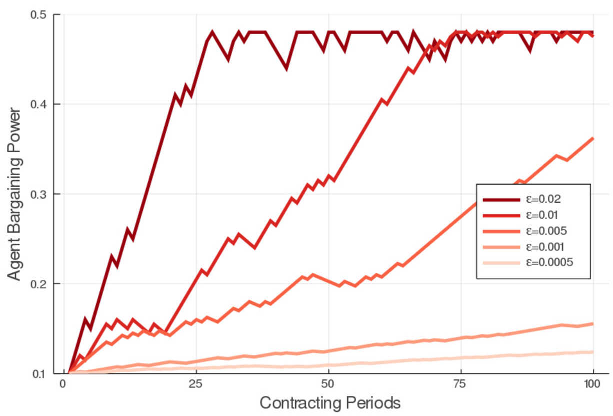
Figure 3.10 tracks the evolution of bargaining power across different initial conditions. Bargaining power consistently increases over time as the agent engages in high-effort strategies. Higher values of ε accelerate the growth of bargaining power, leading to larger fluctuations. When the agent starts with low bargaining power ($\delta_0 = 0.10$), the trajectory is gradual and predictable. However, when initial bargaining power is high ($\delta_0 = 0.40$), growth is more volatile. This pattern suggests that bargaining power is self-reinforcing: Once an agent gains power, subsequent gains become easier, reinforcing their influence in negotiations.

Figure 3.11 presents the evolution of agent compensation over time. When initial bargaining power is low, compensation increases smoothly as bargaining power accumulates. When initial bargaining power is high, compensation fluctuates significantly in later periods, reflecting bargaining power volatility. Compensation is highly path-dependent, meaning that early bargaining power levels shape long-term salary trajectories.

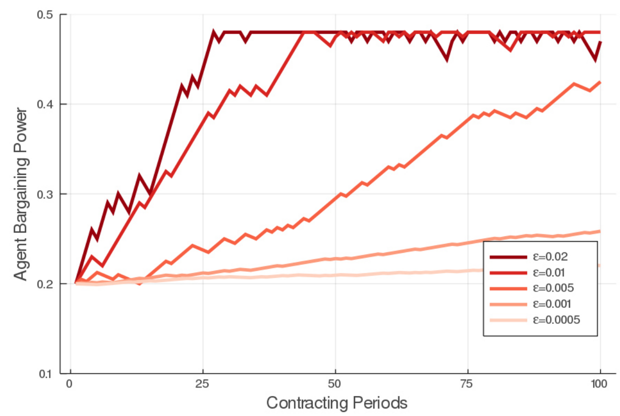
The simulations confirm three key insights. First, bargaining drift ensures that bargaining power grows persistently over time, particularly when firm performance is strong. Second, higher initial bargaining power amplifies future gains but also introduces volatility, making long-term earnings less predictable. Finally, the results suggest that compensation trajectories depend more on accumulated bargaining leverage than on short-term performance, reinforcing the idea that bargaining drift itself serves as an incentive mechanism, distinct from direct pay-for-performance structures.

The findings from both the stationary and simulation analyses lead to four broad conclusions. Bargaining power serves as a substitute for traditional performance-based incentives, meaning that

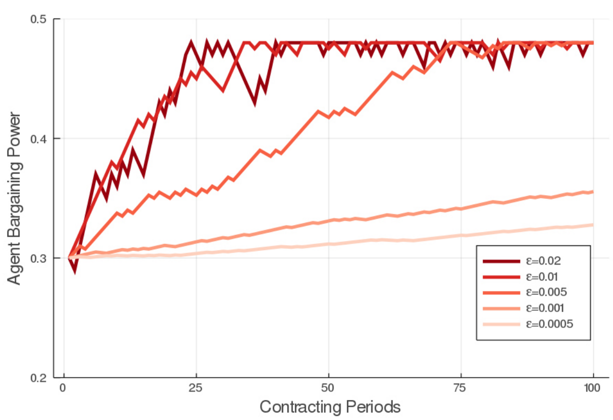
Figure 3.10: Simulation: Agent's bargaining power



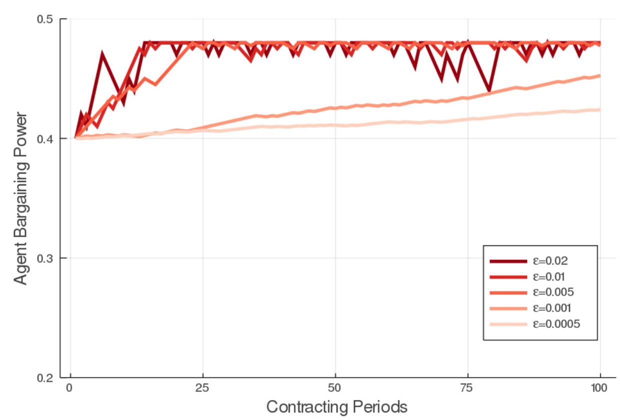
(a) $\delta_0 = 0.10$



(b) $\delta_0 = 0.20$

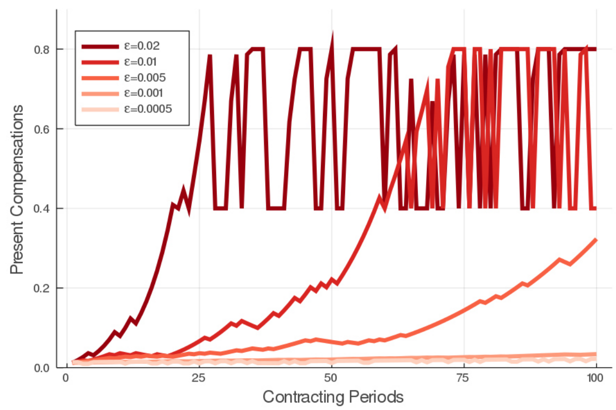


(c) $\delta_0 = 0.30$

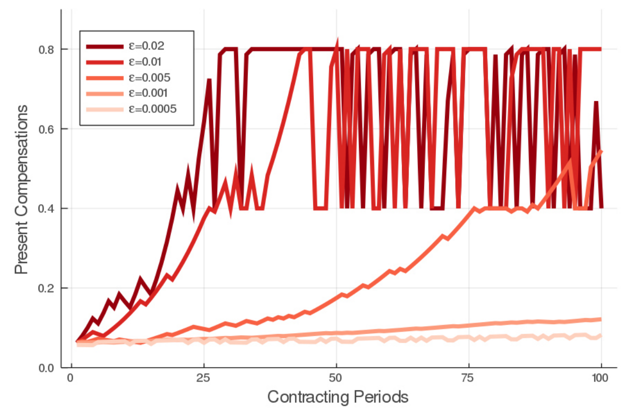


(d) $\delta_0 = 0.40$

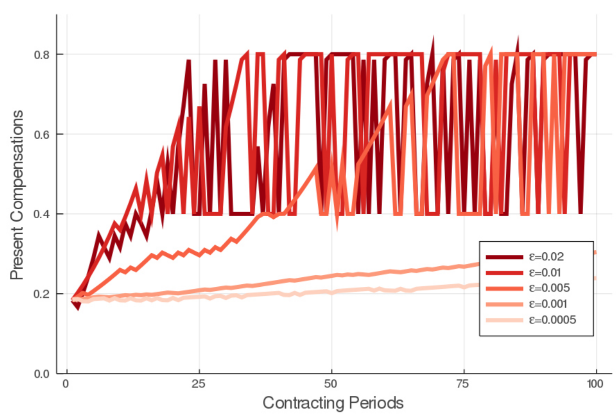
Figure 3.11: Simulation: Agent's compensation



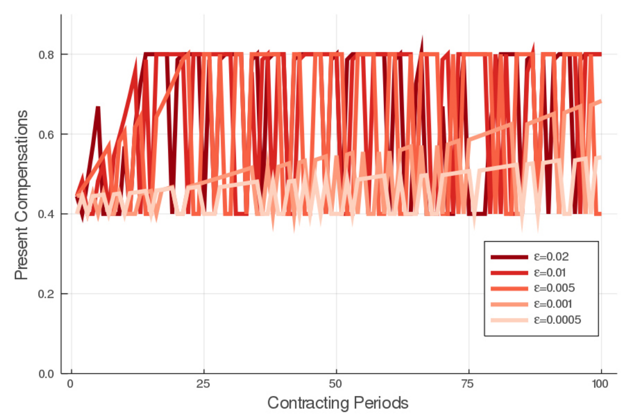
(a) $\delta_0 = 0.10$



(b) $\delta_0 = 0.20$



(c) $\delta_0 = 0.30$



(d) $\delta_0 = 0.40$

pay becomes independent of output once it evolves dynamically. Contract efficiency deteriorates when bargaining power becomes too sensitive to firm performance, as higher ε increases volatility and reduces stability. Risk aversion affects compensation but does not significantly alter bargaining power trajectories. Finally, simulations confirm that bargaining power accumulates predictably over time, reinforcing an agent's influence in future negotiations.

3.6 Econometric Exercise

The numerical results of our model provide a method to estimate the parameter ε in the proposed functional form for the law of motion $z(\delta, y)$, as described in subsection 3.4.2. We interpret this parameter as a measure of the power of incentives in our model: the higher the value of ε,the higher the CEO's level of insurance and the lower the power of incentives.

We begin by considering the results depicted in Figure 3.8, where we plot the agent's current compensation against the corresponding values of the bargaining power parameter. The plot shows that the agent's bargaining power can be represented as a concave function of their current compensation. We assume that both the agent's compensation and their bargaining power are stochastic at any period of time, and that this relationship takes the following form:

$$\delta_t = a + bf(w_t) + e_t \tag{3.16}$$

where $f(w_t)$ is a real, continuous and concave function of w_t, and e_t are independent and normally distributed errors with mean 0 and variance σ^2 for all t.

Furthermore, our numerical results allow us to conclude that when the principal holds all the bargaining power, the agent's compensation will be the lowest possible, which can be normalized to 0, and the parameter a in the bargaining power function takes the value of zero. Similarly, when the agent holds all the bargaining power, they will receive the maximal compensation level, and the parameter b in (3.16) in the bargaining power function takes the value of $1/f(y_H)$. Therefore, given the bargaining power function (3.16) and the results for a and b, we attain the following equation:

$$\delta_t = \frac{1}{f(y_H)} f(w_t) + e_t. \tag{3.17}$$

From the law of motion for the agent's bargaining power, proposed in subsection 3.4.2, we obtain:

$$\delta_{t+1} = z(\delta_t, y) = \begin{cases} \min\{1, \delta_t + \varepsilon \cdot \frac{y_t}{y_H}\} & if \quad y_t = y_H, \\ \max\{0, \delta_t - \varepsilon \cdot \frac{y_t}{y_H}\} & if \quad y_t = y_L. \end{cases}$$

This is equivalent to:

$$\delta_{t+1} - \delta_t = \varepsilon \left((-1)^{I_t^-} \frac{y_t}{y_H} \right),$$

where I_t^- is a dichotomous variable that takes the value 0 when $y_t = y_H$, and the value 1 otherwise.

