Baltimore, Maryland, U. S. A.

Junio 4, 1932

Señor Fernando Torreblanca
Embajada de Mexico
Paris, Francia

Muy querido tío:

Corroborando el cablegrama que te mandé ayer y nuestros deseos que tengo entendido te participó mi papá, quiero suplicarte nos hagas el favor de que se me nombre escribiente en la Embajada de México en Berlín con un sueldo que sea suficiente para nuestros gastos.

Por las cartas que adjunto te podrás dar cuenta que he solicitado ayuda para poder terminar ciertos estudios superiores en física y matemáticas, con el proposito de volver a México así que los haya concluido. Una vez allá deseo identificarme de nuevo con la vida de nuestro país tomando parte activa en el desarrollo social y económico. Espero llevar a cabo estos planes fungiendo como miembro de la Facultad de la Universidad Nacional y como Ingeniero Electricista del Gobierno de México. Para estos planes cuento con tu bondadosa ayuda.

Apesar de todos mis esfuerzos de varios meses y de la carta del Dr. J. M. Puig Casauranc que me tomo la libertad de enviarte, mi solicitud no fue accedida. Por ésto y porque el empleo que tengo en la Universidad de Johns Hopkins como colaborador de investigaciones científicas concluye con el mes de junio, me urge conseguir empleo inmediatamente. Mi esposa y yo estamos dispuestos a volver a México desde luego si fuera posible conseguir un empleo conveniente. Sin embargo, preferiría terminar mis estudios antes de regresar, pues estaría de este modo mejor preparado para llevar a cabo los planes que ahora me animan.

Como escribiente en nuestra embajada en Berlín pienso dedicar al estudio todo el tiempo que me quede libre de mi trabajo asistiendo a varias clases ya sea en la Universidad de Berlín o en alguna otra cercana. Asi dominaré en poco tiempo el idioma alemán, tan indispensable en materias científicas, y llevaré a cabo mis planes de estudio.

Entiendo que estás con tu familia en viaje de descanso y de recreo, y sólo la urgencia del caso me obliga a pedirte ahora este favor. Te agradecería sinceramente me contestaras por cablegrama si contamos con el empleo para el 1º de julio. Nos sería útil saber cuanto antes para poder hacer los preparativos del viaje con tiempo.

Con gusto aprovecho esta oportunidad para expresarte mi agradecimiento por todos tus favores y en particular por este nuevo que te pido.

Esperamos que el viaje les haya sido divertido y provechoso y que muy pronto recobres tu salud. Recibe muy cariñosos saludos de nuestra parte para tí, tu esposa, y niñas. Tu sobrino que te aprecia,

Alfredo Baños Jr.

3

EMBAJADA DE MEXICO

Washington, 8 de febrero de 1932.

Señor Eyler Simpson,
Fundación Guggenheim,
Monte de Piedad 15,
México, D. F.

Muy estimado señor Simpson:

Con una beca que le concedió la Secretaría de Educación Pública, durante mi estancia en dicha dependencia del Ejecutivo por la primera vez, el señor Alfredo Baños, Jr., logró hacer excelentes estudios en la Escuela de Ingeniería de la Universidad de John Hopkins. Después de obtener sus grados se dedicó a especializarse en Geofísica y ha publicado algunos trabajos que demuestran no solo su capacidad sino un vivo interés de investigación.

El señor Baños, Jr., no puede disfrutar ya de la ayuda de México por la reducción que hubo de hacerse a la Partida de Pensiones del presupuesto. Ha solicitado una beca de la Institución Guggenheim, y yo quiero recomendárselo a usted muy empeñosamente para que lo ayude, autorizándolo para enseñar esta carta a los cinco miembros del Comité de Selección, en esa capital.

Anticipando a usted las gracias por la atención que dispense a este asunto, me suscribo una vez más su amigo afectísimo y servidor.

Puig

Dr. J. M. Puig Casauranc

EJD/8.

JOHN SIMON GUGGENHEIM MEMORIAL FOUNDATION

OFICINAS DEL SECRETARIO
DEL COMITE DE SELECCION
EN MEXICO

APARTADO POSTAL 538
MEXICO, D. F.

March 8, 1932

Mr. Alfredo Baños, Jr.,
2608 Maryland Avenue,
Baltimore, Md.,
USA

My dear Mr. Baños:

Your application for a Fellowship on this Foundation was duly presented to the Committee of Selection in Mexico. I regret that they did not find it possible to be of assistance to you.

The Committee desire that I express the highest opinion of your attainments and plans for study. The Fellows appointed, however, seem to be of a type and in a position, such that, after a great deal of difficult comparison, the Committee concluded that perhaps the greatest contributions in the line of the purposes of the Foundation, would be made by them.

