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MONETARY POLICY AND INFORMALITY: HOW ARE MONETARY POLICY SHOCKS TRANSMITTED IN THE PRESENCE OF INFORMAL LABOR MARKETS?

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Abstract

Informality and its implications for the macroeconomy have received little attention from the literature. Although there is a growing consensus on the relevance of informality for explaining business cycles, research is still needed to address several key issues. This thesis aims to understand the mechanisms behind macroeconomic adjustment in a model with informality calibrated for Mexico in the presence of monetary policy shocks. We develop a closed economy New Keynesian model which closely follows Castillo and Montoro (2012) and analyze the dynamics produced by an interest rate shock and a productivity shock. Our findings suggests that with monetary policy shocks formal employment is countercyclical whereas informal employment is procyclical and with productivity shocks formal employment is procyclical whereas informal employment is countercyclical.

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1 Introduction

A key difference that is observed between the economies of developing and developed countries relates to the size and relevance of the informal economy. For example, estimates suggest that for countries in the top quartile of per capita income informality accounts for around 17.3 percent of total economic activity, whereas for countries in the bottom quartile informality accounts for 35.4 percent of total economic activity (La Porta and Shleifer, 2008). In Latin America, the average share of informal employment relative to total non-agricultural employment is estimated by the International Labor Organization at 51%, with countries varying from 40% in Uruguay to 75% in Bolivia. In Mexico, the informal economy is estimated by INEGI to account for a quarter of GDP, with the share of informal employment close to 60%. However, this high rate of informal employment exists in a context with a low rate of open unemployment suggesting that after a formal job loss the alternative of informal employment is the next best alternative. This thesis aims to understand the mechanisms behind macroeconomic adjustment in a model with informality calibrated for Mexico in the presence of monetary policy shocks.

Recently, a small literature has emerged aiming to understand the business cycle implications of informality. Fernández and Meza (2015) develop a RBC small open-economy model with formal and informal labor to analyze the effects of productivity shocks. They find that the informal sector has a strong countercyclicality thus implying a low pass-through level of shocks across sectors. Their model generates formal labor procyclicality higher than that of aggregate labor matching the data. Fernández and Meza (2015) stress that if informal activity was appropriately measured, the volatility of key macroeconomic variables would be lower. Restrepo-Echavarria (2011) develops a RBC small open-economy model with formal and informal sector, the latter not being measured. A key finding is that countries which have higher cyclical volatility of consumption relative to output have large informal sectors, and that substitution between formal and informal goods is higher than usually accounted for. Other recent studies (e.g.,

Christoffel and Linzert (2005), Castillo and Montoro (2012), Sembeta (2013)) have looked at the transmission of demand and productivity in a New-Keynesian framework with informality. Christoffel and Linzert introduce search and matching frictions and real wage rigidities in an otherwise New Keynesian DSGE model and analyze two types of bargaining (Nash and right to manage) finding that real wage rigidities and right to manage bargaining influences marginal costs to adjust slowly obtaining persistence inflation dynamics. Castillo and Montoro (2012) develop their model upon the model of Christoffel and Linzert (2005) but use the frictions in the labor market developed by Blanchard and Gali (2010). They analyze a productivity shock and an aggregate demand shock with a key finding being the buffer effect of informal labor. Castillo and Montoro (2012) argue that due to its lower hiring cost with respect to formal labor, in periods of high demand wholesale firms increase informal employment without incurring in significant increases in marginal costs; however, a key assumption of their framework is the small difference on productivity between the two types of labor. Sembeta (2013) develops a NK small open economy allowing for consumption of tradable and non tradable goods. Tradable domestic goods can be produced either by a formal firm or by an informal firm. Non-tradable goods are produced entirely by informal firms. To the best of our knowledge, it is the only research using the frictions proposed by Blanchard and Gali (2010), modeling informal employment, production and consumption and taking into account monetary policy. Sembeta (2013) calibrates the model according to a sub-saharan economy finding that in low income countries, the existence of an informal sector does not serve as a stabilizing force in the economy after the occurrence of shocks suggesting that there is a higher degree of volatility due to informal activities. However, little attention has been given to the role informality plays in the transmission of monetary policy shocks. The thesis makes a first attempt at filling this gap.

The modeling framework that we employ is based upon Castillo and Montoro (2012). The model extends a standard closed-economy New Keynesian model with search frictions to allow for an informal labor market.[1] Households are assumed to be infinitely-lived and supply labor to both the formal and informal labor markets. The economy is assumed to be comprised of two different goods sectors: a wholesale sector where firms produce intermediate-goods and

[1] See Trigari (2006) for a NK model which integrates labor market search and matching frictions to analyze how these frictions change real marginal costs. Christoffel y Linzert (2005) analyze how real wage rigidities can generate endogenous inflation persistence in the NK model with search frictions. Other examples are Walsh (2003) and Thomas (2008).

a retail sector where firms produce a final good. Wholesale firms produce output using labor as its only production factor offering formal and informal contracts. To hire a worker in the wholesale sector, firms have to pay a hiring cost which is assumed to be higher for formal workers. Formal workers are assumed to be more productive than informal workers and both types of workers engage in a Nash Bargaining process to determine their wages. Firms in the retail sector produce differentiated final goods using wholesale goods as inputs. Each firm in the retail sector sets its price optimally in a staggered price fashion as in Calvo (1983). The central bank conducts monetary policy by using a Taylor-type rule that responds to forward-looking inflation and output.

It is important to emphasis that we make a number of modifications to the framework of Castillo and Montoro (2012) in order to make the model more appropriate for the case of Mexico. First, in our set-up, workers do not receive unemployment benefits from household production. In the original formulation by Castillo and Montoro (2012) , workers receive W^u from household production. With unemployment benefits, the share of the surplus of the job that goes to the worker is lower because of a higher value of being unemployed. However, unemployment insurance benefits in Mexico are yet to be a reality as there are no full coverage in the country but small schemes particularly in Mexico City. Thus, we consider that it is more realistic to assume that the unemployed do not receive any income. Second, the hiring cost for hiring informal workers has been set to zero. Castillo and Montoro (2012) assume a non-zero hiring cost for informal labor that exerts pressure on informal hiring, increasing the rationing of workers, particularly from informal jobs. In the original framework by Castillo and Montoro (2012), the retail marginal cost is determined by the relative difference of cost between hiring a formal worker relative to an informal one. A firm may be in compliance with most tax and labor laws, but prefer working with self-employed workers rather than salaried employees to avoid complying with – and paying for – social security. As self-employment has been considered a good proxy for informality (Fernández and Mesa, 2015), we consider appropriate to set the hiring cost of an informal worker to be zero. Third, monetary policy in our model is given by a forward-looking Taylor rule. Castillo and Montoro (2012) consider a current-looking Taylor rule. Empirical evidence suggests that central banks set the nominal interest rate in a forward-looking manner (see, for example, Clarida *et al.* 1998, 2000; Orphanides 2001,2004; Mihailov 2006).

While Castillo and Montoro (2012) investigate the role of informality under aggregate demand and productivity shocks, this thesis aims to understand how the macroeconomy adjusts to monetary policy shocks in the presence of an informal labor market. To uncover the adjustment mechanism we consider two versions of the model, a formal only version and the model with informality. We also look at productivity shocks to show that the adjustment mechanism behaves differently under monetary policy shocks. Our results our also shown to be robust to a current-looking specification of the Taylor rule.

After an interest rate shock, we find under the benchmark formal model that unemployment in the economy increases due to an increase in formal wages. Wholesale firms pay this higher wage, and therefore choose to reduce employment. With regard to the market tightness, which is a measure of the jobs created relative to the unemployed workers, that is, the probability of being hired at time t, an increase in unemployment leads to a decrease in the probability of finding a job. Thus, the effect of the increase in the interest rate in job finding is that it lowers this probability, due to the effects on the behavior of the firms. As there are only formal workers with a higher wage, wholesale marginal cost increases, thus increasing the retail marginal cost. Taking this dynamics into account, retail firms decrease its production. With lower employment and lower wages, households decrease consumption. Inflation, in turn, decreases due to the tightening in monetary policy and the household's decision to consume less.

In the informality model, the dynamic response of the economy differs significantly to monetary policy shocks. Considering that in equilibrium wholesale firms hire both types of labor such that the marginal cost for both types of production technologies remains the same there are different responses for each type of labor, the adjustment mechanism lays on the flexibility of the informal labor, that is, no hiring costs for informal workers. The increase in formal employment is not enough to offset the decrease in informal employment. Wholesale firms adjusts its formal employment to maintain equilibrium. Formal labor is increased after the shock because of the lower wage and its higher productivity compared to the informal workers. Thus, overall employment is lower in the economy. Wages for both type of labor decrease. As labor is the only input for both technologies, production with informal technology decreases whilst the production with formal technology increases. Higher unemployment in the econ-

omy relates to a lower probability of finding a job in our model, and the results we obtain are consistent with it. Both measures of market tightness (i.e. formal and informal) decrease. Inflation decreases due to the tightening in monetary policy and lower retail marginal costs. Upon comparison, inflation decreases more in the informality model than in the benchmark model. We argue this occurs because of the response of formal and informal labor and the increase in formal employment as opposed to the decrease in formal employment in the benchmark model.

With the possibility of informal contracts, wholesale firms can adjusts its labor employment and marginal costs by adjusting informal labor hired. This occurs because informal labor does not have hiring costs. After a shock, either productivity or monetary, wholesale firms have the flexibility of dismissing informal labor easier and adjust the optimality condition, that is, equal marginal cost for both type of technologies, without incurring in significant costs. Considering the relative difference in productivity between the two types of labor, a decrease in informal employment does not affect wholesale production as much a decrease in formal employment. Our results show that after both shocks firms find profitable to increase formal employment. However, the behavior is different for each type of shock. With a monetary policy shock, formal employment is countercyclical whereas informal employment is procyclical, and with a productivity shock formal employment is procyclical whereas informal employment is countercyclical. In addition, because of the adjustment mechanism in employment, after a productivity shock consumption increases more than after the monetary policy shock because formal wage decreases more in the context of a monetary policy shock. Informal labor decreases more with interest rate shocks; we consider that the effect of the increase in productivity allows informal labor to be a profitable employment option to wholesale firms. This result occurs because both types of technologies in the wholesale sector are subject to the same productivity. An interesting question will be to have imperfect pass-through of productivity shocks between sectors as in Fernández and Mesa (2015). Clearly, unemployment is higher when the economy is shocked by productivity. This occurs because hiring costs are a function of productivity, thus affecting directly the employment and wage setting decisions by wholesale firms and workers. Retail output increases after the productivity shock more than after the interest rate shock. Comparing the response of inflation after the two shocks, we see that is lower after the productivity shock than after the monetary policy shock. Even with tight monetary policy, retail marginal cost decrease more by the productivity shock, and it is enough to take inflation down more than

the interest rate increase can.

The adjustment mechanism under productivity shocks differs because of the substitution of informal labor based on the lower productivity. Partial substitution of informal labor generates a tighter labor market for both types of labor when allowing for informality. Under the benchmark model, unemployment less than in the informality model after the productivity shock. In addition, formal employment is procyclical whereas informal employment is countercyclical because of the reduction of informal workers employed but the increase in retail output. The substitution of informal labor by increasing formal employment occurs of the effect of productivity on hiring costs. With the productivity shock, wholesale firms find profitable to increase formal labor due to its higher productivity. However, two different effects occur while the firm adjusts its employment. Formal and informal wages decrease and at the same time formal hiring cost increase (informal hiring costs is calibrated at zero), Thus in this context firms increase its formal employment without affecting the condition for equal marginal costs.

The thesis is organized as follows: Section 2 discusses the small literature related to this area of research. The model, including the log-linearized version, is developed in Section 3. Section 4 discusses the calibration employed and motivates the choice of values for the mexican economy, while Section 5 presents the results obtained from the impulse response analysis. Finally, Section 6 briefly concludes.

