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MAESTRO EN ECONOMÍA

MONETARY POLICY RULES IN A NEW KEYNESIAN MODEL WITH LIMITED ASSET MARKET PARTICIPATION AND INFORMALITY.

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Abstract

We present a Dynamic Stochastic General Equilibrium (DSGE) model that incorporates Limited Assets Market Participation (LAMP) and informality into a New Keynesian frame work and calibrate the model to match the Mexican economy. Dynamic responses to monetary policy shocks, under Taylor and Wicksellian interest-rate feedback rules, are analyzed using an impulse response analysis. The model reproduces the empirical fact that informal sector is countercyclical.

We show that the monetary policy transmission channel in the presence of LAMP affects the labor market is through the labor market, first the intertemporal substitution effect which affects the agent's labor supply decisions. This makes the economy more sensitive to monetary policy shocks, so the results of Bilbiie (2008) obtained for a one-sector economy also hold in a two-sector framework. Informality loses its countercyclical characteristic as the size of informality in the economy becomes larger. We find that Wicksellian monetary policy rules help reduce the sensitivity of the real interest rate to changes in the nominal interest rate. Consequently, the effect the aggregate demand channel of monetary policy is reduced, and monetary policy shocks have a smaller effect on the real economy.

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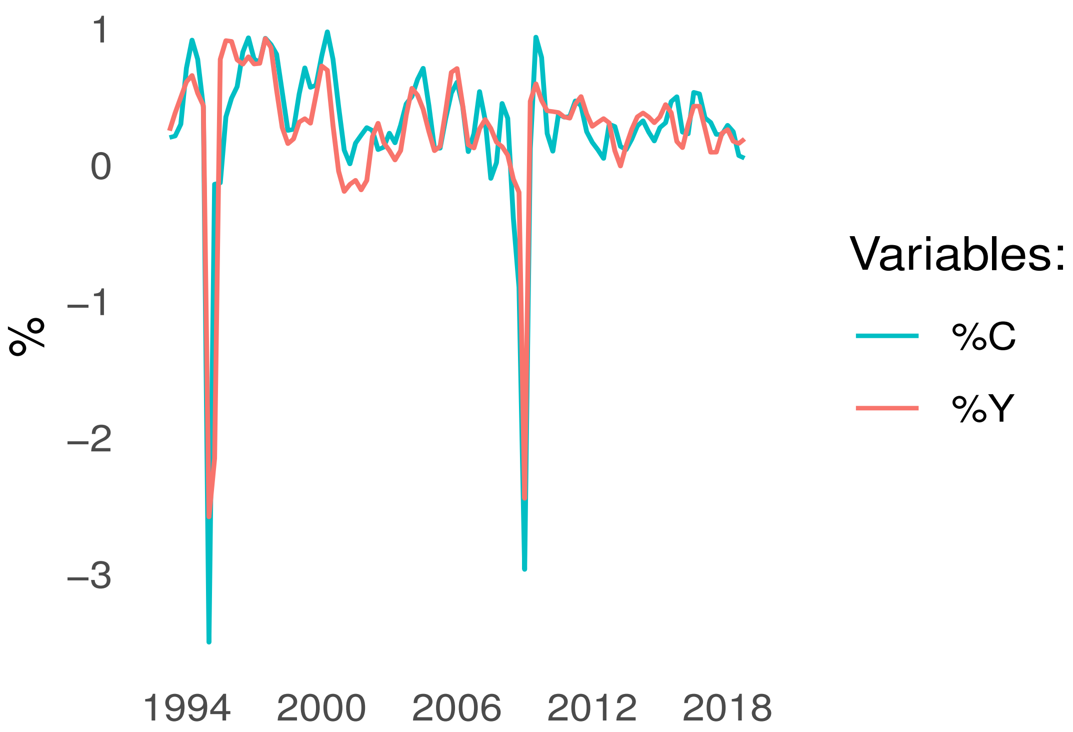
Introduction

According to the *Encuesta Nacional de Inclusión Financieron* ENIF (2018), in Mexico only 31 percent of the people between 18 and 70 years old have, at least, a formal credit; 4 out of ten have, at least, an insurance; and 40 percent have a retirement account. Thus, the postulation in the standard DSGE representative-agent models that there only exists one type of agent that can smooth consumption along their lifetime path using financial assets is not verified. This means that changes in monetary policy, via adjustments in the nominal interest rate, may not affect aggregate consumption in the absence of intertemporal substitution.

Figure 1: Mexico: Private consumption Vs. Output. (1994.01-2018.04)



In the presence of limited asset market participation (LAMP) consumption tracks current income. Without access to financial markets to undertake consumption smoothing, individuals merely consume their current disposable income. Campbell & Mankiw (1989) mention that a large proportion of the US population consume their current income. Moreover, Mexican data suggest a high correlation (98%) between private consumption and output as figure 1 shows.

Given this evidence, this thesis aims to consider the role of LAMP in affecting the transmission mechanism of monetary policy for developing countries like Mexico. Including limited asset participation in the model should describe in a more appropriate way emerging market economies. One might think that the larger the degree of LAMP the lower the effect of monetary policy on the aggregate economy. This is not necessarily true. Bilbiie (2008) investigates the consequences of LAMP on aggregate dynamics through the elasticity of aggregate demand to real interest rates, which depends in a non-linear way on the degree of asset market participation: if participation is restricted below a certain threshold, an increase of participation make the relationship between the real interest rate and aggregate demand stronger; whereas if participation is above the threshold current aggregate output is positively related to the real interest rate.

Another important feature of developing countries is the existence of an informal sector. For example, in Mexico informal labor represents a big share of aggregate labor. According to INEGI (2018) the informal sector represents 22 percent of GDP and it employs 57 percent of the population. Not accounting for informality in macroeconomic models represents a weakness in analyzing the transmission mechanism of monetary policy for developing countries like Mexico. Understanding how informality distorts this transmission mechanism is crucial for the appropriate policy design in these type of economies. Batini describes the formal sector as one that pays taxes, is capital intensive, highly productive and has frictions; whereas the informal sector is untaxed, labor intensive, less productive and frictionless. Informality can be present in the financial market, the labor market, and the goods market. Here we focus only on the latter one. Our model differs from Batini in the sense that we assume that labor is the only production factor, and that formal and informal labor markets are perfectly competitive.

There are are only a few papers that investigate monetary policy in an economy with an informal sector. The study of Batini et al. (2011) who analyze optimal monetary policy and find that assuming stronger frictions on both labor and financial cause the time-inconsistency problem to worsen, so central bank should increase its commitment in economies characterized by a large informal sector. Bandaogo (2018) analyze the conduct of fiscal of monetary policy in an economy with informality and founds that when the country cannot credibly pre-commit to the optimal policy, informality increases the incentive to peg the currency. McKnight & De la O (2016) who show the implications of informality for the determinacy of equilibrium, in particular they show that informality reduces the possibility of multiple equilibria if the interest rate rule reacts to general inflation. Fabrizio & Lorenza (2008) look for optimal monetary policies in a dual economy, they show that the larger the number of firms that belong to the competitive sector, the smaller should be the response of the nominal interest rate to productivity and cost-push shocks; also if the monetary policy reacts to inflationary expectations the optimal monetary policy rule is not affected by the structure of the labor market. Leyva & Urrutia (2018) show that international interest rate shocks affect specifically job creation in the formal sector thus obtaining a counter-cyclical informality rate.

Castillo & Montoro (2012) analyze the effect of informal labor markets on the dynamics of inflation and on the transmission of aggregate demand and supply shocks; their main result is that interest rates are more effective in stimulating output with less impact on inflation. Restrepo-Echavarria (2014) shows that accounting for the informal sector should help understand measured cyclical fluctuations. Fernandez & Meza (2015) found that introducing informal employment into a standard model amplifies the effects of productivity and that imperfect measurement of informal economic activity can translate into stronger variability in aggregate economic activity.

This thesis presents a closed, cashless economy where labor is the only factor of production and is populated by two type of agents: constrained agents who don't have access to financial assets and unconstrained agents who are forward looking and can smooth consumption. The model is a two-sector extension of Bilbiie (2008) to include informality. Formal firms are assumed to be monopolistically competitive and set prices ala Calvo (1983); informal firms are assumed to be perfectly competitive. The labor market in both sectors, formal and informal, is assumed to be perfectly competitive. We focus on the sensitivity of the economy in the presence of nominal interest rates shocks with different types of monetary policy rules. We suppose that the central bank conducts monetary policy by setting the interest rate. We investigate alternative monetary policy rules that can either react to output, inflation, or the price level in the formal economy. We log-linearized the model to solve it using Dynare.

The main results of this thesis can be summarized as follows. First, we conduct a sensitivity analysis of some key parameters in the model relating to the level of informality, the elasticity of substitution between formal goods and the labor supply elasticity. The sensitivity analysis suggest that an increase in the size of the informal sector makes the economy less sensitive to monetary policy shocks; an increase in the elasticity of substitution between formal goods make the economy more sensitive due to a further fall in the labor demand in that sector; an increase in labor elasticity makes the economy less sensitive due to the fact that labor demand has to shift less for supply to meet demand. Second, while the above results are obtained under a Taylor-type interest-rate rule, we show the implications for the model under a Wicksellian monetary policy rule, whereby the nominal interest-rate rule responds to fluctuations in the price level rather than inflation. We find that a key difference between Wicksellian rules and Taylor rules is that the Wicksellian rule reduce the sensitivity of the real interest rate to changes in the nominal interest rate. Thus, the effect of monetary policy on aggregate demand is reduced and monetary policy shocks have a smaller effect on the real economy.

The thesis proceeds as follows. Chapter 1 presents in detail the theoretical model. Chapter 2 shows the derivation of the log-linearized version of the model and discuss the calibration and parameterization of the model. Chapter 3 presents the impulse response analysis of the model under a Taylor-type interest-rate rule which including a sensitivity analysis. Chapter 4 analyzes the dynamics of the model under a Taylor rule that only targets inflation and a Wicksellian interest-rate rule. Finally, some conclusions are presented.

1 A New Keynesian Model with Informality and Limited Asset Market Participation.

This chapter describes the model. The model economy is assumed to be closed to international trade and cashless economy in which labor is the only factor of production and it is populated by heterogeneous infinitely-lived households of measure one. The model is a two-sector extension of Bilbiie (2008) to include informality. There are two types of agents: constrained and unconstrained. By constrained we mean that this type of agent doesn't have access to financial markets: she has zero asset and bond holdings, therefore she does not smooth consumption and consumes only her labor income. Unconstrained agents are households who are forward looking, hold assets and bonds, and hence can smooth consumption. Constrained (unconstrained) agents have a measure of size λ $(1-\lambda)$. Both constrained and unconstrained agents have the same utility function which is an additively separable log-CRRA utility function that depends on consumption and labor. Constrained and unconstrained agents consume a bundle of formal and informal goods and choose the amount of labor they want to supply to each sector.

Formal firms are assumed to be more productive than informal firms and thus the formal real wage is greater than informal real wage.[1] Formal firms are monopolistic competitors who set prices in a staggered basis according to Calvo (1983). Introducing price stickiness into the model generates money non-neutrality and thus a role for monetary policy. Informal firms behave in a competitive way such that the informal sector is characterized by full price flexibility. Informal (formal) firms have a measure of size ρ $(1-\rho)$. We assume that both the size of constrained agents and informal firms are exogenous.

The central bank sets the nominal interest rate as the policy instrument. We consider two popular monetary policy rules: a Taylor-type feedback rule whereby the nominal interest rate responds to changes in formal inflation and output; and a Wicksellian interest-rate rule, whereby the interest-rate responds to fluctuations in the formal aggregate price level.

[1] Moreno Treviño (2007) shows that the average wage difference between formal and informal is about 54.2 percent. According to INEGI (2015) average formal wage is 38 percent more than average informal wage; nevertheless, this difference changes geographically, by gender and by level of education.

1.1 Agreggate Variables:

Aggregate consumption, C_t^A, is a consumption basket which is composed of informal, $C_{i,t}$, and formal, $C_{f,t}$, aggregate consumption bundles of goods of imperfectly substitutable varieties. Let η represents the elasticity of substitution between the two types of goods:

$$C_t^A = [\rho^{\frac{1}{\eta}} C_{i,t}{}^{\frac{\eta-1}{\eta}} + (1-\rho)^{\frac{1}{\eta}} C_{f,t}{}^{\frac{\eta-1}{\eta}}]^{\frac{\eta}{\eta-1}} \tag{1}$$

where:

$$C_{i,t} \equiv \int_0^{\rho} C_{i,t}(i)\ di \tag{2}$$

$$C_{f,t} \equiv \left[\left(\frac{1}{1-\rho} \right)^{\frac{1}{\epsilon}} \int_{\rho}^{1} C_{f,t}(f)^{\frac{\epsilon-1}{\epsilon}} df \right]^{\frac{\epsilon}{\epsilon-1}} \tag{3}$$

Let $C_{i,t}(i)$ be the aggregate demand for an individual informal good; $C_{f,t}(f)$ is the aggregate demand for an individual formal good. Aggregate formal consumption is composed of imperfectly substitutable varieties for the case of formal goods, with elasticity of substitution $\epsilon > 1$. Where ρ is the size of the informal sector and η is the elasticity of substitution between formal and informal goods. Optimal allocation of expenditure within each variety of good are represented by (4) and (5):

$$C_{i,t}(i) = \frac{1}{\rho}\ C_{i,t} \tag{4}$$

$$C_{f,t}(f) = \frac{1}{(1-\rho)} \left(\frac{P_{f,t}(f)}{P_{f,t}} \right)^{-\epsilon} C_{f,t} \tag{5}$$

$P_{f,t}(f)$ is the individual price of a formal good and $P_{f,t}$ is the price sub-index for formal goods.

