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INDETERMINACY AND MONETARY POLICY RULES: THE IMPLICATIONS OF LABOR MOBILITY FOR THE TAYLOR PRINCIPLE

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Indeterminacy and Monetary Policy Rules
The implications of labor mobility for the Taylor principle

Omar Souza Roldan

Abstract

This thesis analyzes the implications of labor mobility for the Taylor principle. The migration decision is introduced based on a setup where labor markets are frictionless. Households compare expected utility flows in the home country versus an alternative abroad. This thesis finds that free movement of labor can strengthen the public finance channel of monetary policy, increasing the likelihood of indeterminacy under a balanced budget fiscal rule. Moreover, with sufficiently high levels of government debt, the Taylor principle is inverted and a passive monetary policy is required for equilibrium determinacy.

Keywords: Equilibrium Determinacy, Taylor Principle, Labor Mobility

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Chapter 1

Introduction

How should central banks set the nominal interest rate when there is free movement of labor? This thesis attempts to answer this question by incorporating the migration decision into a New Keynesian dynamic stochastic general equilibrium (DSGE) model. Recent DSGE-based studies suggest that migration may have significant implications for the macroeconomy. Stalder (2010) argues that key macroeconomic variables like equilibrium unemployment and potential output growth may be affected by labor mobility. Micheli (2020) identifies labor mobility as an important mechanism to cushion adverse demand shocks. However, the implications of migration for the conduct of monetary policy has not yet been examined using DSGE models. The purpose of this thesis is to investigate whether the Taylor principle is still appropriate for inducing equilibrium determinacy with the free mobility of labor. Under the Taylor principle, the central bank conducts monetary policy by adjusting the nominal interest rate by proportionally more than the increase in inflation. Since the real interest rate rises in response to higher inflation, this creates a stabilizing effect on the economy preventing the emergence of welfare-reducing self-fulfilling expectations. This thesis shows that migration can have serious implications on the effectiveness of the Taylor principle in preventing indeterminacy and self-fulfilling expectations. For sufficiently high levels of government debt, we find that an `inverted Taylor principle' policy is recommended to generate determinacy in the presence of migration.

According to Grogger and Hanson (2008), individuals residing outside their country of birth accounted for 3.0% of the world's population as of 2005, compared to the current global estimate of 3.5% stated in the World Migration Report (2020) issued by the International Organization for Migration (IOM). In other words, even though remaining within one's country of birth is still the norm, the increase in international migrants over time is evident. The fact that migration affects equilibrium properties is particularly relevant in the European Union (EU) where national disparities in labor markets have widened since the financial crisis (Huart and Tchakpalla, 2015). In 2020, these disparities are expected to increase due to the COVID-19 pandemic. In Mexico there has been a large level of emigration during the last decade. Data from the IOM (2020) shows that Mexico had the second largest number of migrants living abroad with 11.8 million only after India. Moreover, Mexico was the largest emigration country in Latin America and the Caribbean in 2019.

There are several popular approaches to model the migration decision in DSGE models. One popular approach is to use a framework where the labor market is frictionless. For instance, Mandelman and Zlate (2012) build a two-country DSGE model which allows for endogenous labor migration. Similarly, Chortareas et al. (2008) develop a New Keynesian model with no transaction frictions, but with nominal and real asset holdings while assuming that immigrants and natives are imperfect substitutes. Another popular approach is based on explicit search and matching frictions. Models based on this approach take into account labor market frictions such as bargaining, wage rigidities, and non-market clearing prices, all of which play an important role in the labor market. Chassamboulli and Palivos (2014) analyze the impact of immigration flows in the United States using a search and matching model that allows for skill heterogeneity, where unemployment exists due to search frictions. Likewise, Lozej (2018) explores the effects of migration as an endogenous decision on the labor market in a search and matching framework. A final approach associated with Micheli (2020) models the migration decision based on a comparison of expected utility flow in the home economy to an alternative abroad and assumes that temporary shocks have permanent effects on population size.

The current thesis investigates the implications of migration for the Taylor principle by augmenting a New Keynesian sticky-price model to include labor mobility. Labor mobility is modeled following Micheli (2020) by assuming that the migration decision entirely depends on a comparison of expected utility flow. The model allows for distortionary taxation with microeconomic foundations in which rational agents, like households and firms, make optimizing decisions. The economy is cashless and closed to international trade, households are homogeneous, and for simplicity we assume that labor is the only factor of production (i.e., the model ignores capital and investment). Households supply labor to the firms that produce intermediate goods, which operate under monopolistic competition. The intermediate goods are used as inputs to produce a final good, which is consumed by households. Fiscal policy is given by a balanced-budget rule and monetary policy is characterized by a forward-looking interest-rate feedback rule that responds to inflation and output. In the model, the transmission mechanism of monetary policy operates through two channels. There exists the conventional aggregate demand channel, where changes in the nominal interest rate affect output via changes in the real interest rate from the expectational IS curve, which results in a change in inflation from the New Keynesian Phillips Curve (NKPC). There also exists a public finance channel of monetary policy, where changes in the nominal interest rate, by affecting the real interest rate, raises the future debt obligations of the government. Under a balanced budget fiscal policy, this increases the need for higher taxes, which increases the future real marginal cost of firms, resulting in higher future inflation from the NKPC.

Our main results and key findings are as follows. First, if monetary policy is characterized by an interest-rate feedback rule that reacts only to expected future inflation, we find that for relatively high levels of government debt and migration, determinacy requires the inverted Taylor principle. In this case the central bank should ensure that real interest rates fall in response to higher inflation. To get some intuition, consider the public finance channel of monetary policy. Suppose there is an increase in expected future inflation. Under the Taylor principle, the real interest rate rises which raises future government debt and future taxation, raising real marginal cost, thereby resulting in higher future inflation (a self-fulfilling prophecy). By increasing the need for future taxation, the public finance channel is strengthened under labor mobility, as living abroad becomes relatively more attractive. This increases per capita public debt that can result in further increases of taxes in order to satisfy the balanced budget rule making indeterminacy more likely. Second, we also find that the indeterminacy problem worsens with relatively higher levels of government debt to output ratio and with a relatively lower degree of price stickiness. Third, if monetary policy responds to both expected future inflation and output, we find that the inverted Taylor principle is still needed for determinacy. Under the inverted Taylor principle, the long-run NKPC becomes downward sloping. Thus, permanently higher inflation reduces output in the economy.

This thesis contributes to the literature that investigates the determinacy implications of interest rate rules. Bullard and Mitra (2002) and Woodford (2003) show the ability of the Taylor principle to guarantee equilibrium uniqueness for a variety of popular linear policy feedback rules. McKnight and Mihalov (2015) show that real balance effects in monetary economies can restrict the ability of the Taylor principle to prevent indeterminacy of the rational expectations equilibrium. There are also a few studies that introduce a public finance channel into the New Keynesian model by adding distortionary taxation and government. For instance, Linnemann (2006) assesses the fiscal conditions that give rise to determinacy under an active interest rate policy when the government only has access to distortionary income taxation, whereas McKnight (2017) investigates when the Taylor principle fails to generate determinacy under both income taxation and consumption taxation. However, migration is absent from these models. Our approach to modeling migration is similar to Micheli (2020) who introduces migration into a flexible-price RBC model. However, there is no role for monetary policy in his paper. Overall, to the best of our knowledge, this thesis is the first to investigate the determinacy implications of labor mobility in the setting of monetary policy.

The rest of the thesis is structured as follows. Chapter 2 provides the details of the model. Chapter 3 derives the determinacy conditions. Chapter 4 discusses the implications. Chapter 5 concludes.

Chapter 2

A New Keynesian Model with Labor Mobility

In this chapter we outline the model in detail, which uses a New Keynesian framework with distortionary taxation and augments it by introducing labor mobility. In the economy there is no money (i.e., it is cashless) and it is closed to international trade. Within the economy there exist identical households who supply labor, a continuum of monopolistically competitive firms that produce intermediate goods who set their prices à la Calvo (1983), a firm that produces a final good by using intermediate goods as inputs, which is consumed by the households, and a fiscal and monetary authority. The government taxes household income to finance its spending and is assumed to follow a balanced-budget rule. The monetary authority conducts monetary policy through a simple instrument rule where the nominal interest rate it sets depends on expected inflation and output.

The households decide in period $t-1$ whether to move abroad or not based on comparing expected utility flow in the home economy versus the one abroad. It is assumed that the home economy is small relative to the world economy. Therefore, the former does not affect the latter. Taking into account the population size at period t, denoted by N_t, population growth H_t is:

$$H_t = \frac{N_t}{N_{t-1}} \,. \tag{2.1}$$

The formulation of the migration decision makes the system non-stationary because the population size N_t exhibits a unit root. In this way, other variables such as aggregate consumption and aggregate output also become non-stationary. However, by using population growth H_t and expressing the model in per capita terms relative to the current period's population size N_t we can detrend the system and induce stationarity.

2.1 Households

We assume the economy in period t is populated by N_t infinitely-lived households. The representative household obtains utility from consumption and disutility from working, and chooses consumption C_t and hours of work L_t to maximize expected discounted lifetime utility:

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t) ,$$

$$\implies \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t N_t U(c_t, l_t) = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t N_t \left[\log c_t - \frac{l_t^{1+\varphi}}{1+\varphi} \right] , \tag{2.2}$$

where E_0 is the expectations operator, $\beta \in (0,1)$ is the discount factor, $\varphi \geq 0$ determines the curvature of the disutility of labor (inverse of the elasticity of labor supply), per capita consumption c_t is defined as $\frac{C_t}{N_t}$, and per capita hours worked l_t is defined as $\frac{L_t}{N_t}$. Finally, it is assumed that the period utility function is separable between consumption and leisure and twice differentiable.