2 Literature Review

The dualistic vision of labor markets have been studied since the seminal work of Lewis (1954) of dual economies. So far there are two different views, one which considers informality as an option to avoid tax and law enforcement on regulatory burden (Mitra, 2003, Batini *et al.*, 2011, Leal-Ordoñez, 2014), considering that workers have informal jobs because they are being rationed out of the formal sector as proposed by Harris and Todaro (1970). Firms and workers are assumed to comply with regulations in the formal sector but not in the informal sector. Wages in the formal sector are set exogenously and above the competitive equilibrium level. The labor market is then segmented, with only some workers obtaining jobs in the formal sector while others are queuing for formal jobs. The second view considers informality as a legitimate option in which workers value the benefits of an informal contract versus a formal one electing the informal with full conscience and not because of the rationing they are being subject to (Maloney, 2004, Balan *et al.*, 1973).

Informality and its implications for the economy have received little attention, however, new literature that incorporates informality into DSGE models is increasing. Specifically, RBC modeling is paying increasing attention to the effects of informality into business cycles. Fernández and Meza (2015) develop a RBC small open economy model with both formal and informal labor markets. They analyze several empirical facts, particularly the large share of informal employment and its cyclical properties, and argue that these facts help to explain the labor market dynamics. Fernández and Meza (2015) develop a model with formal and informal labor markets; households derive utility from consumption and leisure and decide how to allocate labor between each market. Consumption is derived from a bundle of formal and informal goods which are highly substitutable. Moreover, households accumulate two different capital stocks, with no adjustment costs, that are market specific and can trade a one-period non-contingent bonds in foreign capital markets. Goods in the formal sector are produced by firms and in the

informal sector people are self-employed. Formal firms pay taxes and have a higher productivity relative to the informal sector that does not pay taxes. Government taxes wages, capital rents and the hiring of labor to finance its purchases of formal goods. Fernández and Meza (2015) consider a labor-augmenting shock and an imperfect pass-through across the sectors to analyze the dynamics of their model. They find that their model matches the variability of labor, output and consumption. The imperfect propagation of the shock through the sectors is a strong driving force behind their results. In addition, the model capture the moderate procyclicality and volatility of total employment.

Restrepo-Echavarría (2011) considers a RBC small open economy where a representative agent decide whether to work formally and paid taxes or informally evading taxes but subject to the risk of a tax audit and consequent punishment. The agents' consumption is a bundle of formal and informal goods and invest in international capital markets. Government's revenue is obtained from labor taxes and uses it to increase law enforcement. The formal sector is perfectly competitive maximizing profits by choosing capital and labor. Households decide how time they devote to formal and informal activites. After they decide the amount of time working in the formal sector, the rest is devoted to the informal sector assuming free mobility between sectors. In this model, the informal sector is the one shocked by productivity. Restrepo-Echavarría (2011) argues that having a relative productivity shock in the informal sector is analogous to having both the formal and the informal sector face a productivity shock that is positively but not perfectly correlated due to financial frictions faced by the informal sector. Restrepo-Echavarría (2011) states that formal and informal goods are close substitutes, therefore the elasticity of substitution between them should be high. An important result is that countries with a large informal sector have a higher cyclical volatility of consumption relative to output. Restrepo-Echavarría (2011) stresses that if aggregate data accounted for informality, consumption would not be more volatile than output.

There is a small literature that is interested in understanding the implications for monetary policy under informality. Christoffel and Linzert (2005) use a New Keynesian model with labor market frictions and wage rigidities to analyze the effect of a monetary policy shock and how this frictions affects the transmission process of monetary policy. In their set-up households derive utility from consumption and leisure. There are two goods in the economy wholesale

and retail. Households consume retail goods and retail firms use wholesale good as the input for retail production. The wholesale firms employ labor as the only input to production. Search frictions are introduced as in Pissarides (2000) and wages are negotiated by the firms and workers in two possible ways: Nash bargaining process and right to manage bargaining. Retail firms buy wholesale goods at marginal cost, transform them into differentiated goods and sell them with a mark-up over marginal costs. In addition. retail firms are subject to price stickiness. Following the empirical evidence on Europe, Christoffel and Linzert (2005) introduce real wage rigidity and analyze the outcome under the Nash bargaining and under right to manage bargaining. A key result is that under right to manage bargaining and assuming a high degree of wage rigidity causes marginal cost to adjust slowly generating more persistence inflation dynamics. Under Nash bargaining , in which marginal costs are determined by hours worked, wage rigidity does not lead to higher inflation persistence.

Blanchard and Gali (2010) propose a model in which they allow frictions in the labor market similar to those in the Diamond-Mortensen-Pissarides (DMP) framework. The DMP framework has hiring costs, such as the job searching of a worker and its matching with the job being offered by a firm. With this frictions, models provide better similarities with the data regarding the unemployment dynamics. However, Blanchard and Gali (2010) use a formulation of their own in order to have a New-Keynesian Model with frictions, to analyze the effects of monetary policy. Addressing the similarities between the DMP tradition and its work, the authors state that

"vacancies are assumed to be filled immediately by paying the hiring cost, which is a function of labor market tightness. By contrast, in the DMP model, the hiring cost is uncertain, with its expected value corresponding to the (per period) cost of posting a vacancy times the expected time to fill it. This expected time is an increasing function of the ratio of vacancies to unemployment, which can be expressed in turn as a function of labor market tightness".
Thus, while there is a different formulation of the frictions in the labor market, both approaches share the characteristic that the hiring cost is an increasing function of the labor market tightness.

Castillo and Montoro (2012) and Sembeta (2013) use the New Keynesian framework with friction as in Blanchard and Gali (2010). Castillo and Montoro (2012) follow the production

structure of Christoffel and Linzert (2005) in which there are two sectors wholesale and retail[1] and the consumption of the retail good by the households, however they do not introduce real wage rigidities. In Castillo and Montoro (2012), firms in the wholesale sector balance the higher productivity and higher costs of formal workers with the lower costs and productivity of the informal workers. Thus, wholesale marginal costs are a function of formal and informal labor in the economy. The frictions in the model are the hiring costs that wholesale firms have to pay to hire either type of labor. They use an aggregate demand shock and a productivity shock, both lasting only one period, to analyze the effects of informality on the economy. Their results states that in periods of high demand, informal sector, and by consequence informal output, expands more than the formal sector because of the lower costs associated with the informal sector even though the formal sector is more productive. They also found that at the margin, wholesale firms can substitute formal labor with informal labor without dramatically changing the wholesale marginal cost; in this case, inflation dynamics does not depend simply on unemployment but in the flows of workers between formal and informal jobs. Sembeta (2013) uses a multi-sector small open economy with dual labor markets to analyze the effects of both external and domestic shocks. In Sembeta (2013) households derive utility from consumption and leisure, and consumption is a bundle of both tradable and non-tradable goods. Tradable goods are domestically or foreign produced with different types of labor. The economy features state-contingent assets (i.e. complete asset markets) trade by households. Tradable goods production is modeled by a two-step production process but as Blanchard and Gali (2010), labor frictions are introduced in the intermediate firm while price stickiness is modeled in the final good. Intermediate goods can be produced either by a formal firm or an informal firm which has a lower productivity compared to its formal counterpart. As opposed to Castillo and Montoro (2012) firms cannot mix the type of contractual arraignments. In Sembeta (2013) intermediate firms are ither formal or informal and the wages have a different setting process. Formal wages are negotiated using a Nash bargaining process whereas informal wages are determined by the common competitive mechanisms as there are no hiring costs for informal firms. In addition, the only firms that are constantly generating vacancies are the formal firms, so exiting the unemployment depends on the vacancies created by the formal sector and the unoccupied jobs in the informal sector. Final tradable goods are produced out of formal and informal intermediate

[1] Krause and Lubik (2003) embed the two sector structure into a single integrated firm employing labor to produce intermediate and final good.

goods using CES technology. Final tradable goods, both domestic and foreign, are modeled with sticky prices, thus inflation in the economy is a combination of domestic and imported inflation. A central result of Sembeta (2013) argues that the view of an informal sector and its role as a stabilizing force of the economy in the event of shocks is not supported when informality is not generated by a mix of contractual arraignments by a firm rather than two different sectors.

3 A Sticky Price Model with Informal Labor

This section discusses the modeling framework used in the thesis, which closely follows the model of Castillo and Montoro (2012). The economy is assumed to be cashless, where there exists infinitely-lived households, a continuum of retail (or final) goods producing firms, wholesale (or intermediate) goods producing firms, and a central bank. Wholesale firms sell homogeneous goods to retail firms who use these intermediate products to create a differentiated final good, which is sold to households for consumption purposes. It is assumed that retail firms are monopolisitically competitive and set prices in a staggered fashion according to Calvo (1983). Wholesale firms are perfectively competitive and hire labor to produce intermediate goods where two different types of labor, formal and informal, are available. Informal labor is assumed to be cheaper and less productive than its formal counterpart, and wholesale firms must pay a hiring cost to recruit both formal and informal workers. Both formal and informal wages are determined by Nash bargaining. Monetary policy is governed by a forward-looking Taylor-type rule, where the nominal interest rate reacts to expected future inflation and output. The economy faces two types of shocks: a productivity shock in the wholesale sector and a monetary policy (i.e., interest rate) shock.

3.1 Households

The economy is populated by infinitely-lived households which choose consumption C_t to maximize utility. Let the period utility function be given by:

$$U_t^j(C, L) = \frac{(C_t^j)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \eta \frac{(L_t^j)^{1+\varphi}}{1+\varphi} \tag{3.1}$$

where σ is intertemporal elasticity of substitution in consumption, φ is the inverse of the Frisch elasticity of labor supply and η is a preference parameter representing the marginal disutility from working. It is assumed that the household can supply labor to both the formal L_F and informal L_I sectors:

$$L_t = L_{F,t} + L_{I,t} \tag{3.2}$$

The household maximizes utility subject to the following period budget constraint (expressed in real terms):

$$C_t^j + b_t^j = w_{F,t}L_{F,t}^j + w_{I,t}L_{I,t}^j + \Pi_t^j + R_{t-1}b_{t-1}^j\frac{P_{t-1}}{P_t} \tag{3.3}$$

where P_t is the price index of retail goods in time t, $w_{F,t} = W_{F,t}/P_t$ is the real wage for formal workers, $w_{I,t} = (W_{I,t}/P_t)$ is the real wage for informal workers, and $L_{k,t}^j$ is the fraction of household labor supply for sector k, where k = F, I (F = formal, I = informal). The household carries b_{t-1} holdings of real bonds into period t, which pay the nominal interest rate R_{t-1} and the household receives real profits Π from the ownership of retail firms.

Consequently, the household maximization problem can be expressed as:

$$\begin{aligned}\max_{C_t^j, B_t^j} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t^j)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \eta\frac{(L_{F,t}^j + L_{I,t}^j)^{1+\varphi}}{1+\varphi} \right) \\ -\tau_t \left[C_t^j + b_t^j - w_{F,t}L_{F,t}^j - w_{I,t}L_{I,t}^j - \frac{\Pi_t^j}{P_t} - R_{t-1}b_{t-1}^j\frac{P_{t-1}}{P_t} \right]\end{aligned} \tag{3.4}$$

where τ_t denotes the Lagrange multiplier. The first order conditions with respect to consumption and bonds are:

$$\beta^t(C_t^j)^{-\frac{1}{\sigma}} - \tau_t = 0 \Rightarrow \beta^t(C_t^j)^{-\frac{1}{\sigma}} = \tau_t \tag{3.4a}$$

$$-\tau_t + \mathbb{E}_t\left[\tau_{t+1}R_t\frac{P_t}{P_{t+1}}\right] = 0 \Rightarrow \tau_t = \mathbb{E}_t\left[\tau_{t+1}R_t\frac{P_t}{P_{t+1}}\right] \tag{3.4b}$$

Forwarding equation (3.4a) by one period:

$\beta^{t+1}(C_{t+1}^j)^{-\frac{1}{\sigma}} - \tau_{t+1} = 0 \Rightarrow \tau_{t+1} = \beta^{t+1}(C_{t+1}^j)^{-\frac{1}{\sigma}}$

and substituting the above into equation (3.4b) and rearranging yields the standard Euler equation:

$$(C_t^j)^{-\frac{1}{\sigma}} = \mathbb{E}_t\left[\beta(C_{t+1}^j)^{-\frac{1}{\sigma}}R_t\frac{P_t}{P_{t+1}}\right] \tag{3.5}$$

The Euler equation equates the cost of delaying consumption with its expected marginal benefit.