Replacing w_{t+1} and w_t from (3.17) in the last expression, the following result is obtained:

$$f(w_{t+1}) = f(w_t) + \varepsilon \left((-1)^{I_t^-} \frac{y_t}{y_H} \right) f(y_H) + u_t, \tag{3.18}$$

with $u_t = (e_t - e_{t+1}) f(y_H)$.

Expression (3.18) could be used to empirically estimate ε using a time series database containing CEO compensations and company revenues.

In expression (3.16) a strong assumption is made—that the errors e_t are assumed to be independent. In a time series setting, it is common for the errors to be serially correlated, are independent. In a time series setting, it is common for the errors to be serially correlated, so the best way to estimate the regression is to use an autoregressive moving average (ARMA) model. This approach accounts for the correlation and provides consistent estimators for ε. Another possibility is to consider panel data with observations from different companies during the same period. Exploiting the structure of panel data could give us more insight into how ε varies from one company to another and how it affects the CEO's contractual packages. The remainder of this section is devoted to evaluating parameter values for which this functional form might provide a good identification.

3.6.1 Testing with Simulation Data

In this subsection, we present an econometric exercise using the simulation data from the previous section. The objective is to identify parameter sets for which the functional form given by equation (3.18) yields accurate estimates of ε. We use the function $\delta_t = f(w_t) = w_t^\beta$ to check whether f is concave by running a simple linear regression to estimate β, f will be concave if $\beta \in (0,1)$. In Table 3.1, we present the results of the linear regression: $log(\delta_t) = \beta \log(w_t)$. Given the parameter values used in our numerical implementation, we expect to obtain estimates of $\hat{\beta}$ close to 0.5. Our results indicate that these estimates are indeed close to this value for several values of ε when using the CRRA utility function in the main numerical exercise. Furthermore, for the additional utility functions mentioned in Figure 3.9, the estimates of $\hat{\beta}$ behave as expected, corroborating the results shown in the figure. Hence, this econometric exercise supports the assumption that the function f is concave and validates our proposed empirical equation for identifying the parameter ϵ.

Next, we run a regression to implement equation (3.18). In Table 3.2, we summarize the results obtained using model-generated data over 100 contractual periods for different values of ε and various initial values of the state variable, δ_0. The first section corresponds to $\epsilon = 0.001$. We observe that when the initial value of the agent's bargaining power is 0.10, the ϵ-estimate matches the real value in the model. Furthermore, when the initial values are within the interval $[0.20, 0.30]$, the ϵ estimates are close to their real value and statistically significant. However, when $\delta_0 = 0.40$, the estimate of ϵ estimate is not a good approximation and even has a negative sign.

These results suggest that when the initial value of the agent's bargaining power is low, our proposed dynamics accurately explain the relationship between the agent's compensation and the firm's performance. Conversely, when the initial value of the agent's bargaining power is high, the dynamics appear to change. From Figure 3.11, we can infer that with a high initial value of the agent's bargaining power, compensation remains in a stationary state; it neither grows because it has reached the maximum possible value, nor decreases because the agent continues to exert high

Table 3.1: Regression results on shape of bargaining power on expected compensation

	Dependent variable:		
	log(Deltas)		
	$\varepsilon = 0.001$	$\varepsilon = 0.005$	$\varepsilon = 0.01$
$v(w,a) = \frac{w^{1-h}}{1-h} - a^2$			
log(exp_comp)	0.554^{***}	0.554^{***}	0.553^{***}
	(0.003)	(0.007)	(0.010)
Observations	901	181	91
R^2	0.969	0.969	0.969
$v(w,a) = -exp(-0.5w) - a^2$			
log(exp_comp)	0.368^{***}	0.363^{***}	0.284^{***}
	(0.008)	(0.017)	(0.031)
Observations	381	77	15
R^2	0.860	0.857	0.860
$v(w,a) = -exp[a - \gamma w]$			
log(exp_comp)	0.453^{***}	0.195^{***}	0.047^{***}
	(0.009)	(0.018)	(0.005)
Observations	481	97	33
R^2	0.839	0.538	0.717
$v(w,a) = \log(w+1) - a^2$			
log(exp_comp)	0.364^{***}	0.358^{***}	0.352^{***}
	(0.010)	(0.021)	(0.030)
Observations	281	57	29
R^2	0.839	0.834	0.827
$v(w,a) = \sqrt{w} - a^2$			
log(exp_comp)	0.407^{***}	0.401^{***}	0.401^{***}
	(0.001)	(0.003)	(0.004)
Observations	1,081	217	109
R^2	0.989	0.988	0.988
Note:			*p<0.1; **p<0.05; ***p<0.01

effort levels. Therefore, the observed results for high values of the state variable are expected.

The second and third sections of the table correspond to the values: $\epsilon = \{0.05, 0.01\}$, respectively. We observe that as the value of ε decreases, the range of the state variable values for which we obtain accurate estimates of ε also decreases. One hypothesis is that this phenomenon is due to the linearity of our dynamics. If the actual dynamics follow a higher-order function as ε increases, the regions where our approximations are accurate and the associated errors also expand. Consequently, our model performs better in scenarios where changes in the agent's bargaining power are relatively small, aligning with the assumptions of linearity in our proposed functional form.

3.7 Conclusions

The dynamic evolution of bargaining power has significant implications for compensation design, corporate governance, and broader economic outcomes. This study contributes to the dynamic principal-agent literature by introducing a novel framework where bargaining power evolves exogenously as a function of firm performance. In our proposed law of motion of bargaining power, the bargaining drift coefficient controls how responsive is the next period bargaining power to current firm performance. Unlike traditional models where incentives are primarily driven by direct pay-for-performance mechanisms, our framework highlights how bargaining drift can act as an alternative source of incentives. The analysis shows that as bargaining power accumulates over time, its role in shaping compensation structures becomes more prominent, potentially reducing reliance on traditional performance-contingent pay. Numerical simulations reveal that higher bargaining drift coefficients lead to greater volatility in bargaining power, reinforcing long-term bargaining leverage while simultaneously introducing unpredictability in compensation trajectories. These findings underscore the importance of balancing bargaining drift effects to maintain contract efficiency and stability in executive compensation design.

The findings extend existing research on executive compensation and bargaining power. The study builds on Pandher and Currie (2013)'s analysis of CEO pay evolution by demonstrating that bargaining power accumulation over time can decouple executive compensation from immedi-

Table 3.2: Regression results using simulated data

	Dependent variable:			
	y			
$\varepsilon = \mathbf{0.001}$	$(\delta_0 = 0.10)$	$(\delta_0 = 0.20)$	$(\delta_0 = 0.30)$	$(\delta_0 = 0.40)$
X	0.001^{***}	0.002^{***}	0.002^{***}	-0.048^{***}
	(0.00002)	(0.00002)	(0.00003)	(0.008)
R^2	0.986	0.985	0.985	0.263
F Statistic (df = 1; 99)	$6,954.480^{***}$	$6,667.812^{***}$	$6,483.073^{***}$	35.355^{***}
$\varepsilon = \mathbf{0.005}$				
X	0.009^{***}	0.008^{***}	-0.035^{***}	-0.116^{***}
	(0.0002)	(0.002)	(0.012)	(0.018)
R^2	0.950	0.202	0.083	0.301
F Statistic (df = 1; 99)	$1,893.272^{***}$	25.006^{***}	9.005^{***}	42.633^{***}
$\varepsilon = \mathbf{0.01}$				
X	-0.037^{***}	-0.059^{***}	-0.069^{***}	-0.099^{***}
	(0.013)	(0.015)	(0.015)	(0.017)
R^2	0.074	0.134	0.177	0.253
F Statistic (df = 1; 99)	7.934^{***}	15.261^{***}	21.302^{***}	33.474^{***}
Observations	100			
Note:				*p<0.1; **p<0.05; ***p<0.01

ate performance, shifting incentives toward long-term strategic positioning rather than short-term financial outcomes. Similarly, this framework complements Lyu et al. (2018)'s work on governance and negotiation by illustrating how bargaining power dynamics interact with governance mechanisms, particularly in settings where executives with accumulated influence rely more on negotiation leverage than on direct performance-based pay adjustments. Additionally, the study aligns with Rajgopal et al. (2011)'s analysis of labor market frictions, showing how the dynamic interaction between bargaining drift and compensation mechanisms can lead to increased contract volatility, affecting both firm stability and managerial incentives. These insights provide a richer understanding of how evolving managerial power shapes executive compensation structures in dynamic settings.

The evolution of bargaining power in a principal-agent relationship directly affects how firms design executive compensation packages. Traditional pay structures assume a relatively static power dynamic between shareholders and executives, but our model demonstrates that managerial power is an evolving state variable. This suggests that firms should adopt adaptive pay structures that respond dynamically to shifts in bargaining power. Firms should tie the vesting of stock options and other incentive-based compensation to both short-term performance and long-term shifts in bargaining power (Bebchuk and Fried (2003)). Executives with higher initial bargaining power may extract higher salaries, requiring firms to increase pay-performance sensitivity accordingly (Edmans and Gabaix (2016)). Contracts should be designed to limit the excessive accumulation of bargaining power, such as through caps on bonuses or clawback provisions (Jenter and Kanaan (2015)).

Corporate governance must also adapt by monitoring power trajectories and implementing oversight mechanisms if managerial influence grows disproportionately. Firms can achieve this by using structured performance evaluations that incorporate bargaining power metrics, such as the frequency of executive influence on board decisions, compensation adjustments, and strategic shifts favoring executive priorities. Regular external audits of executive compensation, benchmarking against industry standards, and requiring transparent disclosures about CEO influence can serve as additional tools. AI-driven analytics and real-time corporate governance dashboards can also help track changes in executive bargaining power over time, providing early warnings for potential governance risks. Predefined power thresholds could serve as triggers for governance checks, reducing the risk of rent extraction and aligning CEO incentives with shareholder interests (Sannikov (2008)).

Beyond corporate settings, our model provides a framework for understanding power dynamics in various high-performance industries, such as sports, academia, and entertainment. Coaches and athletes negotiate contracts based on previous performance, similar to executives who use their bargaining power to obtain better compensation (Gayle and Miller (2009)). High-impact researchers use citation metrics and grant funding as leverage in salary negotiations, reflecting CEO pay dynamics (Binmore et al. (1986)). In the entertainment industry, actors and directors with strong box office performance can demand better contracts over time. Additionally, our findings offer insight into wage inequality. As bargaining power accumulates, the disparities between executive and worker pay increase (Gabaix and Landier (2008)). Regulators and corporate boards could consider power-adjusted pay ratios to prevent excessive divergence between compensation levels (Holmström (1979); Grossman and Hart (1983a)).