Letting P_t denote the aggregate price level, during period t the representative household receives income from real wages w_t and real profits Θ_t. The government levies an income tax τ_t on the household's real income $w_t L_t + \Theta_t$ in order to obtain revenue. The household carries B_{t-1} holdings of one-period nominal government bonds into period t, which pay a nominal interest rate R_{t-1}. The household uses its after-tax resources to purchase one-period nominal bonds B_t and to consume the final good C_t. The period budget constraint in nominal terms takes the form:

$$B_t + P_t C_t \leq R_{t-1} B_{t-1} + P_t (1 - \tau_t)(w_t L_t + \Theta_t) , \tag{2.3}$$

which expressed in per capita terms is:

$$\rho_t + P_t c_t \leq R_{t-1} \frac{\rho_{t-1}}{H_t} + P_t (1 - \tau_t)(w_t l_t + \vartheta_t) , \tag{2.4}$$

where $\rho_t = \frac{B_t}{N_t}$ and $\vartheta_t = \frac{\Theta_t}{N_t}$ are nominal bonds per capita in period t and per capita real profits, respectively.

The current-value Lagrangian for the utility maximization problem is:

$$\mathcal{L} = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t N_t \left\{ \log c_t - \frac{l_t^{1+\varphi}}{1+\varphi} + \lambda_t \left[R_{t-1} \frac{\rho_{t-1}}{H_t} + P_t(1-\tau_t)(w_t l_t + \vartheta_t) - \rho_t - P_t c_t \right] \right\} ,$$

where λ_t denotes the Lagrange multiplier. The first-order conditions for c_t, l_t, and ρ_t are given by:

$$\begin{aligned} \frac{1}{c_t} &= \lambda_t P_t \,, \\ l_t^{\varphi} &= \lambda_t P_t (1-\tau_t) w_t \,, \\ \lambda_t &= \beta R_t \mathrm{E}_t \left\{ \lambda_{t+1} \right\} \,. \end{aligned}$$

Combining the first-order conditions above yields the consumption Euler equation and the optimal labor supply condition:

$$\frac{1}{P_t c_t} \frac{1}{R_t} = \beta \mathrm{E}_t \left\{ \frac{1}{P_{t+1} c_{t+1}} \right\} , \tag{2.5}$$

$$l_t^{\varphi} c_t = (1-\tau_t) w_t \,. \tag{2.6}$$

Also, optimality requires that the following transversality condition is satisfied:

$$\lim_{j \to \infty} \mathrm{E}_t \left\{ \frac{\rho_{t+j}}{\prod_{k=1}^{j} R_{t+k}} \right\} = 0 \,. \tag{2.7}$$

2.2 Firms

In the economy there is a continuum of intermediate-goods producing firms indexed by $i \in [0,1]$. Each firm produces a differentiated good $Y_t(i)$ and they all use identical technology. The final-good producing firm who is perfectly competitive aggregates intermediate goods to produce the final good Y_t using the following CES production function:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} , \tag{2.8}$$

where $\varepsilon > 1$ is the constant elasticity of substitution between intermediate goods. Denoting the price of intermediate good i by $P_t(i)$, the final-good producing firm chooses $Y_t(i)$ to maximize:

$$P_t Y_t - \int_0^1 P_t(i) Y_t(i) di \,,$$

which after using (2.8) can be rewritten as:

$$P_t \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 P_t(i) Y_t(i) di.$$

Solving the profit maximization problem yields:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t \,. \tag{2.9}$$

Equation (2.9) is the demand schedule for each intermediate good, where P_t is the aggregate price index is defined as:

$$P_t = \left[\int_0^1 P_t(i)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}} \,. \tag{2.10}$$

Intermediate-good producing firms produce output in period t hiring labor and using a linear production technology:

$$Y_t(i) = L_t(i) \,. \tag{2.11}$$

The cost minimization problem is:

$$\min_{L_t(i)} P_t w_t L_t(i) \quad \text{s.t. } Y_t(i) = L_t(i) \,.$$

Solving this problem yields the following first-order condition:

$$P_t w_t = MC_t(i) \,, \tag{2.12}$$

where $MC_t(i)$ is the nominal marginal cost of firm i. Since all of the intermediate-goods producing firms face the exact same nominal wage rate $P_t w_t$, equation (2.12) can be rewritten in real terms as:

$$\Longrightarrow \frac{MC_t}{P_t} \equiv mc_t = w_t \,, \tag{2.13}$$

where mc_t is the real marginal cost.

We assume that intermediate sector firms set their prices according to Calvo (1983), where the probability that a firm is able to change its prices $1-\psi$ is the same for every period, no matter when it was the last time it changed its price. Therefore, ψ is the probability that the firm will not be able to change its price. Consequently, ψ measures the degree of nominal stickiness (i.e., price stickiness). In this way, if a firm is selected to change its price at time t, it will choose the price $P_t^*(i)$ to maximize:

$$\max_{P_t^*(i)} \mathrm{E}_t \left\{ \sum_{j=0}^{\infty} \psi^j \Delta_{t,t+j} \left[P_t^*(i) Y_{t+j}(i) - P_{t+j} mc_{t+j} y_{t+j}(i) \right] \right\} ,$$

subject to the demand constraint (2.9), where:

$$\Delta_{t,t+j} \equiv \beta^j \left[\frac{c_t}{c_{t+j}} \right] \left(\frac{P_t}{P_{t+j}} \right) \tag{2.14}$$

is the stochastic discount factor and ψ^j is the probability that the price $P_t^*(i)$ still applies in period $t+j$. Using (2.9) to eliminate $Y_{t+j}(i)$ yields the following objective function:

$$\max_{P_t^*(i)} \mathrm{E}_t \left\{ \sum_{j=0}^{\infty} \psi^j \Delta_{t,t+j} \left[\left(\frac{P_t^*(i)}{P_{t+j}} \right)^{-\varepsilon} (P_t^*(i) - P_{t+j} mc_{t+j}) Y_{t+j} \right] \right\} .$$

Moreover, we can set $P_t^*(i) = P_t^*$ because all firms that get chosen to reset their price will behave in the same way. The first-order condition is:

$$P_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{\mathrm{E}_t \left\{ \sum_{j=0}^{\infty} \psi^j \Delta_{t,t+j} P_{t+j}^{\varepsilon+1} Y_{t+j} mc_{t+j} \right\}}{\mathrm{E}_t \left\{ \sum_{j=0}^{\infty} \psi^j \Delta_{t,t+j} P_{t+j}^{\varepsilon} Y_{t+j} \right\}} , \tag{2.15}$$

and using (2.14), we can express (2.15) as:

$$P_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{\mathrm{E}_t \left\{ \sum_{j=0}^{\infty} (\psi\beta)^j c_{t+j} P_{t+j}^{\varepsilon} Y_{t+j} mc_{t+j} \right\}}{\mathrm{E}_t \left\{ \sum_{j=0}^{\infty} (\psi\beta)^j c_{t+j} P_{t+j}^{\varepsilon-1} Y_{t+j} \right\}} .$$

The optimal price set is a mark up $\frac{\varepsilon}{\varepsilon-1} > 1$ over a weighted average of future real marginal costs. When prices are fully flexible (i.e., $\psi = 0$), the optimal price-setting condition (2.15) simplifies to:

$$P_t^* = \frac{\epsilon}{\epsilon - 1} P_t mc_t \, .$$

In other words, when prices are fully flexible the optimal price set is a mark up $\frac{\varepsilon}{\varepsilon-1}$ over current nominal marginal cost $P_t mc_t$. In this context, since the marginal cost is below the price, the output produced by the intermediate-goods producing firms will be sub-optimally low.

We can rewrite the aggregate price index P_t as:

$$P_t^{1-\varepsilon} = \psi P_{t-1}^{1-\varepsilon} + (1-\psi)(P_t^*)^{1-\varepsilon} \, . \tag{2.16}$$

Dividing both sides of (2.16) by $P_{t-1}^{1-\varepsilon}$ yields the following expression for inflation:

$$\pi_t^{1-\varepsilon} = \psi + (1-\psi)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon} , \tag{2.17}$$

where $\pi_t \equiv \frac{P_t}{P_{t-1}}$ is the inflation rate in period t.

2.3 Fiscal and Monetary Policy

The government purchases G quantity of the final good, which is financed by issuing new nominal debt B_t and from levying taxes on real income $\tau_t Y_t$, where $Y_t = w_t L_t + \Theta_t$. The government budget constraint in nominal terms is given by:

$$P_t G = P_t \tau_t Y_t + B_t - R_{t-1} B_{t-1} \, , \tag{2.18}$$

which in per capita terms can be expressed as:

$$P_t g = P_t \tau_t y_t + \rho_t - R_{t-1} \frac{\rho_{t-1}}{H_t} \, , \tag{2.19}$$

where g is per capita government consumption, which is defined as $g = \frac{G}{N_t}$. The government follows a balanced-budget rule, where the stock of real per capita government debt b_t is fixed at its constant steady state level b, such that:

$$b_t \equiv \frac{B_t}{N_t P_t} = \frac{\rho_t}{P_t} = b\,. \tag{2.20}$$

The central bank sets the nominal interest rate to changes in both expected inflation and per capita output:

$$R_t = R^{ss}\left(\frac{\mathrm{E}_t\left\{\pi_{t+1}\right\}}{\pi^{ss}}\right)^{\mu_\pi}\left(\frac{\mathrm{E}_t\left\{y_{t+1}\right\}}{y^{ss}}\right)^{\mu_y}, \tag{2.21}$$

where π^{ss} and y^{ss} denote steady-state inflation and per capita output, respectively. Similarly, R^{ss} is the steady-state nominal interest rate defined as $R^{ss} = \frac{\pi^{ss}}{\beta} > 1$, $\mu_\pi \geq 0$ is the inflation response coefficient, and $\mu_y \geq 0$ is the output response coefficient. This particular type of interest-rate rule is typically known as a Taylor rule.

2.4 Labor Mobility

Basing ourselves on random growth theory, we assume that temporary shocks have permanent effects on the size of the population. Therefore, population size follows a unit root process:

$$\log\left(\frac{N_t}{N_{t-1}}\right) = \mu_H \mathrm{E}_t\left(\frac{U(c_{t+1})}{U(c*)}\right) = \mu_H \mathrm{E}_t \log\left(\frac{c_{t+1}}{c*}\right), \tag{2.22}$$

where c^* represents consumption abroad, which is assumed to be constant at its steady state. The parameter $\mu_H \geq 0$ is the migration coefficient which can be understood as the effect of different consumption opportunity combinations of migration: a 1 percent increase (decrease) in expected consumption in the next period relative to expected consumption in the alternative abroad results in a μ_H percent increase (decrease) in the next period's domestic population (Micheli, 2020).