3.2 Wholesale (Intermediate) goods producing firms

It is assumed that the economy consists of a large number of intermediate firms producing wholesale goods, $i \in [0, 1]$, who operate under perfect competition. Wholesale firm i produces output using two types of linear production technology: $Y_{F,t}$ and informal $Y_{I,t}$ where it is assumed that there is a bias towards the formal sector represented by the parameter $\gamma_1 \in [0, 0.5)$:

$$Y_t^w(i) = (1 - \gamma_1)Y_{F,t}(i) + (\gamma_1)Y_{I,t}(i) \tag{3.6}$$

The production functions for each sector are given by

$$Y_{F,t}(i) = Z_t L_{F,t}(i) \qquad \text{and} \qquad Y_{I,t}(i) = \omega Z_t L_{I,t}(i) \tag{3.7}$$

where Z_t is the level of productivity following the law of motion: $lnZ_t = \rho_z Z_{t-1} + \epsilon_{z,t}$ and $\rho_z \in (0, 1)$ and $\epsilon_{Z,t}$ as an i.i.d error term distributed as $\epsilon_{Z,t} \sim \mathcal{N}(0, \sigma^2_{\epsilon_Z})$. In addition, the parameter $\omega < 1$ captures the lower productivity of informal sector.
Total employment in the economy is defined as:

$$L_t = L_{F,t} + L_{I,t}$$

Let the evolution of employment be represented as:

$$L_t = (1 - \delta)L_{t-1} + A_t$$

and $0 < \delta < 1$ is the fraction of workers that lose their job and A_t is the fraction of unemployed workers that get a job. Each type of labor has its own motion given by:

$$L_{F,t} = (1 - \delta)L_{F,t-1} + A_{F,t}$$

$$L_{I,t} = (1 - \delta)L_{I,t-1} + A_{I,t}$$

where $A_{F,t}$ denotes newly created formal jobs and $A_{I,t}$ is newly created informal jobs. For simplicity we assume that the job destruction rate δ is the same for both types of labor.

Unemployment can be defined at the beginning of the period:

$$U_t = 1 - (1 - \delta)(L_{F,t-1} + L_{I,t-1}) \tag{3.8}$$

with the following conditions $L_{F,t} \geq 0$, $L_{I,t} \geq 0$, $W_{F,t} \geq W_{I,t}$.

Wholesale firms are required to post vacancies, screen possible employees, fill-out government requirements, etc. Therefore, in line with Blanchard and Gali (2010), we introduce the following cost of hiring formal labor $G_{F,t}A_{F,t}(j)$ and informal labor $G_{I,t}A_{I,t}(j)$, which are expressed in terms of the CES bundle of retail goods. $G_{F,t}(G_{I,t})$ is the hiring cost per worker and is independent of the vacancies created $A_{F,t}(j)(A_{I,t}(j))$ and is taken as given by the individual firm. Formally these costs can be expressed as[1]:

$$G_{F,t} = Z_t \Theta_F X_{F,t}^{\alpha_F} \qquad and \qquad G_{I,t} = Z_t \Theta_I X_{I,t}^{\alpha_I} \tag{3.9}$$

where Z_t is the productivity level, Θ_F and Θ_I are non-negative constants with $\Theta_F > \Theta_I$, $\alpha_F \geq 0$ and $\alpha_I \geq 0$ are parameters with $\alpha_F > \alpha_I$. $X_{F,t}, X_{I,t} \in [0, 1]$ represent the labor market tightness for formal and informal labor. While wholesale firms face a cost to hiring both formal and informal labor, in our parametrization we will assume that $\Theta_I = 0$ so that there are no costs associated with hiring informal labor.

Labor market tightness is defined to be the ratio of aggregate hiring, either formal $A_{F,t}$ or informal $A_{I,t}$, to unemployment U_t given by:

$$X_{F,t} = \frac{A_{F,t}}{U_t} \qquad \text{and} \qquad X_{I,t} = \frac{A_{I,t}}{U_t} \tag{3.10}$$

We assume as Castillo and Montoro (2012) that the labor market tightness is different for each kind of labor and that the overall market tightness X_t can be expressed as: $X_t = X_{F,t} + X_{I,t}$. It is worth noting that the hiring cost is increasing in the labor market tightness. This labor market tightness can be understood as the probability of getting hired as it is the ratio between

[1]Blanchard and Gali (2010, p. 8) explain that *the motivation for the presence of Z_t in the expressions for $G_{F,t}$ and $G_{I,t}$ is to avoid effects of productivity shocks on the cost of hiring relative to the cost of producing.*

the vacancies filled (people hired) to all the people wanting to get a job.

Wholesale firms maximize their expected discounted profits by selecting sequences for the formal labor $L_{F,t}$, informal labor $L_{I,t}$ given by:

$$\mathbb{E}_T \left(\sum_{j=0}^{\infty} \Omega_{t,t+j} \Pi_{t+j}(i) \right) \tag{3.11}$$

where:
$\Omega_{t,t+j} = \beta^j \dfrac{u'(c_{t+j})}{u'(c_t)}$ is the discount factor between periods t and $t+i$.
$\Pi_{t+j}(i)$ is the profit function of the firm in real terms, given by:

$$\Pi_{t+j}(i) = \frac{P_t^w}{P_t} \left[(1-\gamma_1) Z_t L_{F,t} + \gamma_1 \omega Z_t L_{I,t} \right] - \left[w_{F,t} L_{F,t} + G_{F,t} A_{F,t} + w_{I,t} L_{I,t} + G_{I,t} A_{I,t} \right] \tag{3.12}$$

where P_t^w denotes the price of the wholesale goods, P_t is the price of retail goods.

The corresponding first order conditions with respect to L_F and L_I yields the formal and informal labor demand functions:

$$w_{F,t} = \frac{P_t^w}{P_t}(1-\gamma_1) Z_t - G_{F,t} + (1-\delta)\beta \mathbb{E}_t \left[\frac{u'(c_{t+1})}{u'(c_t)} G_{F,t+1} \right] \tag{3.13}$$

and

$$w_{I,t} = \frac{P_t^w}{P_t}\gamma_1 \omega Z_t - G_{I,t} + (1-\delta)\beta \mathbb{E}_t \left[\frac{u'(c_{t+1})}{u'(c_t)} G_{I,t+1} \right] \tag{3.14}$$

All wholesale firms will hire each type of labor up to the point in which the marginal productivity equals the cost of an additional worker, wage and hiring cost, plus the discounted expected saving on the filled vacancy. The difference between this first order condition and the one obtained under a perfect competitive environment is due to hiring costs.

From cost minimization the marginal cost of wholesale firms mc_t^w can be expressed as:

$$\frac{P_t^w}{P_t} = mc_t^w \tag{3.15}$$

with

$$mc_t^w = \frac{w_{F,t} + G_{F,t} - (1-\delta)\beta\mathbb{E}_t\left[\frac{u'(c_{t+1})}{u'(c_t)}G_{F,t+1}\right]}{(1-\gamma_1)Z_t} = \frac{w_{I,t} + G_{I,t} - (1-\delta)\beta\mathbb{E}_t\left[\frac{u'(c_{t+1})}{u'(c_t)}G_{I,t+1}\right]}{\gamma_1\omega Z_t} \tag{3.16}$$

The intuition behind equation (3.16) is as follows. It shows that in equilibrium firms hire both types of labor such that the marginal cost for both types of production technologies remains the same. Castillo and Montoro (2012) argue that in equilibrium labor moves from one sector into the other or in and out of unemployment in a way that makes real marginal costs for each type of labor equal to each other.

3.2.1 Wage Determination

Following Castillo and Montoro (2012) households and firms set wages through a Nash Bargaining process. The value functions of being employed under a formal and informal contract, $V_{F,t}$ and $V_{I,t}$, are respectively:

$$\begin{aligned} V_{F,t} &= w_{F,t} - \eta\left[L_t\right]^{\varphi} C_t^{\frac{1}{\sigma}} \\ &+\mathbb{E}_t\left[\Omega_{t,t+1}\left\{(1-\delta+\delta X_{F,t+1})V_{F,t+1} + \delta X_{I,t+1}V_{I,t+1} + \delta(1-X_{t+1})V_{u,t+1}\right\}\right] \end{aligned} \tag{3.17}$$

$$\begin{aligned} V_{I,t} &= w_{I,t} - \eta\left[L_t\right]^{\varphi} C_t^{\frac{1}{\sigma}} \\ &+\mathbb{E}_t\left[\Omega_{t,t+1}\left\{(1-\delta+\delta X_{I,t+1})V_{I,t+1} + \delta X_{F,t+1}V_{F,t+1} + \delta(1-X_{t+1})V_{u,t+1}\right\}\right] \end{aligned} \tag{3.18}$$

The value of the job to a household employed in the formal sector is the real formal wage net of the marginal rate of substitution between consumption and labor plus the discounted future value of these possible states: maintaining the job with probability $(1-\delta)$ and receive $V_{F,t+1}$, losing the job with probability δ but gaining another formal position with probability $X_{F,t+1}$ and receiving $V_{F,t+1}$, losing their job with probability δ but gaining an informal position with probability $X_{I,t+1}$ and receive $V_{I,t+1}$, and separated from the job with probability δ and remain unemployed receiving $V_{u,t+1}$ while searching for a formal job with probability $(1-X_{F,t+1})$. Similar interpretations can be given for the value function for the informal worker.

The value of an unemployed household member in the is given by:

$$V_{u,t} = \mathbb{E}_t \left[\Omega_{t,t+1} \left\{ X_{F,t+1} V_{F,t+1} + X_{I,t+1} V_{I,t+1} + (1 - X_{t+1}) V_{u,t+1} \right\} \right] \tag{3.19}$$

That is, the value of an unemployed member is the discounted expected value of having a formal job in the next period with probability $X_{F,t+1}$ and receiving $V_{F,t+1}$, obtaining an informal job with probability $X_{I,t+1}$ and receiving $V_{I,t+1}$ or remaining unemployed with probability $1 - X_{t+1}$. In this case we do not assume that the workers receive a payment W_t^u while being unemployed as in Castillo and Montoro (2012).

The bargaining power of workers is given by $\xi \in [0, 1]$ such that:

$$\xi G_{F,t} = V_{F,t} - V_{u,t} \quad \text{and} \quad \xi G_{I,t} = V_{I,t} - V_{u,t}$$

Subtracting the value of being unemployed (3.19) from the value of a job under a formal contract equation (3.17) and the value of being unemployed (3.19) from the value of a job under a informal contract equation (3.18) and solving for $w_{F,t}$ and $w_{I,t}$ we obtain:

$$\begin{aligned} w_{F,t} &= \xi \Theta_F Z_t X_{F,t}^{\alpha} + \eta \left[L_t \right]^{\varphi} C_t^{\frac{1}{\sigma}} \\ -(1-\delta)\xi \mathbb{E}_t \left[\Omega_{t,t+1} \left\{ (1 - X_{F,t+1}) \Theta_F Z_{t+1} X_{F,t+1}^{\alpha_F} - X_{I,t+1} \Theta_I Z_{t+1} X_{I,t+1}^{\alpha_I} \right\} \right] \end{aligned} \tag{3.20}$$

$$\begin{aligned} w_{I,t} &= \xi \Theta_I Z_t X_{I,t}^{\alpha} + \eta \left[L_t \right]^{\varphi} C_t^{\frac{1}{\sigma}} \\ -(1-\delta)\xi \mathbb{E}_t \left[\Omega_{t,t+1} \left\{ (1 - X_{I,t+1}) \Theta_I Z_{t+1} X_{I,t+1}^{\alpha_I} - X_{F,t+1} \Theta_F Z_{t+1} X_{F,t+1}^{\alpha_F} \right\} \right] \end{aligned} \tag{3.21}$$

The interaction between formal and informal labor supply and formal and informal labor demand gives the equilibrium formal and informal wages, and employed workers under both types of contracts.