Optimal allocation of aggregate expenditure between informal and formal bundles are represented in (6) and (7) obtained from solving a standard cost-minimization problem:

$$C_{i,t} = \rho \left(\frac{P_{i,t}}{P_t} \right)^{-\eta} C_t^A \tag{6}$$

$$C_{f,t} = (1-\rho) \left(\frac{P_{f,t}}{P_t} \right)^{-\eta} C_t^A \tag{7}$$

where P_t is the utility-based price index which is defined as the minimum expenditure needed to buy one unit of the composite good C_t^A. Analytically P_t is defined by equation (8):

$$P_t^A \equiv P_t = [\rho P_{i,t}^{1-\eta} + (1-\rho) P_{f,t}^{1-\eta}]^{\frac{1}{1-\eta}} \tag{8}$$

Household's labor supply, $N_t^l \; \forall \; l = \{c, u\}$, is the sum of formal labor supply $N_{f,t}^l$ and informal labor supply $N_{i,t}^l$, where the superscript c denotes variables for the constrained agent, and superscript u denotes variables for unconstrained agent.

$$N_t^c = N_{i,t}^c + N_{f,t}^c \tag{9}$$

$$N_t^u = N_{i,t}^u + N_{f,t}^u \tag{10}$$

1.2 Households:

1.2.1 Unconstrained agent maximization problem, u:

The unconstrained agents of size $(1-\lambda)$ is the typical agent in the New Keynesian literature: these households are forward looking, have access to bonds and are able to trade in financial markets for state-contingent securities. The unconstrained representative agent chooses a path of consumption, labor supply and asset holdings to maximize expected discounted utility:

$$\max \quad E_t \sum_{i=0}^{\infty} \beta^i U({C^u}_{t+i}, {N^u}_{t+i})$$

subject to a sequence of period budget constraints:

$$P_t C_t^u + B_t + \Omega_{t+1} V_t \leq Z_t + W_{f,t} {N^u}_{f,t} + W_{i,t} {N^u}_{i,t} + \Omega_t (V_t + P_t D_t) \tag{11}$$

where $\beta \in (0,1)$ is the discount factor; B_t is the nominal value at end of period t of a portfolio of all state-contingent assets held; $W_{f,t}$ is the nominal wage in the formal sector; $W_{i,t}$ is the nominal wage in the informal sector; Ω_t are share holdings of formal firms; V_t is the average market value at time t shares in formal firms; D_t are real dividend payoffs of formal firm's shares; Z_t is beginning of period wealth not including the payoff of shares.[2] The utility function is assumed to have the following specific functional form:

$$U(C_t^u, N_t^u) = ln(C_t^u) - \omega_i \frac{{N_{i,t}^u}^{1+\varphi}}{1+\varphi} - \omega_f \frac{{N_{f,t}^u}^{1+\varphi}}{1+\varphi} \tag{12}$$

where $\omega_i > 0$ is how working in the informal sector is valued relative to consumption, $\omega_f > 0$ is how working in the formal sector is valued relative to consumption, and $\varphi > 0$ is the inverse labor supply elasticity. The first-order conditions yield a regular consumption Euler equation which tells us that the marginal utility of consuming one unit today, for utility maximization, must be

[2] Absence of arbitrage implies that $B_t = E[\Lambda_{t,t+1} Z_t]$ and $V_t = E_t[\Lambda_{t,t+1}(V_{t+1} + P_{t+1} D_{t+1})]$ and $\frac{1}{R_t} = E_t[\Lambda_{t_t+1}]$

the same as the marginal utility for consuming the same unit in period $t+1$:

$$\beta E_t[\frac{C_t^u}{C_{t+1}^u}] = \frac{1}{R_t} E_t[\frac{P_{t+1}}{P_t}] \tag{13}$$

The remaining first-order conditions yields a labor-consumption marginal rate of substitution that means that the marginal utility of working in each sector has to be equal to the real value, in terms of the wage obtained in this sector, of the marginal utility of consumption.

$$\omega_f (N_{f,t}^u)^{\varphi} = \frac{1}{C^u{}_t} \frac{W_{f,t}}{P_t} \tag{14}$$

$$\omega_i (N_{i,t}^u)^{\varphi} = \frac{1}{C^u{}_t} \frac{W_{i,t}}{P_t} \tag{15}$$

1.2.2 Constrained households:

Constrained households, of size λ, do not solve an intertemporal problem. Instead at each date they solve the following problem since they merely consume labor income as they have no access to financial markets. The representative constrained household chooses consumption at date t, and labor supply for both sectors, given wages and prices:

$$\begin{aligned} \text{max} \quad & U(C_t^c, N_t^c) = ln(C_t^c) - \omega_i \frac{N_{i,t}^c{}^{1+\varphi}}{1+\varphi} - \omega_f \frac{N_{f,t}^c{}^{1+\varphi}}{1+\varphi} \\ \text{subject to} \quad & C_t^c = \frac{W_{i,t}}{P_t} N_{i,t}^c + \frac{W_{f,t}}{P_t} N_{f,t}^c \end{aligned}$$

The first order-conditions yield three optimality conditions. Equations (16) and (17) have the same interpretation as the optimal labor supply conditions for the unconstrained agent. Optimal condition (18) which is the budget constraint emphasizes that the constrained agent simply consumes her labor income.

$$N^c_{i,t}: \quad \omega_i({N^c}_{i,t})^{\varphi} = \frac{1}{C^c_t}\frac{W_{i,t}}{P_t} \tag{16}$$

$$N^c_{f,t}: \quad \omega_f({N^c}_{f,t})^{\varphi} = \frac{1}{C^c_t}\frac{W_{f,t}}{P_t} \tag{17}$$

$$C^c_t = \frac{W_{i,t}}{P_t}N^c_{i,t} + \frac{W_{f,t}}{P_t}N^c_{f,t} \tag{18}$$

1.3 Consumption and Labor Supply Aggregation:

Aggregate demand by type of firm in terms of the type of agent is the sum of constrained and unconstrained consumption weighted by their respectively size. Aggregate demand by sector is the aggregation over each one of the firms of their respectively sector as (19) and (20) suggest.

- Informal sector.

$$\begin{gathered} C_{i,t}(i) = \lambda C^c_{i,t}(i) + (1-\lambda)C^u_{i,t}(i) \\ \int_0^{\rho} C_{i,t}(i)\ di = \int_0^{\rho} (\lambda C^c_{i,t}(i) + (1-\lambda)C^u_{i,t}(i))\ di \\ C_{i,t} = \lambda C^c_{i,t} + (1-\lambda)C^c_{i,t} \end{gathered} \tag{19}$$

- Formal sector.

$$\begin{gathered} C_{f,t}(f) = \lambda C^c_{f,t}(f) + (1-\lambda)C^h_{f,t}(f) \\ \int_{\rho}^{1} C_{f,t}(f)\ df = \int_{\rho}^{1} (\lambda C^c_{f,t}(f) + (1-\lambda)C^u_{f,t}(f))\ df \\ C_{f,t} = \lambda C^c_{f,t} + (1-\lambda)C^c_{f,t} \end{gathered} \tag{20}$$

Aggregate labor supply is the sum of aggregate constrained labor supply and aggregate unconstrained labor supply weighted by their relative sizes.

$$N_t = \lambda N_t^u + (1-\lambda) N_t^c \tag{21}$$

1.4 Informal firms:

Informal goods are produced by a perfectly competitive firm i using a linear technology given by:

$$Y_{i,t}(i) = A_{i,t} N_{i,t}(i) \tag{22}$$

where $A_{i,t} > 0$ is the constant informal labor productivity and $N_{i,t}(i)$ is the quantity of labor used by the informal firm. The cost minimization problem implies that the nominal marginal cost is equated to the nominal wage divided by productivity:

$$MC_{i,t} = \frac{W_{i,t}}{A_{i,t}} \tag{23}$$

Given that informal firms behave competitively, profit maximization for all firms implies:

$$P_{i,t} = MC_{i,t} \tag{24}$$

1.5 Formal firms:

Formal goods are produced by a monopolistically competitive firm f. All formal firms use the same linear technology given by:

$$Y_{f,t}(f) = A_{f,t} N_{f,t}(f) - \frac{m}{(1-\rho)} \tag{25}$$

where $\frac{m}{(1-\rho)}$ is a fixed cost common to all formal firms. Following Bilbiie (2008), this parameter governs the degree of returns to scale. It is assumed that formal firms are more productive than informal ones which implies that $A_{f,t} > A_{i,t}$. From the cost minimization problem, nominal marginal cost for formal firms is given by:

$$MC_{f,t} = \frac{W_{f,t}}{A_{f,t}} \tag{26}$$

Formal firms behave monopolistically competitive and set prices *a-la* Calvo (1983). Firms in the formal sector adjust their prices infrequently, where ϕ is the constant probability of keeping their prices unchanged. The parameter ϕ represents the degree of nominal rigidity. A formal firm that can change its price with probability $(1-\phi)$ must choose $P^*_{f,t}(f)$ to maximize the discounted sum of future nominal profits:

$$E_t \left\{ \sum_{j=0}^{\infty} \phi^j \Lambda_{t,t+j} [P^*_{f,t}(s) Y_{f,t+j}(f) - MC_{f,t+j} Y_{f,t+j}(f)] \right\}$$

subject to the aggregate demand constraint that the firm faces (7). Where $\Lambda_{t,t+1}$ is the stochastic market discount factor. The optimal price for the formal firm is:

$$P^*_{f,t}(f) = \left(\frac{\epsilon}{\epsilon - 1} \right) \frac{E_t \sum_{j=0}^{\infty} \phi^j \Lambda_{t,t+j} P_{t+j}{}^{\epsilon} \; Y_{f,t+j}}{E_t \left\{ \sum_{j=0}^{\infty} \phi^j \Lambda_{t,t+j} P^{\epsilon}_{t+j} \; Y_{f,t+j} \right\}} MC_{f,t+j} \tag{27}$$

Note that when prices are fully flexible $\phi = 0$, so

$$P^*_{f,t}(f) = \left(\frac{\epsilon}{\epsilon - 1} \right) MC_{f,t}$$

where $\left(\frac{\epsilon}{\epsilon-1}\right)$ is the markup for formal firms.

1.5.1 Sub-price index for formal goods:

From the definition of aggregate formal demand (3), the subprice index is:

$$P_{f,t}^{1-\epsilon} = \left(\frac{1}{1-\rho}\right) \int_{\rho}^{1} P_{f,t}(f)^{1-\epsilon}\ df \tag{28}$$

Using the fact that in equilibrium all firms, that with probability $(1-\phi)$ can change their prices, will choose the same price and hence the same level of output, the last expression becomes:

$$P_{f,t}^{1-\epsilon} = \phi P_{f,t-1}^{1-\epsilon} + (1-\phi) {P_{f,t}^{*}}^{1-\epsilon} \tag{29}$$

1.5.2 Real profits of formal firms:

The aggregated profit function for formal firms is given by:

$$D_t = Y_{f,t} \left[\frac{P_{f,t}}{P_t}\right] \left[1 - \frac{MC_{f,t}}{P_{f,t}} (\left(\frac{1}{1-\rho}\right) \triangle_{f,t} - \frac{m}{Y_{f,t}})\right] \tag{30}$$

where the term $\triangle_{f,t} \equiv \int_{\rho}^{1} \left[\frac{P_{f,t}(f)}{P_{f,t}}\right]^{-\epsilon} df$ is the relative price dispersion in the formal sector.

1.6 Monetary Policy

The central bank implements monetary policy through the control of the nominal interest rate. The central bank adjusts the interest rate in response to changes in expected formal inflation, and deviations of formal output respect from it's flexible price level and it is susceptible for shocks (ξ_t). As McKnight & Mihailov (2015) says, when designing simple feedback rules in a forward looking way, central bank ensure a determinate equilibrium.

$$R_t = R\left(E_t \frac{\Pi_{f,t+1}}{\Pi_f}\right)^{\Phi_\pi} \left(\frac{Y_{f,t}}{Y_f}\right)^{\Phi_x} \xi_t \tag{31}$$

where $\chi_t \equiv \frac{\xi_t - \xi_{ss}}{\xi_{ss}}$ is supposed to follow an AR(1) process:

$$\chi_t = \kappa_\chi \chi_{t-1} + \varepsilon_t^\chi$$

with $\kappa_\chi \in (0,1)$ and ε_t^χ is an $i.i.d$ shock such that $\varepsilon_t^\chi \sim N(0, \sigma_\chi^2)$.

1.7 Market clearing:

Labor market clearing requires that labor demand equals total labor supply in each sector:

$$N_{f,t} = (1-\lambda)N_{f,t}^c + \lambda N_{f,t}^u \tag{32}$$

$$N_{i,t} = (1-\lambda)N_{i,t}^c + \lambda N_{i,t}^u \tag{33}$$

State-contingent assets and equity shares are in zero net supply (markets are complete and agents trading in them are identical)

$$B_t = 0 \tag{34}$$

$$\Omega_t = (1-\lambda)^{-1} \tag{35}$$

Market clearing in the goods market:

- **Informal sector**

$$Y_{t,i}(i) = C^A_{i,t}(i) = \lambda C^c_{i,t}(i) + (1-\lambda)C^u_{i,t}(i)$$

Aggregating across firms using (4) and (19):

$$Y_{i,t} = C^A_{i,t} = \left[\lambda C^c_{i,t} + (1-\lambda)C^u_{i,t}\right] \tag{36}$$

- **Formal sector**

$$Y_{f,t}(f) = C^A_{f,t}(f) = \lambda C^c_{f,t}(f) + (1-\lambda)C^u_{f,t}(f)$$

Aggregating across firms and using (5) and (20).

$$Y_{f,t} = C^A_{f,t} = \left[\lambda C^c_{f,t} + (1-\lambda)C^u_{f,t}\right] \tag{37}$$

- **Aggregate Good's Market Clearing:**

  From (11), imposing market clearing conditions (36) and (37) including (18); and finally imposing market clearing conditions (32) and (33) yields:

$$(1-\lambda)C^u_t + \lambda C^c_t = \left[\frac{P_{i,t}}{P_t}\right] Y_{i,t} + Y_{f,t}\left[\frac{P_{f,t}}{P_t}\right] \tag{38}$$

defining,

$$C^A_t = (1-\lambda)C^u_t + \lambda C^c_t \tag{39}$$

and using the resource constraint:

$$C^A_t = Y_t \tag{40}$$

therefore:

$$Y_t = \left[\frac{P_{i,t}}{P_t}\right] Y_{i,t} + Y_{f,t}\left[\frac{P_{f,t}}{P_t}\right] \tag{41}$$

- **Aggregate version of the production function:**

  Using (36) and (37) in demand conditions (4) and (5) respectively and market clearing conditions (32), (33), we can express aggregate output for both sectors in terms of a production function. The denominator in (42) implies that an increase in price dispersion reduces output:

$$Y_{f,t} = \frac{A_{f,t}N_{f,t} - f}{\left(\frac{1}{1-\rho}\right)\triangle_{f,}} \tag{42}$$

$$Y_{i,t} = A_{i,t}N_{i,t} \tag{43}$$

1.8 Definition of equilibrium

A rational expectation equilibrium is a set of 28 endogenous variables:

- Sequence of prices: $\{W_{i,t}, W_{f,t}, MC_{i,t}, MC_{f,t}, P_{i,t}, P^*_{f,t}, P_{f,t}, P_t, \triangle_{f,t}\}$;
- Sequence of allocations: $\{C^A_t, C^u_t, C^c_t, C_{i,t}, C_{f,t}, N^u_{i,t}, N^u_{f,t}, N^c_{i,t}, N^c_{f,t}, N_{f,t}, N_{i,t}, N_t, B_t, \Omega_t, Y_{f,t}, Y_{i,t}, Y_t, D_t\}$
- A monetary policy: $\{R_t\}$

satisfying the following 28 equilibrium conditions:

- The first-order conditions of the constrained and unconstrained agents (13), (14), (15), (16), (17)
- Period budgets constraints: (11) and (18)
- The first-order condition of intermediate formal and informal firms: (23) and (26)

- Real profits of formal firms: (30)
- Price setting rules: (24), (27), (29)
- CPI price index: (8)
- Interest rate rule: (31)
- Market-Clearing conditions: (32), (33), (34) (35), (36), (37)
- Law of motion for price dispersion: $\triangle_{f,t}$
- Aggregate version of the formal and informal production functions: (42), (43)
- Aggregate output: (41)
- Aggregate consumption: (39)
- Aggregate labor: (21)
- Resource constraint: (40)

2 Log-Linearized Model and Calibration.

To solve the non-linear model developed in the previous chapter the equilibrium conditions are log-linearized around the steady state using a Taylor approximation following the methodology of Uhlig (1998). The interpretation is simple, they represent percentage deviations from the steady state. In appendix B we carefully explain all the steps and equations used to get the following linearized system. The complete log linearized system of equations is summarized in Table 1.