If the migration coefficient was negative (i.e., $\mu_H < 0$), then a 1 per cent increase (decrease) in expected consumption in the next period relative to expected consumption in the alternative abroad would result in a μ_H percent decrease (increase) in the next period's domestic population. In other words, individuals would choose to migrate even though they would have higher expected discounted utility by staying in their country of birth, which is not consistent with optimizing behavior. Also, note that labor mobility can be excluded from the model by setting $\mu_H = 0$ so that the system collapses to the New Keynesian model of McKnight (2017).

2.5 Market Clearing

Market clearing in the labor and final goods market requires that $L_t = \int_0^1 L_t(i)di$ and

$$Y_t = C_t + G\,, \tag{2.23}$$

which in per capita terms is:

$$y_t = c_t + g\,. \tag{2.24}$$

The aggregate production function can be obtained using the production function and the demand schedule for the intermediate goods:

$$\left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t = L_t(i)\,,$$
$$\Longrightarrow\ Y_t \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} d_i = \int_0^1 L_t(i)di\,,$$

and since $\int_0^1 L_t(i)di = L_t$, then it follows that:

$$Y_t = \frac{L_t}{d_t}\,, \tag{2.25}$$

which in per capita terms is:

$$y_t = \frac{l_t}{d_t}\,, \tag{2.26}$$

where d_t is defined as $d_t = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} di$, and measures how dispersed are prices across intermediate goods. It can also be shown that this term can be expressed as:

$$d_t = \psi \pi_t^{\varepsilon} d_{t-1} + (1-\psi)\left(\frac{P_t^*}{P_t}\right)^{-\varepsilon}\,, \tag{2.27}$$

which is the law of motion for price dispersion.

2.6 Equilibrium

Given the initial allocation b_{t_0-1}, the constant g, and an initial condition d_{t_0-1}, a rational expectations equilibrium consists of a sequence of prices $\{ w_t, mc_t, P_t, P_t^*, d_t \}$, a sequence of allocations $\{ c_t, y_t, l_t, b_t, H_t \}$, a fiscal policy $\{ \tau_t \}$ and a monetary policy $\{ R_t \}$ satisfying:

i. the optimality conditions of the representative household (2.5) and (2.6), and the transversality condition (2.7);

ii. the optimality condition of intermediate-goods producing firms (2.13), the price-setting rules (2.15) and (2.16), the aggregate production function (2.26), and the law of motion for price dispersion (2.27);

iii. the government budget constraint (2.19), the balanced-budget rule (2.20), and the monetary policy rule (2.21);

iv. the clearing of the final goods market (2.24);

v. and the equation for population growth (2.22).

Chapter 3

Equilibrium Dynamics

In this chapter, we study the local dynamics of the model. To do this, we first take a linear approximation of the equilibrium conditions around the zero-inflation, deterministic steady state. Then, we explain the baseline parameter values we will use to illustrate the conditions for determinacy. Next, we derive the necessary and sufficient conditions for determinacy first where the interest-rate rule depends only on expected future inflation, and then when it depends on both expected future inflation and output.

3.1 The Steady State

In what follows, let all variables with a ss superscript denote steady-state values. In a zero-inflation steady state, it follows that:

$$\pi^{ss} \equiv \frac{P_t^{ss}}{P_{t-1}^{ss}} = 1 \, , \tag{3.1}$$

$$\Longrightarrow \; P_t^{ss} = P_{t-1}^{ss} \, ,$$

and the expression for inflation (2.17) in the steady state implies:

$$1 = \psi + (1-\psi)\left(\frac{(P_t^*)^{ss}}{P_{t-1}^{ss}}\right)^{1-\varepsilon} \, ,$$

$$\Longrightarrow \; (P_t^*)^{ss} = P_{t-1}^{ss} \, ,$$

$$\Longrightarrow \; (P_t^*)^{ss} = P_t^{ss} = P_{t-1}^{ss} \, . \tag{3.2}$$

Next, in the steady state, the stochastic discount factor (2.14) is equal to:

$$\Delta^{ss} = \beta^j \, , \tag{3.3}$$

and since prices are "flexible" in the steady state, i.e. $\psi = 0$, it follows from (2.13) and (2.15) that:

$$(P_t^*)^{ss} = \frac{\varepsilon}{\varepsilon - 1} MC^{ss} ,$$

$$\Longrightarrow \; w^{ss} = mc^{ss} = \frac{\varepsilon - 1}{\varepsilon} > 0 . \tag{3.4}$$

It follows from (2.27) and (3.2) that there is no price dispersion in the steady state ($d^{ss} = 1$), and (2.24) and (2.26) imply:

$$y^{ss} = l^{ss} = c^{ss} + g^{ss} . \tag{3.5}$$

From the consumption Euler equation (2.5), it follows that:

$$R^{ss} = \frac{1}{\beta} > 1 , \tag{3.6}$$

and from the optimal labor supply condition (2.6):

$$(l^{ss})^{\varphi} c^{ss} = (1 - \tau^{ss}) w^{ss} . \tag{3.7}$$

3.2 Log-Linearized New Keynesian model with Labor Mobility

In this section, we log-linearize the equations of the model around the zero-inflation (i.e., $\pi^{ss} = 1$) steady state outlined above. In what follows, a variable $\hat{X}_t$ denotes the log-deviation from its steady state X^{ss} (i.e., $\hat{X}_t = \ln X_t - \ln X^{ss}$). The linearized consumption Euler equation (2.5) is given by:

$$\widehat{c}_t = \mathrm{E}_t \widehat{c}_{t+1} - \widehat{R}_t + \mathrm{E}_t \widehat{\pi}_{t+1} , \tag{3.8}$$

where $\mathrm{E}_t \hat{\pi}_{t+1} \equiv \mathrm{E}_t \hat{P}_{t+1} - \hat{P}_t$. The linearized aggregate resource constraint (2.24) is:

$$\widehat{y}_t = s_c \widehat{c}_t , \tag{3.9}$$

where $s_c = \frac{c^{ss}}{y^{ss}}$ is the steady state consumption share in output. Using (3.9), we can rewrite (3.8) as:

$$\widehat{y}_t = \mathrm{E}_t \widehat{y}_{t+1} - s_c (\widehat{R}_t - \mathrm{E}_t \widehat{\pi}_{t+1}) . \tag{3.10}$$

Equation (3.10) is the intertemporal IS equation. In period t, per capita output $\hat{y}_t$ depends on expected future per capita output $\mathrm{E}_t\hat{y}_{t+1}$ and the real interest rate $\hat{r}_t \equiv \hat{R}_t - \mathrm{E}_t\hat{\pi}_{t+1}$. A higher $\hat{r}_t$ has contractionary effects on aggregate demand and reduces current period output $\hat{y}_t$.

The optimal labor supply condition (2.6) is linearized as:

$$\varphi\widehat{l}_t + \widehat{c}_t = \widehat{w}_t - \frac{\tau^{ss}}{1-\tau^{ss}}\widehat{\tau}_t \,. \tag{3.11}$$

The linearized cost-minimization condition of intermediate goods producers (2.13) is:

$$\widehat{w}_t = \widehat{mc}_t \,. \tag{3.12}$$

The linearized aggregate production function (2.26) is:

$$\widehat{y}_t = \widehat{l}_t \,. \tag{3.13}$$

Combining (3.11) - (3.13) yields the following expression for real marginal cost:

$$\varphi\widehat{y}_t + \frac{1}{s_c}\widehat{y}_t = \widehat{mc}_t - \frac{\tau^{ss}}{1-\tau^{ss}}\widehat{\tau}_t \,,$$

$$\implies \widehat{mc}_t = \left(\varphi + \frac{1}{s_c}\right)\widehat{y}_t + \left(\frac{\tau^{ss}}{1-\tau^{ss}}\right)\widehat{\tau}_t \,. \tag{3.14}$$

Log-linearizing the price setting equations (2.15) and (2.16) yields the NKPC:

$$\widehat{\pi}_t = \beta\mathrm{E}_t\widehat{\pi}_{t+1} + \lambda\widehat{mc}_t \,, \tag{3.15}$$

where the slope coefficient $\lambda \equiv \frac{(1-\psi)(1-\beta\psi)}{\psi} > 0$ is the real marginal cost elasticity of inflation and $0 < \psi < 1$ is the degree of price stickiness. Using equation (3.14), we can rewrite the NKPC as:

$$\widehat{\pi}_t = \beta\mathrm{E}_t\widehat{\pi}_{t+1} + \lambda\left(\varphi + \frac{1}{s_c}\right)\widehat{y}_t + \lambda\left(\frac{\tau^{ss}}{1-\tau^{ss}}\right)\widehat{\tau}_t \,. \tag{3.16}$$

Log-linearizing the equation for population growth (2.22) yields:

$$\widehat{H}_t = \mu_H\mathrm{E}_t\widehat{c}_{t+1} \,,$$

$$\implies \widehat{H}_t = \frac{\mu_H}{s_c}\mathrm{E}_t\widehat{y}_{t+1} \,. \tag{3.17}$$

To linearize the government budget constraint (2.19), we first express it in real terms:

$$g = \tau_t y_t + b_t - R_{t-1}\frac{b_{t-1}}{H_t\pi_t} \,,$$

and then linearizing:

$$\tau^{ss}\widehat{\tau}_t + \tau^{ss}\widehat{y}_t + s_b\widehat{b}_t = \frac{s_b}{\beta}(\widehat{R}_{t-1} + \widehat{b}_{t-1} - \widehat{H}_t - \widehat{\pi}_t)\,, \tag{3.18}$$

where $s_b = \frac{b^{ss}}{y^{ss}}$ is the ratio of steady state government debt rate to output. Linearizing the balanced-budget rule (2.20) and the monetary policy rule (2.21) yields:

$$\widehat{b}_t = 0\,, \tag{3.19}$$

$$\widehat{R_t} = \mu_\pi \mathrm{E}_t\widehat{\pi}_{t+1} + \mu_y \mathrm{E}_t\widehat{y}_{t+1}. \tag{3.20}$$

The complete log-linearized equilibrium system is given by four equations, which are shown in Table 3.1. The IS equation (3.10) determines the level of aggregate demand, while aggregate supply is determined by the NKPC (3.16). To close the model we need the government budget constraint (3.18) after imposing the fiscal policy rule (3.19) and the monetary policy rule given by (3.20).