I may add that the Committee also expressed its opinion that your case for a Fellowship as one of the Latin American Exchange Fellows from Mexico will be much stronger after you have returned to Mexico and have had time to identify yourself once more with the life of your native country.

With regrets of my inability to be of service to you, I am,

Yours very sincerely,

Eyler N. Simpson

Eyler N. Simpson

P.S. Under separate cover I am returning to you the various publications which you were kind enough to submit with your application.

Baltimore, Maryland, U. S. A.
Junio 6, 1932

Señor Fernando Torreblanca
Embajada de México
París, Francia

Muy querido tío:

Discutiendo ayer con papá la cuestión de mi empleo como escribiente en la Embajada de México en Berlín me dijo que cuando él te participó nuestros planes te expresaste favorablemente acerca de ellos.

Tanto papá como nosotros teníamos entendidos que mi presente empleo continuaría hasta el 1º de agosto, pero habiendo sido notificado definitivamente que tan sólo cuento con este mes y no teniendo en lo absoluto recursos con que sostenernos, me atrevo a molestarte una vez mas rogándote que por cablegrama arregles los trámites de mi nombramiento.

De nuevo quiero recordarte que estamos deseosos de regresar a México desde luego, en dado caso que no me sea posible terminar mis estudios en Berlín ó en alguna otra capital europea, ya sea París, Roma ó Londres.

Con saludos cariñosos para tí y los tuyos recibe nuestro sincero agradecimiento. Tu sobrino que te quiere,

Alfredo Baños, Jr.

TELEGRAMME

VIA WESTERN UNION

FERNANDO TORREBLANCA

EMBAJADA DE MEXICO

PARIS

6

CIRCUIT

GE 881

BALTIMORE 23 3

1807

Signification des principales indications éventuelles pouvant figurer en tête de l'adresse

D... = Urgent.
AR. = Remettre contre reçu.
PC. = Accusé de Réception.
RP. = Réponse payée.
TC. = Télégramme collationné.
MP. = Remettre en mains propres.
XPx ..... = Exprès payé.
NUIT.... = Remettre même pendant la nuit.
JOUR ... = Remettre seulement pendant le jour.
OUVERT = Remettre ouvert.

Via WESTERN UNION

SUPLICOTE FAVOR ARREGLES NOMBRAMIENTO ESCRIBIENTE EMBAJADA BERLIN CUANTO ANTES ENVIOTE PLANES ESTUDIO CARTA GRACIAS

ALFREDO BANOS JR

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SUPLICOTE FAVOR ARREGLES NOMBRAMIENTO ESCRIBIENTE EMBAJADA

BERLIN CUANTO ANTES ENVIOTE PLANES ESTUDIO CARTA GRACIAS

ALFREDO BANOS JR

Via WESTERN UNION

PARIS 106 ... 17 20 -4 VI 1932

Signification des principales indications éventuelles pouvant figurer en tête de l'adresse

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TC. = Télégramme collationné.	JOUR ... = Remettre seulement pendant le jour.
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PC. = Accusé de Réception.	XP ... = Exprès payé.
AR. = Remettre contre reçu.	
D ... = Urgent.	

TÉLÉGRAMME

64 **VIA WESTERN UNION** 106

FERNANDO TORREBLANCA

EMBAJADA DE MEXICO

PARIS

LE PORT EST GRATUIT. Le facteur doit délivrer un récépissé à so

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PARTICULAR. París, junio 21 de 1932.

Señor Alfredo Baños, Jr.
2608 Maryland Avenue
Baltimore, Md.,
USA.

Mi querido sobrino:

Recibí tus telegramas y tus afectuosas cartas de fechas 4 y 6 de los corrientes junto con los anexos que me enviaste.

Con toda pena te diré que, desde aquí, me es imposible atender tu petición pero tan pronto como regrese a México, trataré el asunto y me esforzaré en obsequiar tus deseos.

Mi esposa y yo les agradecemos a ti y a tu señora sus buenos deseos y cariñosos saludos que les retornamos.

Tu tío que te aprecia.

CIRCUIT

8

AFT 941

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Signification des principales indications éventuelles pouvant figurer en tête de l'adresse

D...	= Urgent.	XPx.....	= Exprès payé.
AR.	= Remettre contre reçu.	NUIT....	= Remettre même pendant la nuit.
PC.	= Accusé de Réception.	JOUR...	= Remettre seulement pendant le jour.
RP.	= Réponse payée.	OUVERT	= Remettre ouvert.
TC.	= Télégramme collationné.		
MP.	= Remettre en mains propres.		