Table 1: **Log-Linearized model**

Description	Equation
Euler Equation, U	$E_t[\hat{c}^u_{t+1}] - \hat{c}^u_t = \hat{r}_t - E_t[\hat{\pi}_{t+1}]$
Formal Labor Supply, U	$\varphi \hat{n_{f,t}}^u = \hat{w}^r_{f,t} - \hat{c}^u_t$
Informal Labor Supply, U	$\varphi \hat{n_{i,t}}^u = \hat{w}^r_{i,t} - \hat{c}^u_t$
Aggregate Labor Supply, U	$\hat{n}^u_t = \frac{N^u_i}{N^u}\hat{n}^u_{i,t} + \frac{N^u_f}{N^u}\hat{n}^u_{f,t}$
Budget Constraint, U	$\hat{c}^u_t = \rho(\hat{w}^r_{i,t} + \hat{n}^u_{i,t}) + (1-\rho)(\hat{w}^r_{f,t} + \hat{n}^u_{f,t}) + \frac{1}{(1-\lambda)}\hat{d}_t$
Formal Labor Supply, C	$\varphi \hat{n_{f,t}}^c = \hat{w}^r_{f,t} - \hat{c}^c_t$
Informal Labor Supply, C	$\varphi \hat{n_{i,t}}^c = \hat{w}^r_{i,t} - \hat{c}^c_t$
Aggregate Labor Supply, C	$\hat{n}^c_t = \frac{N^c_i}{N^c}\hat{n}^c_{i,t} + \frac{N^c_f}{N^c}\hat{n}^c_{f,t}$
Budget Constraint, C	$\hat{c}^c_t = \rho(\hat{w}^r_{i,t} + \hat{n}^c_{i,t}) + (1-\rho)(\hat{w}^r_{f,t} + \hat{n}^c_{f,t})$
Formal Production Function	$\hat{y}_{f,t} = (1-\mu)(\hat{a}_{f,t} + \hat{n}_{f_t})$
Informal Production Function	$\hat{y}_{i,t} = \hat{a}_{i,t} + \hat{n}_{i_t}$
Formal Real Marginal Cost	$\hat{mc}_{f,t} = \hat{w}_{f,t} - \hat{a}_{f,t}$
Informal Real Marginal Cost	$\hat{mc}_{i,t} = \hat{w}_{i,t} - \hat{a}_{i,t}$
Real profits of the formal firms	$\hat{d}_t = -\hat{mc}_{f,t} + \frac{\mu}{(1-\mu)}\hat{y}_{f,t}$
NKPC	$\hat{\pi}_{f,t} = \beta E_t[\hat{\pi}_{t+1}] + \psi \hat{mc}_{i,t}$
Aggregate Inflation	$\hat{\pi}_t = (1-\rho)\hat{\pi}_{f,t} + \rho\hat{\pi}_{i,t}$
Monetary Policy	$\hat{r}_t = \Phi_{\hat{\pi}} E_t[\hat{\pi}_{f,t+1}] + \Phi_{\hat{y}}\hat{y}_{f,t}$
Formal Labor Market Clearing Conditions	$\hat{n}_{f,t} = (1-\lambda)\hat{n}^u_{f,t} + \lambda\hat{n}^c_{f,t}$
Informal Labor Market Clearing Conditions	$\hat{n}_{i,t} = (1-\lambda)\hat{n}^u_{i,t} + \lambda\hat{n}^c_{i,t}$
Formal Good's Market Clearing Conditions	$\hat{y}_{f,t} = \hat{c}_{f,t}$
Informal Good's Market Clearing Conditions	$\hat{y}_{i,t} = \hat{c}_{i,t}$
Aggregate Consumption	$\hat{c}^A_t = (1-\lambda)\ \hat{c}^u_t + \lambda\ \hat{c}^c_t$

Table 2: **Log-Linearized (continued).**

Description	Equation
Aggregate Labor	$\hat{n}_t^A = (1-\lambda)\ \hat{n}_t^u + \lambda\ \hat{n}_t^c$
Resource Constraint	$\hat{y}_t = \hat{c}_t^A$
Aggregate Output	$\hat{y}_t = \rho\ \hat{y}_{i,t} + (1-\rho)\ \hat{y}_{f,t}$
Informal Real Wage	$\hat{w}_{i,t}^r = \hat{w}_{i,t} + \hat{p}_{i,t} - \hat{p}_t$
Formal Real Wage	$\hat{w}_{f,t}^r = \hat{w}_{f,t} + \hat{p}_{f,t} - \hat{p}_t$

The results and simulations are obtained using the parameter calibration summarized in Table 3. The period length is one quarter. Following Galí (2015) we assumed $\phi = 0.75$, which means that on average firms change their prices once per year and is a value commonly used in the New Keynesian literature. We also set $\Phi_{\hat{\pi}_{f,t}} = 1.5$ and $\Phi_{\hat{Y}_{f,t}} = 0.125$ common values used in the literature and are the ones proposed by Taylor (1993) in his seminal paper.[3]

We set $\varphi = 0.3125$ which implies a Frisch elasticity labor supply of 3.2 as reported by Leyva & Urrutia (2018) and is in the range proposed by Whalen & Reichling (2017). For the elasticity of substitution between formal goods, we set $\epsilon = 2.5$ so the markup is $\frac{\epsilon}{\epsilon-1} = 1.66$ in line with the findings of Loecker and De Loecker & Eeckhout (2018) for the Mexican economy. Following Fernandez & Meza (2015) we set $\beta = 0.9976$ which implies a steady state real return on bonds of around 4 percent, other authors set different values for the discount factor for example Aguiar & Gopinath (2004) set $\beta = 0.98$. The fraction of population that do not have access to financial markets is $\lambda = 0.7$, and the fraction of firms that are informal is $\rho = 0.227$ using information from ENIF (2018) and INEGI (2018), respectively. Finally, it is assumed a medium persistence of the monetary policy shock $k_m = 0.5$ which is standard in the literature.

For the steady state values, it is assumed that $\mu = \frac{f}{Y} = \frac{1}{\epsilon-1}$ hence profits are 0 in the steady state. Although it seems that this assumption is non-verifiable for the Mexican economy, it is consistent with the arguments presented by Rotemberg & Woodford (1993) and the idea that the number of formal firms is fixed in the long run. $\omega_f = 1$ and $\omega_i = 0.524$ were calibrated so the model

[3] Which are very similar values to the ones that Moura & de Carvalho (2010) estimate empirically for the Mexican economy.

Table 3: **Calibration**

Parameter	**Description**	**Value**
β	Subjective discount factor	0.9976
φ	Frisch elasticity (Inverse labor supply elasticity)	0.3125
ϵ	Elasticity of substitution between formal goods	2.5
λ	Fraction of population that is constrained	0.7
ρ	Fraction of firms that are informal	0.227
ϕ	Degree of price stickiness	0.75
$\Phi_{\hat{\Pi}_t}$	Central bank degree of response to inflation	1.5
$\Phi_{\hat{Y_{f,t}}}$	Central bank degree of response to output	0.125
k_m	Monetary shock persistance	0.5

Table 4: Steady State Values

Parameter	**SS value**	**Empirical estimates**	**Source**
$\frac{N_i^u}{N^u} = \frac{N_i^c}{N^c} = \frac{Ni}{N}$	0.57	0.57	INEGI (2018)
$\frac{N_f^u}{N^u} = \frac{N_f^c}{N^c} = \frac{N^f}{N}$	0.43	0.43	INEGI (2018)
$\frac{A_f}{A_f}$	2.33	2.1901	Fernandez and Meza (2014)
$\frac{W_f}{W_i}$	1.75	1.54	Moreno (2007)
$\mu = \frac{f}{Y} = \frac{1}{\epsilon - 1}$	0.66	0.55	Loecker and Eeckhout (2018)

matches the Mexican labor data, that 57 percent of the population are employed in the formal sector. We impose that the ratio of formal wage to informal wage in the steady state is 1.75 close to the value presented by Moreno Treviño (2007). The ratio of the formal productivity to informal productivity is a function of the elasticity between formal goods and the ratio of formal wage to informal wage. This parameters yield a ratio of 2.33 between formal and informal productivity, a similar value is proposed by Fernandez & Meza (2015).

3 Model Dynamics

We solve the log linearized model in using Dynare. The code of the baseline model is presented in appendix D. In this chapter we analyze the dynamic responses of a negative monetary policy shock. First we present the transmission mechanism of a negative monetary policy shocks by conducting an impulse response analysis of the model in which we include limited asset participation and informality (Model I); then we conduct a parameter sensitivity analysis for the size of informality (ρ), elasticity of substitution between formal goods (ϵ) and elasticity of labor supply (φ); finally we compare it to the benchmark model which includes informality but no limited asset participation, $\lambda \to 0$, named (Model II). The model II is presented in the appendix C. We focused in the sensitivity of the economy to a monetary policy shock. Steady state values are presented in appendix A.

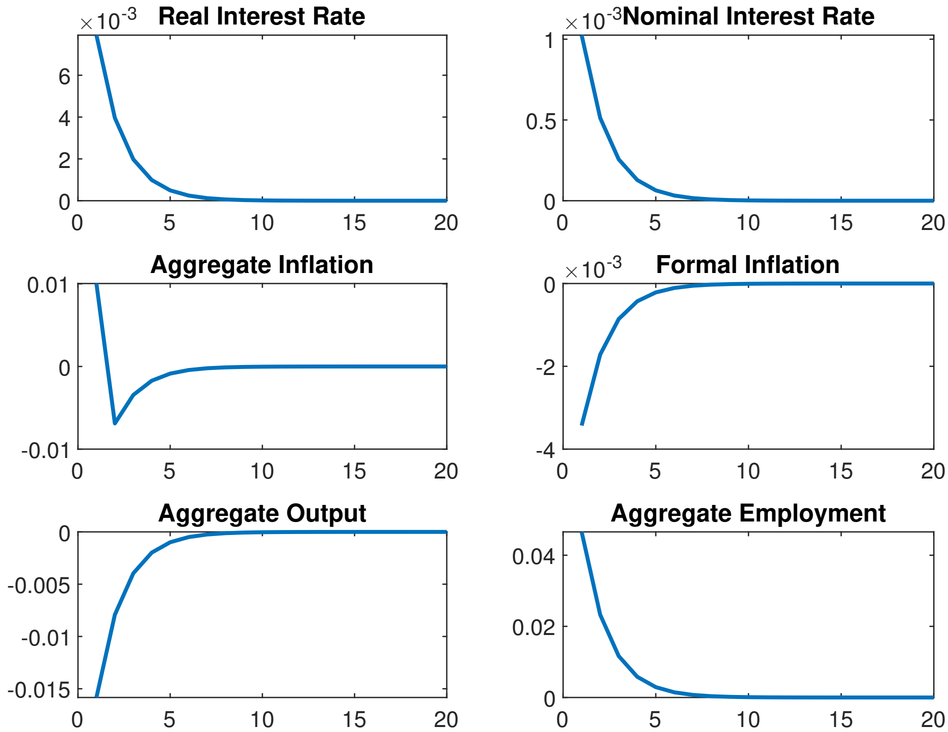
3.1 Monetary Policy Shock.

As we can see in figure 2, a negative monetary policy shock i.e, an increase in the nominal interest rate, results in a fall of aggregate output; and an increase in total employment and aggregate inflation. Aggregate output falls because the increase in informal output doesn't compensate the fall in formal output since the size of informal sector is relatively small. The increase in total employment comes from the increase in informal labor supply, considering that the share of the informal employment in the aggregate is relatively high. Figure 3 presents the impluse response function (IRF) by sector.

The intuition behind this analysis is that a negative monetary shock results in a fall of unconstrained consumption because of the intertemporal substitution represented by the consumption Euler equation.[4]Consider the formal sector, unconstrained agents are willing to work more at the same real formal wage so the formal labor supply curves shifts outwards making real wages in both sectors fall. Since constrained agents merely consume their wage income, they also shift labor supply curves to the right. Now consider labor demand, here price stickiness plays a major

[4]Lenders are willing to sacrifice present consumption in order to consume more in the future and for borrowers present consumption becomes more expensive.

Figure 2: IRF: Aggregate Variables.



role. In the formal market, since real wages fall, prices should fall too but not all prices can do this adjustment. Given the degree of price stickiness, some of the formal firms can't change their prices so they are inefficiently high reducing the demand of this good, shifting the labor demand the left for the supply of the good meets its demand.