Table 3.1: The complete log-linear model.

$\widehat{y}_t = \mathrm{E}_t\widehat{y}_{t+1} - s_c(\widehat{R}_t - \mathrm{E}_t\widehat{\pi}_{t+1})$	IS equation
$\widehat{\pi}_t = \beta\mathrm{E}_t\widehat{\pi}_{t+1} + \lambda\left(\varphi + \frac{1}{s_c}\right)\widehat{y}_t + \lambda\left(\frac{\tau^{ss}}{1-\tau^{ss}}\right)\widehat{\tau}_t$	NKPC
$\tau^{ss}\widehat{\tau}_t + \tau^{ss}\widehat{y}_t = \frac{s_b}{\beta}(\widehat{R}_{t-1} - \frac{\mu_H}{sc}\mathrm{E}_t\widehat{y}_{t+1} - \widehat{\pi}_t)$	Government budget constraint
$\widehat{R_t} = \mu_\pi\mathrm{E}_t\widehat{\pi}_{t+1} + \mu_y\mathrm{E}_t\widehat{y}_{t+1}$	Taylor rule

In this model, monetary policy is transmitted via two channels. The first channel is the conventional aggregate demand channel of monetary policy. Under the Taylor principle, the nominal interest rate rises by proportionally more than the increase in inflation (i.e., $\mu_\pi > 1$). Therefore, the real interest rate $\hat{r}_t$ will also increase, which reduces aggregate demand and output $\hat{y}_t$ from the IS equation. Lower output puts downward pressure on real marginal cost $\widehat{mc}_t$, which reduces inflation $\hat{\pi}_t$ from the NKPC. The second channel of monetary policy is the public finance channel of monetary policy. Under the Taylor principle, an increase in the real interest $\hat{r}_t$ rate raises future government debt repayments and future taxation $\mathrm{E}_t\hat{\tau}_{t+1}$ which increases future real marginal cost and hence future inflation from the next-period NKPC. By inspection of the IS equation, migration has no effect on the aggregate demand channel of monetary policy. However, by affecting the government budget constraint, migration affects the public finance channel of monetary policy.

3.3 Parameterization

To help illustrate the conditions for (in)determinacy, we employ the following values for the model parameters, summarized in Table 3.2. The time interval is assumed to be a quarter. We set $\beta = 0.99$ which is standard in the literature. The parameter φ is taken from Woodford (2003) and following Taylor (1999), we set the parameter $\psi = 0.75$. This implies that prices are fixed on average one year and a real marginal cost elasticity of inflation $\lambda \approx 0.086$. However, there is disagreement about the exact measure of price ridigidty in the economy (Benhabib and Eusepi, 2005). Thus, to investigate the robustness of the results we will also consider different values for ψ.

Table 3.2: Baseline parameter values.

Parameter		Value
β	Discount factor	0.99
φ	Inverse of the elasticity of labor supply	0.47
ψ	Degree of price stickiness	0.75
λ	Real marginal cost elasticity of inflation	0.086
τ^{ss}	Steady state income tax rate	0.2, 0.3, or 0.4
s_b	Steady state ratio of government debt to output	$0 \leq s_b \leq 3$
μ_π	Inflation response coefficient	$0 \leq \mu_\pi \leq 4$
μ_y	Output response coefficient	$0 \leq \mu_y \leq 3$
μ_H	Migration coefficient	$0 \leq \mu_H \leq 1$

Since tax rates and government debt vary depending on the country, we consider three different values for the income tax rate $\tau^{ss} = 0.2, 0.3, 0.4$ and we consider values for the steady-state government debt to output ratio within the range $s_b \in [0,3]$. The consumption share in output s_c can be calculated from the government budget constraint (2.19) given values for the tax rate τ^{ss} and the ratio of government debt to output s_b. In order to determine the values for the migration parameter μ_H, we must make an assumption on the way in which the population size reacts to different utility flows. We follow Micheli (2020) and consider different values of $\mu_H \in [0,1]$. For the policy parameters μ_π and μ_y, we consider the range $0 \leq \mu_\pi \leq 4$ and $0 \leq \mu_y \leq 3$ which cover most empirical estimates.

3.4 Determinacy Analysis

We first consider a strict inflation-targeting policy, where the interest-rate rule only responds to inflation with no policy response to output (i.e., $\mu_y = 0$). Then, we will consider a flexible inflation-targeting policy, where the interest-rate rule reacts to both inflation and output (i.e, $\mu_y > 0$).

3.4.1 Strict Forward-looking Interest-Rate Rules

Suppose the interest-rate rule only reacts to expected future inflation:

$$\widehat{R}_t = \mu_\pi \mathrm{E}_t \widehat{\pi}_{t+1} \,. \tag{3.21}$$

Combining (3.10), (3.16), (3.18), (3.19), and (3.21) allows us to reduce the linearized model to the following system of difference equations:

$$\widehat{y}_t = \mathrm{E}_t \widehat{y}_{t+1} - s_c[(\mu_\pi - 1)\mathrm{E}_t \widehat{\pi}_{t+1}] \,, \tag{3.22}$$

$$\begin{aligned} \widehat{\pi}_t \left(1 - \frac{s_b \lambda(\mu_\pi - 1)}{\beta(1-\tau^{ss})}\right) = \left[\beta + \lambda s_c \left(\frac{\tau^{ss}}{1-\tau^{ss}}\right)(\mu_\pi - 1)\right] \mathrm{E}_t \pi_{t+1} + \\ \lambda\left(\varphi + \frac{1}{s_c}\right)\widehat{y}_t - \left[\frac{s_b \lambda \mu_H}{\beta s_c(1-\tau^{ss})} + \lambda\left(\frac{\tau^{ss}}{1-\tau^{ss}}\right)\right] \mathrm{E}_t \widehat{y}_{t+1} \,. \end{aligned} \tag{3.23}$$

In matrix notation we have $\mathbf{V}_1 z_{t+1} = \mathbf{B}_1 z_t$, where:

$$\mathbf{V}_1 = \begin{bmatrix} 1 & -s_c(\mu_\pi - 1) \\ -\frac{s_b \lambda \mu_H}{\beta s_c(1-\tau^{ss})} - \lambda\left(\frac{\tau^{ss}}{1-\tau^{ss}}\right) & \beta + \lambda s_c\left(\frac{\tau^{ss}}{1-\tau^{ss}}\right)(\mu_\pi - 1) \end{bmatrix},$$
$$\mathbf{B}_1 = \begin{bmatrix} 1 & 0 \\ -\lambda\left(\varphi + \frac{1}{s_c}\right) & 1 - \frac{\lambda s_b(\mu_\pi - 1)}{\beta(1-\tau^{ss})} \end{bmatrix},$$

with $z_t = \begin{bmatrix} \hat{y}_t \\ \hat{\pi}_t \end{bmatrix}$. The set of linearized equations can be reduced to a two-dimensional system:

$$z_{t+1} = \mathbf{A}_1 z_t\,,$$

where:

$$\mathbf{A}_1 = \begin{bmatrix} \frac{\beta - \lambda s_c(\mu_\pi - 1)\left(\varphi + \frac{1}{s_c} - \frac{\tau^{ss}}{1-\tau^{ss}}\right)}{\beta - \frac{\lambda s_b \mu_H(\mu_\pi - 1)}{\beta(1-\tau^{ss})}} & \frac{s_c(\mu_\pi - 1)}{\beta - \frac{\lambda s_b \mu_H(\mu_\pi - 1)}{\beta(1-\tau^{ss})}}\left[1 - \frac{\lambda s_b(\mu_\pi - 1)}{\beta(1-\tau^{ss})}\right] \\ \frac{\frac{\lambda s_b \mu_H}{\beta s_c(1-\tau^{ss})} - \lambda\left(\varphi + \frac{1}{s_c} - \frac{\tau^{ss}}{1-\tau^{ss}}\right)}{\beta - \frac{\lambda s_b \mu_H(\mu_\pi - 1)}{\beta(1-\tau^{ss})}} & \frac{1}{\beta - \frac{\lambda s_b \mu_H(\mu_\pi - 1)}{\beta(1-\tau^{ss})}}\left[1 - \frac{\lambda s_b(\mu_\pi - 1)}{\beta(1-\tau^{ss})}\right] \end{bmatrix}.$$

RESULT 1. *If the central bank follows a strict forward-looking interest-rate rule, the necessary and sufficient conditions for local equilibrium determinacy are as follows:*

Case I:

$$\beta - \left[\beta - \frac{\lambda s_b \mu_H(\mu_\pi - 1)}{\beta(1-\tau^{ss})}\right]^2 > \frac{\lambda s_b(\mu_\pi - 1)}{(1-\tau^{ss})}\left(1 + \frac{\mu_H}{\beta}\left[1 - \frac{\lambda s_b(\mu_\pi - 1)}{\beta(1-\tau^{ss})}\right]\right), \tag{3.24}$$

$$\lambda s_c(\mu_\pi - 1)\left(\varphi + \frac{1}{s_c} - \frac{\tau^{ss}}{1-\tau^{ss}}\right) - \frac{\lambda s_b \mu_H(\mu_\pi - 1)}{\beta(1-\tau^{ss})} > 0, \tag{3.25}$$

$$(1+\beta)2 - \frac{\lambda s_b \mu_H(\mu_\pi - 1)}{\beta(1-\tau^{ss})} > \lambda(\mu_\pi - 1)\left(s_c\left[\varphi + \frac{1}{s_c} - \frac{\tau^{ss}}{1-\tau^{ss}}\right] + \frac{2 s_b}{\beta(1-\tau^{ss})}\right). \tag{3.26}$$