Although the proposed framework serves as a first approximation, it lays a foundation for future research. Enhancements could include refining the law of motion that governs the bargaining drift coefficient, incorporating empirical validation, and exploring industry-specific variations or alternative performance metrics. These extensions would further enrich the theoretical and practical understanding of bargaining dynamics and governance structures. In conclusion, this study advances the analysis of the dynamics of bargaining power in the relationships between principals and agent by addressing a critical gap in the literature. By integrating insights from related literature, such as Pandher and Currie (2013), Lyu et al. (2018), and Rajgopal et al. (2011), it provides a comprehensive framework for understanding the interaction between managerial influence, executive pay, and governance mechanisms. The results offer actionable guidance for scholars, practitioners, and policy makers to better navigate the complexities of CEO compensation and bargaining power, particularly in dynamic settings where the bargaining drift coefficient plays a defining role.

Conclusions

The findings of this thesis underscore the pivotal role of bargaining power in shaping contract structures under moral hazard in both static and dynamic settings. Chapter 2 demonstrates that bargaining power determines the Pareto-efficient frontier of contracts, with compromise solutions ensuring flexibility while maintaining efficiency. Chapter 3 highlights the influence of hidden actions, outside options, and power asymmetries on contract design, leading to deviations from standard Nash solutions when bargaining power is more evenly distributed. Chapter 4 extends this analysis to a dynamic context, illustrating how bargaining power evolves endogenously and impacts compensation structures, particularly in CEO pay-performance sensitivity.

By challenging the traditional one-sided principal-agent framework, this research establishes that bargaining dynamics are integral to optimal contract design. The findings offer a refined perspective on incentive structures, contributing to contract theory and organizational economics.

This thesis advances classical moral hazard models by incorporating bargaining power as an endogenous element, demonstrating its essential role in contract efficiency. By establishing the equivalence between compromise solutions, Pareto weights, and Nash bargaining, it provides a unifying framework for incentive design. Furthermore, the development of a novel dynamic model reveals how bargaining power shifts over time, offering new insights into long-term contract evolution.

From a practical standpoint, these findings have significant implications across various fields. In labor markets, wage negotiations should account for evolving power dynamics rather than relying on static bargaining conditions. In corporate governance, CEO contracts should be designed to reflect long-term bargaining evolution, moving beyond simplistic pay-for-performance schemes. Similarly, in financial contracts, greater bargaining flexibility between lenders and borrowers can enhance contract efficiency. By bridging theory and practice, this research provides valuable guidance for policymakers and organizations in designing incentive-compatible contracts.

Despite its contributions, this thesis has certain limitations that present avenues for further research. Incorporating additional empirical validation using real-world data from labor markets, corporate governance, and financial contracts would reinforce the theoretical findings. Future research addressing these aspects would further refine the insights presented in this study.

This thesis underscores the fundamental role of bargaining power dynamics in contract efficiency and incentive structures. By integrating compromise solutions, Pareto weights, and dynamic modelling, it contributes to contract theory and deepens the understanding of economic negotiations. The findings challenge conventional principal-agent models and highlight the importance of flexible, evolving contract structures. Future research should build upon these insights through empirical validation and policy applications, further enhancing our understanding of bargaining power in economic contracts and its broader implications for markets and institutions.

A Appendix of Chapter 2

A.1 Proof of Proposition 4

Proof. The demonstration is straightforward. To establish the desired optimality condition, one simply differentiates the manager's expected utility with respect to effort and sets the result to zero.

For the second part of the proposition, it is shown that effort e increases with the share of the output a. This is verified by calculating the second derivative of the manager's expected utility with respect to effort, expressed as:

$$\bar{y}''_{ee}(e) - c''_{ee}(e) - \frac{1}{2} r a^2 \sigma''_{ee}(e).$$

Since we are maximizing the manager's expected utility, we assume that this second derivative is negative. Using the first-order condition derived in the initial part of the proposition and applying the implicit function theorem, we obtain:

$$\frac{de}{da} = -\frac{\bar{y}'_e(e) - r a \sigma'_e(e)}{\bar{y}''_{ee}(e) - c''_{ee}(e) - \frac{1}{2} r a^2 \sigma''_{ee}(e)}.$$

This expression is positive because the numerator is positive—based on the conditions imposed on the mean output and its variation—and the denominator is negative, as assumed earlier. Hence, the effort e is positively related to the share of output a. ■

A.2 Proof of Proposition 5

Proof. Let λ denote the Lagrange multiplier associated with the limited liability constraint. The following results are directly obtained from the associated Karush-Kuhn-Tucker problem.

$$\max_{\{a,b\}} \left[\rho \, \mathbb{E}U(a,b,e^*) + (1-\rho) \, \mathbb{E}V(y(e^*),a,b)\right] \quad \text{s.t.} \quad b \geq 0,$$

the associated Lagrangian function is:

$$\mathcal{L} = \rho \mathbb{E}U(a,b,e^*) + (1-\rho)\mathbb{E}V(y(e^*),a,b) + \lambda b;$$

The first-order condition with respect to b is given by:

$$\rho[1] + (1-\rho)[-1] + \lambda = 0.$$

Therefore, $\lambda = 1 - 2\rho$. If λ is binding; that is, $b = 0$, then $\rho < \frac{1}{2}$. If λ is non-binding, $b > 0$, and the optimization problem can be written as:

$$\begin{aligned} \mathcal{L} &= \rho\,\mathbb{E}U(a,b,e^*) + (1-\rho)\mathbb{E}V(y(e^*),a,b) \\ &= \rho b + \rho\,\mathbb{E}U(a,0,e^*) + (1-\rho)(-b) + (1-\rho)\mathbb{E}V(y(e^*),a,0) \\ &= [2\rho-1]b + [\rho\mathbb{E}U(a,0,e^*) + (1-\rho)\mathbb{E}V(y(e^*),a,0)]\,. \end{aligned}$$

Therefore, to maximize the above expression, it is necessary that $\rho \geq \frac{1}{2}$. ■

A.3 Proof of Proposition 6

Proof. Given the Karush-Kuhn-Tucker problem

$$\max_{\{a,b,\}} [\,\rho\,\mathbb{E}U(a,b,e^*) \;+\; (1-\rho)\,\mathbb{E}V(y(e^*),a,b)\,] \qquad s.t.$$

$$\mathbb{E}U(a,b,e^*) \geq \bar{U},$$

$$\mathbb{E}V(y(e^*),a,b) \geq \bar{V},$$

$$b \geq 0.$$

Consider λ, $\mu_{\bar{U}}$, and $\mu_{\bar{V}}$ as the Lagrange multipliers for the limited liability constraint, the participation constraint for the manager, and the participation constraint for the owner, respectively. Therefore, the Lagrangian is the following:

$$\mathcal{L} = \rho\mathbb{E}(\bar{U}) + (1-\rho)\mathbb{E}(\bar{V}) + \lambda b - \mu_{\bar{U}}\left[\bar{U} - \mathbb{E}(\bar{U})\right] - \mu_{\bar{V}}\left[\bar{V} - \mathbb{E}(\bar{V})\right].$$

In Region I, we consider that the limited liability constraint is binding, meaning $b = 0$. Therefore, if $\mu_{\bar{U}}$ is binding and $\mu_{\bar{V}}$ is not, then $\mathbb{E}(\bar{U}) = \bar{U}$. This scenario corresponds to the principal-agent model, where the owner is responsible for designing the contractual scheme. In this case, the owner only needs to ensure that the manager's participation constraint is satisfied. To summarise, this scenario occurs when $\rho \leq \rho_L$, and a^* must be the solution to $a\bar{y}(e) - c(e) - \frac{1}{2}ra^2\sigma(e) = \bar{U}$.

On the other hand, when $\mu_{\bar{U}}$ is not binding and $\mu_{\bar{V}}$ is binding, then $\mathbb{E}(\bar{V}) = \bar{V}$. Here, the solution corresponds to the managerial model, where the manager is responsible for designing the contractual scheme and ensuring that the owner's participation constraint is satisfied. This scenario occurs when $\rho_H \leq \rho$. Then, a^* must be the solution to $(1-a)\bar{y}(e) = \bar{V}$.

In the final scenario, where $\rho_L < \rho < \rho_H$, neither $\mu_{\bar{U}}$ nor $\mu_{\bar{V}}$ is binding, and a^* must be the solution of $\rho[\bar{y}(e) - ra\sigma(e)] + (1-\rho)[(1-a)\bar{y}'_e(e)e'_a(a) - \bar{y}(e)] = 0$, it is derived from the first-order condition with respect to a. ■

A.4 Proof of Proposition 7

Proof. In Region II, since $\lambda = 0$, it follows that $b \geq 0$. According to the Karush-Kuhn-Tucker conditions from the proof of proposition (6), the first-order condition with respect to b gives $\rho = \frac{1}{2}$. For the first-order condition with respect to a, we obtain $(1-a)\bar{y}'_e(e)e'_a(a) - ra\sigma(e) = 0$, which provides the optimal criterion for choosing a. When $\rho < \frac{1}{2}$, we are in a situation where $\mu_{\bar{U}}$ is binding and $\mu_{\bar{V}}$ is not. To ensure the owner's participation constraint, set $b = (1-a^*)\bar{y}(e^*) - \bar{V}$. Conversely, when $\rho > \frac{1}{2}$, $\mu_{\bar{V}}$ is binding and $\mu_{\bar{U}}$ is not. To satisfy the manager's participation constraint, use $b = \bar{U} - a\bar{y}(e) + c(e) + \frac{1}{2}ra^2\sigma(e)$. ■

A.5 Proof of Proposition 8

Proof. In this region, as the name suggests, it represents a mix of the two previous regions. The proof is as follows, based on the cases presented in proofs of propositions 6 and 7. ■

A.6 Bargaining's Extreme Values

Given the outside option for the manager and the owner, $(\bar{U}, \bar{V})$, respectively, to calculate the high bargaining power, ρ_H we recuperated from the owner reserve utility.

$$\bar{V} = \frac{1}{c}\frac{(1-\rho_H)^2 - \rho_H(1-\rho_H)(1-rc\overline{\sigma})}{[2(1-\rho_H) - \rho_H(1-rc\overline{\sigma})]^2}$$