3.3 Retail Firms

The retail firms, indexed by $l \in [0, 1]$ use wholesale products to create a differentiated final good with the following technology:

$$Y_t^R(l) = Y_t^w(l) \tag{3.22}$$

Given the assumption of perfect competition in the wholesale (intermediate) goods sector, the marginal cost for the retailer firm is equal to the marginal cost of the wholesale firm:

$$mc_t^R = \frac{P_t^w}{P_t} = mc_t \tag{3.23}$$

As C_t is the composite good basket for consumption over the $j \in [0,1]$ varieties, it can expressed using the Dixit-Stiglitz aggregator:

$$C_t = \left[\int_0^1 (C_t^j)^{\frac{\phi-1}{\phi}} dj\right]^{\frac{\phi}{\phi-1}} \tag{3.24}$$

where ϕ is the elasticity of substitution between varieties.
Consequently, the demand condition for variety j is:

$$C_t^j = \left(\frac{P_t(j)}{P_t}\right)^{-\phi} C_t \tag{3.25}$$

3.3.1 Price-setting behavior of retail firms

Following Blanchard and Gali (2010), we introduce price-stickiness in the final goods sector rather than the standard procedure of introducing sticky price in the intermediate goods sector. " *The motivation for the separation between final goods producers with monopoly power and perfectly competitive intermediate goods producers is to avoid interactions between price setting and wage bargaining at the firm level* " Blanchard and Gali (2010, p.6).

It is assumed that retail firms set prices according to Calvo (1983), where there is a fraction $1-\lambda$ of firms that have the possibility to adjust their prices and the remaining fraction λ that does not have the possibility to adjust their prices. Given the demand function (3.25), the optimal price-setting problem can be expressed as follows, where the firm must choose $\bar{P}_t$ to maximize:

$$\sum_{t=0}^{\infty} \lambda^i \Omega_{t,t+i} \left[\left(\frac{P_t(j)}{P_{t+i}}\right) - mc_{t+i}\right] \left[C_{t+i}^d(j)\right] \tag{3.26}$$

where:
$\Omega_{t,t+i} = \beta^i \dfrac{u'(c_{t+i})}{u'(c_t)}$ is the discount factor between period t and $t+i$ and it is equal to the intertemporal marginal rate of substitution since all firms are owned by the domestic agents, and

$C^d_{t+i}(j)$ is the demand function (3.25).

Substituting the aggregate demand curve (3.25) into equation (3.26)

$$\sum_{t=0}^{\infty} \lambda^i \Omega_{t,t+i} \left[\left(\frac{P_t(j)}{P_{t+i}} \right)^{1-\phi} - mc_{t+i} \left(\frac{P_t(j)}{P_{t+i}} \right)^{-\phi} \right] [C_{t+i}]$$

Let $\bar{P}_t(j)$ denote the optimal price chosen to maximize profits, and obtaining the first order condition:

$$\sum_{t=0}^{\infty} \lambda^i \Omega_{t,t+i} \left[\frac{(1-\phi)}{P_{t+i}} \left(\frac{\bar{P}_t(j)}{P_{t+i}} \right)^{-\phi} + \frac{\phi}{P_{t+i}} mc_{t+i} \left(\frac{\bar{P}_t(j)}{P_{t+i}} \right)^{-\phi-1} \right] [C_{t+i}] = 0$$

Multiplying by $\bar{P}_t$ and $\frac{1}{1-\phi}$:

$$\sum_{t=0}^{\infty} \lambda^i \Omega_{t,t+i} \left[\left(\frac{\bar{P}_t(j)}{P_{t+i}} \right)^{1-\phi} + \left(\frac{\phi}{1-\phi} \right) mc_{t+i} \left(\frac{\bar{P}_t(j)}{P_{t+i}} \right)^{-\phi} \right] [C_{t+i}] = 0$$

$$\sum_{t=0}^{\infty} \lambda^i \Omega_{t,t+i} \left[\left(\frac{\bar{P}_t(j)}{P_{t+i}} \right)^{1-\phi} \right] [C_{t+i}] =$$
$$\left(\frac{\phi}{\phi-1} \right) \sum_{t=0}^{\infty} \lambda^i \Omega_{t,t+i} \left[mc_{t+i} \left(\frac{\bar{P}_t(j)}{P_{t+i}} \right)^{-\phi} \right] [C_{t+i}]$$

Dividing and multiplying the right hand side by $\frac{\bar{P}_t(j)}{P_{t+i}}$ yields:

$$\sum_{t=0}^{\infty} \lambda^i \Omega_{t,t+i} \left[\left(\frac{\bar{P}_t(j)}{P_{t+i}} \right)^{1-\phi} \right] [C_{t+i}] =$$
$$\left(\frac{\phi}{\phi-1} \right) \sum_{t=0}^{\infty} \lambda^i \Omega_{t,t+i} \left[\frac{MC_{t+i}}{P_{t+i}} \left(\frac{\bar{P}_t(j)}{P_{t+i}} \right)^{-\phi} \right] [C_{t+i}]$$

Further arraignments generate:

$$\bar{P}_t(j) = \left(\frac{\phi}{\phi-1} \right) \frac{\sum_{t=0}^{\infty} \lambda^i \Omega_{t,t+i} \left[MC_{t+i} \left(\frac{1}{P_{t+i}} \right)^{-\phi} \right] [C_{t+i}]}{\sum_{t=0}^{\infty} \lambda^i \Omega_{t,t+i} \left[\left(\frac{1}{P_{t+i}} \right)^{1-\phi} \right] [C_{t+i}]} \tag{3.27}$$

or equivalently:

$$\bar{P}_t(j) = \left(\frac{\phi}{\phi - 1}\right) \sum_{t=0}^{\infty} X_{t,t+i} MC_{t+i} \tag{3.28}$$

with

$$X_{t,t+i} = \frac{\lambda^i \Omega_{t,t+i} \left(\frac{1}{P_{t+i}}\right)^{-\phi} [C_{t+i}]}{\sum_{t=0}^{\infty} \lambda^i \Omega_{t,t+i} \left[\left(\frac{1}{P_{t+i}}\right)^{1-\phi}\right] [C_{t+i}]}$$

From equation (3.28) it is possible to observe that the optimal price set is a markup over a weighted average of current and future nominal marginal costs. If prices are fully flexible, such that $\lambda = 0$, it follows from equation (3.28) that:

$$\bar{P}_t(j) = \left(\frac{\phi}{\phi - 1}\right) MC_t \tag{3.29}$$

At time t, all firms in the economy are facing the problem of price setting, so all firms have the same optimal price in a flexible price environment: $\bar{P}_t(j) = \bar{P}_t$. Thus, each firm setting its new price chooses the same price as any other firm. Then:

$$P_t = \left[(1 - \lambda)(\bar{P}_t)^{1-\phi} + \lambda (P_{t-1})^{1-\phi}\right]^{\frac{1}{1-\phi}} \tag{3.30}$$

Equation (3.30) is the overall retail price.

3.3.2 Retail marginal cost

In this subsection, I focus in the marginal cost for retail firms. We follow Castillo and Montoro (2012) in order to obtain the retail marginal cost. Subtracting informal labor demand (3.14) from formal labor demand (3.13) and rearranging yields:

$$w_{F,t} - w_{I,t} = [(1 - \gamma_1) - \gamma_1 \omega] Z_t mc_t^R - (G_{F,t} - G_{I,t}) + (1 - \delta)\beta \mathbb{E}_t \left\{ \left(\frac{C_t}{C_{t+1}}\right)^{\frac{1}{\sigma}} (G_{F,t+1} - G_{I,t+1}) \right\} \tag{3.31}$$

Subtracting informal labor supply (3.21) from formal labor supply (3.20) generates:

$$w_{F,t} - w_{I,t} = \xi \left[(G_{F,t} - G_{I,t})) - (1-\delta)\beta \mathbb{E}_t \left\{ \left(\frac{C_t}{C_{t+1}} \right)^{\frac{1}{\sigma}} (G_{F,t+1} - G_{I,t+1}) \right\} \right] \tag{3.32}$$

With this system of equations formed by (3.31) and (3.32), we solve for the retail marginal cost:

$$\left[(1-\gamma_1) - \gamma_1 \omega \right] Z_t mc_t^R = (1+\xi) \left[(G_{F,t} - G_{I,t}) - (1-\delta)\beta \mathbb{E}_t \left\{ \left(\frac{C_t}{C_{t+1}} \right)^{\frac{1}{\sigma}} (G_{F,t+1} - G_{I,t+1}) \right\} \right] \tag{3.33}$$

By inspection of (3.33), marginal cost depends on the difference in cost between hiring a formal worker and an informal worker divided by the gain in productivity of hiring an informal worker.

3.4 Monetary Policy Rules

In setting the nominal interest rate, the Central Bank is assumed to conduct monetary policy using a Taylor-type rule. For this thesis we will evaluate two different Taylor rules: a current-looking specification that reacts to contemporaneous inflation and (retail) output, and a forward-looking specification that responds to expected future inflation and output.

- Forward Looking Taylor Rule

$$R_t = \mathbb{E}_t \left[\bar{R} \left(\pi_{t+1} \right)^{\phi_\pi} \left(\frac{Y_{t+1}^R}{\bar{Y}^R} \right)^{\phi_Y} \right] \epsilon_{R,t}$$

- Current Looking Taylor Rule

$$R_t = \left[\bar{R} \left(\pi_t \right)^{\phi_\pi} \left(\frac{Y_t^R}{\bar{Y}^R} \right)^{\phi_Y} \right] \epsilon_{R,t}$$

The interest rate shock is modeled as an autoregressive process AR(1) in order to signal the time it takes monetary policy to have any effect in the economy.

3.5 Market Clearing

Regarding the labor market we know that total employment equals the number of workers employed under a formal contract plus the number of workers employed under an informal contract:

$$L_t = L_{F,t} + L_{I,t} = \int_0^1 L_{F,t}^j dj + \int_0^1 L_{I,t}^i di \tag{3.34}$$

Aggregating the household budget constraint (3.3), and imposing bond market clearing we obtain:

$$C_t = w_{F,t} L_{F,t} + w_{I,t} L_{I,t} + \Pi_t \tag{3.35}$$

Given the assumption of perfect competition from the intermediate goods sector we know that:

$$\left(\frac{P_t^w}{P_t}\right) Y_t^w = w_{F,t} L_{F,t} + w_{I,t} L_{I,t} + G_{F,t} A_{F,t} + G_{I,t} A_{I,t} \tag{3.36}$$

Consequently, real profits in the retail sector are given by:

$$\Pi_t = Y_t^R - \left(\frac{P_t^w}{P_t}\right) Y_t^w \tag{3.37}$$

Substituting equation (3.36) into equation (3.37) yields:

$$\Pi_t = Y_t^R - [w_{F,t} L_{F,t} + w_{I,t} L_{I,t} + G_{F,t} A_{F,t} + G_{I,t} A_{I,t}] \tag{3.38}$$

Finally, substituting equation (3.38) into equation (3.35) leads to

$$Y_t^R = C_t + G_{F,t} A_{F,t} + G_{I,t} A_{I,t} \tag{3.39}$$

As explained in the wholesale firms section, we assume that the hiring costs of the firms $G_{F,t}$ and $G_{I,t}$ are paid in terms of the CES bundle of retail goods. Therefore, after aggregation, retail production equals retail consumption and the relevant cost of hiring workers. And using equation (3.25) and aggregating over retail producers we can obtain

$$Y_t^w = \Delta Y_t^R \tag{3.40}$$

where $\Delta = \left[\int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-(\phi-1)} dj\right]^{\frac{\phi}{\phi-1}}$ is a measure of price dispersion.

3.6 Equilibrium

Equilibrium can be defined as as the set of sequences:
$\{w_{F,t}, w_{I,t}, mc_t^R, mc_t^w, P_t^w, P_t^R, C_t, L_t, L_{F,t}L_{I,t}, Y_{F,t}, Y_{I,t}, Y_t^w, Y_t^R, R_t\}$
such that for $t \geq 0$ the optimality condition for the household (3.5), optimality conditions for the wholesale firm, (3.13) and (3.14), the aggregate version of (3.6), marginal cost for the retail sector, equation (3.33), formal and informal labor supply, (3.20) and (3.21), the monetary policy rule, the consumption demand (3.25), the price index (3.30) and the market clearing condition is satisfied (3.39).