Case II:

$$\beta + \left[\beta - \frac{\lambda s_b \mu_H(\mu_\pi - 1)}{\beta(1-\tau^{ss})}\right]^2 < \frac{\lambda s_b(\mu_\pi - 1)}{(1-\tau^{ss})}\left(1 + \frac{\mu_H}{\beta}\left[1 - \frac{\lambda s_b(\mu_\pi - 1)}{\beta(1-\tau^{ss})}\right]\right), \tag{3.27}$$

$$\lambda s_c(\mu_\pi - 1)\left(\varphi + \frac{1}{s_c} - \frac{\tau^{ss}}{1-\tau^{ss}}\right) - \frac{\lambda s_b \mu_H(\mu_\pi - 1)}{\beta(1-\tau^{ss})} < 0, \tag{3.28}$$

$$(1+\beta)2 - \frac{\lambda s_b \mu_H(\mu_\pi - 1)}{\beta(1-\tau^{ss})} < \lambda(\mu_\pi - 1)\left(s_c\left[\varphi + \frac{1}{s_c} - \frac{\tau^{ss}}{1-\tau^{ss}}\right] + \frac{2 s_b}{\beta(1-\tau^{ss})}\right). \tag{3.29}$$

3.4.2 Flexible Forward-looking Interest-Rate Rules

Now, we consider the determinacy implications of a policy response to output when the interest rate rule reacts both to expected future inflation and output:

$$\widehat{R_t} = \mu_\pi \mathrm{E}_t \widehat{\pi}_{t+1} + \mu_y \mathrm{E}_t \widehat{y}_{t+1} \,. \tag{3.30}$$

The new system is now given by the following two difference equations:

$$\widehat{y}_t = \mathrm{E}_t \widehat{y}_{t+1}(1 - s_c \mu_y) - s_c \mathrm{E}_t \widehat{\pi}_{t+1}(\mu_\pi - 1) \,, \tag{3.31}$$

$$\begin{aligned} \widehat{\pi}_t \left(1 - \frac{s_b \lambda(\mu_\pi - 1)}{\beta(1 - \tau^{ss})}\right) &= \left[\beta + \lambda s_c \left(\frac{\tau^{ss}}{1 - \tau^{ss}}\right)(\mu_\pi - 1)\right] \mathrm{E}_t \pi_{t+1} + \\ \lambda \left[\left(\varphi + \frac{1}{s_c}\right) + \frac{s_b \mu_y}{\beta(1 - \tau^{ss})}\right] \widehat{y}_t &- \left[\frac{s_b \lambda \mu_H}{\beta s_c (1 - \tau^{ss})} + \lambda \left(\frac{\tau^{ss}}{1 - \tau^{ss}}\right)(1 - s_c \mu_y)\right] \mathrm{E}_t \widehat{y}_{t+1} \,. \end{aligned} \tag{3.32}$$

In matrix notation, we have $\mathbf{V}_2 z_{t+1} = \mathbf{B}_2 z_t$, where the matrices:

$$\mathbf{V}_2 = \begin{bmatrix} 1 - s_c \mu_y & -s_c(\mu_\pi - 1) \\ -\frac{s_b \lambda \mu_H}{\beta s_c (1-\tau^{ss})} - \lambda \left(\frac{\tau^{ss}}{1-\tau^{ss}}\right)(1 - s_c \mu_y) & \beta + \lambda s_c \left(\frac{\tau^{ss}}{1-\tau^{ss}}\right)(\mu_\pi - 1) \end{bmatrix} \,,$$

$$\mathbf{B}_2 = \begin{bmatrix} 1 & 0 \\ -\lambda \left[\left(\varphi + \frac{1}{s_c}\right) + \frac{s_b \mu_y}{\beta(1-\tau^{ss})}\right] & 1 - \frac{\lambda s_b(\mu_\pi - 1)}{\beta(1-\tau^{ss})} \end{bmatrix} \,.$$

Once again, the set of linearized equations can be reduced to a two-dimensional system:

$$z_{t+1} = \mathbf{A}_2 z_t \,,$$

where:

$$\mathbf{A}_2 = \begin{bmatrix} \frac{\beta - \lambda s_c(\mu_\pi - 1)\left(\varphi + \frac{1}{s_c} - \frac{\tau^{ss}}{1-\tau^{ss}} + \frac{s_b \mu_y}{\beta(1-\tau^{ss})}\right)}{\beta(1 - s_c \mu_y) - \frac{\lambda s_b \mu_H (\mu_\pi - 1)}{\beta(1-\tau^{ss})}} & \frac{s_c(\mu_\pi - 1)}{\beta(1 - s_c \mu_y) - \frac{\lambda s_b \mu_H (\mu_\pi - 1)}{\beta(1-\tau^{ss})}} \left[1 - \frac{\lambda s_b(\mu_\pi - 1)}{\beta(1-\tau^{ss})}\right] \\ \frac{\frac{\lambda s_b \mu_H}{\beta s_c(1-\tau^{ss})} - \lambda(1 - s_c \mu_y)\left(\varphi + \frac{1}{s_c} - \frac{\tau^{ss}}{1-\tau^{ss}} + \frac{s_b \mu_y}{\beta(1-\tau^{ss})}\right)}{\beta(1 - s_c \mu_y) - \frac{\lambda s_b \mu_H (\mu_\pi - 1)}{\beta(1-\tau^{ss})}} & \frac{1 - s_c \mu_y}{\beta(1 - s_c \mu_y) - \frac{\lambda s_b \mu_H (\mu_\pi - 1)}{\beta(1-\tau^{ss})}} \left[1 - \frac{\lambda s_b(\mu_\pi - 1)}{\beta(1-\tau^{ss})}\right] \end{bmatrix} .$$

RESULT 2. *If the central bank follows a flexible forward-looking interest-rate rule, the necessary and sufficient conditions for local equilibrium determinacy are as follows:*

Case III:

$$\beta(1-s_c\mu_y) - \left[\beta(1-s_c\mu_y) - \frac{\lambda s_b \mu_H(\mu_\pi - 1)}{\beta(1-\tau^{ss})}\right]^2 > \frac{\lambda s_b(\mu_\pi - 1)}{(1-\tau^{ss})}\left((1-s_c\mu_y) + \frac{\mu_H}{\beta}\left[1 - \frac{\lambda s_b(\mu_\pi - 1)}{\beta(1-\tau^{ss})}\right]\right), \quad (3.33)$$

$$\frac{s_c\mu_y(1-\beta)}{1-s_c\mu_y} + \frac{\lambda s_c(\mu_\pi - 1)}{1-s_c\mu_y}\left(\varphi + \frac{1}{s_c} - \frac{\tau^{ss}}{1-\tau^{ss}}\right) - \frac{\lambda s_b \mu_H(\mu_\pi - 1)}{\beta(1-\tau^{ss})(1-s_c\mu_y)} > 0, \quad (3.34)$$

$$(1+\beta)\left[2 + \frac{s_c\mu_y}{1-s_c\mu_y}\right] - \frac{\lambda s_b \mu_H(\mu_\pi - 1)}{\beta(1-\tau^{ss})(1-s_c\mu_y)} > \frac{\lambda(\mu_\pi - 1)}{1-s_c\mu_y}\left(s_c\left[\varphi + \frac{1}{s_c} - \frac{\tau^{ss}}{1-\tau^{ss}}\right] + \frac{2s_b}{\beta(1-\tau^{ss})}\right). \quad (3.35)$$

Case IV:

$$\beta(1-s_c\mu_y) - \left[\beta(1-s_c\mu_y) - \frac{\lambda s_b \mu_H(\mu_\pi - 1)}{\beta(1-\tau^{ss})}\right]^2 < \frac{\lambda s_b(\mu_\pi - 1)}{(1-\tau^{ss})}\left((1-s_c\mu_y) + \frac{\mu_H}{\beta}\left[1 - \frac{\lambda s_b(\mu_\pi - 1)}{\beta(1-\tau^{ss})}\right]\right), \quad (3.36)$$

$$\frac{s_c\mu_y(1-\beta)}{1-s_c\mu_y} + \frac{\lambda s_c(\mu_\pi - 1)}{1-s_c\mu_y}\left(\varphi + \frac{1}{s_c} - \frac{\tau^{ss}}{1-\tau^{ss}}\right) - \frac{\lambda s_b \mu_H(\mu_\pi - 1)}{\beta(1-\tau^{ss})(1-s_c\mu_y)} < 0, \quad (3.37)$$

$$(1+\beta)\left[2 + \frac{s_c\mu_y}{1-s_c\mu_y}\right] - \frac{\lambda s_b \mu_H(\mu_\pi - 1)}{\beta(1-\tau^{ss})(1-s_c\mu_y)} < \frac{\lambda(\mu_\pi - 1)}{1-s_c\mu_y}\left(s_c\left[\varphi + \frac{1}{s_c} - \frac{\tau^{ss}}{1-\tau^{ss}}\right] + \frac{2s_b}{\beta(1-\tau^{ss})}\right). \quad (3.38)$$

Chapter 4

Discussion

This chapter discusses the determinacy results derived in the previous chapter. If the interest-rate feedback rule reacts only to future expected inflation and there is no labor mobility (i.e., $\mu_H = 0$), indeterminacy can arise under the Taylor principle (i.e., $\mu_\pi > 1$) when there is government debt (i.e., $s_b > 0$). Case I and Case II of Result 1 would collapse to the New Keynesian model of McKnight (2017). To see this, in the model without labor mobility it follows from our parameter values that $\varphi + \frac{1}{s_c} - \frac{\tau^{ss}}{1-\tau^{ss}} > 0$, and thus Case II of Result 1 would never apply. Condition (3.25) would only require the Taylor principle to be satisfied (i.e., $\mu_\pi > 1$), and conditions (3.24) and (3.26) would yield the upper bounds:

$$\mu_\pi < 1 + \frac{\beta(1-\beta)(1-\tau^{ss})}{\lambda s_b}, \tag{4.1}$$

$$\mu_\pi < 1 + \frac{2(1+\beta)}{\lambda\left[s_c\left(\varphi + \frac{1}{s_c} - \frac{\tau^{ss}}{1-\tau^{ss}}\right) + \frac{2s_b}{\beta(1-\tau^{ss})}\right]}. \tag{4.2}$$

When there is no government debt (i.e., $s_b = 0$), the empirically-relevant upper bound on μ_π is given by (4.2). Employing the baseline parameter values, the upper-bound in the inflation response coefficient is so high that indeterminacy is never a serious problem under the Taylor principle, as shown in Table 4.1.