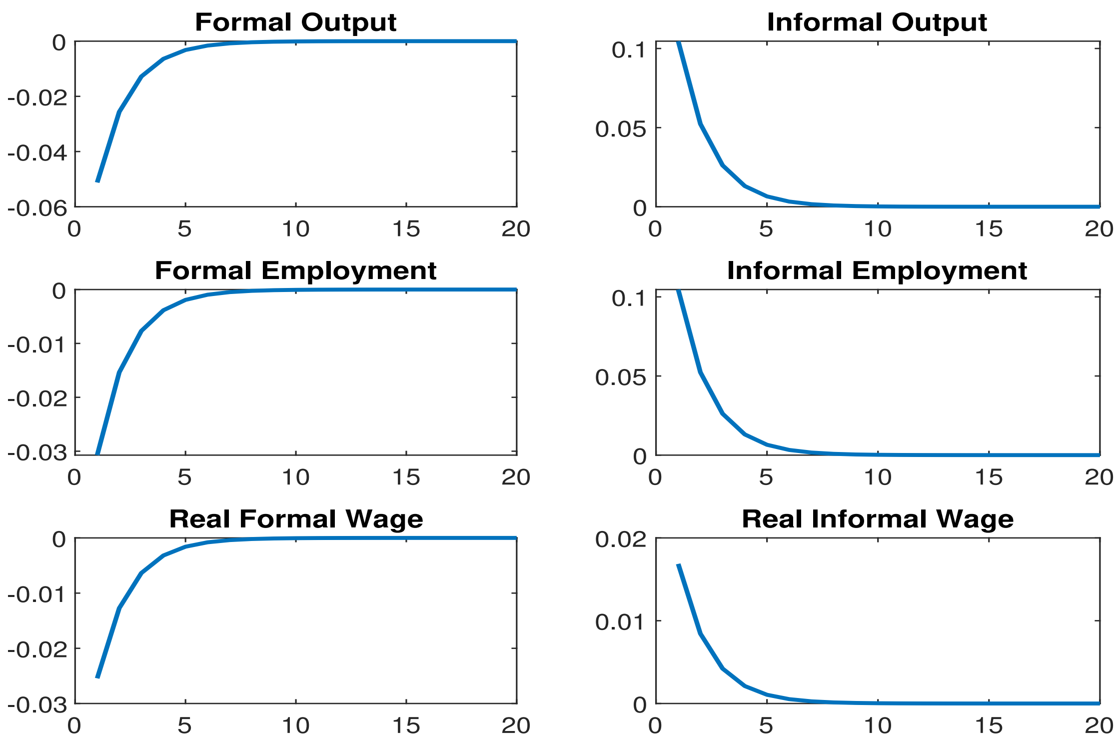
In the informal market labor increases, hence informal output, since there is no price stickiness. For the equilibrium to be consistent, informal wage should raise to consume the informal output excedent, so labor demand should shift right. This increment in the informal output does not offset the fall in the formal one because the size of the informal sector is not big enough and not productive enough.

To see why the informal sector is countercyclical, consider the difference in real wages between sectors. Making the change in profits $\hat{d}_t = 0$, if we substitute the log-linearized versions of the budget constraint in the informal and formal log-linearized labor supply equations we get, for both agents:

$$\hat{n}_{f,t} = \frac{\rho}{\varphi + (1-\rho)} \left[\hat{w}^r_{f,t} - \hat{w}^r_{i,t} - \hat{n}_{i,t}\right]$$
$$\hat{n}_{i,t} = \frac{(1-\rho)}{\varphi + \rho} \left[\hat{w}^r_{i,t} - \hat{w}^r_{f,t} - \hat{n}_{f,t}\right] \tag{44}$$

In equation (44) we can see how the counter cyclical feature of the informal sector depends on the difference between real informal and formal wages. As the difference becomes greater more labor is supplied to the informal sector so the counter cyclical effect is greater.

Figure 3: IRF by sector.

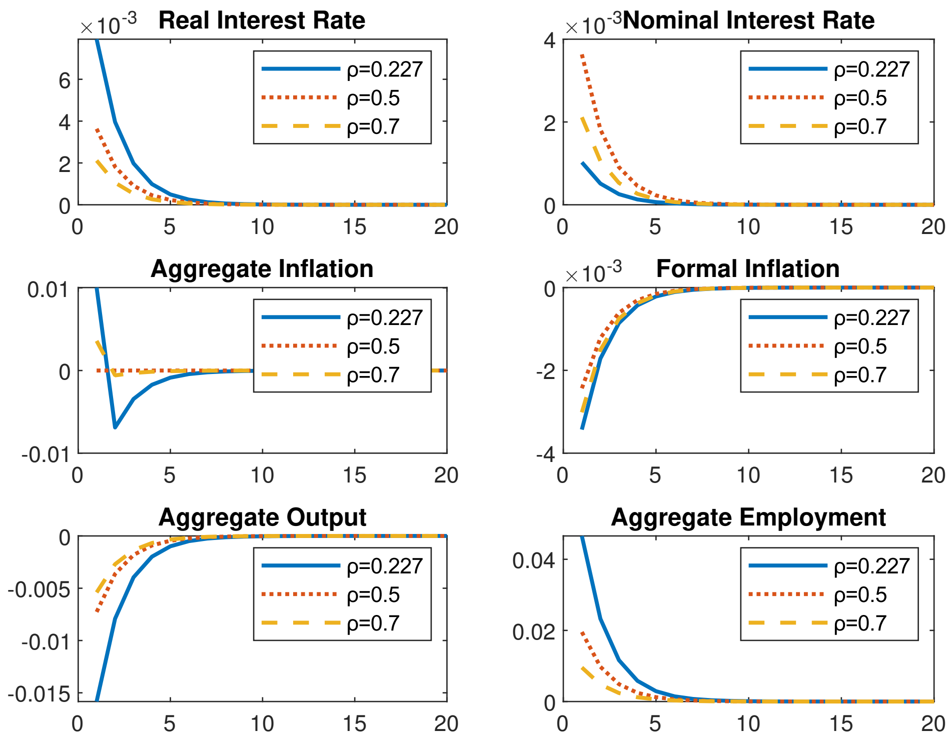


3.2 Sensitivity Analysis.

3.2.1 Informality

Since the size of informality, ρ, is a key variable for developing countries, a reasonable question is how informality affects the transmission of a monetary policy shock. The results presented in the last section indicate that, informal real wages and informal employment is counter cyclical; one should think that increasing the size of the informal sector, $\rho \to 1$, would offset the negative effect of the formal sector on aggregate output, since now informal sector accounts for a bigger share in aggregate output. This is not the case. One of the key assumptions is that the informal sector behaves perfectly competitively so prices are fully flexible. If $\rho \to 0$ the model collapses to the one developed by Bilbiie (2008), and aggregate inflation collapses to the NKPC since now more weight is put on formal inflation in the log linearized equation of aggregate inflation. The real interest rate becomes more sensitive to changes in the nominal interest rate and thus aggregate demand falls further.

Figure 4: IRF: Degree of informality.



In figure 4 we present the model with three different sizes of informality: 0.227 (baseline), 0.5

and 0.7. Changes in the nominal interest rate affects the economy from the consumption Euler equation via influencing the real interest rate. As we can see in figure 4, small values of informality in the economy becomes more sensitive to monetary policy shocks as a result of price stickiness and for greater values of informality the economy is less sensitive due to flexible prices.

Figure 5: IRF: Degree of informality by sector.

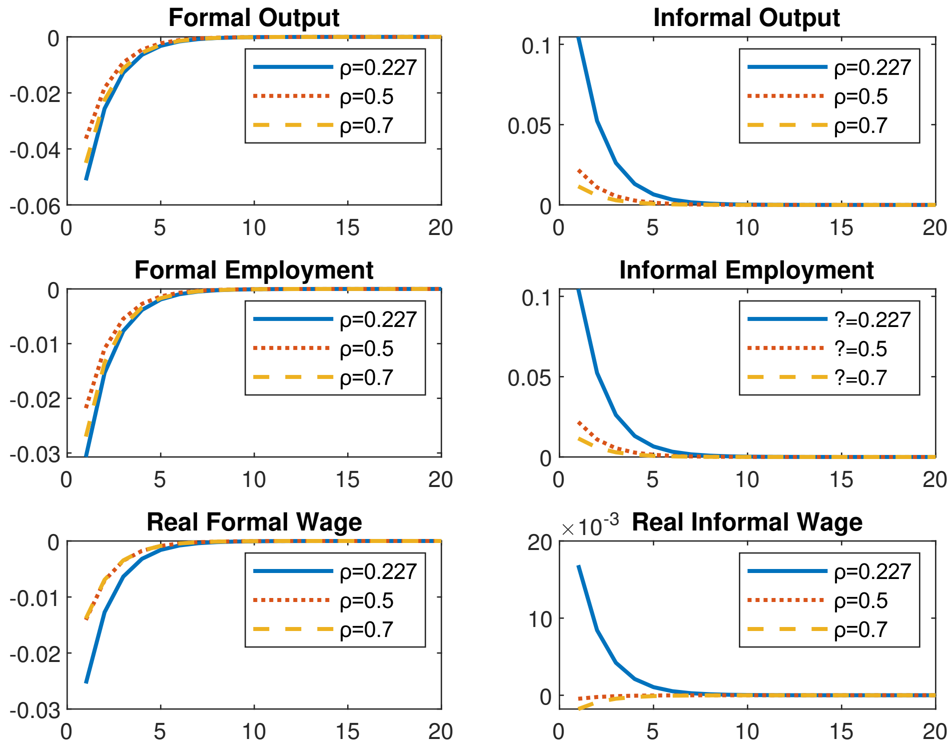
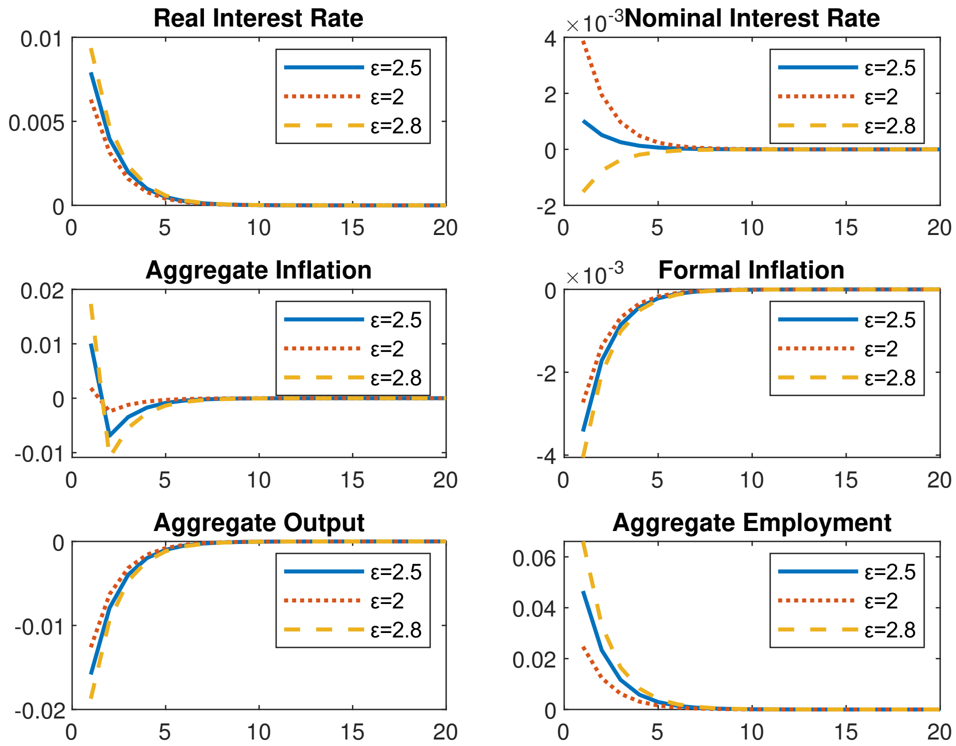


Figure 5 shows the impulse response functions by sector for different values of ρ. Given the intuition presented in the last section, for larger values of ρ the formal real wage still falls but since the informal real wage doesn't change much, agents do not offer as much informal labor as in the baseline case, so informal output doesn't increase as much. Notice that as $\rho \to 1$ informal real wages loses its counter cyclical effect and less weight is put, in the log linearized equation of aggregate output, on formal output; for this reason, larger values of ρ never offsets, but softens, the fall in the formal sector.

3.2.2 Elasticity of substitution between formal goods

Figure 6: IRF: Elasticity of substitution.

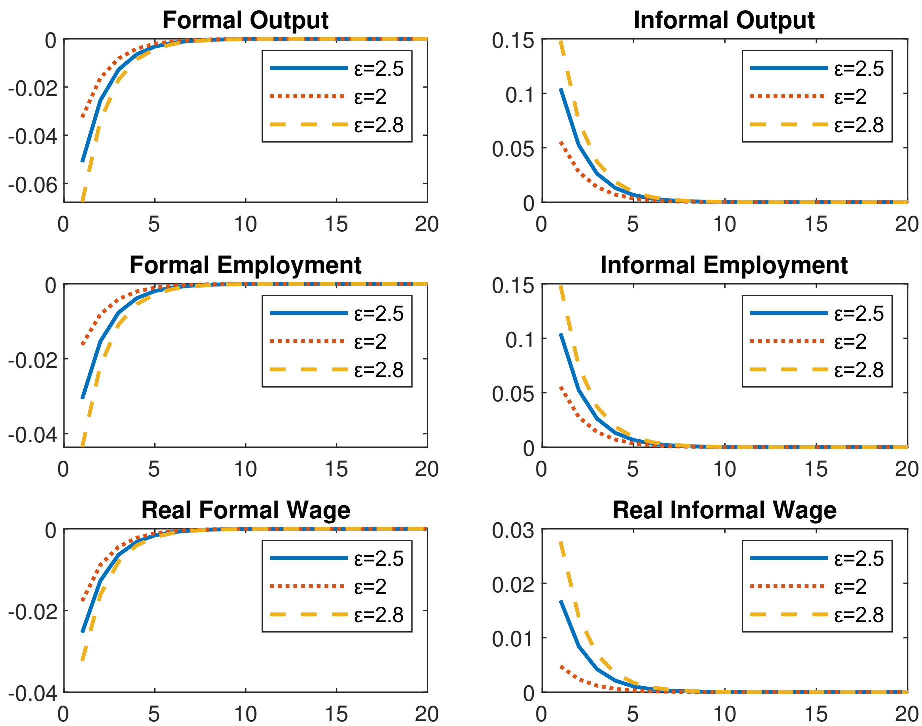


We now investigate the sensitivity of the baseline results to changes in the elasticity of substitution between formal goods. One of the key assumptions is that formal sector set prices according to Calvo (1983). Optimal behavior implies that formal firms set their prices according to (27). Recall the log-linearized version of the aggregate formal production function using the definition of μ.

$$\hat{y}_{f,t} = \left(\frac{\epsilon}{\epsilon - 1} \right) (\hat{a}_{f,t} + \hat{n}_{f_t}) \tag{45}$$

As the markup increases, formal output becomes more sensitive to changes in work. However, if the substitution between formal goods is inelastic enough, $\epsilon \to 1$, the fall in the good's demand is lower. Hence, formal real wages, formal employment, and formal output do not fall as much as for greater values of ϵ.

Figure 7: IRF: Elasticity of substitution by sector.