3.7 Log-linearized version of the model

Due to difficulties in computation of non-linear models, to further proceed we need to log-linearize the equilibrium conditions of the model. Log-linearization can be explained as the log-deviation around a steady state value. If x^{ss} denotes the steady state value of the variable x_t. Defining the log-deviation of variable x_t from its steady state x^{ss} as:

$$\tilde{x}_t \equiv ln(x_t) - ln(x^{ss})$$

Log deviations of x_t from its steady state value are approximately equal to the percentage difference between x_t and its steady state value. This approximation is appropriate in a small neighborhood around the steady state.
The log linearized version of the relevant equations is:

1. Euler equation

$$(C_t^j)^{-\frac{1}{\sigma}} = \mathbb{E}_t\left[\beta(C_{t+1}^j)^{-\frac{1}{\sigma}} R_t \frac{P_t}{P_{t+1}}\right]$$
$$\rightarrow \mathbb{E}_t[\tilde{C}_{t+1}] = \tilde{C}_t + \sigma\mathbb{E}_t[\tilde{R}_t - (\tilde{\pi}_{t+1})]$$

2. Aggregate demand condition

$$Y_t^R = C_t + G_{F,t}A_{F,t} + G_{I,t}A_{I,t}$$
$$\rightarrow \tilde{Y}_t^R = \frac{C^{ss}}{Y^{ss}}\tilde{C}_t + \frac{G_F^{ss}A_F^{ss}}{Y^{ss}}(\tilde{G}_{F,t} + \tilde{A}_{F,t}) + \frac{G_I^{ss}A_I^{ss}}{Y^{ss}}(\tilde{G}_{I,t} + \tilde{A}_{I,t})$$

$$\rightarrow \tilde{Y}_t^R = \frac{C^{ss}}{(Y^R)^{ss}}\tilde{C}_t + \frac{\Theta_F Z^{ss}(\delta L_F^{ss})^{1+\alpha_F}}{(Y^R)^{ss}(U^{ss})^{\alpha_F}}\left(\tilde{Z}_t + \alpha_F\tilde{X}_{F,t} + \frac{\tilde{L}_{F,t}}{\delta} - \frac{(1-\delta)}{\delta}\tilde{L}_{F,t-1}\right)$$
$$+\frac{\Theta_I Z^{ss}(\delta L_I^{ss})^{1+\alpha_I}}{(Y^R)^{ss}(U^{ss})^{\alpha_I}}\left(\tilde{Z}_t + \alpha_I\tilde{X}_{I,t} + \frac{\tilde{L}_{I,t}}{\delta} - \frac{(1-\delta)}{\delta}\tilde{L}_{I,t-1}\right)$$

3. Wholesale sector output

$$Y_t^w = (1-\gamma_1)Y_{F,t} + \gamma_1 Y_{I,t}$$
$$\rightarrow \tilde{Y}_t^w = (1-\gamma_1)\frac{Y_F^{ss}}{(Y^w)^{ss}}\tilde{Y}_{F,t} + \gamma_1\frac{Y_I^{ss}}{(Y^w)^{ss}}\tilde{Y}_{I,t}$$

4. Formal labor production function

$$Y_{F,t} = Z_t L_{F,t}$$
$$\rightarrow \tilde{Y}_{F,t} = \tilde{Z}_t + \tilde{L}_{F,t}$$

5. Informal labor production function

$$Y_{I,t} = \omega_I Z_t L_{I,t}$$
$$\rightarrow \tilde{Y}_{I,t} = \tilde{Z}_t + \tilde{L}_{I,t}$$

6. Aggregate labor constraint

$$L_t = L_{F,t} + L_{I,t}$$
$$\rightarrow L^{ss}(1+\tilde{L}_t) = L_F^{ss}(1+\tilde{L}_{F,t}) + L_I^{ss}(1+\tilde{L}_{I,t})$$
$$\rightarrow \tilde{L}_t = \frac{L_F^{ss}}{L^{ss}}\tilde{L}_{F,t} + \frac{L_I^{ss}}{L^{ss}}\tilde{L}_{I,t}$$

7. Productivity process

$$lnZ_t = \rho Z_{t-1} + \epsilon_{Z,t}$$
$$\rightarrow ln(Z) * e^{\tilde{Z}_t} = \rho ln(Z) * e^{\tilde{Z}_{t-1}} + \epsilon_{Z,t}$$
$$\rightarrow ln(Z) + \tilde{Z}_t = \rho(ln(Z) + \tilde{Z}_{t-1}) + \epsilon_{Z,t}$$

In the steady state $ln(Z) = \rho ln(Z)$

$$\rightarrow \tilde{Z}_t = \rho\tilde{Z}_{t-1} + \epsilon_{Z,t}$$

8. Formal labor market tightness

$$X_{F,t} = \frac{A_{F,t}}{U_t}$$

$$\rightarrow ln(X_{F,t}) + \tilde{X}_{F,t} = ln(L_F^{ss} - (1-\delta)L_F^{ss}) + \frac{L_F^{ss}}{(L_F^{ss} - (1-\delta)L_F^{ss})}\tilde{L}_{F,t} - \frac{(1-\delta)L_F^{ss}}{(L_F^{ss} - (1-\delta)L_F^{ss})}\tilde{L}_{F,t-1} - ln(U_t) - \tilde{U}_t$$

$$\rightarrow ln(X_{F,t}) + \tilde{X}_{F,t} = ln(L_F^{ss}) + \frac{L_F^{ss}}{\delta L_F^{ss}}\tilde{L}_{F,t} - \frac{(1-\delta)L_F^{ss}}{\delta L_F^{ss}}\tilde{L}_{F,t-1} - ln(U_t) - \tilde{U}_t$$

$$\rightarrow \tilde{X}_{F,t} = \frac{1}{\delta}\tilde{L}_{F,t} - \frac{(1-\delta)}{\delta}\tilde{L}_{F,t-1} - \tilde{U}_t$$

9. Informal labor market tightness.

$$X_{I,t} = \frac{A_{I,t}}{U_t}$$

$$\rightarrow ln(X_{I,t}) + \tilde{X}_{I,t} = ln(L_F^{ss} - (1-\delta)L_I^{ss}) + \frac{L_I^{ss}}{(L_I^{ss} - (1-\delta)L_I^{ss})}\tilde{L}_{I,t} - \frac{(1-\delta)L_I^{ss}}{(L_I^{ss} - (1-\delta)L_I^{ss})}\tilde{L}_{I,t-1} - ln(U_t) - \tilde{U}_t$$

$$\rightarrow ln(X_{I,t}) + \tilde{X}_{I,t} = ln(L_I^{ss}) + \frac{L_I^{ss}}{\delta L_I^{ss}}\tilde{L}_{I,t} - \frac{(1-\delta)L_I^{ss}}{\delta L_I^{ss}}\tilde{L}_{I,t-1} - ln(U_t) - \tilde{U}_t$$

$$\rightarrow \tilde{X}_{I,t} = \frac{1}{\delta}\tilde{L}_{I,t} - \frac{(1-\delta)}{\delta}\tilde{L}_{I,t-1} - \tilde{U}_t$$

10. Unemployment dynamics

$$U_t = 1 - (1-\delta)(L_{F,t-1} + L_{I,t-1})$$

$$\rightarrow ln(U_t) = ln(1 - (1-\delta)(L_{F,t-1} + L_{I,t-1}))$$

$$\rightarrow ln(U_t) + \tilde{U}_t = ln(1-(1-\delta)(L_F^{ss}+L_I^{ss})) - \frac{(1-\delta)}{1-(1-\delta)(L_F^{ss}+L_I^{ss})}\left[L_F^{ss}\tilde{L}_{F,t-1} + L_I^{ss}\tilde{L}_{I,t-1}\right]$$

$$\rightarrow \tilde{U}_t = -\frac{(1-\delta)}{1-(1-\delta)(L_F^{ss}+L_I^{ss})}\left[L_F^{ss}\tilde{L}_{F,t-1} + L_I^{ss}\tilde{L}_{I,t-1}\right]$$

11. Aggregate supply condition (New-Keynesian Phillips Curve)

$$P_t = \left[(1-\lambda)(\bar{P}_t)^{1-\phi} + \lambda(P_{t-1})^{1-\phi}\right]^{\frac{1}{1-\phi}}$$

$$\rightarrow \tilde{\pi}_t = \kappa \tilde{mc}_t^R + \beta \tilde{\pi}_{t+1}$$

with

$$\kappa = \frac{(1-\lambda)(1-\lambda\beta)}{\lambda}$$

12. Labor demand for formal workers

$$w_{F,t} = (1-\gamma_1) mc_t^R Z_t - G_{F,t} + \beta(1-\delta)\mathbb{E}_t\left[\frac{u'(c_{t+1})}{u'(c_t)} G_{t+1}\right]$$

$$\begin{aligned}\rightarrow \tilde{w_{F,t}} = \frac{1}{w_F^{ss}}\Big((1-\gamma_1)(mc^R)^{ss} Z^{ss}(\tilde{mc}_t^R + \tilde{Z}_t) - \Theta_F Z^{ss}(X_F^{ss})^{\alpha_F}(\tilde{Z}_t + \alpha_F \tilde{X}_{F,t})\Big) \\ + \frac{1}{w_F^{ss}}(1-\delta)\beta\Theta_F Z^{ss}(X_F^{ss})^{\alpha_F}\mathbb{E}_t\left[(\tilde{Z}_{t+1} + \alpha_F \tilde{X}_{F,t+1} - \frac{1}{\sigma}(\tilde{C}_{t+1} - \tilde{C}_t))\right]\end{aligned}$$

With $(mc^R)^{ss} = \frac{\phi-1}{\phi}$

13. Labor supply of formal workers

$$w_{F,t} = \eta\left[L_t\right]^{\varphi} C_t^{\frac{1}{\sigma}} + \xi\left(G_{F,t} - (1-\delta)\mathbb{E}_t\left[\Omega_{t,t+1}\left\{(1-X_{F,t+1})G_{F,t+1} - X_{I,t+1}G_{I,t+1}\right\}\right]\right)$$

$$\begin{aligned}\rightarrow \tilde{w}_{F,t} = \frac{1}{w_F^{ss}}\left(\eta(L^{ss})^{\varphi}(C^{ss})^{\frac{1}{\sigma}}(\varphi\tilde{L}_t + \frac{1}{\sigma}\tilde{C}_t)\right) \\ + \frac{1}{w_F^{ss}}\left(\xi G_F^{ss}\tilde{G}_{F,t} - (1-\delta)\beta\xi G_F^{ss}(\frac{1}{\sigma}(\tilde{C}_t - \tilde{C}_{t+1}) + \tilde{G}_{F,t+1})\right) \\ + \frac{1}{w_F^{ss}}\xi\beta(1-\delta)\left[X_F^{ss}G_F^{ss}(\frac{1}{\sigma}(\tilde{C}_t - \tilde{C}_{t+1}) + \tilde{X}_{F,t+1} + \tilde{G}_{F,t+1})\right] \\ + \frac{1}{w_F^{ss}}\xi\beta(1-\delta)\left[X_I^{ss}G_I^{ss}(\frac{1}{\sigma}(\tilde{C}_t - \tilde{C}_{t+1}) + \tilde{X}_{I,t+1} + \tilde{G}_{I,t+1})\right]\end{aligned}$$

14. Labor demand for informal workers

$$w_{I,t} = mc_t^R \gamma_1 \omega Z_t - G_{I,t} + (1-\delta)\beta\mathbb{E}_t\left[\frac{u'(c_{t+1})}{u'(c_t)} G_{I,t+1}\right]$$

$$\begin{aligned}\rightarrow \tilde{w_{I,t}} = \frac{1}{w_I^{ss}}\Big((\gamma_1)(mc^R)^{ss} Z^{ss}(\tilde{mc}_t^R + \tilde{Z}_t) - \Theta_I Z^{ss}(X_I^{ss})^{\alpha_I}(\tilde{Z}_t + \alpha_I \tilde{X}_{I,t})\Big) \\ + \frac{1}{w_I^{ss}}(1-\delta)\beta\Theta_I Z^{ss}(X_I^{ss})^{\alpha_I}\mathbb{E}_t\left[(\tilde{Z}_{t+1} + \alpha_I \tilde{X}_{I,t+1} - \frac{1}{\sigma}(\tilde{C}_{t+1} - \tilde{C}_t))\right]\end{aligned}$$