Therefore,

$$\rho_H = \frac{[8c\bar{V} - (1-4c\bar{V})(1-rc\overline{\sigma}) - 1] \pm \sqrt{[-8c\bar{V} + (1-4c\bar{V})(1-rc\overline{\sigma}) + 1]^2 - 4[4c\bar{V} + (1-4c\bar{V})(1-rc\overline{\sigma}) + c\bar{V}(1-rc\overline{\sigma})][4c\bar{V} - 1]}}{2[4c\bar{V} + (1-4c\bar{V})(1-rc\overline{\sigma}) + c\bar{V}(1-rc\overline{\sigma})]}$$

On the other hand, to calculate the low bargaining power, ρ_L we recuperated from the manager reserve utility.

$$\bar{U} = \frac{1}{2c}\frac{(1-\rho_L)^2(1-rc\overline{\sigma})}{[2(1-\rho_L) - \rho_L(1-rc\overline{\sigma})]^2}$$

Therefore,

$$\rho_L = \frac{[16c\bar{U} + 8c\bar{U}(1-rc\overline{\sigma}) - 2(1-rc\overline{\sigma})] \pm \sqrt{[-16c\bar{U} - 8c\bar{U}(1-rc\overline{\sigma}) + 2(1-rc\overline{\sigma})]^2 - 4[8c\bar{U} + 8c\bar{U}(1-rc\overline{\sigma}) + 2c\bar{U}(1-rc\overline{\sigma})^2 - (1-rc\overline{\sigma})][8c\bar{U} - (1-rc\overline{\sigma})]}}{2[8c\bar{U} + 8c\bar{U}(1-rc\overline{\sigma}) + 2c\bar{U}(1-rc\overline{\sigma})^2 - (1-rc\overline{\sigma})]}$$

A.7 Comparative Statics in the Region I: Share of Output

We present the derivatives related to the comparative statics of the share of output allocated to managerial compensation in Region I, for each case. On the left-hand side, we have the case where ρ lies within critical thresholds, ρ_L and ρ_H. On the right side, we present the case for $\rho < \rho_L$. Finally, we examine the case when $\rho > \rho_H$.

$$\frac{\partial}{\partial \rho}\left[\frac{1-\rho}{2((1-\rho)-\rho(1-rc\overline{\sigma})}\right] = \frac{1-rc\overline{\sigma}}{[2((1-\rho)-\rho(1-rc\overline{\sigma})]^2} > 0$$

$$\frac{\partial}{\partial c}\left[\frac{1-\rho}{2((1-\rho)-\rho(1-rc\overline{\sigma})}\right] = -\frac{(1-\rho)\rho r\overline{\sigma}}{[2((1-\rho)-\rho(1-rc\overline{\sigma})]^2} > 0$$

$$\frac{\partial}{\partial r}\left[\frac{1-\rho}{2((1-\rho)-\rho(1-rc\overline{\sigma})}\right] = -\frac{(1-\rho)\rho c\overline{\sigma}}{[2((1-\rho)-\rho(1-rc\overline{\sigma})]^2} > 0$$

$$\frac{\partial}{\partial \overline{\sigma}}\left[\frac{1-\rho}{2((1-\rho)-\rho(1-rc\overline{\sigma})}\right] = -\frac{(1-\rho)\rho rc}{[2((1-\rho)-\rho(1-rc\overline{\sigma})]^2} > 0$$

$$\frac{\partial}{\partial \bar{U}}\sqrt{\frac{2c\bar{U}}{1-rc\overline{\sigma}}} = \frac{c}{(1rc\overline{\sigma})\sqrt{\frac{2c\bar{U}}{1-rc\overline{\sigma}}}} > 0$$

$$\frac{\partial}{\partial c}\sqrt{\frac{2c\bar{U}}{1-rc\overline{\sigma}}} = \frac{\bar{U}}{(1-rc\overline{\sigma})\sqrt{\frac{2c\bar{U}}{1-rc\overline{\sigma}}}} > 0$$

$$\frac{\partial}{\partial r}\sqrt{\frac{2c\bar{U}}{1-rc\overline{\sigma}}} = \frac{\bar{U}c^2\overline{\sigma}}{(1-rc\overline{\sigma})\sqrt{\frac{2c\bar{U}}{1-rc\overline{\sigma}}}} > 0$$

$$\frac{\partial}{\partial \overline{\sigma}}\sqrt{\frac{2c\bar{U}}{1-rc\overline{\sigma}}} = \frac{\bar{U}c^2 r}{(1-rc\overline{\sigma})\sqrt{\frac{2c\bar{U}}{1-rc\overline{\sigma}}}} > 0$$

$$\frac{\partial}{\partial \bar{V}}\left[\frac{1-\sqrt{1-4c\bar{V}}}{2}\right] = \frac{c}{\sqrt{1-4c\bar{V}}} > 0$$

$$\frac{\partial}{\partial c}\left[\frac{1-\sqrt{1-4c\bar{V}}}{2}\right] = \frac{\bar{V}}{\sqrt{1-4c\bar{V}}} > 0$$

A.8 Comparative Statics Region II: Share of Output and Optimal Effort

The derivatives associated with the comparative statics of the output share assigned to managerial compensation (on the left-hand side) and the optimal effort (on the right-hand side) in Region II are presented. Recall that in this region, they do not depend on bargaining power. For $\rho > 1/2$ is examined on the left side and for $\rho < 1/2$ on the right side.

$$\frac{\partial}{\partial r}\left[\frac{1}{1+r c\bar{\sigma}}\right] = -\frac{c\bar{\sigma}}{(1+rc\bar{\sigma})^2} < 0 \qquad \frac{\partial}{\partial r}\left[\frac{1}{c(1+r c\bar{\sigma})}\right] = -\frac{\bar{\sigma}}{(1+rc\bar{\sigma})^2} < 0$$

$$\frac{\partial}{\partial c}\left[\frac{1}{1+r c\bar{\sigma}}\right] = -\frac{r\bar{\sigma}}{(1+rc\bar{\sigma})^2} < 0 \qquad \frac{\partial}{\partial c}\left[\frac{1}{c(1+r c\bar{\sigma})}\right] = -\frac{1+2rc\bar{\sigma}}{c^2(1+rc\bar{\sigma})^2} < 0$$

$$\frac{\partial}{\partial \bar{\sigma}}\left[\frac{1}{1+r c\bar{\sigma}}\right] = -\frac{rc}{(1+rc\bar{\sigma})^2} < 0 \qquad \frac{\partial}{\partial \bar{\sigma}}\left[\frac{1}{c(1+r c\bar{\sigma})}\right] = -\frac{r}{(1+rc\bar{\sigma})^2} < 0$$

A.9 Comparative Statics Region II: Base Compensation

In this appendix, we present the comparative statics of the base compensation, which serves as the incentive instrument in this section, to achieve the outside option of either the owner or the manager in Region II.

$$\frac{\partial}{\partial \bar{V}}\left[\frac{rc\bar{\sigma}}{c(1+rc\bar{\sigma})^2} - \bar{V}\right] = -1 < 0 \qquad \frac{\partial}{\partial \bar{U}}\left[\bar{U} - \frac{1-rc\bar{\sigma}}{2c(1+rc\bar{\sigma})^2}\right] = 1 > 0$$

$$\frac{\partial}{\partial r}\left[\frac{rc\bar{\sigma}}{c(1+rc\bar{\sigma})^2} - \bar{V}\right] = \frac{\bar{\sigma}(1-rc\bar{\sigma})}{(1+rc\bar{\sigma})^3} > 0 \qquad \frac{\partial}{\partial r}\left[\bar{U} - \frac{1-rc\bar{\sigma}}{2c(1+rc\bar{\sigma})^2}\right] = -\frac{\bar{\sigma}(rc\bar{\sigma}-3)}{2(1+rc\bar{\sigma})^3} > 0$$

$$\frac{\partial}{\partial c}\left[\frac{rc\bar{\sigma}}{c(1+rc\bar{\sigma})^2} - \bar{V}\right] = -\frac{2r^2\bar{\sigma}^2}{(1+rc\bar{\sigma})^3} < 0 \qquad \frac{\partial}{\partial c}\left[\bar{U} - \frac{1-rc\bar{\sigma}}{2c(1+rc\bar{\sigma})^2}\right] = \frac{1+rc\bar{\sigma}(3-2rc\bar{\sigma})}{2c^2(1+rc\bar{\sigma})^3} > 0$$

$$\frac{\partial}{\partial \bar{\sigma}}\left[\frac{rc\bar{\sigma}}{c(1+rc\bar{\sigma})^2} - \bar{V}\right] = \frac{r(1-rc\bar{\sigma})}{(1+rc\bar{\sigma})^3} > 0 \qquad \frac{\partial}{\partial \bar{\sigma}}\left[\bar{U} - \frac{1-rc\bar{\sigma}}{2c(1+rc\bar{\sigma})^2}\right] = -\frac{r(rc\bar{\sigma}-3)}{2(1+rc\bar{\sigma})^3} > 0$$

B Appendix of Chapter 3

B.1 Proof of Proposition 9

Proof. Let δ be arbitrary fixed. $\mathcal{W}(\delta)$ is bounded. We need to prove that $\mathcal{W}(\delta)$ is also closed. Let $\{(U^n(\delta), V^n(\delta))\} \in \mathcal{W}(\delta)$ such that $\underset{n\longrightarrow\infty}{lim}\{(U^n(\delta), V^n(\delta))\} = \{(U^\infty(\delta), V^\infty(\delta))\}$. We have to show that $\{(U^\infty(\delta), V^\infty(\delta))\} \in \mathcal{W}(\delta)$, or that there exists a contract $\sigma^{\delta,\infty}$ that satisfies (3.1), (3.2), (3.3), (3.4), $U(\sigma^{\delta,\infty} \mid h^0) = U^\infty(\delta)$, and $V(\sigma^{\delta,\infty} \mid h^0) = V^\infty(\delta)$. We construct this optimal contract $\sigma^{\delta,\infty}$. The definition of $\mathcal{W}(\delta)$ allows us to say that there exists a sequence of contracts $\{\sigma^{\delta,n}\} = \{a_t^{\delta,n}(h^{t-1}), w_t^{\delta,n}(h^t)\}$ that satisfies (3.1), (3.2), (3.3), and (3.4), $\forall n$. Hence,

$$U^\infty(\delta) = \underset{n\longrightarrow\infty}{lim}\sum_{t=0}^{\infty}\beta^t \int_Y (y_t - w_t^{\delta,n}(h^t) f(y_t; a_t^{\delta_0,n}(h^{t-1}))dh^t,$$

$$V^{\infty}(\delta) = \underset{n\longrightarrow\infty}{lim}\sum_{t=0}^{\infty}\beta^t \int_Y (v(w_t^{\delta,n}(h^t), a_t^{\delta,n}(h^{t-1}))f(y_t; a_t^{\delta,n}(h^{t-1}))dh^t.$$

At $t = 1$, $\sigma_1^{\delta} = \{a_1^{\delta,n}(h^0), w_1^{\delta,n}(h^1)\}$ is a finite collection of bounded sequences, so there exists a collection of subsequences $\{a_1^{\delta,n_q}(h^0), w_1^{\delta,n_q}(h^1)\}$ that satisfy:

$$\underset{n_q\longrightarrow\infty}{lim} a_1^{\delta,n_q}(h^0) = a_1^{\delta,\infty}(h^0), \quad \text{and}$$

$$\underset{n_q\longrightarrow\infty}{lim} w_1^{\delta,n_q}(h^1) = w_1^{\delta,\infty}(h^1).$$

The following law of motion must hold:

$$\delta_1^{\delta,n_q}(h^0) = z(h^0).$$

Also, $(U^{\infty}(\delta), V^{\infty}(\delta))$ must be equal to $(U(\sigma_1^{\delta} \mid h^0), V(\sigma_1^{\delta} \mid h^0))$. If $V^{\infty}(\delta) < V(\sigma_1^{\delta} \mid h^0)$, the agent would not accept this contract given that the principal is obtaining $U(\sigma_1^{\delta} \mid h^0)$, and if $V^{\infty}(\delta) > V(\sigma_1^{\delta} \mid h^0)$, $V^{\infty}(\delta)$ would not belong to $\mathcal{W}(\delta)$ because it does violate the principle of Pareto optimality, given that the principal is getting $U(\sigma_1^{\delta_0} \mid h^0)$. A similar argument proves that $U^{\infty}(\delta) = U(\sigma_1^{\delta} \mid h^0)$.