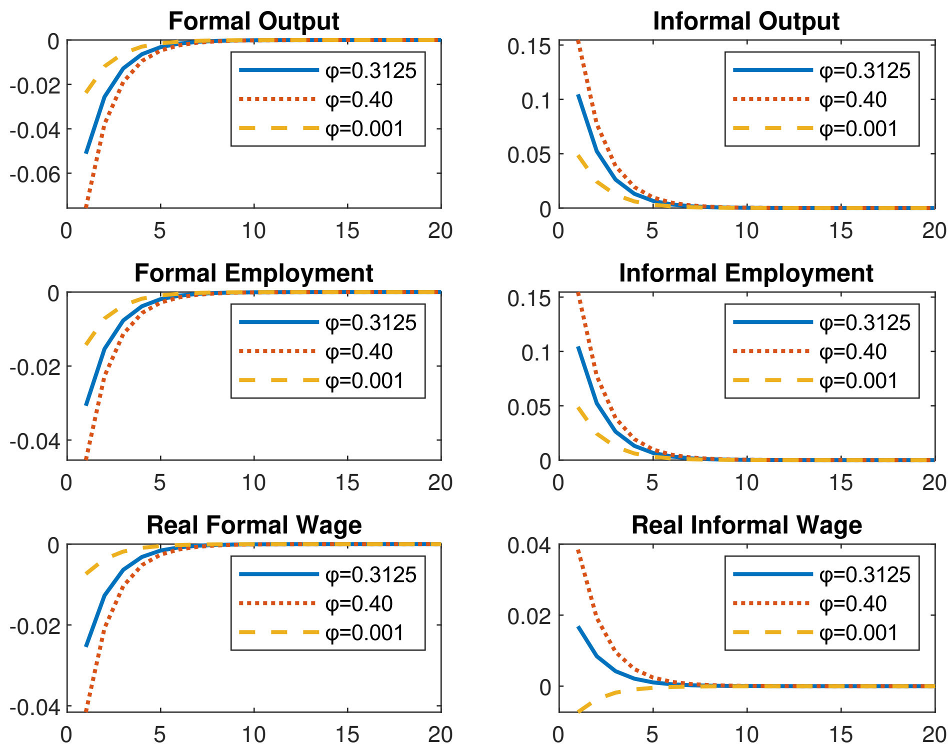


Figures 6 and 7 show how shocks to monetary policy affect the economy at different values of ϵ. As we can see, if the elasticity of substitution between formal goods is relatively elastic the economy becomes more sensitive to monetary policy shocks.

3.2.3 Labor elasticity

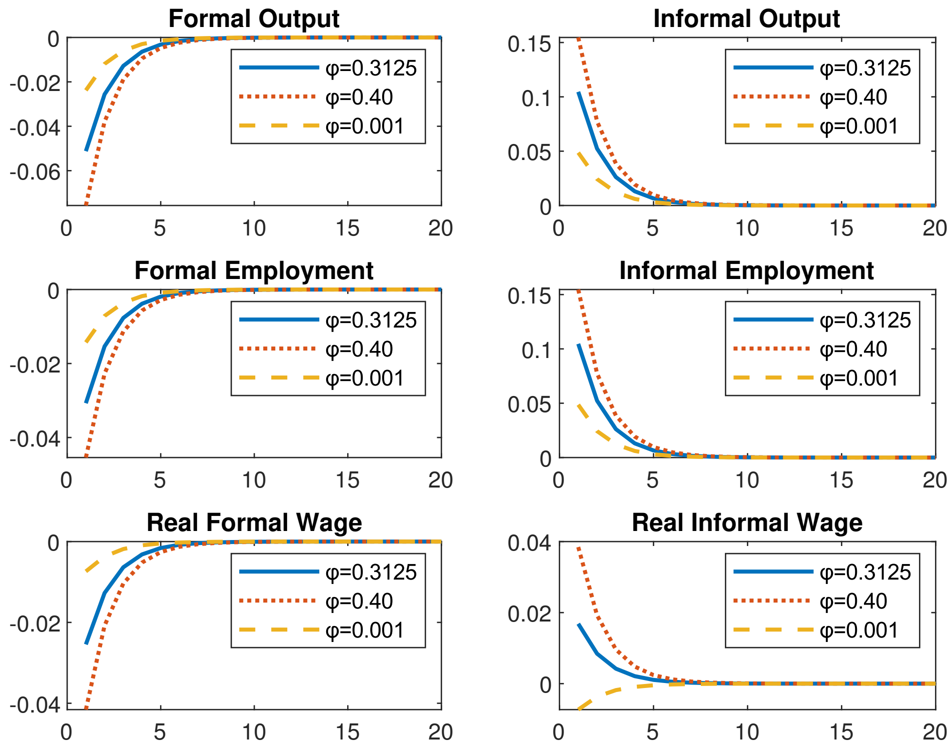
As in Bilbiie (2008), the labor supply elasticity parameter plays a crucial role in the sensitivity of shocks to monetary policy. Consider two cases, in the formal sector, with two different types of elasticity: one relatively elastic, $\varphi \to 0$, and one relatively inelastic $\varphi \to \infty$. In the face of a monetary policy shock, labor demand has to shift further to the left, in the case of a relatively inelastic labor supply, to adjust output to demand. This shift makes formal real wage decrease more so prices are *more* inefficiently high so the fall in the goods demand is larger. In the case of an elastic labor supply, labor demand can adjust output without a further decrease in real formal wages.

Figure 8: IRF: Labor elasticity



Figures 8 and 9, presents IRF for different values of labor supply elasticity. The baseline economy represents the case in which labor supply is relatively inelastic. As we can see, for greater values of φ, the economy is more sensitive to monetary policy shocks. Informal employment loses its countercyclical feature as labor supply becomes more elastic because the formal wage doesn't fall too much and agents are not willing to offer as much informal labor.

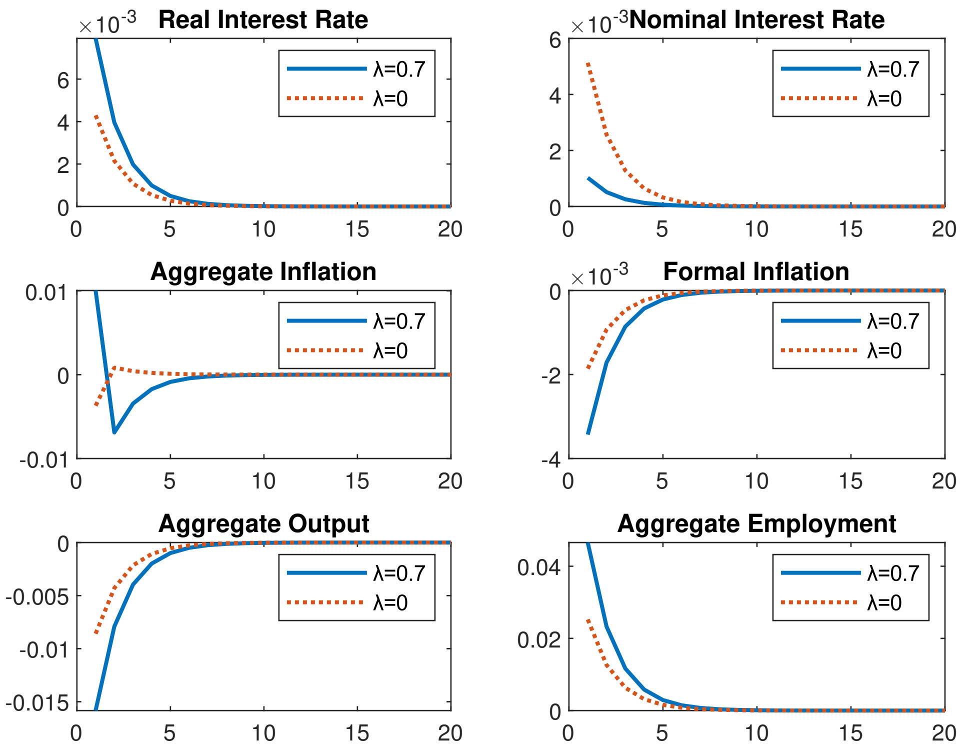
Figure 9: IRF: Labor elasticity by sector.



3.3 The role of LAMP

We now consider a version of the model without limited asset market participation, $\lambda \to 0$, to highlight the role of λ. As we can see in figure 10, the real interest rate and aggregate output from the benchmark model are less responsive to monetary policy shocks; and aggregate inflation has contrary dynamics in comparison to the baseline model.

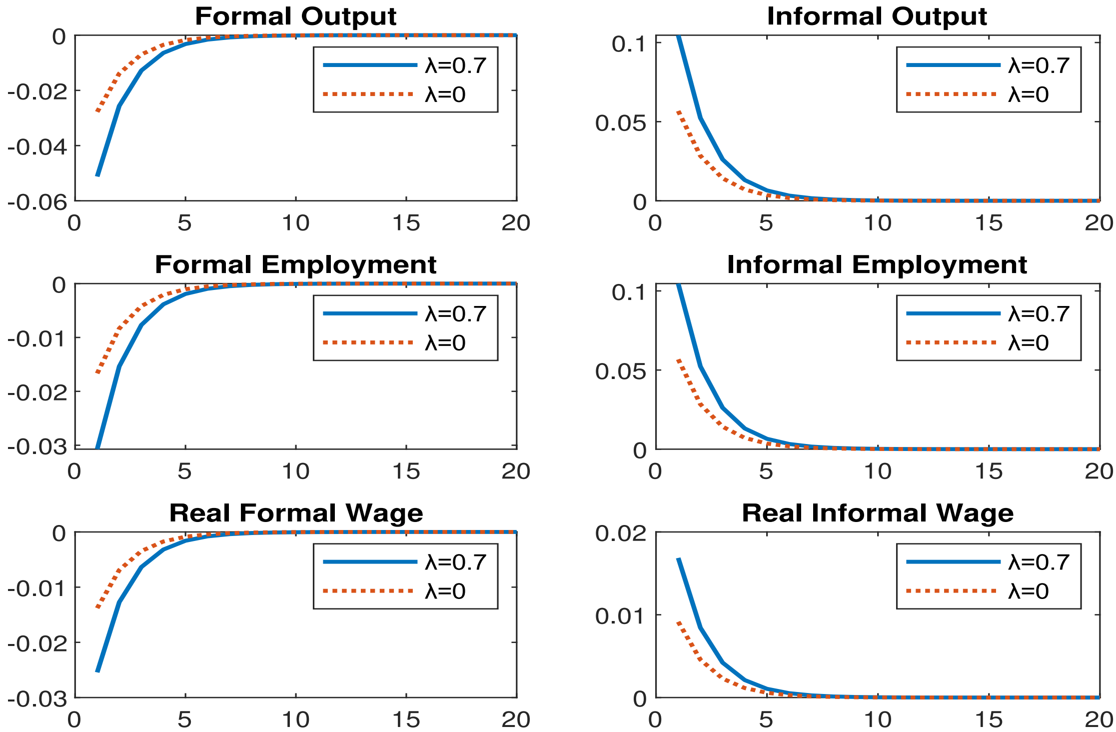
Figure 10: IRS: Aggregate Variables. The role of λ



Bilbiie (2008) performs a similar exercise comparing the model he develops (a one-sector including limited asset market participation) and the standard full participation version. He finds that including frictions in the asset market makes the transmission mechanism of monetary policy more powerful:

"The fall in real wage brought about by intertemporal substitution of asset holders now means a further fall in demand, since non-asset holders merely consume their wage income. This generates a further shift [**to the left**] in labor demand, so the new equilibrium is one with even lower (compared to the full participation one) output, consumption, hours and real wage."

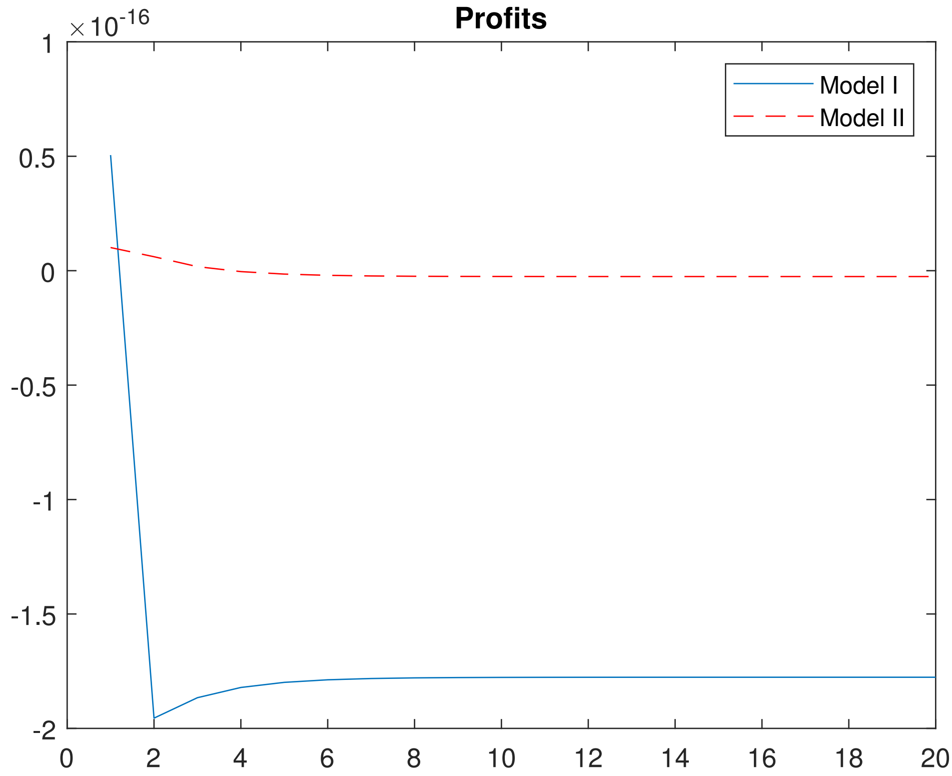
Figure 11: IRS by sector.



But, as he explains, there is a non-monotonicity of this effect. In fact, there are two regions: the standard region in which an increase in the share of non-asset holders makes the aggregate demand more elastic to the interest rate, maintaining the negative relationship; and the inverted region in which this elasticity decreases in absolute value and a positive relationship arises. The reason for this is that profits plays a major role the model. Consider the profits part of the log-linearized version of the unconstrained budget constraint, if $\lambda \to 1$ then $\frac{1}{(1-\lambda)} \to \infty$. A slight increase in profits, increases unconstrained consumption by a larger proportion. As Bilbiie (2008) adds:

'In particular, if labor supply is inelastic enough and/if asset market participation is limited enough (..) an increase in profits would occur that would generate a positive income effect on asset holders. This expansionary effect contradicts both the initial 'intertemporal substitution' effect on labor supply of asset holders and the contractionary effect of monetary policy on their demand. (...) This is an equilibrium whereby consumption, output, hours and the real wage increase- hence 'expansionary monetary contractions'.'