Via WESTERN UNION

HOPKINS POSITION ENDS JULY FIRST ANXIOUS TO KNOW DATE OF EMPLOYMENT EMBASSY BERLIN PLEASE CABLE TH~~E~~ANKS

ALFREDO BANOS JR

PARIS

MEXICAN EMBASSY

NLT FERNANDO TORREBLANCA

VIA WESTERN UNION

TÉLÉGRAMME

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AFT 941

BALTIMORE MD

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Signification des principales indications éventuelles pouvant figurer en tête de l'adresse

Via WESTERN UNION

HOPKINS POSITION ENDS JULY FIRST ANXIOUS TO KNOW DATE OF EMPLOYMENT EMBASSY BERLIN PLEASE CABLE THANKS

ALFREDO BAROS JR

9

Baltimore, Maryland, U. S. A.
Junio 17, 1932

Señor Fernando Torreblanca
Embajada de México
París, Francia

Muy querido tío:

Acabo de recibir por conducto de papá el cablegrama en que me dices que por correo contestas a mi petición, lo cual te agradecemos sinceramente, pues ya bien sabes las razones por las que estamos ansiosos de definir nuestra situación.

Quiero aprovechar esta ocasión para recordarte que hace cuatro años recibí mi título de ingeniero electricista; desde entonces, practicando mi profesión he adquirido experiencia y conocimientos que no dudo me capacitan para dar instrucción en la Universidad Nacional y en la Escuela de Ingeniería sobre asignaturas pertenecientes a mi ramo. Con gusto te envío adjunto copias de dos artículos que conciernen mis trabajos en la Universidad de Johns Hopkins; asimismo te participo que en noviembre del año pasado tuve el honor de leer ante una convención de ingenieros que tuvo verificativo en la Universidad de Harvard un artículo que describe parte de mis estudios.

Al mencionar estos hechos quiero hacer patente que no olvido que me ha sido posible completar mi carrera y hacer estos estudios gracias a la ayuda financiera que como pensionado de la Secretaría de Educación Pública recibí por varios años. Esto constituye una deuda que deseo pagar dedicando el fruto de mis estudios en bien de nuestro país. Para mejor llevar a cabo estos planes quisiera perfeccionarme en el idioma alemán y completar ciertos estudios que me permitan obtener el título de Doctor en Filosofía (especializando en física y matemáticas). Creo de este modo estar mejor preparado para identificarme una vez más con la vida nacional. Para consumar esto que sinceramente anhelo solicito encarecidamente tu ayuda.

Con cariñosos saludos de nuestra parte para tí y los tuyos recibe de nuevo nuestro agradecimiento. Tu sobrino,

Alfredo Baños, Jr.

Predetermination of the A-C. Characteristics of Dielectrics

BY J. B. WHITEHEAD* and ALFREDO BAÑOS, JR.†

Fellow, A.I.E.E. Associate, A.I.E.E.

Synopsis.—From the charge and discharge currents of any type of dielectric under continuous potential it is possible to predict accurately the loss, power factor and capacity at 60 cycles. This is done by an empirical determination of the equation for the relaxation function of the dielectric at a given temperature, followed by the application of Von Schweidler's method. The method developed is available at any frequency, provided that the continuous potential charge and discharge currents may be measured over initial time intervals comparable with the alternating period.

It is shown that agreement of the relaxation function with the t^{-n} expression is not general, but on the contrary, it is confined to dielectrics showing negligible or no irreversible conduction. The inadequacy of the t^{-n} expression is proved on experimental and analytical reasons. Chief among them is the failure of the t^{-n} expression to predict accurately the variation of dielectric loss with frequency.

A convenient and sufficient expression for the relaxation function is shown to be a sum of three exponentials. Further, experiment and analysis prove that the method of three exponentials predicts accurately the a-c. behavior of a dielectric at 60 cycles.

The usual forms of irreversible conduction encountered are defined and classified. The case where the initial constant current does not obey Ohm's law has been considered analytically in its contribution to the a-c. behavior.

The manner in which the complete a-c. behavior of a dielectric is accurately predicted from suitable d-c. measurements is illustrated by three typical sets of experimental data; paper, oil, and impregnated paper.

* * * * *

INTRODUCTION

THE purpose of this paper is to give the experimental proof that the a-c. behavior of a dielectric at 60 cycles may be accurately predicted from suitable d-c. measurements. The fact that dielectric loss may be explained in terms of reversible absorption, or residual charge, and anomalous conduction, as observed under continuous potential, has led to several well known extensions to the alternating case of various theories of dielectric absorption. Notable among these are Wagner's[1] extension of Maxwell's stratified dielectric and von Schweidler's[2] extension of Pellat's theory. Von Schweidler has deduced the expression for the dielectric loss in an alternating field from the equation of the empirical absorption current-time relation as observed under continuous potential. The work of F. Tank,[3] 1915, apparently represents the first effort made using von Schweidler's method for a direct check between measured and computed losses. Since then other checks on this method have been made by Whitehead,[4] Benedict,[5] and others. This paper discusses further the possibilities and limitations.