Table 4.1: Intervals of inflation response coefficients that induce determinacy with no labor mobility or government debt.

τ^{ss}	Interval
0.2	$1 < \mu_\pi < 40.35$
0.3	$1 < \mu_\pi < 45.97$
0.4	$1 < \mu_\pi < 53.47$

However, when there is government debt (i.e., $s_b > 0$), the relevant upper bound on μ_π is now given by (4.1). For instance, for only a very small value of $s_b = 0.1$ this upper bound is likely to bind, as shown in Table 4.2.

Table 4.2: Intervals of inflation response coefficients that induce determinacy with government debt but no labor mobility.

τ^{ss}	$s_b = 0.1$
0.2	$1 < \mu_\pi < 1.92$
0.3	$1 < \mu_\pi < 1.81$
0.4	$1 < \mu_\pi < 1.69$

Figure 4.1 depicts these results for $\tau^{ss} = 0.2$ on the left-hand-side and $\tau^{ss} = 0.4$ on the right-hand-side. The top half of Figure 4.1 graphs the (in)determinacy regions for combinations of μ_π and s_b with $\psi = 0.75$. The bottom half graphs the (in)determinacy regions for combinations of μ_π and ψ, with $s_b = 2.0$[1].

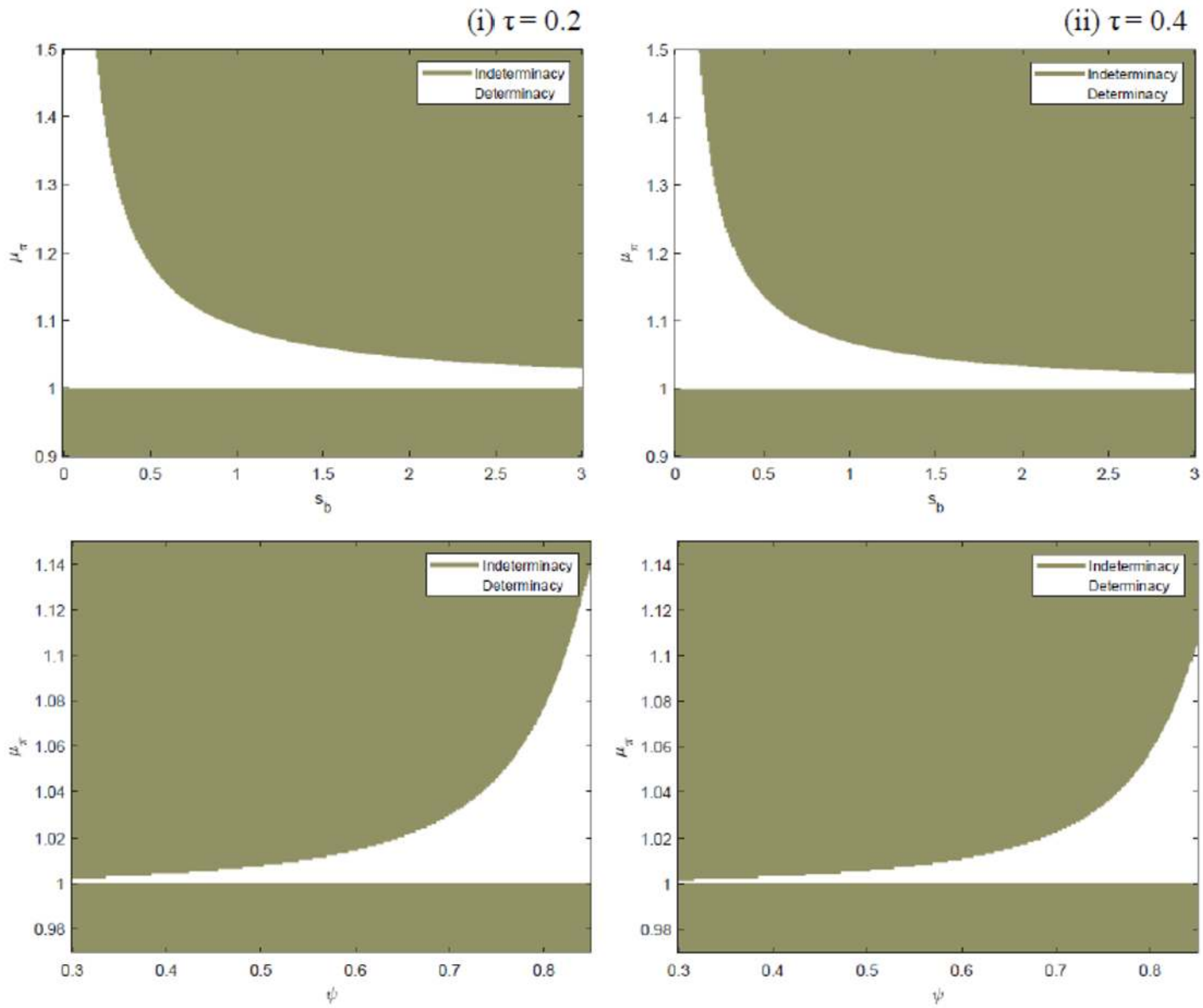

Figure 4.1: Regions of (in)determinacy under a strict future-inflation-targeting policy for $\tau = 0.2, 0.4$ in the absence of migration.

[1] This implies a yearly government debt-output ratio of 50%.

Figure 4.1 shows that for different values of τ, the determinacy region remains practically the same. It is clear that the problem of indeterminacy becomes more severe with low levels of price stickiness ψ and high values of government debt s_b.

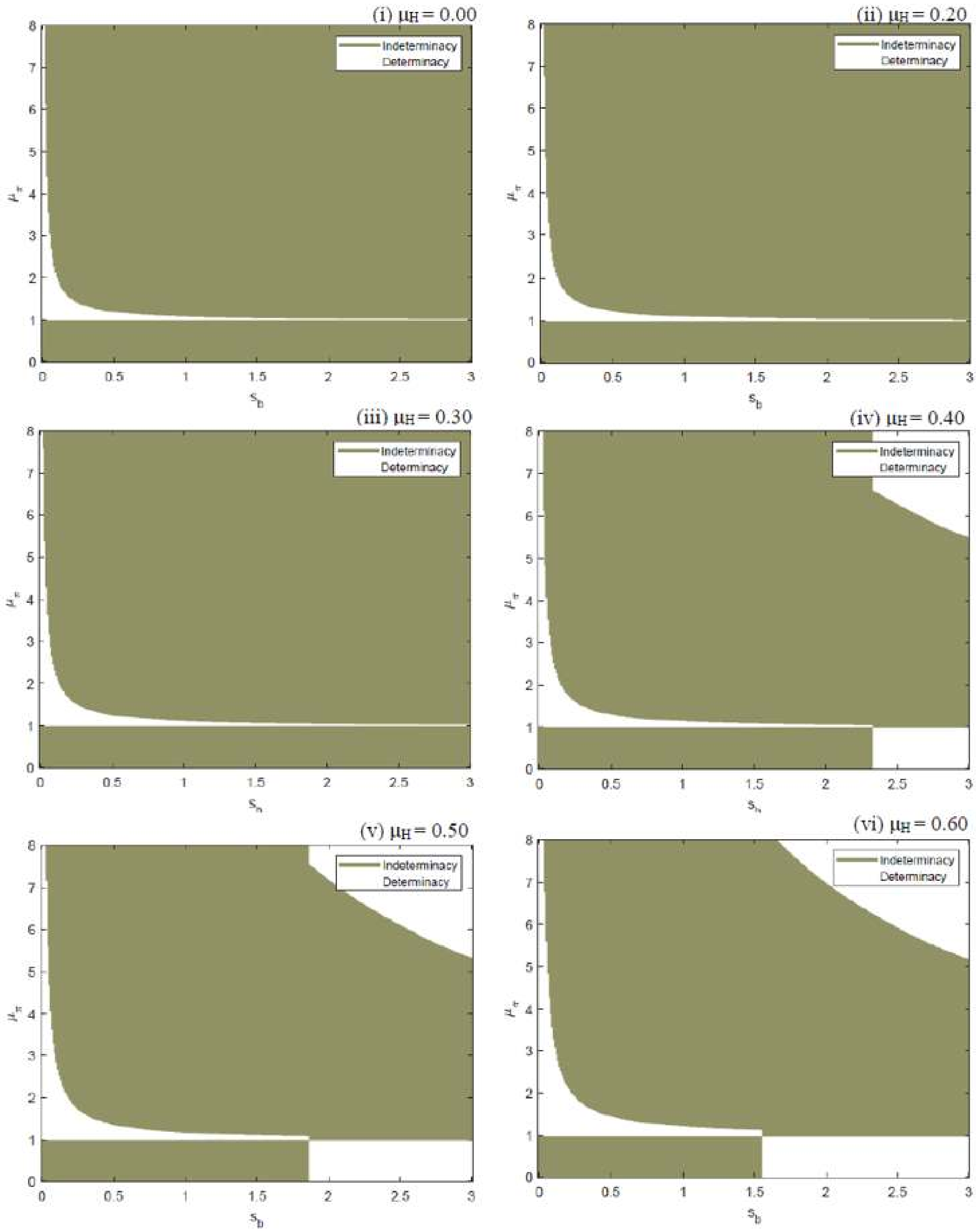

Figure 4.2: Regions of (in)determinacy under a strict future-inflation-targeting policy and labor mobility for $\mu_H = 0.0, 0.2, 0.3, 0.4, 0.5, 0.6$.