With $(mc^R)^{ss} = \frac{\phi-1}{\phi}$

15. Labor supply for informal workers

$$w_{I,t} = \eta\left[L_t\right]^{\varphi} C_t^{\frac{1}{\sigma}} + \xi\left(G_{I,t} - (1-\delta)\mathbb{E}_t\left[\Omega_{t,t+1}\left\{(1-X_{I,t+1})G_{I,t+1} - X_{F,t+1}G_{F,t+1}\right\}\right]\right)$$

$$\rightarrow \tilde{w}_{I,t} = \frac{1}{w_I^{ss}}\left(\eta(L^{ss})^{\varphi}(C^{ss})^{\frac{1}{\sigma}}(\varphi\tilde{L}_t + \frac{1}{\sigma}\tilde{C}_t)\right)$$
$$+\frac{1}{w_I^{ss}}\left(\xi G_I^{ss}\tilde{G}_{I,t} - (1-\delta)\beta\xi G_I^{ss}(\frac{1}{\sigma}(\tilde{C}_t - \tilde{C}_{t+1}) + \tilde{G}_{I,t+1})\right)$$
$$+\frac{1}{w_I^{ss}}\xi\beta(1-\delta)\left[X_I^{ss}G_I^{ss}(\frac{1}{\sigma}(\tilde{C}_t - \tilde{C}_{t+1}) + \tilde{X}_{I,t+1} + \tilde{G}_{I,t+1})\right]$$
$$+\frac{1}{w_I^{ss}}\xi\beta(1-\delta)\left[X_F^{ss}G_F^{ss}(\frac{1}{\sigma}(\tilde{C}_t - \tilde{C}_{t+1}) + \tilde{X}_{F,t+1} + \tilde{G}_{F,t+1})\right]$$

16. Formal hiring cost

$$G_{F,t} = Z_t\Theta_F X_{F,t}^{\alpha}$$
$$\rightarrow \tilde{G}_{F,t} = \tilde{Z}_t + \alpha\tilde{X}_{F,t}$$

17. Informal Hiring Cost

$$G_{I,t} = Z_t\Theta_I X_{I,t}^{\alpha}$$
$$\rightarrow \tilde{G}_{I,t} = \tilde{Z}_t + \alpha\tilde{X}_{I,t}$$

18. Marginal cost of retail firms

$$\left[(1-\gamma_1) - \gamma_1\omega\right] Z_t mc_t^R = (1+\xi)\left[(G_{F,t} - G_{I,t}) - (1-\delta)\beta\mathbb{E}_t\left\{\left(\frac{C_t}{C_{t+1}}\right)^{\frac{1}{\sigma}}(G_{F,t+1} - G_{I,t+1})\right\}\right]$$

Let $\Phi = \left[(1-\gamma_1) - \gamma_1\omega\right]$ and knowing that $(mc^R)^{ss} = \frac{\phi-1}{\phi}$ the log linearized version of the marginal cost becomes:

$$\rightarrow \tilde{mc}_t = \frac{1}{(mc^R)^{ss}}\frac{1}{\Phi}\frac{1}{Z^{ss}}\left[(1+\xi)\left\{(G_F^{ss}\tilde{G}_{F,t} - G_I^{ss}G_{I,t}^{ss})\right\}\right]$$
$$-\frac{1}{(mc^R)^{ss}}\frac{1}{\Phi}\frac{1}{Z^{ss}}(1+\xi)(1-\delta)\beta\mathbb{E}_t\left(G_F^{ss}(\sigma^{-1}(\tilde{C}_t - \tilde{C}_{t+1}) + \tilde{G}_{F,t+1}\right)$$
$$+\frac{1}{(mc^R)^{ss}}\frac{1}{\Phi}\frac{1}{Z^{ss}}(1+\xi)(1-\delta)\beta\mathbb{E}_t\left(G_I^{ss}(\sigma^{-1}(\tilde{C}_t - \tilde{C}_{t+1}) + \tilde{G}_{I,t+1})\right) - \tilde{Z}_t$$

Taylor Rule

19. Forward Looking

$$R_t = \mathbb{E}_t\left[\bar{R}\left(\pi_{t+1}\right)^{\phi_\pi}\left(\frac{Y_{t+1}^R}{\bar{Y}^R}\right)^{\phi_Y}\right]\epsilon_{R,t}$$
$$\rightarrow \tilde{R}_t = \phi_\pi\mathbb{E}_t\left[\tilde{\pi}_{t+1}\right] + \phi_y\mathbb{E}_t\left[\tilde{Y}_{t+1}^R\right] + \epsilon_{R,t}$$

20. Current Looking

$$R_t = \left[\bar{R} \left(\pi_t \right)^{\phi_\pi} \left(\frac{Y_t^R}{\bar{Y}^R} \right)^{\phi_Y} \right] \epsilon_{R,t}$$

$$\rightarrow \tilde{R}_t = \phi_\pi \tilde{\pi}_t + \phi_y \tilde{Y}_t^R + \epsilon_{R,t}$$

4 Calibration

In order to solve the model we use the DYNARE1 [1] package, which is a free toolkit for Matlab, primarily to solve, simulate, and estimate DSGE models. After the equations are log-linearized I use this toolkit to generate impulse-response functions to help understand the dynamic behavior of the economy after a shock. The parameters are especially chosen to calibrate the model for Mexico. The calibration employed is as follows.

The time period is assumed to be a quarter. As is standard in the literature, we set the discount factor $\beta = 0.99$, the intertemporal elasticity of substitution in consumption $\sigma = 1$ and the inverse of Frisch elasticity of labor supply $\varphi = 1$. With no direct estimate of the parameter of lower productivity in the informal sector ω, we calibrate it at $\omega = 0.5$ in line with the estimates of Fernández and Meza (2015) of $A^F/A^I = 2.1901$ with A^F being the productivity of the formal sector and A^I being the productivity of the informal sector.

For simplicity we set $\eta = 1$. Regarding the job destruction rate, Bosch and Esteban-Pretel (2012) estimate that the job destruction rate δ varies from 0.03 to 0.10 in developing countries. Kaplan *et al.* (2003), using data from I.M.S.S.[2], estimates that the average job destruction rate for formal workers in Mexico during the period 1994-2000 is 0.135 and 0.132 for the year 2000. Blanchard and Gali (2010) use the job finding rate (x) and the unemployment rate (u) to find the job destruction rate. Let δ be the job destruction rate and define it as $\delta = \frac{ux}{(1-u)(1-x)}$. Ball *et al.* (2013) estimate the long run unemployment rate of Mexico at $u = 3.28$ and Bosch and Maloney (2007) estimate the job finding rate in Mexico at $x = 0.6$ implying a job destruction rate $\delta = 0.0508$ which relates to the findings of Bosch and Esteban-Pretel (2012). Consequently, we set $\delta = 0.05$

[1] DYNARE (version 4.4.3) is used to solve the model in this thesis.http://www.dynare.org

[2] IMSS stands for Instituto Mexicano del Seguro Social, which creates mandatory insurance for formal workers paid in part by the employer and the employee.

Regarding the elasticities of hiring costs α_F and α_I, if we assume a matching function of the form $H = ZU^{\vartheta}V^{1-\vartheta}$, we have $V/H = Z^{\frac{1}{\vartheta-1}}(H/U)^{\frac{\vartheta}{1-\vartheta}}$. With a simple mapping between the model proposed by Blanchard and Gali (2010) and the standard DMP model we can obtain the value of $\alpha_F = \vartheta/(1-\vartheta)$. With estimates from Petrongolo and Pissarides (2001), we set $\vartheta = 0.5$ implying therefore that $\alpha_F = 1$. However, this is only for the relevant elasticity for formal jobs. Lacking any direct evidence or source to use to estimate the elasticity of hiring cost regarding informal jobs α_I, we set $\alpha_I = 0.75$.

Bargaining power is also relevant in the determination of wages. Botero *et al.* (2004) estimate bargaining power at $\xi = 0.5774$. The parameter value used in our simulations is $\xi = 0.6$ in line with the value from Botero *et al.* (2004) and the original values from Castillo and Montoro (2012).

For the elasticity of substitution between retail goods ϕ, estimates vary from 7 (Batini *et al.*, 2011) to 11 (Céspedes, *et al.*, 2005). We will choose $\phi = 11$ implying $\mathcal{M} = 1.1$ as the gross steady state markup.

Regarding the parameters of the hiring costs, Θ_F and Θ_I, to the best of our knowledge no direct empirical estimates exist. Therefore we follow Castillo and Montoro (2012) and set $\Theta_F = 2.5$ for the value of the hiring cost for formal contracts. In what follows, we set $\Theta_I = 0$, implying that there is no hiring cost for informal labor.

Ramos-Francia and Torres (2006), using monthly data of the Mexican economy for the period January 1992 to June 2006, investigate the degree of price rigidity in Mexico. Using this data, they estimate $\lambda = 0.858$ implying that on average prices remain fixed for 2.3 quarters. Therefore, we choose $\lambda = 0.858$ in our calibration. We impose $\gamma_1 = 0.20$ as we lack of any direct evidence of this value.

In addition, interest rate shock is modeled as an AR(1) process, therefore we set the $\rho_r = 0.7$ as the value for the coefficient for the first lag. Productivity is modeled also as an AR(1) process, thus we set the value of $\rho_z = 0.70$ for the coefficient. For the Central Bank policy rule, we set $\phi_\pi = 1.5$ and $\phi_y = 0.3$.

The complete list of the values used in the calibration of the model is summarized in Table 4.1.

Table 4.1: Parameter values used for calibration

β	0.99	Discount factor
σ	1	Intertemporal elasticity of substitution in consumption
φ	1	Inverse of Frisch elasticity of labour supply
η	1	Marginal disutility from working
δ	0.05	Job separation rate
ϕ	11	Elasticity of substitution of retail goods
γ_1	0.2	Informal technology bias
ω	0.50	Informal productivity parameter
ρ_Z	0.70	Productivity shock persistence
Θ_F	2.5	Formal hiring cost parameter
Θ_I	0	Informal hiring cost parameter
α_F	1	Labor market tightness parameter for formal hiring cost
α_I	0.75	Labor market tightness parameter for informal hiring cost
ξ	0.6	Bargaining power
λ	0.86	Degree of price stickiness
ϕ_π	1.5	Taylor rule inflation response coefficient
ϕ_y	0.3	Taylor rule output response coefficient
ρ_r	0.7	Interest Rate Shock persistence coefficient

5 Results - impulse response analysis

In this section I explore the transmission mechanism of the two exogenous shocks, by means of an impulse response analysis. I investigate two different versions of the model: a benchmark model, in which I will only focus in an economy where the only possibility for hiring a worker is by means of a formal contract. However, in order to fulfill its obligations, the firm must pay the appropriate hiring cost. For the model with informality, wholesale forms have the possibility of hiring informal labor. The aim is to try to understand the role of informality for the macroeconomy and how the mechanism of adjustment differs in economies with large informal sectors.