Figure 12: IRF: Profits in the Baseline model vs the model without LAMP.



Comparing the baseline model against the model in the absence of LAMP, with the parameters described in table 3, changes in profits around the steady state are nearly 0 as we can see in figure 12. This result suggest, under the logic presented by Bilbiie for this particular parameterization, that there is no *reverse demand logic* in the model with two sectors.

4 Wicksellian Rules Vs. Taylor Rules

In this chapter we compare the results if the monetary policy rule is characterized by a pure inflation targeting rule or a Wicksellian interest rate rule. In the baseline model, the central bank implements monetary policy using a Taylor-type rule, whereby the nominal interest rate adjusts to deviations of inflation in the formal sector and deviations of formal output from the stead state.

$$R_t = R\left(\frac{E_t[\Pi_{f,t+1}]}{\Pi_f}\right)^{\Phi_\pi}\left(\frac{Y_{f,t}}{Y_f}\right)^{\Phi_{Y_f}} \tag{46}$$

Another type of monetary policy rule a pure inflation-targeting rule where $\Phi_{Y_f} = 0$. Monetary policy only targets deviations of formal inflation around its steady state value:

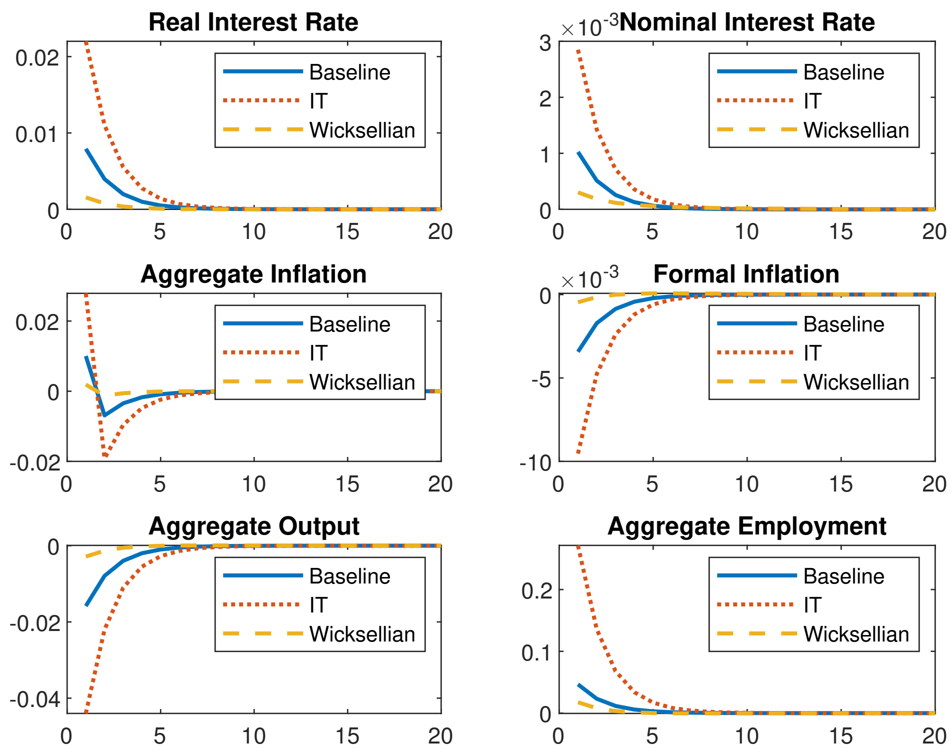
$$R_t = R\left(\frac{E_t[\Pi_{f,t+1}]}{\Pi_f}\right)^{\Phi_\pi} \tag{47}$$

An alternative to inflation-targeting rules are Wicksellian interest-rate rules, where the monetary policy rule reacts to fluctuations in the aggregate price level

$$R_t = R\left(\frac{E_t[P_{f,t+1}]}{P_f}\right)^{\Phi_\pi}\left(\frac{Y_{f,t}}{Y_f}\right)^{\Phi_{Y_f}} \tag{48}$$

Equation (48) is the Wicksellian monetary policy rule. There are other reasons to consider Wicksellian rules. McKnight (2018) shows that Taylor principle is irrelevant under Wicksellian monetary policy rules because it solves the problem of indeterminacy even if output also enters into the monetary policy rule. We set $\Phi_\pi = 1$ and $\Phi_{Y_f} = 1$ for simplicity.

Figure 13: IRF: Monetary Policy Rules.

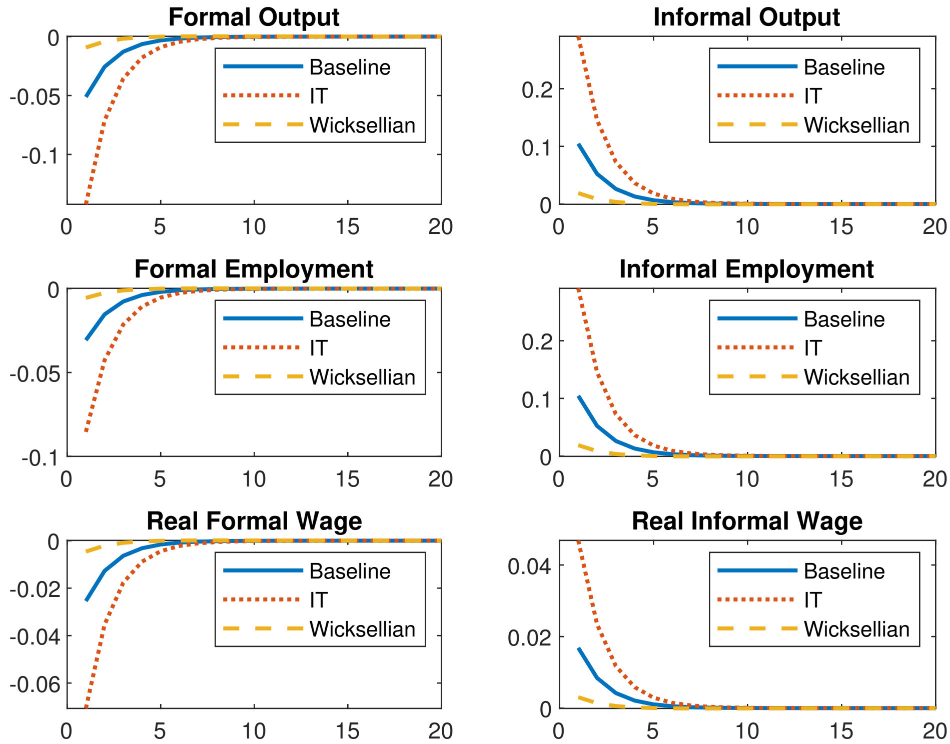


As we can see from figure 13, the Taylor rule that only reacts to expected future formal inflation makes the economy more sensitive to shocks to monetary policy. In the Inflation Targeting (IT) case, the upper-left panel of figure 13 shows how aggregate inflation raises more in the initial periods than in the baseline case. This makes the IT monetary policy rule raise the nominal interest rate further because output deviations are not being considered: if the monetary policy rule also reacts to output, then the initial deviations from its steady state should be compensated with a minor increase in the nominal interest rate which is the case of the baseline model.

Wicksellian monetary policy rules makes the price level return to its steady state level. In this case a negative monetary policy shock does not make the nominal interest rate increase as much as under Taylor rules because formal prices fall and this reduces the effect of the shock in the monetary policy if it targets formal price level.

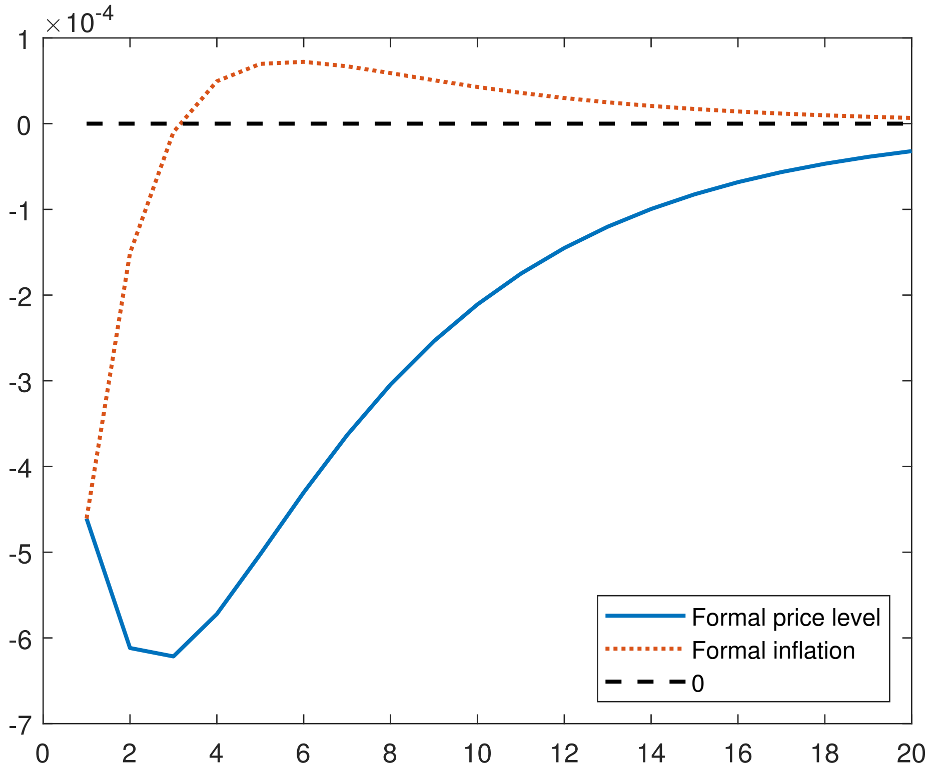
Figure 15 shows how Wicksellian monetary policy rules work. The initial fall in formal prices is compensated with an increase of formal inflation in the future. As formal prices returns to old steady state level, formal inflation becomes positive. The key difference between Wicksellian rules

Figure 14: IRF: Monetary Policy Rules by Sector.



and Taylor rules is that Wicksellian rules reduce the sensitivity of the real interest rate to changes in the nominal interest rate. Thus, the effect of monetary policy in aggregate demand is reduced and monetary policy shocks have a smaller effect on the real economy.

Figure 15: IRF: Formal prices in the Wicksellian case



5 Conclusions

In this thesis we have presented a DSGE model that incorporates limited asset market participation and informality into the New Keynesian framework and calibrate the model to obtain a steady state equilibrium with realistic steady state set of outcomes for the Mexican economy. We focused on the sensibitiviy of the economy in the presence of negative monetary policy shocks. We assumed that the central bank conducts monetary policy by setting the nominal interest rate. The thesis considered different monetary policy rules: Taylor interest-rate rules and Wicksellian policy rules.

The key findings of the thesis can be summarized as follows. First, informal real wages are counter cyclical, making agents increase their informal labor supply and reducing their formal labor supply. Thus, formal output decreases and informal output increases. This countercyclical characteristic of the informal sector is a special empirical feature of emerging economies. The increase of the size of the informal sector, $\rho \to 1$, reduces the countercyclical effect in the informal real wage because it became unresponsive to shocks in monetary policy. If $\rho \to 0$ the real interest rate become more sensitive to changes in the nominal interest thus aggregate demand falls further.

Second , an increase in the parameter of the elasticity of substitution between formal goods make the economy more sensitive due to a further fall in the labor demand in that sector; and an increase in the labor elasticity parameter makes the economy less sensitive due to the fact that labor demand has to shift less to supply meet the demand in the good's market.

Third, the inclusion of LAMP makes the monetary policy shock have an additional effect since constrained agents merely consume their labor income and the fall in formal real wage makes aggregate demand fall further. Under this particular parameterization, changes in profits around the steady state are nearly 0 so it seems that there is no *reverse aggregate demand logic* as the results presented in Bilbiie (2008). Nevertheless, *reverse aggregate demand logic* occurs with indeterminacy so for future research indeterminacy analysis under LAMP and informality is a relevant and interesting question to answer.

Finally, we present monetary policy shocks under three different types of rule: a Taylor rule, a Taylor rule that only targets deviations of expected formal inflation, and a Wicksellian rule that targets deviations of the formal price level and formal output deviations. First, we show that a Taylor rule that only targets deviations makes the economy more sensitive to monetary policy shocks because output deviations are not being considered: if the monetary policy also react to output nominal interest rates should increase less. Making the comparison between Taylor and Wicksellian rules, the last one makes the economy less sensitive to monetary policy shocks. The key difference is that Wicksellians rules reduce the sensitivity of the real interest rate to changes in the nominal interest rate.

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A Steady State

- from (13), where $R \equiv 1 + r$:

$$R_{ss} = \beta^{-1} \tag{49}$$

- Defining $\mu \equiv \frac{m}{Y_f} \equiv (\epsilon - 1)^{-1}$ from (32):

$$D_{ss} = 0 \tag{50}$$

- In a steady state with zero inflation:

$$\begin{aligned} \pi_{f,t}^{ss} &= \frac{P_{f,t}^{ss}}{P_{f,t-1}^{ss}} = 1 \\ \Longrightarrow P_{f,t}^{ss} &= P_{f,t-1}^{ss} \end{aligned} \tag{51}$$

- From (30):

$$P_{f,t}^{ss} = P_{f,t}^{*\ ss} \tag{52}$$

- Since prices are "flexible" in the steady state, it follows from (29) that:

$$P_{f,t-1}^{ss} = P_{f,t}^{ss} = P_{f,t}^{*\ ss} = \left(\frac{\epsilon}{\epsilon - 1}\right) MC_{f,t}^{ss} \tag{53}$$

- From (27) using (26) real formal wage in the steady state is:

$$\frac{W_{f,t}^{ss}}{P_{f,t}^{ss}} = A_{f,t}^{ss} \left(\frac{\epsilon - 1}{\epsilon}\right) \tag{54}$$

And informal real wage:

$$\frac{W_{i,t}^{ss}}{P_{t}^{ss}} = A_{i,t}^{ss} \tag{55}$$

- Since preferences are the same across groups and profits are 0 in the steady state, agents consume and offer labor in the same amount:

$$
\begin{aligned}
N_{i,t}^{u,ss} &= Nc,ss_{i,t} = Nss_{i,t} \\
N_{f,t}^{u,ss} &= Nc,ss_{f,t} = Nss_{f,t} \\
N_{t}^{u,ss} &= Nc,ss_{t} = Nss \\
C_{t}^{u,ss} &= C_{t}^{c,ss} = C_{t}^{A,ss}
\end{aligned}
\tag{56}
$$

- In the stady state, we suppose that:

$$
P_i = P = P_f \tag{57}
$$

B Log-linearized model

We take a log-linear approximation of the equilibrium conditions around the steady state using Uhlig (1999) log-linear approximations where $X_t = X^{ss}e^{\hat{x}_t}$. Where X_t denotes a variable in levels at time t; X^{ss} is the variable at steady state (we will avoid using the superscript ss and instead omit the subscript t). Also, we are using the approximation $e^{\hat{x}_t + a\hat{z}_t} \approx 1 + \hat{x}_t + a\hat{z}_t$. Also, we suppose that $\hat{\triangle}_{f,t} \approx 0$.