Repeating this procedure for $t = 2, ..., \infty$, and letting:

$$\sigma^{\delta,\infty} = \{a_t^{\delta,\infty}(h^{t-1}), w_t^{\delta,\infty}(h^t)\},$$

we obtain the desired contract $\sigma^{\delta,\infty}$. ■

B.2 Proof of Proposition 10

Proof. Let δ be arbitrary fixed. First, we show that $\Gamma(U^*, V^*)(\delta) \leq (U^*(\delta), V^*(\delta))$. This is true if there exists σ^{δ} that is feasible and incentive compatible such that $(U(\sigma^{\delta} \mid h^0), V(\sigma^{\delta} \mid h^0)) = \Gamma(U^*(\delta), V^*(\delta))$. We construct this contract σ^{δ} by letting $a(\delta)$, $w(\delta, y)$, $\overline{U}(\delta, y)$, and $\overline{V}(\delta, y)$ be the solution to $\Gamma(U^*(\delta), V^*(\delta))$, and by letting:

$$a_1(h^0) = a(\delta), \text{ and } \; w_1(h^1) = w(\delta, y), \quad \forall h^0, h^1.$$

For a given $y_1 \in Y$, there exists $\sigma^\delta_{y_1}$ such that the principal receives $\overline{U}(\delta, y_1)$ and the agent receives $\overline{V}(\delta, y_1)$. Let

$$\sigma^\delta \mid h^1 = \sigma^\delta_{y_1}, \quad \forall h^1.$$

Notice that $\sigma^\delta_{y_1}$ complies with Pareto optimality, because $\overline{U}(\delta, y_1) = U^*(\sigma^\delta_{y_1} \mid h^1)$, and $\overline{V}(\delta, y_1) = V^*(\sigma^\delta_{y_1} \mid h^1)$. So, there is no other contract $\varphi^\delta_{y_1}$ that is Pareto optimal such that $U^*(\varphi^\delta_{y_1} \mid h^1)$ and $V^*(\varphi^\delta_{y_1} \mid h^1)$ dominate $U^*(\sigma^\delta_{y_1} \mid h^1)$ and $V^*(\sigma^\delta_{y_1} \mid h^1)$; that is, $U^*(\varphi^\delta_{y_1} \mid h^1) \prec U^*(\sigma^\delta_{y_1} \mid h^1)$ and $V^*(\varphi^\delta_{y_1} \mid h^1) \prec V^*(\sigma^\delta_{y_1} \mid h^1)$. So, $\sigma^\delta_{y_1}$ is the contract we need, and $\Gamma(U^*, V^*)(\delta) \leq (U^*(\delta), V^*(\delta))$.

The second part of the proof shows that $(U^*(\delta), V^*(\delta)) \leq \Gamma(U^*, V^*)(\delta)$. Let $\sigma^{\delta *}$ be an optimal contract. Hence,

$$U^*(\delta) = U(\sigma^{\delta *} \mid h^0) = \int_Y [y_1 - w^{\delta *}(y_1) + \beta U(\sigma^{\delta *} \mid h^1)] f(y_1; a^{\delta *}(h^0)) dy_1,$$

$$V^*(\delta) = V(\sigma^{\delta *} \mid h^0) = \int_Y [v(w^{\delta *}(y_1), a^{\delta *}(h^0)) + \beta V(\sigma^{\delta *} \mid h^1)] f(y_1; a^{\delta *}(h^0)) dy_1,$$

and

$$(U^*(\delta), V^*(\delta)) \leq \Gamma(U^*, V^*)(\delta);$$

if we set $a(\delta) = a^{\delta *}(h^0)$, $w(\delta, y_1) = w^{\delta *}(y_1)$, $\overline{U}(\delta, y_1(\delta_1)) = U^*(\sigma^{\delta *} \mid y_1)$, and $\overline{V}(\delta, y_1(\delta_1)) = V^*(\sigma^{\delta *} \mid y_1)$, for $y_1 \in Y$; for (7), (8), and (9) are satisfied. It must be noted that $U^*(\delta) = \overline{U}(\delta, y_1)$ and $V^*(\delta) = \overline{V}(\delta, y_1(\delta_1))$ because, $\sigma^{\delta *}$ is Pareto optimal; and there is no other contract $\varphi^{\delta *}$ that is also Pareto optimal. ■

B.3 Proof of Proposition 11

We follow the same strategy used by Clementi et al. (2010) for proving their Proposition 3.

Lemma 1

Lemma 1. (a) D is self generating. (b) If M is self-generating, then $B(D) \subseteq D$.

Proof. First, we prove (a). Let $\delta \in D$. There exists a contract $\sigma(a_t(h^{t-1}), w_t(h^t))$ and a sequence $\delta_t(h^t) = z(h^t, y)$ that satisfy (3.7), (3.8), (3.9), and (3.10). If $a(\delta) = a_1(h^0)$, $w(\delta, y) = w_1(h^1)$ and $\delta'(\delta, y) = z(h^0, y)$ for all y; then $a(\delta)$, $w(\delta, y)$, and $\delta'(\delta, y)$ satisfy (3.7), (3.8), (3.9), and (3.10). So, $\delta \in B(D)$. Next, we prove (b). Let an arbitrary $\hat{\delta} \in B(D)$. Given that: $\hat{\delta} \in B(D)$, $\exists\, a^*(\hat{\delta}), w(\hat{\delta}, y), \bar{V}(\hat{\delta}, y), \bar{U}(\hat{\delta}, y)$ that satisfy (3.7), (3.8), (3.9), (3.10), and $\delta' \in D\, \forall y$. We need to construct a contract $\sigma^{\hat{\delta}}$ that satisfies (3.1), (3.2), (3.3), and (3.4). We construct such contract by recursion. By defining $\delta_0 = \hat{\delta}$, $h^0 = \{\delta_0\}$ and $h^1 = \{(\delta_0, y(\delta_0))\}$, we get: $a_1(h^0)$, which by (3.7) fulfills (3.4) and (3.1); $w_1(h^1)$, which by (3.8) satisfies (3.2); and $\delta_1(h^1) = z(h^0, y)$, which by (3.9) fulfills (3.3). At $t = 2$, by defining $h^2 = \{(\delta_0, y(\delta_0)), (\delta_2, y(\delta_1))\}$, we get: $a_2(h^1)$, which by (3.7), then (3.4) and (3.1) hold; $w_2(h^2)$, which by (3.8), then (3.2) holds; and $\delta_2(h^2) = z(h^1, y)$, which by (3.9), then (3.3) holds. We continue for $t = 3, 4, ...$ Hence, we have obtained $\sigma^{\hat{\delta}}$ that satisfies (3.1), (3.2), (3.3), and (3.4). Therefore, $\hat{\delta} \in D$. ■

Proposition 11

Proof. Part (a) follows from Lemma 1. To prove (b), first we need to demonstrate that the sequence $\{M_n\}$ is convergent. It is true that $B(M_0) \subseteq M_0$, and using the operator B on both sides, we have that: $M_{n+1} = B(M_n) \subseteq M_n$ for all n because the operator B is assumed to be monotonically increasing. Therefore, the sequence $\{M_n\}$ is bounded and monotonically decreasing with $M_\infty = \lim_{n\to\infty} M_n = \cap_n^\infty M_n$. Now, we demonstrate that $D \subseteq M_\infty$. Because of the monotonicity of B, we can say that $B(D) \subseteq B(M_0)$. By (a) we have that $D = B(D)$, and $B(M_0) = M_1$ by construction, so it follows that $B(D) \subseteq M_1$. Therefore, by iterating and taking the limit, we have that $D \subseteq M_\infty$.To prove that $M_\infty \subseteq D$, we use the fact that $M_\infty = \cap_{n=1}^\infty M_n \subseteq D$. To prove that $M_\infty \subseteq D$, we can say that because of the properties of the sequence $\{M_n\}$, the following equality must hold: $B(M_\infty) = M_\infty$. Finally, M_∞ is self-generating and $M_\infty = B(M_\infty) \subseteq D$. ■

B.4 Proof of Proposition 12

Definition of the Static Problem

Define the Standard Static Principal-Agent Model (SSPA) as follows:

$$\max_{w(y)} \mathbb{E}[y - w(y, \hat{V})]$$

subject to

$$a \in argmax_{a'} \quad \mathbb{E}[v(w(y), a')], \quad \text{Incentive Compatible;}$$

$$\mathbb{E}[v(w(y), a')] = \hat{V}, \qquad \text{Individual Rationality;}$$

$$a \in A, \qquad \text{Feasible Effort;}$$

$$0 \leq w(y) \leq y \qquad \text{for all } y, \qquad \text{Limited Liability;}$$

Define the multiobjective Static Principal-Agent Model (MOSPA1) as follows:

$$\max_{w(y)} \{\mathbb{E}[y - w(y, \hat{V})], \mathbb{E}[v(w(y), a')]\}$$

subject to

$$a \in argmax_{a'} \quad \mathbb{E}[v(w(y), a')], \quad \text{Incentive Compatible;}$$

$$a \in A, \qquad \text{Feasible Effort;}$$

$$0 \leq w(y) \leq y \qquad \text{for all } y, \qquad \text{Limited Liability;}$$

Lemma 2

Lemma 2. Let $\hat{V} \leq \max \mathbb{E}[v(w(y), a)]$, let $\{w^*(y), a^*\}$ solve the problem (SSPA), and assume that, if $\{w^*(y), a^*\}$ is not unique, then $\{w^*(y), a^*\}$ is an optimal solution of problem (SSPA) with maximal $\mathbb{E}[v(w(y), a')]$ value. Then $\{w^*(y), a^*\}$ solves problem (MOSPA1).