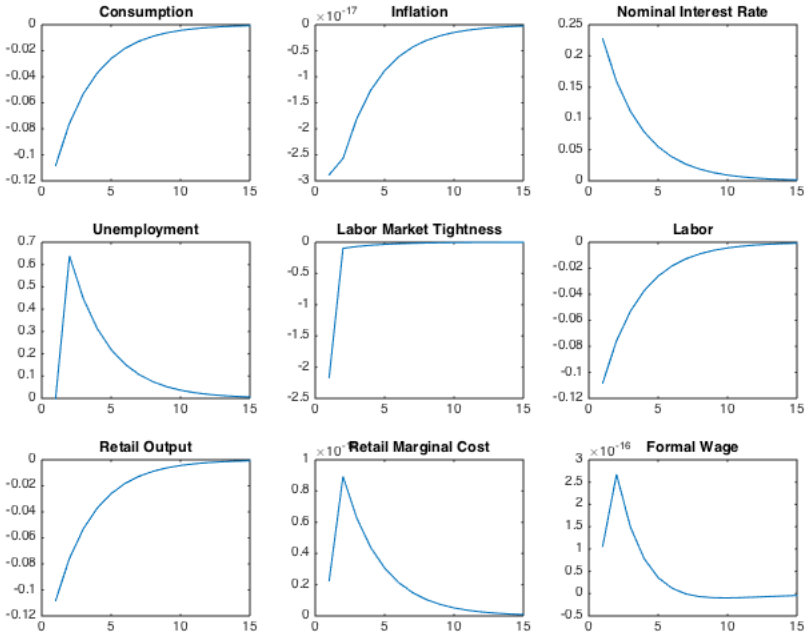
I first consider the effects of the interest-rate shock and then I study the implications of a productivity shock. Both shocks are analyzed under the forward-looking version of Taylor rule.

5.1 Interest rate shocks

5.1.1 Baseline Model

After the interest-rate shock, a tightening of monetary policy generates a fall in aggregate demand resulting in lower consumption, retail output, and inflation. Retail marginal costs increase due to higher formal wages. Given the increase in the cost of labor and the decrease in demand, it is optimal for firms to reduce labor as is shown in Figure 5.1, which increases unemployment in the economy. This results in an increase in labor market tightness: with higher unemployment, the vacancies the firm are filling are less relative to the number of unemployed workers willing to incorporate into jobs. Consequently there is a reduction in the probability of getting hired.

Figure 5.1: Interest rate shock in the benchmark model.



In this basic framework, where there are only formal labor workers and wholesale firms have to pay the cost for hiring formal labor, the dynamics of the labor market are not adjusted by hours worked, as in the basic New Keynesian model (see for example Gali, 2010), but by the number of workers employed, that is, at the extensive margin. With the interest rate shock, wholesale firms have to pay a higher wage. This higher wage causes the wholesale firm to adjust labor employed and puts pressure in its marginal cost. To adjust wholesale marginal costs, wholesale firms readjust in the short-run its production, and as labor is the only factor of production, formal labor decrease. Retail firms buy the wholesale good at its marginal price. With an increase in wholesale marginal cost, retail marginal costs increases as well, so retail firms adjusts downward their production.

We also know that the firms and workers share the surplus of the vacancy filled, that is, firms will not pay in a future period the cost of hiring a worker. Wages are set by the Nash bargaining process and they are above the perfectly competitive level. After the shock and the increase in wages and the adjustment in wholesale marginal costs, unemployment increases and more workers are expecting to find a job, thereby there is a lower probability of finding a job for every unemployed worker. This is observed in the decrease of the labor market tightness, which is

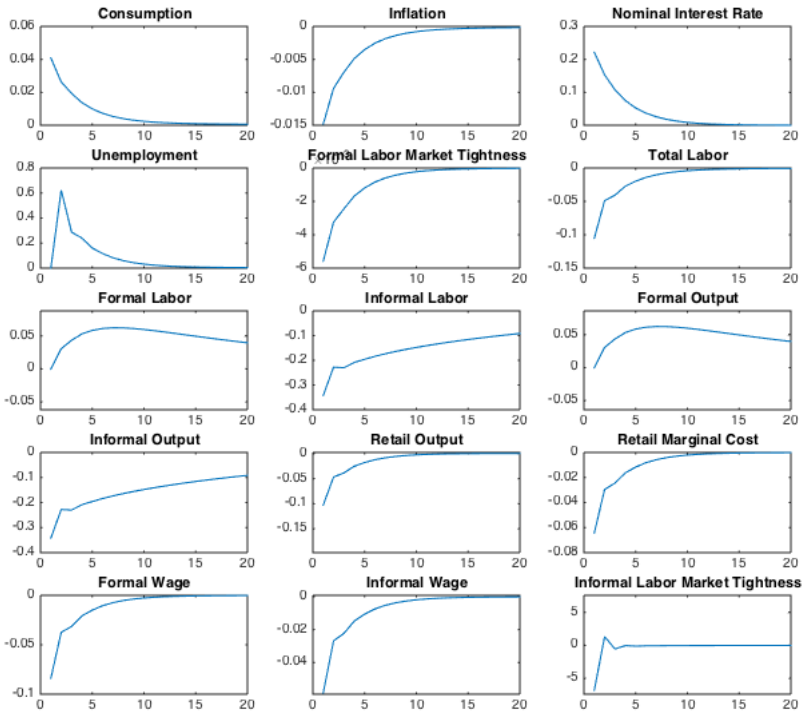
our measure of hiring relative to unemployment. We also observe that although wages are higher, consumption decreases by a higher unemployment. Then, we argue that the monetary policy shock has a negative effect in the economy as firms, both wholesale and retail, decrease its production, households consume less and the fraction of household members employed is lower.

5.1.2 Informality Model

Allowing informal hiring changes the dynamics and analyzing its behavior reveals new mechanisms. By inspection of Figure 5.2, wholesale firms respond by increasing formal labor employment. We can observe also that both informal employment and informal wage is lower. This could be considered contradictory, but in our set-up formal workers are more productive than informal workers, and by the dynamics of the interest shock, we can argue that firms increase its formal employment due to the fact of higher productivity and lower wages for both kinds of labor.

The increase in formal employment does not offset the fall in informal employment, thus increasing unemployment in the economy. Then, by the link between aggregate formal hiring and aggregate informal hiring and unemployment, market tightness for both types of labor decreases. As explained before, it is possible to consider the labor market tightness as the probability of getting hired. Given that unemployment increased, there are more workers queuing for job opportunities. We can discuss about the form of the impulse-response function for informal labor market tightness. We consider that the first peak occurs at the same time when unemployment has its maximum. After the economy recovers, unemployment, and by definition informal labor market tightness, returns to its steady state. The reason for a quick recovery could be considered because of the recovery of total employment which is composed of a steep increase and slow decrease of formal employment and a steady recovery of informal employment.

Figure 5.2: Interest rate shock in the informality model.



A key difference between the baseline model and the informality model is the behavior of consumption which increases in response to higher interest rates when allowing for informal employment. We consider this effect occurs due to the fact that employment of formal workers increases whereas employment of informal workers falls. This could be considered because although both formal and informal wage falls, the formal wage is still higher than the informal wage allowing for higher consumption. This result suggests that formal labor is countercyclical after an interest rate shock.

We argue that the decision of wholesale firms to hire a combination of formal and informal labor such that in equilibrium marginal cost for both type of technologies remains equal is the driving force behind the dynamics of the economy after the monetary policy shock.

Informal wages are lower, thus marginal cost for the informal technology. Wholesale firms adjusts its formal employment to maintain equilibrium. Formal labor is increased after the shock because of the lower wage and its higher productivity compared to the informal workers. Thus, by balancing the marginal cost for each type of technology, wholesale firms drives the mecha-

nism and dynamics of the economy.

Both formal and informal wages decrease, thereby wholesale marginal cost decreases. The increase in output produced by formal labor is not enough to offset the decrease the output produced using informal labor, and as wholesale firms try to maintain in equilibrium the marginal cost for both technologies, the reduction in informal employment is higher than the increase in formal employment, causing wholesale output and marginal cost to decrease after the shock.

Retail firms buy wholesale goods at their marginal price for producing its retail good variety. Because of the reduction in wholesale production and marginal cost, retail production and marginal cost decreases. Consumption increases and we argue that this occurs owing to the fact that formal employment increased.

With regard to the effects on unemployment under both models, we observe that the effect of the shock on unemployment is almost the same in both models. This can be considered possible because of the substitution of informal labor from formal labor. However, inflation decreases more in the informality model than in the benchmark model. We argue this occurs because of the response of formal and informal labor and the increase in formal employment as opposed to the decrease in formal employment in the benchmark model.

Our results are robust to a current looking Taylor rule, suggesting that the underlying mechanism discussed, that is, the equilibrium condition on marginal cost of the formal and informal technologies helps explaining the dynamics. But there are small differences worth noting. Under a current looking regime[1] unemployment is affected less by the change in employment composition due to the central bank acting at the time of the shock. However, under a current looking Taylor rule, formal employment after the shock is higher than in the forward looking version rather than a decrease less steep of informal employment. As formal labor is the only production input for the formal technology, its output increases more when the central bank follows a current looking Taylor rule. Consumption, in turn, increases more under the current looking version because of the higher formal employment when compared to the forward looking Taylor rule.

[1] See the appendix for the impulse response function under a current looking Taylor rule.

We believe that it is important to analyze the dynamics of the model under a productivity shock to understand the novelty of your findings. In the next subsection we present the effects of a positive one-standard-deviation productivity shock on our model with a forward looking Taylor rule.

5.2 Productivity shock

In this section we consider a one-standard-deviation positive productivity shock to analyze the mechanisms around its steady state.

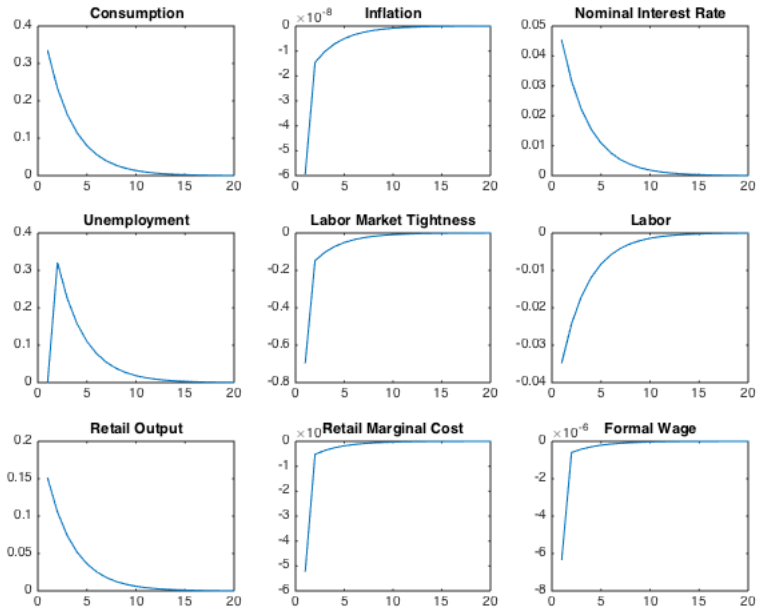
5.2.1 Baseline Model

When compared to the textbook New Keynesian model (see for example Gali, 2010), this model offers two possible states: either being employed under a formal contract or being unemployed as opposed to adjustment in the hours worked. As workers become more productive real wages fall but by a very small amount and there is less employment. This occurs because the hiring cost is a function of the productivity. When productivity increases, hiring costs increase, thus causing firms to delay hiring as the cost of hiring surpasses the expected saving on hiring costs of not having to fill that vacancy.

Retail marginal cost is lower due to lower wages in the wholesale sector, and as workers in that sector are more productive, production in the wholesale sector increases, thus increasing retail output. Regarding the effect on workers, with higher unemployment at the shock affects the economy, the labor market tightness decreases.

In the case of a productivity shock, dynamics affect directly labor demand and supply from wholesale firms. Specifically, labor demand is affected due to optimization taking into account the hiring costs. Labor becomes more productive, and given the transitory nature of the productivity shock, firms find profitable to delay hiring due to lower productivity in the future translating into lower hiring costs. Wholesale firms require less workers to maintain and increase its production so they lower their labor demand and adjust labor at the extensive margin. The outcome of the shock in the labor market is an increase in unemployment, thereby de-

Figure 5.3: Productivity shock in the benchmark model.