When we introduce labor mobility into the model (i.e., $\mu_H > 0$), Case I of Result 1 remains the necessary and sufficient conditions for local equilibrium determinacy. Conditions (3.25) and (3.26) simplify to:

$$(\mu_\pi - 1)\left[\lambda s_c\left(\varphi + \frac{1}{s_c} - \frac{\tau^{ss}}{1-\tau^{ss}}\right) - \frac{\lambda s_b \mu_H}{\beta(1-\tau^{ss})}\right] > 0\,, \tag{4.3}$$

$$\mu_\pi < 1 + \frac{2(1+\beta)}{\lambda\left[s_c\left(\varphi + \frac{1}{s_c} - \frac{\tau^{ss}}{1-\tau^{ss}}\right) + \frac{s_b(2+\mu_H)}{\beta(1-\tau^{ss})}\right]}\,. \tag{4.4}$$

By inspection of (4.3), if the parameter pair $s_b\mu_H$ is relatively large then the inverse of the Taylor principle (i.e. $\mu_\pi < 1$) would become necessary in order for condition (4.3) to hold. In other words, the inverted Taylor principle becomes a necessary condition for determinacy under forward-looking monetary policy when $s_b\mu_H$ becomes sufficiently large[2]. The higher the level of debt s_b and the migration coefficient μ_H, the more likely $\lambda s_c\left(\varphi + \frac{1}{s_c} - \frac{\tau^{ss}}{1-\tau^{ss}}\right) - \frac{\lambda s_b \mu_H}{\beta(1-\tau^{ss})} < 0$ and the inverted Taylor principle is needed for determinacy.

Figure 4.2 graphs the (in)determinacy regions for combinations of s_b and μ_π, using six alternative values of the migration coefficient $\mu_H = 0.0, 0.2, 0.3, 0.4, 0.5, 0.6$ and setting $\tau^{ss} = 0.2$. By inspection of Figure 4.2, for low levels of μ_H the determinacy region remains narrow under the Taylor principle. It is not until μ_H reaches relatively high values that we begin to see that determinacy requires the inverted Taylor principle for relatively high values of s_b. The Taylor principle gets reversed for $s_b > 2.3$ when $\mu_H = 0.40$, $s_b > 1.8$ for $\mu_H = 0.50$, and $s_b > 1.55$ for $\mu_H = 0.60$. Note that for relatively high levels of s_b and μ_H, we can still get determinacy under the Taylor principle, but only for high levels of μ_π, which are much larger than most empirical estimates of this parameter.

Figure 4.3 illustrates the determinacy implications for combinations of μ_H and μ_π, setting $\tau^{ss} = 2.0$ and using four alternative values for government debt $s_b = 0.0, 1.0, 2.0, 3.0$. By inspection, when $s_b = 1.0$ the determinacy region remains extremely narrow even for relatively high levels of μ_H. However, for $s_b = 2.0$ and $s_b = 3.0$ the Taylor principle gets reversed for $\mu_H > 0.47$ and $\mu_H > 0.31$, respectively.

[2] However, when there is no government debt (i.e., $s_b = 0$), condition (4.3) would only require the Taylor principle to hold.

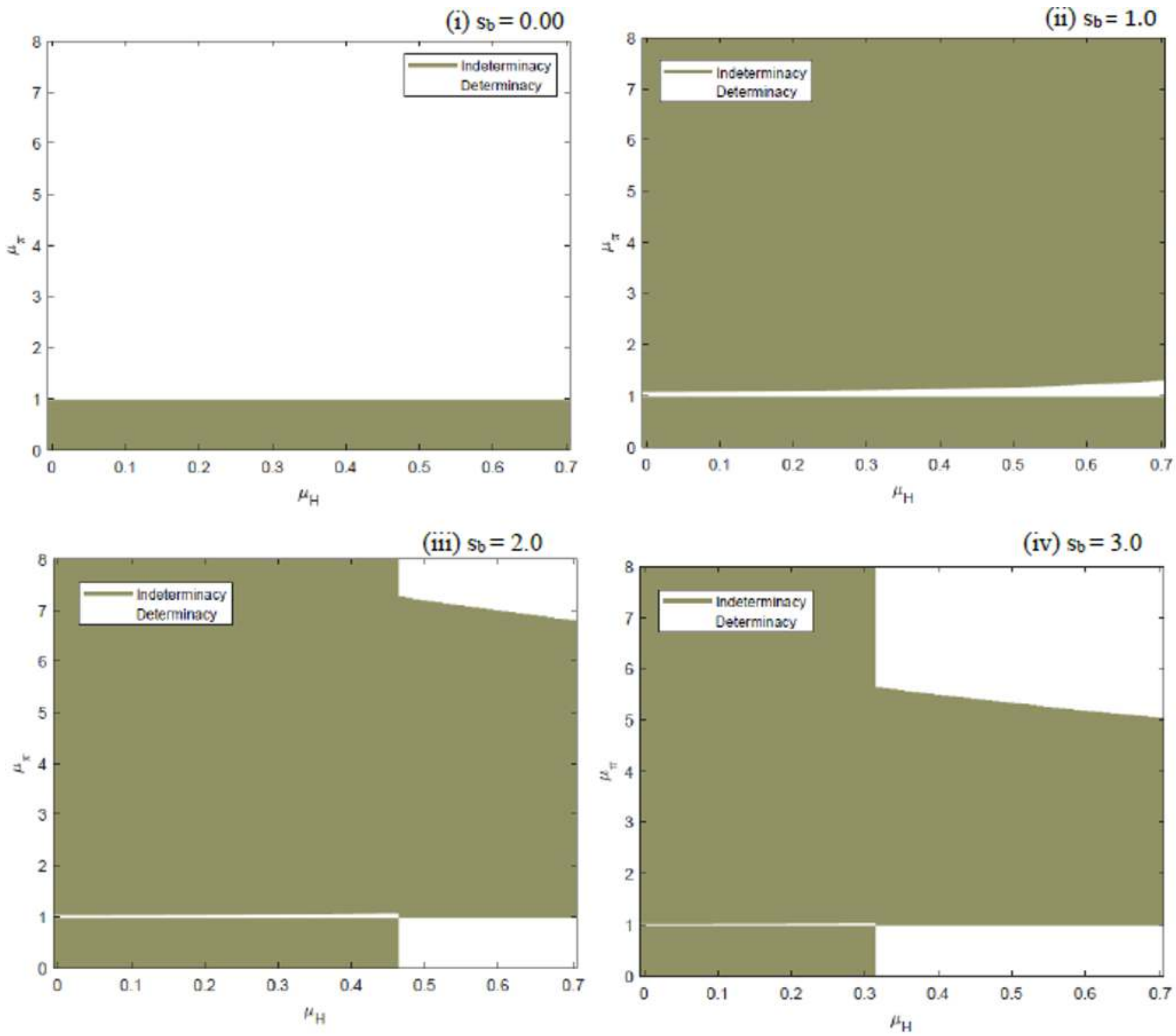

Figure 4.3: Regions of (in)determinacy under a strict future-inflation-targeting policy and labor mobility for $s_b = 0.0, 1.0, 2.0, 3.0$.

If the interest-rate feedback rule reacts to both future expected inflation and output, and there is no labor mobility (i.e., $\mu_H = 0$), condition (3.34) would collapse to the long-run version of the Taylor principle:

$$\mu_\pi + \alpha_y^1 \mu_y > 1 \,, \tag{4.5}$$

where α_y^1 is the slope of the long-run NKPC:

$$\alpha_y^1 \equiv \frac{(1-\beta)}{\lambda\left(\varphi + \frac{1}{s_c} - \frac{\tau^{ss}}{1-\tau^{ss}}\right)} \,, \tag{4.6}$$

where $\varphi + \frac{1}{s_c} - \frac{\tau^{ss}}{1-\tau^{ss}} > 0$ under our baseline parameter values. According to the long-run version of the Taylor principle, the higher μ_y is, the lower the value for μ_π that is needed for determinacy. For all parameter values employed in the numerical analysis the slope will be positive and the long-run Taylor principle would be a necessary condition. Thus, Case IV of Result 2 would never apply.

However, when there is labor mobility (i.e., $\mu_H > 0$), the slope α_y^2 is given by:

$$\alpha_y^2 \equiv \frac{(1-\beta)}{\lambda\left[\left(\varphi + \frac{1}{s_c} - \frac{\tau^{ss}}{1-\tau^{ss}}\right) - \frac{s_b \mu_H}{\beta(1-\tau^{ss})s_c}\right]}. \tag{4.7}$$

By inspection of (4.7), if there is a sufficiently high level of government debt s_b and of labor mobility μ_H, the slope of the long-run NKPC may become negative. Note that if the slope becomes negative, permanently higher inflation reduces output from the long-run NKPC. In other words, with a policy response to output, the slope in (4.7) will become negative and μ_π must be set sufficiently greater than 1 to induce determinacy.

Figure 4.4 illustrates the determinacy implications for combinations of μ_π and μ_y, setting $s_b = 2.0$, $\tau^{ss} = 0.3$, and $\psi = 0.75$ for different values of μ_H. By inspection, the bound pivots clockwise around $\mu_\pi = 1$ gets inverted for relatively high values of μ_H and gets steeper as μ_H increases, making the indeterminacy problem worse.