Proof. The proof follows from the equivalence theorem in Haimes et al. (1971) given that $y-w(y)$, and $v(y)$ are twice continuously differentiable functions of y.

Assume $\{w^*(y), a^*\}$ does not solve problem (MOSPA1). Then there exists a solution to (MOSPA1) $\{\hat{w}(y), \hat{a}\}$ that Pareto dominates $\{w^*(y), a^*\}$, i.e. such that either:

$$\mathbb{E}[y-\hat{w}(y)] \geq \mathbb{E}[y-w^*(y)], \quad \mathbb{E}[v(\hat{w}(y), \hat{a})] > \mathbb{E}[v(w^*(y), a^*)] \tag{A1}$$

$$\mathbb{E}[y-\hat{w}(y)] > \mathbb{E}[y-w^*(y)], \quad \mathbb{E}[v(\hat{w}(y), \hat{a})] \geq \mathbb{E}[v(w^*(y), a^*)] \tag{A2}$$

But (A1) contradicts the fact $\{w^*(y), a^*\}$ solves (SSPA), while (A2) contradicts the hypothesis that $\{w^*(y), a^*\}$ is the solution with the maximum value for the expected utility of the agent. Hence, the lemma is proved. ■

Proof of Proposition 12

Proof. By Lemma 2 and the static variational structure of the dynamic multiobjective principal-agent problem, then we say that the proposition is proved. ■

B.5 Proof of Proposition 13

Proof. We consider a multiobjective principal-agent model, where the principal maximizes their discounted expected utility:

$$U(\delta) = \int_Y [y - w(\delta, y) + \beta\overline{U}(\delta, y)] f(y \mid a^*(\delta)) dy,$$

and the agent maximizes their expected utility:

$$V(\delta) = \int_Y [v(w(\delta, y), a^*(\delta)) + \beta\overline{V}(\delta, y)] f(y \mid a^*(\delta)) dy.$$

Given a Pareto Weight $\delta \in [0, 1]$, we have already established that this problem can be expressed as follows:

$$\max_{w(\delta,y)} [\ \delta\, V(\delta) + (1-\delta)\, U(\delta)\ ]$$

The Pareto-optimal contract must satisfy both incentive compatibility and feasibility constraints. The agent's bargaining power evolves according to:

$$\delta' = z(\delta, y) \in [0, 1],$$

where $z(\delta, y)$ is a deterministic law of motion.

The necessary conditions from Proposition 5 ensure that the agent's optimal compensation $w * (\delta, y$ satisfies the risk-sharing condition, given by:

$$\frac{1}{v_w(w(\delta, y), a^*(\delta))} = \frac{\delta}{1-\delta} - \frac{\mu(\delta)}{1-\delta} \frac{f_a(y \mid a^*(\delta))}{f(y \mid a^*(\delta))},$$

and the first order conditions regarding the incentive compatibility condition, given by:

$$(1-\delta) \left[\int_Y [y - w(\delta, y) + \beta \overline{U}(\delta, y)] f_a(y \mid a^*(\delta)) dy \right] +$$

$$\mu(\delta) \left[\int_Y [v(w(\delta, y), a^*(\delta)) + \beta \overline{V}(\delta, y)] f_{aa}(y \mid a^*(\delta)) dy - v_{aa}(w(\delta, y), a^*(\delta)) \right] = 0,$$

ensures that the agent exerts optimal effort.

To show sufficiency, we conclude that the weighted utility function is concave. This follows from the fact that the principal's utility is linear, the agent's utility is strictly concave, and their convex combination preserves concavity. Additionally, the effort choice set A is compact, ensuring bounded solutions. The law of motion $\delta' = z(\delta, y)$ keeps bargaining power within the feasible set $[0, 1]$. Following Spear and Srivastava (1987), the concavity of the continuation utilities $\overline{V}$ and $\overline{U}$ is preserved under optimality, ensuring that future payoffs do not create incentives for deviation from the contract. Furthermore, the contract satisfies the Pareto condition, implying that no alternative feasible contract can enhance one party's utility without diminishing the other's. Since all these assumptions hold, the solution characterized by the first-order conditions uniquely determines the

Pareto-optimal contract, thereby establishing both necessity and sufficiency. ■

B.6 Computational Algorithm

The first step of the algorithm is to compute the set of admissible values of the agent's bargaining power. The objective is to discretize the range of the admissible values of the state variable, δ in n steps, such that n is sufficiently large given ε.

For this, we define the current expected utilities of the agent and the principal, respectively, given the exertion of the high and low effort levels on the part of the agent, respectively:

$$EV_h(w_H, w_L) = \left[f(y_H; a_H) \left(\frac{w_H^{1-h}}{1-h} - a_H^2 \right) + f(y_L; a_H) \left(\frac{w_L^{1-h}}{1-h} - a_H^2 \right) \right],$$

$$EV_l(w_H, w_L) = \left[f(y_H; a_L) \left(\frac{w_H^{1-h}}{1-h} - a_L^2 \right) + f(y_L; a_L) \left(\frac{w_L^{1-h}}{1-h} - a_L^2 \right) \right],$$

$$EU_h(w_H, w_L) = \left[f(y_H; a_H) \left(y_H - w_H \right) + f(y_L; a_H) \left(y_L - w_L \right) \right],$$

$$EU_l(w_H, w_L) = \left[f(y_H; a_L) \left(y_H - w_H \right) + f(y_L; a_L) \left(y_L - w_L \right) \right].$$

Then, for the minimum bargaining power δ_{min} that guarantees interior solutions, we solve two optimization problems, one to incentivize the agent to exert low effort and one for high effort.

First, we set $\delta_0 = 0$ and solve for the high effort model, as follows:

$$\max_{w_H, w_L} \left\{ \delta_0 EV_h(w_H, w_L) + (1 - \delta_0) EU_h(w_H, w_L) \right\} \tag{HE}$$

subject to:

$$EV_h(w_H, w_L) \geq EV_l(w_H, w_L),$$

$$0 \leq w_H, w_L \leq y \qquad \text{for all } y.$$

Now, we do the same for the case of the low effort model:

$$\max_{w_H, w_L} \{\delta_0 EV_l(w_H, w_L) + (1-\delta_0) EU_l(w_H, w_L)\} \tag{LE}$$

subject to:

$$EV_l(w_H, w_L) \geq EV_h(w_H, w_L),$$

$$0 \leq w_H, w_L \leq y \qquad \text{for all } y.$$

From solving these two problems, we obtain the corner solution: $EV^{**} = \{EV_i | \delta_0 EV_i^* + (1-\delta_0)EU_i^* \geq \delta_0 EV_j^* + (1-\delta_0)EU_j^*; i,j = H, L\}$.

At this point, we iterate $\delta_t = \delta_{t-1} + \Delta$, for $t = 1, 2, ...$; where Δ is an arbitrarily small and positive number, and solve the same two problems that we solved for δ_0, (HE) and (LE), just varying δ_t.

If the solution: $EV^* = \{EV_i | \delta_t EV_i^* + (1-\delta_t)EU_i^* \geq \delta_t EV_j^* + (1-\delta_t)EU_j^*; i,j = H, L\}$, when $EV^* \neq EV^{**}$ is satisfied, the iteration stops, and we set $\delta_{min} = \delta_t$.

For the maximum admissible bargaining power, we do something similar, this time we set $\delta_0 = 1$, solve (HE) and (LE), and define EV^{**} as before. Afterwards, we iterate $\delta_t = \delta_{t-1} - \Delta$, $t = 1, 2, ...$, then solve (HE) and (LE). Again, the iteration stops when $EV^* \neq EV^{**}$, then $\delta_{max} = \delta_t$.

To finish the first step, we compute the set of admissible values of the agent's bargaining power: $\mathbb{D} = \{D(K), K = 1, ..., N\}$; where $N = \frac{(\delta_{max} - \delta_{min}) \times 2}{\varepsilon + 1}$. Notice that this characterization of N ensures the step between two consecutive bargaining powers is equal to $\frac{\varepsilon}{2}$, and:

$$D(K) = \delta_{min} + \frac{K-1}{N-1}[\delta_{max} - \delta_{min}].$$

The second step is to find the stationary solution of the Bellman Equation. We start with an all-zero guess for the value function $S_0(K) = 0, \forall K = 1, ..., N$; and initialize all-zero vectors for the agent and principal expected discounted utilities $U_0(K) = 0, V_0(K) = 0, \forall K = 1, ..., N$. Define:

$$EV_t^H(K; w_H, w_L) = \left[f(y_H; a_H) \left(\frac{w_H^{1-h}}{1-h} - a_H^2 + \beta V_{t-1}(P) \right) + f(y_L; a_H) \left(\frac{w_L^{1-h}}{1-h} - a_H^2 + \beta V_{t-1}(Q) \right) \right],$$

$$EV_t^L(K; w_H, w_L) = \left[f(y_H; a_L) \left(\frac{w_H^{1-h}}{1-h} - a_L^2 + \beta V_{t-1}(P) \right) + f(y_L; a_L) \left(\frac{w_L^{1-h}}{1-h} - a_L^2 + \beta V_{t-1}(Q) \right) \right],$$

$$EU_t^H(K; w_H, w_L) = [f(y_H; a_H)(y_H - w_H + \beta U_{t-1}(P)) + f(y_L; a_H)(y_L - w_L + \beta U_{t-1}(Q))],$$

$$EU_t^L(K; w_H, w_L) = [f(y_H; a_L)(y_H - w_H + \beta U_{t-1}(P)) + f(y_L; a_L)(y_L - w_L + \beta U_{t-1}(Q))];$$

where $P = \min(K + 2, N)$, $Q = \max(K - 1, 0)$. Notice that the value for P (Q) gives us the index for δ' in $\mathbb{D}$ when output y_H or y_L is observed.

Suppose that we are now at the t-th iteration, $t \geq 1$:

$$S_t(K) = \max\{S_t^i(K); i = H, L\},$$

where

$$S_t^H(K) = \max_{w_H, w_L} \left\{ D(K) EV_t^H(K; w_H, w_L) + (1 - D(K)) EU_t^H(K; w_H, w_L) \right\};$$

subject to:

$$EV_t^H(w_H, w_L) \geq EV_t^L(w_H, w_L),$$

$$0 \leq w_H, w_L \leq y \qquad \text{for all } y,$$

and

$$S_t^L(K) = \max_{w_H, w_L} \left\{ D(K) EV_t^L(K; w_H, w_L) + (1 - D(K)) EU_t^L(K; w_H, w_L) \right\};$$

subject to:

$$EV_t^L(w_H, w_L) \geq EV_t^H(w_H, w_L),$$

$$0 \leq w_H, w_L \leq y \qquad \text{for all } y.$$

The algorithm stops once we find the stationary solution; that is, $S_t(K) = S_{t-1}(K)$, for all

$K = 1, ..., N$.