1. **Euler equation**:

   From (13):

$$\beta E_t[\frac{C_t^u}{C_{t+1}^u}] = \frac{1}{R_t} E_t[\frac{P_{t+1}}{P_t}]$$
$$\beta E_t[\frac{C^u}{C^u}(1 + \hat{c}_t^u - \hat{c}_{t+1}^u)] = \frac{1}{R} E_t[\frac{P}{P}(1 + \hat{p_{t+1}} - \hat{p}_t - \hat{r}_t)]$$
$$\beta E_t[(1 + \hat{c}_t^u - \hat{c}_{t+1}^u)] = \frac{1}{R} E_t[(1 + \hat{p_{t+1}} - \hat{p}_t - \hat{r}_t)]$$
$$E_t[\hat{c}_t^u - \hat{c}_{t+1}^u] = E_t[\hat{p_{t+1}} - \hat{p}_t - \hat{r}_t]$$

   Defining $\hat{\pi}_t = \hat{p_{t+1}} - \hat{p}_t$

$$E_t[\hat{c}_{t+1}^u] - \hat{c}_t^u = \hat{r}_t - E_t[\hat{\pi}_t] \tag{58}$$

2. **Labor supply** From (14), (15), (16), (17)

$$\omega_f(N_{f,t}^u)^\varphi = \frac{1}{{C^u}_t}\frac{W_{f,t}}{P_t}$$
$$\omega(N_f^u)^\varphi(1 + \varphi \hat{n_{f,t}}^u) = \frac{1}{C^u}\frac{W_f}{P}(1 + \hat{w_{f,t}} - \hat{c}_t^u) \tag{59}$$
$$\varphi \hat{n_{f,t}}^u = \hat{w_{f,t}} - \hat{c}_t^u$$

$$\begin{aligned}
\omega_i (N^u_{i,t})^{\varphi} &= \frac{1}{{C^u}_t}\frac{W_{i,t}}{P_t} \\
\omega (N^u_i)^{\varphi}(1+\varphi \hat{n_{i,t}}^u) &= \frac{1}{C^u}\frac{W_f}{P}(1+\hat{w_{i,t}} - \hat{c}^u_t) \\
\varphi \hat{n_{i,t}}^u &= \hat{w_{i,t}} - \hat{c}^u_t
\end{aligned} \tag{60}$$

$$\begin{aligned}
\omega_f (N^c_{f,t})^{\varphi} &= \frac{1}{{C^c}_t}\frac{W_{f,t}}{P_t} \\
\omega (N^c_f)^{\varphi}(1+\varphi \hat{n_{f,t}}^c) &= \frac{1}{C^c}\frac{W_f}{P}(1+\hat{w_{f,t}} - \hat{c}^c_t) \\
\varphi \hat{n_{f,t}}^c &= \hat{w_{f,t}} - \hat{c}^c_t
\end{aligned} \tag{61}$$

$$\begin{aligned}
\omega (N^c_{i,t})^{\varphi} &= \frac{1}{{C^c}_t}\frac{W_{i,t}}{P_t} \\
\omega (N^c_i)^{\varphi}(1+\varphi \hat{n_{i,t}}^c) &= \frac{1}{C^c}\frac{W_i}{P}(1+\hat{w_{i,t}} - \hat{c}^c_t) \\
\varphi \hat{n_{i,t}}^c &= \hat{w_{i,t}} - \hat{c}^u_t
\end{aligned} \tag{62}$$

3. **Budget Constrain, c**

From (18)

$$\begin{aligned}
\hat{c}^c_t &= \left[\frac{W_i N^c_i}{C^c}\right](\hat{w}_{i,t} + \hat{n}^c_{i,t}) + \left[\frac{W_f N^c_f}{C^c}\right](\hat{w}_{f,t} + \hat{n}^c_{f,t}) \\
\hat{c}^c_t &= \left[\frac{W_i N^c_i}{Y}\right](\hat{w}_{i,t} + \hat{n}^c_{i,t}) + \left[\frac{W_f N^c_f}{Y}\right](\hat{w}_{f,t} + \hat{n}^c_{f,t}) \\
\hat{c}^c_t &= \left[\frac{A_i}{A_i}\frac{W_i N^c_i}{Y}\right](\hat{w}_{i,t} + \hat{n}^c_{i,t}) + \left[\frac{A_f}{A_f}\frac{W_f N^c_f}{Y}\right](\hat{w}_{f,t} + \hat{n}^c_{f,t}) \\
\hat{c}^c_t &= \left[\frac{MC_i Y_i}{Y}\right](\hat{w}_{i,t} + \hat{n}^c_{i,t}) + \left[\frac{MC_f (Y_f + f)}{Y}\right](\hat{w}_{f,t} + \hat{n}^c_{f,t}) \\
\hat{c}^c_t &= \left[\frac{Y_i}{Y}\right](\hat{w}_{i,t} + \hat{n}^c_{i,t}) + \left[\frac{\frac{\epsilon-1}{\epsilon} Y_f (1+f_y)}{Y}\right](\hat{w}_{f,t} + \hat{n}^c_{f,t}) \\
\hat{c}^c_t &= \left[\frac{Y_i}{Y}\right](\hat{w}_{i,t} + \hat{n}^c_{i,t}) + \left[\frac{\frac{\epsilon-1}{\epsilon} Y_f (\frac{\epsilon}{\epsilon-1})}{Y}\right](\hat{w}_{f,t} + \hat{n}^c_{f,t}) \\
\hat{c}^c_t &= \left[\frac{Y_i}{Y}\right](\hat{w}_{i,t} + \hat{n}^c_{i,t}) + \left[\frac{Y_f}{Y}\right](\hat{w}_{f,t} + \hat{n}^c_{f,t})
\end{aligned}$$

$$\hat{c}^c_t = \rho(\hat{w}_{i,t} + \hat{n}^c_{i,t}) + (1-\rho)(\hat{w}_{f,t} + \hat{n}^c_{f,t}) \tag{63}$$

4. **Budget Constrain, u**

From (11):

$$\hat{c}^u_t = \left[\frac{W_i N^u_i}{C^u}\right](\hat{w}_{i,t} + \hat{n}^u_{i,t}) + \left[\frac{W_f N^u_f}{C^u}\right](\hat{w}_{f,t} + \hat{n}^u_{f,t}) + \frac{1}{(1-\lambda)}\frac{D_t}{C^u}$$

Define $\hat{d}_t \equiv \frac{D_t}{Y}$ since profits in the steady state are 0.

$$\hat{c}^u_t = \left[\frac{W_i N^u_i}{C^u}\right](\hat{w}_{i,t} + \hat{n}^u_{i,t}) + \left[\frac{W_f N^u_f}{C^u}\right](\hat{w}_{f,t} + \hat{n}^u_{f,t}) + \frac{1}{(1-\lambda)}\hat{d}_t \tag{64}$$

5. **Marginal Cost:**

From (23):

$$\hat{mc}_{i,t} = \hat{w}_{i,t} - \hat{a}_{i,t} \tag{65}$$

From(26):

$$\hat{mc}_{f,t} = \hat{w}_{f,t} + \hat{a}_{f,t} \tag{66}$$

Formal marginal cost is in terms of formal prices and informal marginal cost is nominal.

6. **Inflation in the formal market**

Divide (29) by $P^{1-\epsilon}_{f,t}$ and log-linearize it:

$$1 = (1-\phi)\left(\frac{P^*_{f,t}}{P_{f,t}}\right)^{1-\epsilon} + \phi\left(\frac{P_{f,t-1}}{P_{f,t}}\right)^{1-\epsilon}$$

$$1 = (1-\phi)[1 + (1-\epsilon)(\hat{p}^*_{f,t} - \hat{p}_{f,t})] + \phi[1 + (1-\epsilon)(\hat{p}_{f,t-1} - \hat{p}_{f,t})]$$

$$0 = (1-\phi)[(1-\epsilon)(\hat{p}^*_{f,t} - \hat{p}_{f,t})] + \phi[(1-\epsilon)(\hat{p}_{f,t-1} - \hat{p}_{f,t})]$$

$$0 = (1-\phi)(\hat{p}^*_{f,t} - \hat{p}_{f,t}) + \phi(\hat{p}_{f,t-1} - \hat{p}_{f,t})$$

$$(1-\phi)\hat{p}^*_{f,t} = \hat{p}_{f,t} - \phi\hat{p}_{t-1} \tag{67}$$

Now, since every intermidiate firm set the same price $P^*_{f,t}(i) = P^*_{f,t}$:

$$P^*_{f,t} = \left(\frac{\epsilon}{\epsilon-1}\right)\frac{E_t\sum_{j=0}^{\infty}\phi^j\Lambda_{t,t+j}P_{f,t+j}{}^{\epsilon}\ Y_{f,t+j}}{E_t\left\{\sum_{j=0}^{\infty}\phi^j\Lambda_{t,t+j}P^{\epsilon}_{f,t+j}\ Y_{f,t+j}\right\}}MC_{f,t}$$

Divide by $P_{f,t}$.

$$\frac{P^*_{f,t}}{P_{f,t}}\ E_t\left\{\sum_{j=0}^{\infty}\phi^j\Lambda_{t,t+j}P^{\epsilon}_{f,t+j}\ Y_{f,t+j}\right\} = \frac{1}{P_{f,t}}E_t\left\{\sum_{j=0}^{\infty}\phi^j\Lambda_{t,t+j}\left(\frac{\epsilon}{\epsilon-1}\right)MC_{f,t}P_{f,t+j}{}^{\epsilon}\ Y_{f,t+j}\right\}$$

Log-linearized it. Left hand:

$$\sum_{j=0}^{\infty}\phi^j\Lambda P^{\epsilon}_f\ Y_f[1+\hat{p}^*_{f,t}-\hat{p}_{f,t}+E_t\hat{\Lambda}_{t+j}+E_t\hat{y}_{f,t+j}+\epsilon\ E_t\hat{p}_{f,t+j}] \tag{68}$$

Right hand[5]:

$$\begin{aligned}\frac{\epsilon}{\epsilon-1}\frac{MC_f}{P_{f,t}}\sum_{j=0}^{\infty}\phi^j\Lambda P^{\epsilon}_f\ Y_f[1-\hat{p}_{f,t}+E_t\hat{\Lambda}_{t+j}+E_t\hat{y}_{f,t+j}+E_t\epsilon\hat{p}_{f,t+j}+E_t[\hat{mc}_{f,t+j}+\hat{p}_{f,t+j}]]\\ \sum_{j=0}^{\infty}\phi^j\Lambda P^{\epsilon}_{f,t}\ Y_f[1-\hat{p}_{f,t+j}+E_t\hat{\Lambda}_{t+j}+E_t y_{f,t+j}+\epsilon\ E_t\hat{p}_{f,t+j}+E_t[\hat{mc}_{f,t+j}+\hat{p}_{f,t+j}]]\end{aligned} \tag{69}$$

Noticing that $\Lambda = \beta^i$ and $\sum_{j=0}^{\infty}\phi^j\beta^j = \frac{1}{1-\phi\beta}$

$$\hat{p}^*_{f,t} = (1-\phi\beta)\sum_{j=0}^{\infty}\phi^j\beta^j E_t[\hat{mc}_{f,t+j}+\hat{p}_{f,t+j}]$$

$$(1-\phi)\hat{p}^*_t = (1-\phi)(1-\phi\beta)\sum_{j=0}^{\infty}\phi^j\beta^j E_t[\hat{mc}_{f,t+j}+\hat{p}_{f,t+j}]$$

[5]Using marginal cost in terms of formal price instead of nominal.

Spliting $(1-\phi)(1-\phi\beta)\sum_{j=0}^{\infty}\phi^j\beta^j E_t[mc_{f,t+j}+p_{f,t+j}]$

$$\hat{p}_{f,t}-\hat{p}_{f,t-1}=\frac{(1-\phi\beta)}{\phi}\hat{mc}_{f,t}+\beta E_t[\hat{p}_{f,t+1}-\hat{p}_{f,t}]$$

Defining $\pi_{f,t}\equiv\hat{p}_{f,t}-\hat{p}_{f,t-1}$ and $\psi\equiv\frac{(1-\phi\beta)}{\phi}$:

$$\pi_{f,t}=\beta E_t[\pi_{f,t+1}]+\psi mc_{f,t}$$

7. **General Inflation:**

From (8), dividing by $P_{f,t}$ and defining $T_t\equiv\frac{P_{i,t}}{P_{f,t}}$

$$\begin{aligned}\left[\frac{P_t}{P_{f,t}}\right]^{1-\theta}&=\rho(T_t)^{1-\theta}+(1-\rho)\\ \left[\frac{P}{P_f}\right]^{1-\theta}(1+(1-\theta)(\hat{p}_t-\hat{p}_{f,t}))&=\rho(T)^{1-\theta}(1+(1-\theta)\hat{T}_t)+(1-\rho)\\ (\hat{p}_t-\hat{p}_{f,t})&=\rho(T)^{1-\theta}\hat{T}\\ \hat{p}_t&=\hat{p}_{f,t}+\rho(T)^{1-\theta}\hat{T}_t\end{aligned}\tag{70}$$

Subtracting one period lagged.

$$\begin{aligned}(\hat{p}_t-\hat{p}_{t-1})&=(\hat{p}_{f,t}-\hat{p}_{f,t-1})+\rho(T)^{1-\theta}(\hat{T}_t-\hat{T}_{t-1})\\ \hat{\pi}_t&=\hat{\pi}_{f,t}+\rho(T)^{1-\theta}(\hat{T}_t-\hat{T}_{t-1})\end{aligned}$$

But, $\hat{T}_t=\hat{p}_{i,t}-\hat{p}_{f,t}$, so $\hat{T}_t-\hat{T}_{t-1}=\hat{\pi}_{i,t}-\hat{\pi}_{f,t}$. It follows that:

$$\hat{\pi}_t=(1-\rho T^{1-\theta})\hat{\pi}_{f,t}+\rho T^{1-\theta}\hat{\pi}_{i,t}$$

$$\hat{\pi}_t=(1-\rho)\hat{\pi}_{f,t}+\rho\hat{\pi}_{i,t}\tag{71}$$

Where $T=1$

8. **Monetary Policy**

From (31):

$$\hat{r}_t = \Phi_{\hat{\pi}} E_t \hat{\pi}_{f,t} + \Phi_{\hat{y}} E_t \hat{y}_t \tag{72}$$