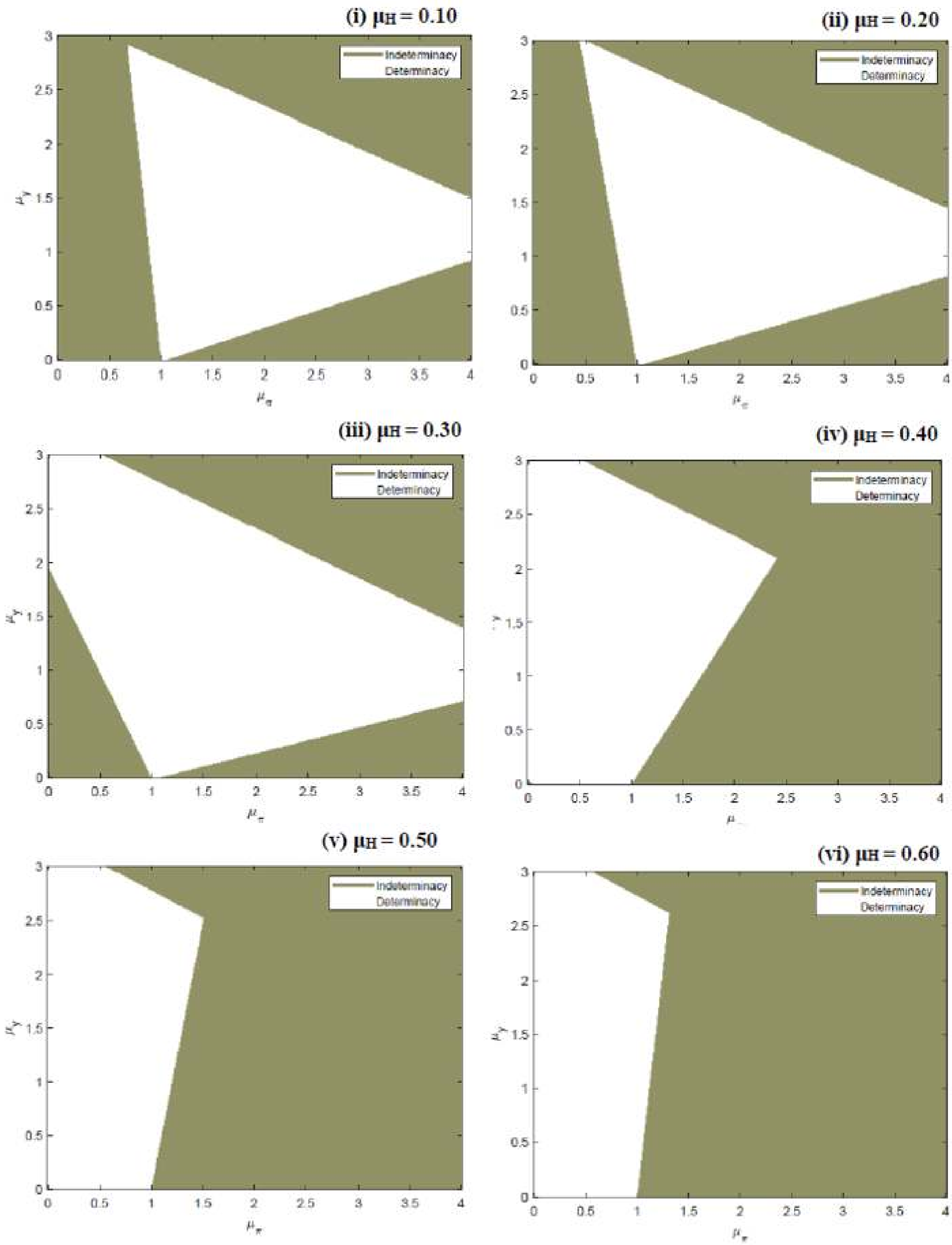

Figure 4.4: Regions of (in)determinacy under a flexible future-inflation-targeting policy and labor mobility for $\mu_H = 0.10, 0.20, 0.30, 0.40, 0.50, 0.60$ when setting $\psi = 0.75$.

Figure 4.5 illustrates the determinacy implications if the interest-rate rule also reacts to future output (i.e., $\mu_y > 0$). They depict the regions of (in)determinacy for combinations of μ_π and μ_y. We set $\tau^{ss} = 0.3$ and $s_b = 2.0$, together with four different values for $\psi = 0.50, 0.75, 0.85, 0.95$. Figure 4.5 implies that indeterminacy is less likely the higher ψ is.

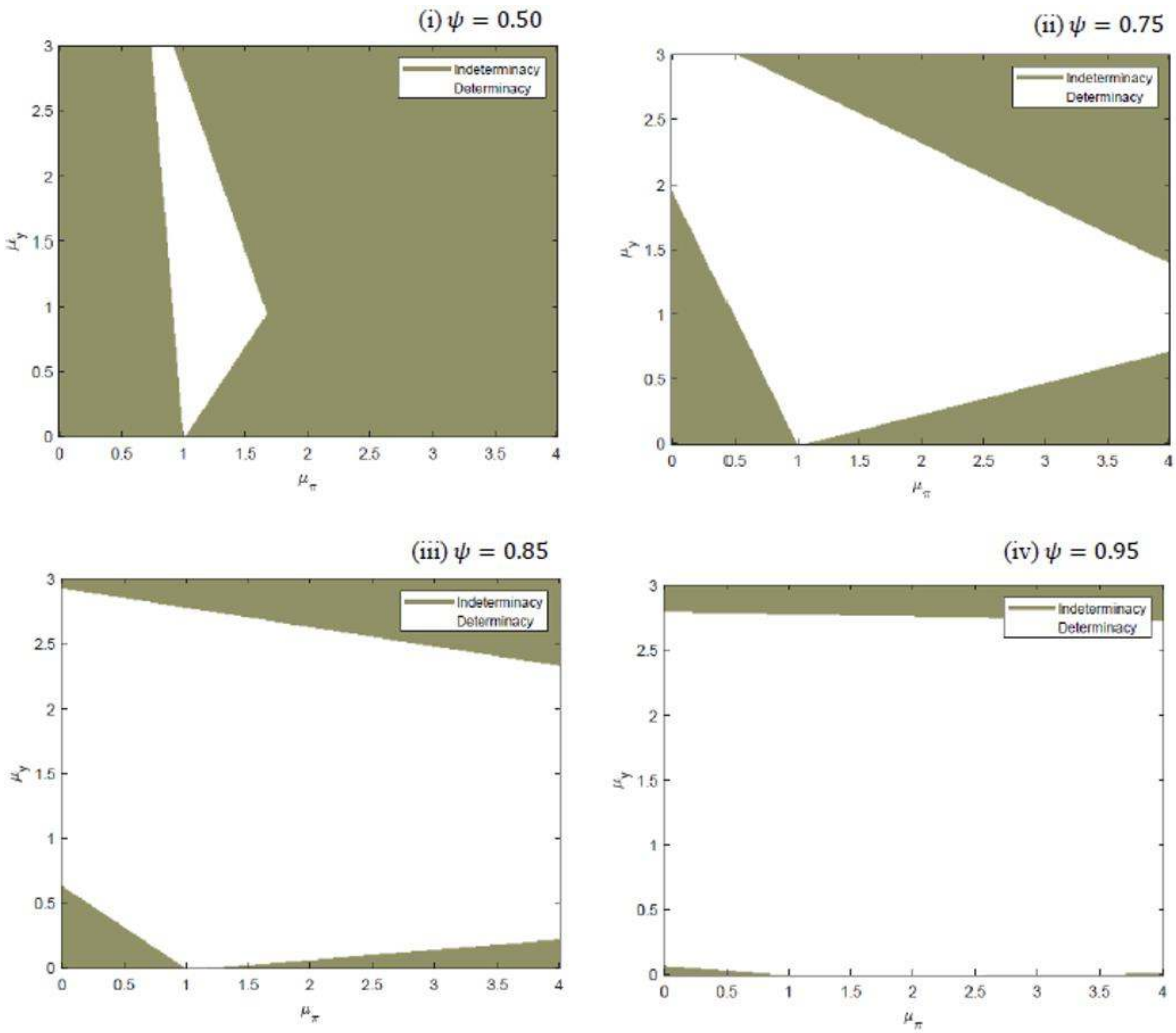

Figure 4.5: Regions of (in)determinacy under a flexible future-inflation-targeting policy and labor mobility for $\psi = 0.50, 0.75, 0.85, 0.95$ when setting $\mu_H = 0.30$.

The intuition behind these results lies in the public finance channel of monetary policy. To see this, consider an increase in the nominal interest rate R_t. Under a balanced budget rule, if the monetary authority follows the Taylor principle, the real interest rate increases, which increases future government debt and future taxation. Higher future taxation $\hat{\tau}_{t+1}$ increases the future real marginal cost $\widehat{mc}_{t+1}$ through (3.14), which in turn will result in a higher future inflation $\hat{\pi}_{t+1}$ through (3.16). Also, migration plays a very important role. To see this, consider a rise in the nominal interest rate $\hat{R}_t$ in the presence of government debt (i.e., $s_b > 0$). Since this reduces output per capita $\hat{y}_t$ and can raise future taxation $\hat{\tau}_{t+1}$, living abroad becomes relatively more attractive, which results in people moving abroad. This leaves the remaining population with the burden of servicing the existing debt, increasing per capita public debt, which results in further tax increases in order to satisfy the balanced budget rule. Migration can strengthen the public finance channel of monetary policy so much that the inverted Taylor principle is needed for determinacy.

Chapter 5

Conclusions

This thesis has examined the determinacy implications of migration by augmenting labor mobility to a New Keynesian sticky-price model with distortionary taxation. The thesis considered the role played by migration in affecting the determinacy properties of forward-looking interest-rate rules. We have showed that migration can have important implications for equilibrium determinacy by affecting the public finance channel of monetary policy. We find that for a combination of sufficiently large government debt and migration, the Taylor principle can induce indeterminacy, and the inverted Taylor principle is required to prevent self-fulfilling expectations.

The intuition behind our results is as follows. In the absence of government debt, the public finance channel of monetary policy is absent and monetary policy is transmitted through the conventional aggregate demand channel. In this case determinacy requires the central bank to adopt the Taylor principle. With labor mobility and government debt, real interest rate increases make moving abroad relatively more attractive because of the need for higher taxation, resulting in emigration. This leaves the country with a higher level of per capita public debt resulting in the need for further tax increases. Consequently, inflation expectations become self-fulfilling under the Taylor principle. In summary, in the presence of low levels of government debt and migration, the Taylor principle remains only a necessary condition for determinacy. The remaining conditions impose an upper bound on the inflation response coefficient. But when government debt and migration reach sufficiently high levels, the slope of the long-run NKPC gets inverted. Consequently, the Taylor principle needs to be inverted in order to achieve determinacy.

Although the aim of this thesis was not to come up with a sophisticated way of modelling the migration decision, we were able to develop a rich and significantly tractable model that allowed us to study the determinacy implications of labor mobility. For future research, it would be interesting to investigate the robustness of the results in different modeling frameworks, such as when the economy is open to international trade or when there is capital and investment spending. Alternative approaches to model labor mobility would also be interesting to study. Overall, further research is needed to understand the implications of migration for the conduct of monetary policy.

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