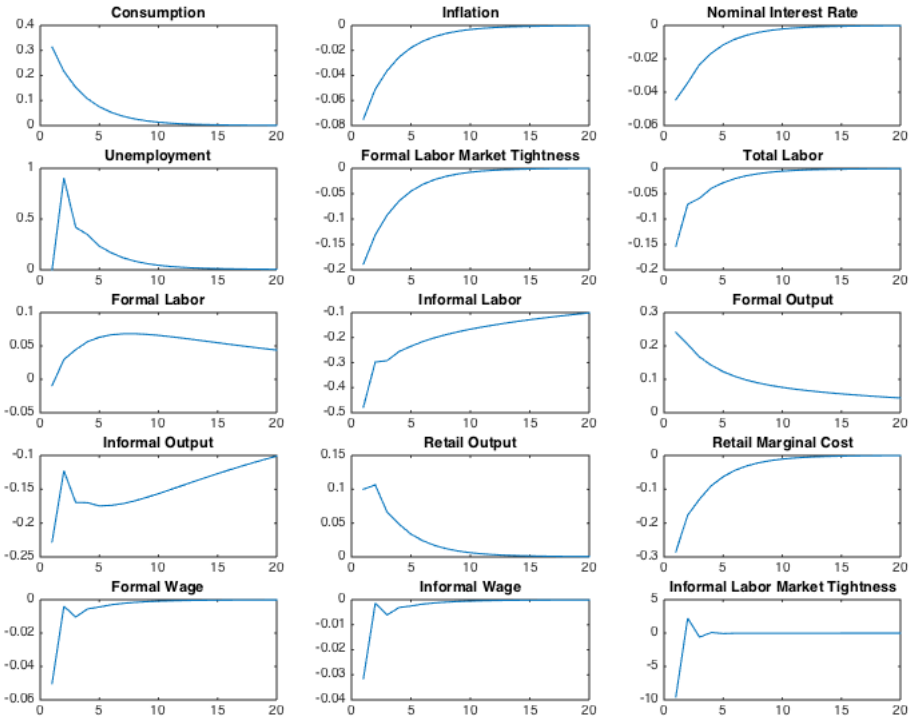
creasing the value of the labor market tightness, that is, reducing the probability of finding a job. The wage setting process that sets the labor supply is affected by productivity reducing the formal wage paid by wholesale firms and their marginal cost. With the productivity shock, the wholesale supply curve shifts downward so that at every price, the supply is higher. Retail firms pay the wholesale goods at their reduced marginal cost, and by the effect of the productivity shock retail marginal costs are lower. Due to lower marginal cost in the retail sector, caused by the mechanism exposed above, inflation in the economy decreases.

An interesting difference between the adjustment of a monetary policy shock and a productivity shock in the benchmark model occurs because of hiring costs being a function of productivity. In both cases unemployment increases but this happens with different mechanisms. With a productivity shock, firms find profitable to delay hiring and dismissing workers although the wage is lower but the hiring cost increases; in the case of an interest rate shock formal wage increases and this causes the wholesale firms to adjust its employment. This effects in turn cause the decrease in the retail marginal cost after a productivity shock and the increase in the retail marginal cost after the central bank tightens its monetary policy.

5.2.2 Informality Model

With the increase in productivity firms find more profitable to substitute informal labor with formal labor. However, informal employment decreases more than the increase in formal employment. Thus, unemployment in the economy is higher affecting labor market tightness for formal and informal employment. Informal labor market tightness is recovering faster than its formal counterpart because of the absence of hiring costs in informal employment.

Figure 5.4: Productivity shock in the informality model.



Wholesale firms have a lower marginal cost due to the lower wage obtained by Nash bargaining. This occurs after an increase in unemployment and the effect of productivity. In turn, retail marginal cost is lower but production increases. The increase in unemployment affects the labor market tightness in both sectors. As explained before, this market tightness can be considered as the probability of getting hired, then, a decrease in the labor market tightness, or in this case, a negative percentage deviation from its steady state, is considered a negative effect on the labor market.

Retail output is produced from wholesale production, which in turn uses two types of technologies: a formal one and an informal one depending on the type of labor contractual arrangements. Therefore, retail output does not increase by the same amount as the increase in the production using the formal technology because of the diminishing informal technology output. In addition, retail marginal cost is lower because of lower wages in both formal and informal labor. The lower wage in formal sector partially helps the economy to increase the formal employment. The results suggest that formal employment is now procyclical whereas informal employment is countercyclical.

We consider that the dynamics observed in the informality model is best explained because of the hiring costs being a direct function of the productivity and the effect on labor demand and supply by the wholesale firms. From the labor demand side, after the transitory increase in productivity, hiring costs for formal labor increase. If this effect was isolated on the formal labor employment, it will decrease as it was shown using the benchmark model. The increase in productivity causes hiring formal costs to increase. Wholesale firms balance the expected savings of hiring a worker now, which are not enough to offset the higher hiring cost at the time of the shock, and the lower formal wage due to the effect of productivity on the Nash bargaining process. While the informal wage decreases making more attractive to hire informal workers, formal wage decreases more, and considering the higher productivity of formal workers, wholesale firms reallocate resources towards formal labor maintaining the marginal costs of both type of technologies in equilibrium. Regarding the labor market, the increase in unemployment affects the formal and informal labor market tightness lowering the probability of finding a job.

While the main force driving the response of the economy after the productivity shock and the interest rate shock is the decision to maintain in equilibrium the marginal costs of both type of technologies in the wholesale sector, the mechanisms differ by the effect of hiring costs. Hiring costs are increasing in productivity and are affected directly by it, thus causing the dynamics seen in the model. Although unemployment is higher after the productivity shock and causes that formal labor market tightness to decrease more when compared to the effect of the interest rate shock, this is not enough to offset the effect of the increase in productivity in increasing hiring costs. Wages fall more after the interest rate shock allowing formal labor employment

to increase more with respect to the result of the productivity shock. Another key macroeconomic variable is inflation, which decreases more after the interest rate shock. In this sense, formal employment benefits of its higher productivity after both shocks and the substitution of informal labor is stronger in the informality model after a productivity shock. We consider that both the choice of $\omega = 0.5$, as it relates to increase the breach in employment after the shocks, and the allocation of labor between technologies based on the costs of each type of labor by the wholesale firm can account to the differences in the dynamics between the two types of shocks.

6 Conclusions

The aim of this thesis was to investigate how the macroeconomy adjusts after a monetary policy shock in the presence of an informal labor market.

Using the model of Castillo and Montoro (2012) the thesis introduced informality into a sticky-price DSGE model. In this model, infinitely-lived households derive utility from consumption and determine the fraction of supply labor they provide to each sector. The retail sector firms buy wholesale goods at marginal cost, transform them into a differentiated final good, which is sold to household for consumption purposes. Retail firms operate in a monopolistic competition environment and set prices in a staggered fashion according to Calvo (1983). Wholesale firms are perfectly competitive and hire labor, deciding between formal and informal contracts, to produce intermediate goods. Informal labor is assumed to be cheaper due and less productive than its formal counterpart, and wholesale firms must pay a hiring cost to recruit both formal and informal workers. In our set-up, informal hiring costs are calibrated to zero. Both formal and informal wages are determined by Nash bargaining. Following empirical evidence (Clarida *et al*. (1998), (2000); Orphanides (2001),(2004); Mihailov (2006)), we assumed that the central bank uses a forward-looking Taylor-type rule, where the nominal interest rate reacts to expected future inflation and output.

The model proposed in this exercise allowed to include informality in the economy. However, informality arises from informal hiring of the wholesale firms rather than allowing for an informal sector in the economy. In this model, by hiring informal workers, the firm does not pay any costs. However this set-up generates dynamics that allow substitution of informal labor with formal labor as it was shown. This effect occurs of a lower productivity of informal workers, a fact that is documented in the literature. In addition, informal employment is a consequence of the rationing exercised by the wholesale firms because of the hiring costs.

The model of Castillo and Montoro (2012) predicts that adding the informal sector results in different implications for certain important macro variables depending on whether the shock is monetary or productivity. After an interest rate shock unemployment rises, labor market tightness falls but formal wages and retail marginal costs increase. However, with a positive productivity shock, unemployment rises and labor market tightness falls as in the monetary policy shock. But in this case formal wages and retail marginal costs fall. We consider that the Nash Bargaining process allow us to propose an explanation. Workers and firms take into account current and one-period ahead hiring costs and labor market tightness. Due to a higher unemployment in period t, hiring cost decreases due to lower labor market tightness, but in the next period as economy returns to its steady state hiring cost will increase again. Balancing this effects generates a higher wage rate, thus increasing retail marginal costs. The same bargaining process can be used to analyze the effect after the productivity shock. However, after a productivity shock unemployment increases less than after an interest rate shock, then labor market tightness decreases less in the context of a productivity shock. In Castillo and Montoro (2012) firms employ both types of labor which allows firms to immediately change their labor input by increasing or decreasing the informal component of employment. However, this seems to be an inappropriate assumption for Mexico.

Another interesting result is the different behavior of formal and informal employment after the different shocks. With monetary policy shocks, formal employment is countercyclical whereas informal employment is procyclical. This suggests that the Castillo and Montoro (2012) model is not so good for Mexico. Business cycle statistics for Mexico suggest that over the cycle informal employment is countercyclical but formal employment is cyclical (see Fernandez and Mesa, 2015). There is also the issue of different consumption dynamics. More investigation is needed to see if such behavior is found in the business cycle data for countries with large informal sectors.

With productivity shocks formal labor is procyclical whereas informal employment is countercylical. I consider that the different mechanisms given the type of the shocks depends on the calibration for the relative difference of productivity between the formal and informal sector and that hiring cost parameter in the in- formal employment is zero. In addition, unemployment

in both models increase in the same quantity after the interest rate shock, but after the productivity shock unemployment increases more in the informality model than in the benchmark model.

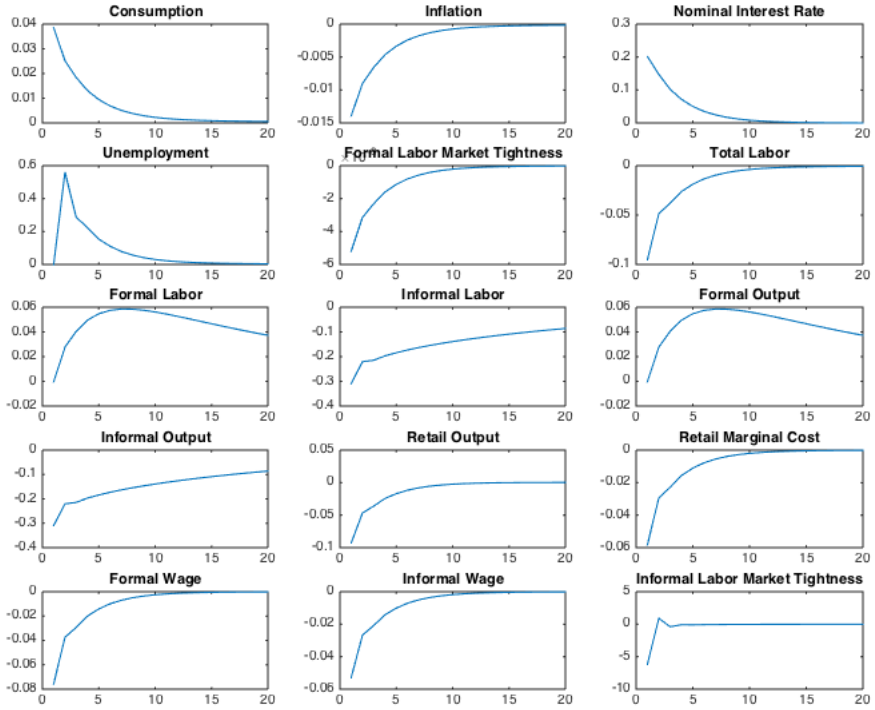
Further analysis will use a different calibration and undertake sensitivity analysis. We could assume non-zero hiring cost parameter to see the different dynamics. Another possibility could be allowing consumption of both informal and informal goods, which are documented to be highly sustitutable (Restrepo-Echavarría 2011), and modeling an informal sector rather than allowing informal hiring in the wholesale sector.

7 Appendix

7.1 Interest rate shock. Informality model

To reinforce the robustness of our findings we present the impulse-response function of the informality model under a current-looking version of the Taylor-type rule.

Figure 7.1: Interest rate shock in the informality model with current looking Taylor Rule.



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