9. **Labor Market clearing conditions:**

From (32) and (33)

- **Formal:**

$$\hat{n}_{f,t} = (1-\lambda)\hat{n}^u_{f,t} + \lambda \hat{n}^c_{f,t} \tag{73}$$

- **Informal:**

$$\hat{n}_{i,t} = (1-\lambda)\hat{n}^u_{i,t} + \lambda \hat{n}^c_{i,t} \tag{74}$$

10. **Goods Market clearing conditions:**

From (36) and (36):

$$\hat{y}_{i,t} = \hat{c}^A_{i,t} \tag{75}$$

$$\hat{y}_{f,t} = \hat{c}^A_{f,t} \tag{76}$$

11. **Aggregate version of the Formal output**

From (42):

$$Y_{f,t} = \frac{A_{f,t} N_{f,t} - f}{\left(\frac{1}{1-\rho}\right) \triangle_{f,}}$$

$$Y_f(1 + \hat{y}_{f,t}) = A_f N_f (1 + \hat{a}_{f,t} + \hat{n}_{f,t}) - f$$

$$
\begin{aligned}
Y_f(\hat{y}_{f,t}) &= A_f N_f(\hat{a}_{f,t} + \hat{n}_{f,t}) \\
Y_f(\hat{y}_{f,t}) &= (Y_f + f)(\hat{a}_{f,t} + \hat{n}_{f,t}) \\
\hat{y}_{f,t} &= (1+\mu)(\hat{a}_{f,t} + \hat{n}_{f,t})
\end{aligned} \tag{77}
$$

12. **Aggregate version of the Informal output**

    From (43):

$$
\hat{y}_{i,t} = \hat{a}_{i,t} + \hat{n}_{i_t} \tag{78}
$$

13. **Profits**

    From (30):

$$
\begin{aligned}
D_t &= \frac{P_{f,t}}{P_t} Y_{f,t} - \frac{MC_{f,t}}{P_t}\left(\frac{1}{(1-\rho)} \triangle_{f,t} Y_{f,t} + m\right) \\
-D_t + Y_f(1 + \hat{p}_{f,t} - \hat{p}_t + \hat{y}_{f,t}) &= \frac{\epsilon - 1}{\epsilon} Y_f(1 + \hat{MC}_{f,t} - \hat{p}_t + \hat{y}_{f,t}) + \frac{\epsilon - 1}{\epsilon}(\hat{MC}_{f,t} - \hat{p}_t) \\
-d_t + \hat{p}_{f,t} - \hat{p}_t + \hat{y}_{f,t} &= \frac{\epsilon - 1}{\epsilon}(\hat{MC}_{f,t} - \hat{p}_t + \hat{y}_{f,t}) + \frac{1}{\epsilon}(\hat{MC}_{f,t} - \hat{p}_t) \\
\hat{d}_t &= -\hat{mc}_{f,t} + \frac{\mu}{(1-\mu)} \hat{y}_{f,t}
\end{aligned} \tag{79}
$$

14. **Aggregation of output:**

    From (41)

$$
\begin{aligned}
Y_t &= \left[\frac{P_{i,t}}{P_t}\right] Y_{i,t} + Y_{f,t} \left[\frac{P_{f,t}}{P_t}\right] \\
Y(1 + \hat{y}_t) &= \left[\frac{P_i}{P}\right] Y_i(1 + (\hat{p}_{i,t} - \hat{p}_t) + \hat{y}_{i,t}) + \left[\frac{P_f}{P}\right] Y_f(1 + (\hat{p}_{f,t} - \hat{p}_t) + \hat{y}_{f,t}) \\
\hat{y}_t &= \left[\frac{Y_i}{Y}\right]((\hat{p}_{i,t} - \hat{p}_t) + \hat{y}_{i,t}) + \left[\frac{Y_f}{Y}\right]((\hat{p}_{f,t} - \hat{p}_t) + \hat{y}_{f,t}) \\
\hat{y}_t &= \rho \hat{y}_{i,t} + (1-\rho)\hat{y}_{f,t}
\end{aligned} \tag{80}
$$

15. **Aggregation of Consumption:**

From (39)

$$
\begin{aligned}
C_t^A &= (1-\lambda)C_t^u + \lambda C_t^c \\
C^A(1+\hat{c}_t^A) &= (1-\lambda)C^U(1+\hat{c}_t^U) + \lambda C^C(1+\hat{c}_t^C) \\
\hat{c}_t^A &= (1-\lambda)\frac{C^U}{C^A}\ \hat{c}_t^U + \lambda\frac{C^C}{C^A}\ \hat{c}_t^C \\
\hat{c}_t^A &= (1-\lambda)\hat{c}_t^U + \lambda\hat{c}_t^c
\end{aligned} \tag{81}
$$

16. **Aggregation of Labor:**

From (9)

$$
\begin{aligned}
N_t^A &= (1-\lambda)N_t^u + \lambda N_t^c \\
N^A(1+\hat{n}_t^A) &= (1-\lambda)N^U(1+\hat{n}_t^U) + \lambda N^C(1+\hat{n}_t^C) \\
\hat{n}_t^A &= (1-\lambda)\ \hat{n}_t^U + \lambda\ \hat{n}_t^C
\end{aligned} \tag{82}
$$

17. **Aggregate Market Clearing:**

From (40)

$$
\hat{y}_t = \hat{c}_t^A \tag{83}
$$

C Log-linearized summary for Model II

Making $\lambda \to 0$ the model reduces to the classical new Keynesian framework with two sectors which is our benchmark model.

C.1 Definition of equilibrium:

A rational expectation equilibrium is a set of 23 endogenous variables:

- Sequence of prices: $\{W_{i,t}, W_{f,t}, MC_{i,t}, MC_{f,t}, P_{i,t}, P^*_{f,t}, P_{f,t}, P_t, \triangle_{f,t}\}$;
- Sequence of allocations: $\{C_t, C_{i,t}, C_{f,t}, N^s_{i,t}, N^s_{f,t}, N^d_{f,t}, N^d_{i,t}, N_t, B_t, \Omega_t, Y_{f,t}, Y_{i,t}, Y_t\}$;
- A monetary Policy: $\{R_t\}$

Satisfying the following 23 equilibrium conditions:

- FOC of the two representative agents +3
- Period budgets constrains: +1
- Intermediate firms FOC +2
- Price setting rules: +3
- CPI Price Index: +1
- Interest rate rule: +1
- Market-Clearing conditions: +6
- Law of motion for price dispersion: $\triangle_{f,t}$ +1
- Aggregate version of the production function: +2
- Aggregation of output: +1

- Aggregate Labor +1

- Resource Constrain +1

Table 5: Log-Linearized Model II.

Description	**Equation**
Euler Equation	$\hat{c}_t - E_t[\hat{c}_{t+1}] = E_t[\hat{\pi_{t+1}}] - \hat{r}_t$
Formal Labor Supply	$\hat{n_{f,t}}^s = \hat{w_{f,t}} - \hat{c}_t$
Informal Labor Suply	$\hat{n_{i,t}}^s = \hat{w_{i,t}} - \hat{c}_t$
Budget Constrain	$\hat{c}_t = \rho(\hat{w}_{i,t} + \hat{n}^s_{i,t}) + (1-\rho)(\hat{w}_{f,t} + \hat{n}^s_{f,t})$
Formal Production Function	$\hat{y}_{f,t} = (1+\mu)(\hat{a}_{f,t} + \hat{n}^s_{f_t})$
Informal Production Function	$\hat{y}_{i,t} = \hat{a}_{i,t} + \hat{n}^s_{i_t}$
Marginal Cost in terms of formal price	$\hat{mc}_{f,t} = \hat{w}_{f,t} - \hat{a}_{f,t}$
Informal Nominal Marginal Cost	$\hat{mc}_{i,t} = \hat{w}_{i,t}\hat{a}_{i,t}$
Formal Inflation	$\hat{\pi}_{f,t} = \beta E_t \hat{\pi}_{t+1} + \psi \hat{mc}_{i,t}$
General Inflation	$\hat{\pi}_t = (1-\rho)\hat{\pi}_{f,t} + \rho\hat{\pi}_{i,t}$
Monetary Policy	$\hat{r}_t = \Phi_{\hat{\pi}} E_t \hat{\pi}_{f,t} + \Phi_{\hat{y}} E_t \hat{y}_t$
Formal Labor Market Clearing Conditions	$\hat{n}^d_{f,t} = \hat{n}^s_{f,t}$
Informal Labor Market Clearing Conditions	$\hat{n}^d_{i,t} = \hat{n}^s_{i,t}$
Formal Good's Market Clearing Conditions	$\hat{y}_{f,t} = \hat{c}_{f,t}$
Informal Good's Market Clearing Conditions	$\hat{y}_{i,t} = \hat{c}_{i,t}$
Aggregate Labor	$\hat{n}_t = \frac{N_i}{N}\ \hat{n}^s_{i,t} + \frac{N_f}{N}\ \hat{n}^s_{f,t}$
Aggregate Output	$\hat{y}_t = \rho\hat{y}_{i,t} + (1-\rho)\hat{y}_{f,t}$
Resource Constrain	$\hat{y}_t = \hat{c}_t$

D Matlab Code for the baseline model:

/* * This file correspond to the model which includes informality and limited asset participation * with zero profits in the steady state. * u, corresponds to unconstrained representative agent. * c, corresponds to constrained representative agent. */

var wi wf mci mcf Pi pi pf P p ca cu cc cai caf nu nui nuf nc nci ncf nf ni n yf yi y r rr ai af v d;

varexo epsAi epsAf epsV;

parameters mu epsilon laambda rho phi omegga omeggo pssi kappa betta mupi muy rhoAf rhoAi rhoV N Ni Nf;

epsilon= 2.5; /* Elasticity of substitution between formal goods*/
mu=(1/(epsilon-1)); /* Steady state net markup as defined by Bilbiie (2008) */
laambda=0.7; /* Fraction of population that is constrained, ENIF (2018) */
rho= 0.227; /* Fraction of firms that are informal, INEGI (2018) */
phi=0.3125; /* Inverse labor supply elasticity, Leyva Urrutia (2018)*/
omegga=0.52259; /* Value of leisure relative to consumption, i; calibrated */
omeggo=1; /* Value of leisure relative to consumption, f */
betta= 0.9976; /* Discount factor, Fernandez and Meza (2014)*/
pssi=0.75; /* Degree of price stickiness, Gali (2015) */
kappa=(1-pssi)*(1-pssi*betta)*(1/pssi); /* parameter in NKPC */
mupi=1.5; /* inflation response coefficient, Gali (2015) */
muy=0.125; /* output response coefficient, Gali (2015) */
rhoAf=0.9; /* autocorrelation of productivity shocks */
rhoAi=0.9; /* autocorrelation of productivity shocks */
rhoV=0.5; /* autocorrelation of monetary policy shocks */
wfwi=1.75; /* Ratio of formal wage to informal wage */
Ai=(1/wfwi)*((epsilon-1)/epsilon); /* Informal productivity parameter in the production function */ C=((((epsilon-1)/(omeggo*epsilon))(1/phi))+((Ai/omegga)(1/phi)))(phi/(phi+1)); /* Consumption in the steady state */

Nf= ((1/omeggo)*((epsilon-1)/epsilon)*(1/C))(1/phi); /* Formal labor supply in the steady state for both, u c*/

Ni= ((1/omegga)*Ai*(1/C))(1/phi); /* Formal labor supply in the steady state for both, u c*/

N=Ni+Nf;

model(linear);

/* LOG-LINEARISED EQS */

/*E1 Euler eq.*/ cu(+1)=cu+(r-p(+1));

/*E2 Labor supply u, f */ phi*(nuf)=(wf+pf-P)-cu;

/*E3 Labor supply u, i */ phi*(nui)=(wi-P)-cu;

/*E4 Labor supply u, a */ nu=(Ni/N)*nui+(Nf/N)*nuf;

/*E5 Labor supply c, f */ phi*(ncf)=(wf+pf-P)-cc;

/*E6 Labor supply c, i */ phi*(nci)=(wi-P)-cc;

/*E7 Labor supply c */ nc=(Ni/N)*nci+(Nf/N)*ncf;

/*E8 Budget Constrain u*/ cu=(1-rho)*((wf+pf-P)+nuf)+rho*(wi-P+nui)+(1/(1-laambda))*d;

/*E9 Budget Constrain c*/ cc=(1-rho)*(wf+pf-P+ncf)+rho*(wi-P+nci);

/*E10 Marginal Cost f*/ mcf=wf-af;

/*E11 Marginal Cost i*/ mci=wi-ai;

/*E12 NKPC f*/ pf-pf(-1)=kappa*(mcf)+betta*(pf(+1)-pf);

/*E13 Inflation i*/ pi=mci-mci(-1);

/*E14 General inflation*/ p= (1-rho)*pi- (rho)*pi;

```
/*E15 Taylor rule*/ r=mupi*(pf(+1)-pf)+muy*yf+v;

/*E16 Labor MCC f */ nf=(1-laambda)*nuf+laambda*ncf;

/*E17 Labor MCC i */ ni=(1-laambda)*nui+(laambda)*nci;

/*E18 Good's MCC f */ yf=caf;

/*E19 Good's MCC i */ yi=cai;

/*E20 Formal output */ yf=(1+mu)*(af+nf);

/*E21 Informal output */ yi=ai+ni;

/*E22 Agregation of labor */ n=(1-laambda)*nu+laambda*nc;

/*E23 Aggregation of consumption */ ca= (1-laambda)*cu+(laambda)*cc;

/*E24 Agregation of output */ y=rho*yi+(1-rho)*yf;

/*E25 Resource constrain */ y=ca;

/*E26 Productivity shock f*/ af=rhoAf*af(-1)+epsAf;

/*E27 Productivity shock i*/ ai=rhoAi*ai(-1)+epsAi;

/*E28 Monetary policy shock*/ v=rhoV*v(-1)+epsV;

/*E29 Price i */ Pi=mci;

/*E30 Price */ P=rho*Pi+(1-rho)*pf;

/*E31 Profits */ d= -mcf + (mu/(1+mu))*yf;

/*E32 Real Interest rate*/ rr=r-p(+1);

end;
```

```
steady;

check;

shocks; var epsV; stderr 0.01; end;

stochsimul(irf=20, hpfilter=1600);
```