B.7 Benchmark Models

We use three benchmark or reference models in our numerical implementation. Here we present them:

The Standard Dynamic Principal-Agent Model (SDPA)

A first reference model is the standard dynamic principal-agent model. In particular, we adapt the model of Wang (1997), which is a model formulated as the maximization of the expected discounted utility of the principal subject to the participation constraint, the incentive compatibility constraint, and feasibility constraints. The value function is the principal's expected discounted utility is given by:

$$U(\hat{V}) = \mathbb{E}[y - w(y, \hat{V}) + \beta \overline{U}(y, \hat{V})],$$

where y is the observed output, $w(y, \hat{V})$ is the compensation given the observed output y and β is the discount factor of the principal and the agent. The agent's lifetime discounted expected utility is given by :

$$V(\hat{V}) = \mathbb{E}[v(w(y, \hat{V}), a(\hat{V})) + \beta \overline{V}(y, \hat{V})],$$

where $v(w(y, \hat{V}))$ is the temporary utility function of the agent, and $a \in A$ is the effort exerted by the agent. In addition, $\overline{V}(y, \hat{V})$ is the agent future discounted utility, which is the promised expected utility from tomorrow on. $\hat{V} \in \mathbf{V}$ is the model's state variable, and it is the agent's reservation utility. This is an important difference with respect to our multiobjective dynamic models given that our models' state variable is the agent's initial bargaining power.

The dynamic maximization program is:

$$\max_{w(y,\hat{V}), \overline{V}(y,\hat{V})} \mathbb{E}[y - w(y, \hat{V}) + \beta \overline{U}(y, \hat{V})]$$

subject to

$$a(\hat{V}) \in argmax_{a'} \quad \hat{V}(a', \hat{V}), \qquad \text{Incentive Compatible;}$$

$$V(\hat{V}, a(\hat{V})) = \hat{V}, \qquad \text{Individual Rationality;}$$

$$a(\hat{V}) \in A, \qquad \text{Feasible Effort;}$$

$$0 \leq w(y, \hat{V}) \leq y \qquad \text{for all } y, \qquad \text{Limited Liability;}$$

$$\overline{V}(y, \hat{V}) \in \mathcal{V} \qquad \text{for all y,} \qquad \text{Feasible and Incentive compatible } \overline{V}.$$

Let $\mathcal{V}(\hat{V})$ and $\mathcal{U}(\hat{V})$ be the set of feasible and incentive compatible expected discounted utilities of the agent and principal, respectively. Wang (1997) demonstrates that $\mathcal{U}(\hat{V})$ is compact. Therefore, by virtue of the Bellman equation, there exists a principal's maximal expected discounted utility that is feasible and incentive compatible.

The Multiobjective Static Model (MOSPA2)

A second reference model is the multiobjective static principal-agent model. In this setting, the static contracting problem is to choose an action $a \in A$ and a compensation scheme $w(y, \delta) \in [0, y]$, $\forall y \in Y$, to maximize the Pareto Weights function of the expected utility of the principal and that of the agent; that is:

$$\max_{a(\delta), w(y,\delta)} [\delta v(w(y, \delta), a(\delta)) + (1 - \delta) u(y, w(y, \delta))],$$

subject to

$$\int_Y v(w(y, \delta), a(\delta)) f(y; a) dy \geq \int_Y v(w(y, \delta), a'(\delta)) f(y; a'(\delta)) dy \quad \forall a'(\delta) \in A,$$

$$0 \leq w(y, \delta) \leq y \quad \forall y \in Y.$$

The Multiobjective Dynamic Model With No Bargaining Power Dynamics (MODPA1)

A third reference model is the multiobjective dynamic principal-agent model with no dynamics of bargaining power. In particular, we analyze the optimal contractual arrangements having the agent's bargaining power, δ, as the model's state variable, but without the implications of explicitly including a law of motion for δ. That is, in this version of the model we analyze the maximization of the Pareto Weights function of the expected discounted utility of the principal and that of the agent, subject to the feasibility and incentive compatible constraints. This problem is formulated as follows:

$$\max_{a(\delta),w^{\delta}(y^{\delta},W),\overline{V}^{\delta}(y^{\delta},W),\overline{U}^{\delta}(y^{\delta},W)} [\delta V^{\delta} + (1-\delta)U^{\delta}],$$

where:

$$U(\delta) = \int_Y [y - w(y,\delta) + \beta\overline{U}(y,\delta)] f(y; a(\delta)) dy,$$

$$V(\delta) = \int_Y [v(w(y,\delta), a(\delta)) + \beta\overline{V}(y,\delta)] f(y; a(\delta)) dy;$$

subject to

$$a(V) \in argmax_{\,a'} \quad \hat{V}(a', V),$$

$$0 \leq w(y,\delta) \leq y \quad \forall y \in Y,$$

$$\delta \in [0,1],$$

$$(\overline{U}(y,\delta), \overline{V}(y,\delta)) \in \mathcal{W}(\delta') \quad \forall y \in Y.$$

Bibliography

Dilip Abreu, David Pearce, and Ennio Stacchetti. Toward a Theory of Discounted Repeated Games with Imperfect Monitoring. *Econometrica*, 58(5):1040–1063, 1990. ISSN 0012-9682.

Daron Acemoglu and Alp Simsek. Moral Hazard and Efficiency in General Equilibrium with Anonymous Trading, April 2010.

Philippe Aghion and Jean Tirole. The Management of Innovation. *The Quarterly Journal of Economics*, 109(4):1185–1209, November 1994. ISSN 0033-5533. doi: 10.2307/2118360.

Gerhard Aust and Udo Buscher. Cooperative advertising models in supply chain management: A review. *European Journal of Operational Research*, 234(1):1–14, April 2014. ISSN 03772217. doi: 10.1016/j.ejor.2013.08.010.

Martin Neil Baily. Wages and Employment under Uncertain Demand. *The Review of Economic Studies*, 41 (1):37–50, 1974. ISSN 0034-6527. doi: 10.2307/2296397. Publisher: [Oxford University Press, Review of Economic Studies, Ltd.].

Dieter Balkenborg. How Liable Should a Lender Be? The Case of Judgment-Proof Firms and Environmental Risk: Comment. *American Economic Review*, 91(3):731–738, June 2001. ISSN 0002-8282. doi: 10.1257/aer.91.3.731.

Robert G. Bartle and Donald R. Sherbert. *Introduction to Real Analysis*. Wiley, August 2011. ISBN 9781118135860.

Thorsten Bayindir-Upmann and Anke Gerber. The Kalai-Smorodinsky Solution in Labor-Market Negotiations. *CESifo Working Paper Series*, 2003.

Lucian Bebchuk and Jesse M. Fried. Executive Compensation as an Agency Problem. *Journal of Economic Perspectives*, 17(3):71–92, 2003.

Lucian A. Bebchuk, Jesse M. Fried, and David I. Walker. Managerial Power and Rent Extraction in the Design of Executive Compensation, June 2002.

Brian Bell, Simone Pedemonte, and John Van Reenen. Ceo Pay and the Rise of Relative Performance Contracts: A Question of Governance? *Journal of the European Economic Association*, 19(5):2513–2542, 2021.

Benjamin Bental and Dominique Demougin. Declining labor shares and bargaining power: An institutional explanation. *Journal of Macroeconomics*, 32(1):443–456, March 2010. ISSN 0164-0704. doi: 10.1016/j.jmacro.2009.09.005.

Ken Binmore, Ariel Rubinstein, and Asher Wolinsky. The Nash Bargaining Solution in Economic Modelling. *The RAND Journal of Economics*, 17(2):176–188, 1986. ISSN 0741-6261. doi: 10.2307/2555382. Publisher: [RAND Corporation, Wiley].

Philip Bond and Armando Gomes. Multitask principal–agent problems: Optimal contracts, fragility, and effort misallocation. *Journal of Economic Theory*, 144(1):175–211, January 2009. ISSN 0022-0531. doi: 10.1016/j.jet.2008.05.008.

Yanbin Chen, Pu Chen, Yumei Guo, Sanxi Li, and Dongmin Yao. Bargaining to Design Contracts under Moral Hazard. *Journal of Institutional and Theoretical Economics*, 175(4):1–22, August 2019. ISSN 1614-0559. doi: 10.1628/jite-2019-0039.

Chongwoo Choe, Gloria Y. Tian, and Xiangkang Yin. CEO power and the structure of CEO pay. *International Review of Financial Analysis*, 35:237–248, 2014.

Gian Luca Clementi, Thomas F. Cooley, and Sonia Di Giannatale. A theory of firm decline. *Review of Economic Dynamics*, 13(4):861–885, October 2010. ISSN 1094-2025. doi: 10.1016/j.red.2010.03.002.

Itza Tlaloc Quetzalcoatl Curiel-Cabral, Sonia Di Giannatale, and Giselle Labrador-Badía. Risk Aversion, Reservation Utility and Bargaining Power: An Evolutionary Algorithm Approximation of Incentive Contracts. *Computational Economics*, December 2022. ISSN 1572-9974. doi: 10.1007/s10614-022-10349-0.

Kaniṣka Dam and Daniel Ruiz Pérez. On the existence of share contracts under limited liability. *Economics Letters*, 117(3):552–555, December 2012. ISSN 0165-1765. doi: 10.1016/j.econlet.2012.07.035.

Dominique Demougin and Carsten Helm. Moral Hazard and Bargaining Power. *German Economic Review*, 7(4):463–470, 2006. ISSN 1468-0475. doi: 10.1111/j.1468-0475.2006.00130.x. _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.1468-0475.2006.00130.x.

Dominique Demougin and Carsten Helm. Job matching when employment contracts suffer from moral hazard. *European Economic Review*, 55(7):964–979, October 2011. ISSN 0014-2921. doi: 10.1016/j.euroecorev.2011.04.002.

Marcus Dittrich and Silvio Städter. Moral hazard and bargaining over incentive contracts. *Research in Economics*, 69, October 2014. doi: 10.1016/j.rie.2014.10.002.

Alex Edmans and Xavier Gabaix. Executive Compensation: A Modern Primer. *Journal of Economic Literature*, 54(4):1232–1287, December 2016. ISSN 0022-0515. doi: 10.1257/jel.20161153.

Alex Edmans, Xavier Gabaix, and Dirk Jenter. Executive Compensation: A Survey of Theory and Evidence. In Benjamin E. Hermalin and Michael S. Weisbach, editors, *The Handbook of the Economics of Corporate Governance*, volume 1 of *The Handbook of the Economics of Corporate Governance*, pages 383–539. North-Holland, January 2017. doi: 10.1016/bs.hecg.2017.11.010.

Qi Feng and Lauren Xiaoyuan Lu. Supply Chain Contracting Under Competition: Bilateral Bargaining vs. Stackelberg. *Production and Operations Management*, 22(3):661–675, 2013. ISSN 1937-5956. doi: 10.1111/j.1937-5956.2012.01417.x.

Xavier Gabaix and Augustin Landier. Why has CEO Pay Increased So Much?*. *The Quarterly Journal of Economics*, 123(1):49–100, February 2008. ISSN 0033-5533. doi: 10.1162/qjec.2008.123.1.49.

George-Levi Gayle and Robert A. Miller. Has Moral Hazard Become a More Important Factor in Managerial Compensation? *American Economic Review*, 99(5):1740–1769, December 2009. ISSN 0002-8282. doi: 10.1257/aer.99.5.1740.

Radhakrishnan Gopalan, Todd Milbourn, and Fenghua Song. Strategic Flexibility and the Optimality of Pay for Sector Performance. *Review of Financial Studies*, 23(5):2060–2098, 2010.

Sanford J. Grossman and Oliver D. Hart. An Analysis of the Principal-Agent Problem. *Econometrica*, 51 (1):7–45, 1983a. ISSN 0012-9682. doi: 10.2307/1912246. Publisher: [Wiley, Econometric Society].

Sanford J. Grossman and Oliver D. Hart. Implicit Contracts Under Asymmetric Information. *The Quarterly Journal of Economics*, 98:123–156, 1983b. ISSN 0033-5533. doi: 10.2307/1885377. Publisher: Oxford University Press.

Paul A. Grout. Investment and Wages in the Absence of Binding Contracts: A Nash Bargaining Approach. *Econometrica*, 52(2):449–460, 1984. ISSN 0012-9682. doi: 10.2307/1911498. Publisher: [Wiley, Econometric Society].

Hongmei Guo, Shuiliang Gu, and Yingsheng Su. Bargaining Power Choices with Moral Hazard in a Supply Chain. *Discrete Dynamics in Nature and Society*, 2018:1–9, 2018. ISSN 1026-0226, 1607-887X. doi: 10.1155/2018/2068287.

Y. Haimes, L. Lasdon, and D. Dang. On a Bicriterion Formulation of the Problems of Integrated System Identification and System Optimization. *IEEE Transactions on Systems, Man, and Cybernetics*, SMC-1 (3):296–297, July 1971. ISSN 2168-2909. doi: 10.1109/TSMC.1971.4308298.

Onésimo Hernández-Lerma and Rosario Romera. The Scalarization Approach to Multiobjective Markov ControlProblems: Why Does It Work? *Applied Mathematics and Optimization*, 50(3):279–293, October 2004. ISSN 1432-0606. doi: 10.1007/s00245-004-0804-4.

Bengt Holmstrom and Paul Milgrom. Aggregation and Linearity in the Provision of Intertemporal Incentives. *Econometrica*, 55(2):303–328, 1987. ISSN 0012-9682. doi: 10.2307/1913238.

Bengt Holmström. Moral Hazard and Observability. *The Bell Journal of Economics*, 10(1):74–91, 1979. ISSN 0361-915X. doi: 10.2307/3003320. Publisher: [RAND Corporation, Wiley].

Roman Inderst. Contract design and bargaining power. *Economics Letters*, 74(2):171–176, 2002. ISSN 01651765. doi: 10.1016/S0165-1765(01)00545-6.

Robert D Innes. Limited liability and incentive contracting with ex-ante action choices. *Journal of Economic Theory*, 52(1):45–67, October 1990. ISSN 0022-0531. doi: 10.1016/0022-0531(90)90066-S.

Michael C. Jensen and Kevin J. Murphy. Performance Pay and Top-Management Incentives. *Journal of Political Economy*, 98(2):225–264, 1990. ISSN 0022-3808.

Dirk Jenter and Fadi Kanaan. CEO Turnover and Relative Performance Evaluation. *The Journal of Finance*, 70(5):2155–2184, 2015. ISSN 1540-6261. doi: 10.1111/jofi.12282.

Richard E. Kihlstrom and Alvin E. Roth. Risk Aversion and the Negotiation of Insurance Contracts. *The Journal of Risk and Insurance*, 49(3):372–387, 1982. ISSN 0022-4367. doi: 10.2307/252493. Publisher: [American Risk and Insurance Association, Wiley].

Jean-Jacques Laffont and David Martimort. *The Theory of Incentives: The Principal-Agent Model*. Princeton University Press, 2002. ISBN 9780691091846. doi: 10.2307/j.ctv7h0rwr.

Guy Laroque and Bernard Salanié. Salaire minimum et emploi en présence de négociations salariales. *Annales d'Économie et de Statistique*, 73(73):1–22, 2004. ISSN 0769-489X. doi: 10.2307/20079066. Publisher: [GENES, ADRES].

Sanxi Li, Hao Xiao, and Yao Dongmin. Contract Bargaining with a Risk-Averse Agent. *The B.E. Journal of Theoretical Economics*, 13(1):285–301, May 2013. ISSN 1935-1704. doi: 10.1515/bejte-2013-0041.

Sanxi Li, Hailin Sun, Jianye Yan, and Xundong Yin. Risk aversion in the Nash bargaining problem with uncertainty. *Journal of Economics*, 115(3):257–274, 2015. ISSN 0931-8658. Publisher: Springer.

Ping Liu and Yuhai Xuan. The Contract Year Phenomenon in the Corner Office: An Analysis of Firm Behavior During CEO Contract Renewals, September 2020.

Xinjun Lyu, Christopher Decker, and Jinlan Ni. Compensation negotiation and corporate governance: the evidence from China. *Journal of Chinese Economic and Business Studies*, 16(2):193–213, 2018.

Inés Macho-Stadler and J. David Pérez-Castrillo. *An Introduction to the Economics of Information: Incentives and Contracts*. Oxford University Press, 2001. ISBN 9780199243259. Google-Books-ID: ywGXqL9nZz4C.

Albert Marcet and Ramon Marimon. Recursive Contracts. *Econometrica*, 87(5):1589–1631, 2019. ISSN 0012-9682. Publisher: [Wiley, The Econometric Society].

Silvia Martinez-Gorricho and Miguel Sanchez Villalba. Incentives, ability and disutility of effort. *SERIEs*, 12(3):453–487, September 2021. ISSN 1869-4195. doi: 10.1007/s13209-021-00236-6.

Antonio Mele. Repeated moral hazard and recursive Lagrangeans. *Journal of Economic Dynamics and Control*, 42:69–85, May 2014. ISSN 0165-1889. doi: 10.1016/j.jedc.2014.03.007.

Kevin J. Murphy. Chapter 38 Executive compensation. In *Handbook of Labor Economics*, volume 3, pages 2485–2563. Elsevier, January 1999. doi: 10.1016/S1573-4463(99)30024-9.

John F. Nash. The Bargaining Problem. *Econometrica*, 18(2):155–162, 1950. ISSN 0012-9682. doi: 10.2307/1907266. Publisher: [Wiley, Econometric Society].

Gurupdesh Pandher and Russell Currie. CEO compensation: A resource advantage and stakeholder-bargaining perspective. *Strategic Management Journal*, 34:22–41, 2013.

Rohan Pitchford. How Liable Should a Lender Be? The Case of Judgment-Proof Firms and Environmental Risk. *The American Economic Review*, 85(5):1171–1186, 1995. ISSN 0002-8282.

Rohan Pitchford. Moral hazard and limited liability: The real effects of contract bargaining. *Economics Letters*, 61(2):251–259, November 1998. ISSN 0165-1765. doi: 10.1016/S0165-1765(98)00141-4.

Shivaram Rajgopal, Daniel J. Taylor, and Mohan Venkatachalam. Frictions in the CEO Labor Market: The Role of Talent Agents in CEO Compensation, February 2011.

William P. Rogerson. The First-Order Approach to Principal-Agent Problems. *Econometrica*, 53(6):1357–1367, 1985. ISSN 0012-9682. doi: 10.2307/1913212.

Sani Saidu. Theoretical and Conceptual Review of CEO Power. *International Journal of Academic Management Science Research*, December 2018.

Yuliy Sannikov. A Continuous-Time Version of the Principal–Agent Problem. *Review of Economic Studies*, 75(3):957–984, July 2008. ISSN 0034-6527, 1467-937X. doi: 10.1111/j.1467-937X.2008.00486.x.

Y. Sawaragi, H. Nakayama, and T. Tanino. *Theory of Multiobjective Optimization*. Academic Press, September 1985. ISBN 9780126203707.

Patrick W. Schmitz. Workplace surveillance, privacy protection, and efficiency wages. *Labour Economics*, 12(6):727–738, December 2005. ISSN 0927-5371. doi: 10.1016/j.labeco.2004.06.001.

Jiuh-Biing Sheu and Xiao-Qin Gao. Alliance or no alliance—Bargaining power in competing reverse supply chains. *European Journal of Operational Research*, 233(2):313–325, March 2014. ISSN 0377-2217. doi: 10.1016/j.ejor.2013.09.021.

Stephen E. Spear and Sanjay Srivastava. On Repeated Moral Hazard with Discounting. *The Review of Economic Studies*, 54(4):599–617, 1987. ISSN 0034-6527. doi: 10.2307/2297484. Publisher: [Oxford University Press, Review of Economic Studies, Ltd.].

William Thomson. Chapter 35 Cooperative models of bargaining. In *Handbook of Game Theory with Economic Applications*, volume 2, pages 1237–1284. Elsevier, January 1994. doi: 10.1016/S1574-0005(05)80067-0.

Cheng Wang. Incentives, CEO Compensation, and Shareholder Wealth in a Dynamic Agency Model. *Journal of Economic Theory*, 76(1):72–105, September 1997. ISSN 0022-0531. doi: 10.1006/jeth.1997.2293.

Zhiyong Yao. Bargaining over incentive contracts. *Journal of Mathematical Economics*, 48(2):98–106, March 2012. ISSN 0304-4068. doi: 10.1016/j.jmateco.2012.01.003.

Árpád Ábrahám, Sebastian Koehne, and Nicola Pavoni. On the first-order approach in principal - agent models with hidden borrowing and lending. *Journal of Economic Theory*, 146(4):1331–1361, July 2011. ISSN 0022-0531. doi: 10.1016/j.jet.2011.03.002.

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