The continuous potential measurements, using the amplifier-oscillograph, give records on photographic films of the charge and discharge current-time relations for the interval of time 0.001 to 0.040 sec. after the application of voltage or of short-circuit. The geometric transient, both on charge and discharge, confined by choice of circuit constants to extremely short intervals of time is, therefore, absent from such records. This permits the accurate determination of the reversible absorption current directly from the discharge records and of the irreversible conduction current as the difference between the corresponding ordinates of charge and discharge.[6] The usual measurements of charge and discharge current taken with a galvanometer and beginning about a minute after the application of voltage or of short-circuit, give values which in most cases have no bearing on the a-c. behavior at commercial frequencies. The condition for suitable d-c. measurements for the accurate prediction of the a-c. behavior is that the range of time covered by the d-c. measurements must be of the same order of magnitude as the period of the alternating wave.

VON SCHWEIDLER'S ANALYSIS

The several components of alternating current in a dielectric at a given frequency may conveniently be represented in a vector diagram. Fig. 1 shows the familiar Wagner diagram for a capacitor exhibiting the anomalous properties of reversible absorption and irreversible conduction in addition to the normal property of specific inductive capacity. Von Schweidler's method permits the computation from the curves of d-c. charge and discharge currents of each of the separate components, thereby giving dielectric loss, phase difference, and the increase in capacity due to absorption. Under continuous potential the irreversible conduction current is usually characterized by a constant initial conductivity which is independent of the applied stress for low stresses and which is almost invariably larger than the final or normal conductivity. The contribution of the initial irreversible conduction to the alternating case is discussed in a later section. Reversible absorption under continuous potential is characterized by a decaying current function of time

$$i_r(t) = EC_0\phi(t) \tag{1}$$

where E is the applied voltage, C_0 the geometric ca-

*Dean, School of Engg., Johns Hopkins University.

†Research Assistant, Johns Hopkins University.

Presented at the Winter Convention of the A.I.E.E., New York, N. Y., January 25-29, 1932.

pacitance, and $\phi(t)$ the relaxation function corresponding to the dielectric at a given temperature; it is measured directly in the d-c. discharge curve. Equation (1) embodies Curie's[8] well known laws; namely, that the ordinates of the absorption current-time relation be directly proportional to the applied voltage and to the dimensions of the capacitor. If these laws are obeyed then the Hopkinson[7]-Curie[8] principle of superposition is applicable to the determination of the current under a

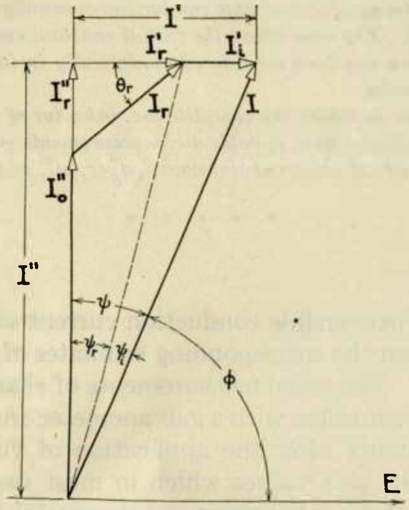

FIG. 1—VECTOR DIAGRAM OF A CAPACITOR SHOWING REVERSIBLE ABSORPTION AND IRREVERSIBLE CONDUCTION IN ADDITION TO SPECIFIC INDUCTIVE CAPACITY

Current Vector	Contribution of
I''_o	Geometric Capacitance
I_r	Reversible Absorption
I'_i	Irreversible Conduction

varying voltage. Von Schweidler's fundamental equation for the current, as dependent on any e.m.f. variation $E(\tau)$ in the past, and as based on the principle of superposition, is

$$i(t) = E(o)\phi(t) + \int_o^t \frac{d}{d\tau} E(\tau)\phi(t-\tau)\,d\tau \qquad (2)$$

which he applies to determine the steady state for the case of the alternating sinusoidal applied potential. The additional admittance contributed to a capacitor by reversible absorption, in complex notation, is shown to be:

$$Y_r = \omega C_0(B + iA) \qquad (3)$$

where $\omega = 2\pi \times$ frequency and

$$\left.\begin{aligned} A &= \int_o^\infty \cos \omega t \,.\, \phi(t) \,.\, dt \\ B &= \int_o^\infty \sin \omega t \,.\, \phi(t) \,.\, dt \end{aligned}\right\} \qquad (4)$$

The Relaxation Function

Hopkinson, Curie, Tank, and others have reported experimental agreement of the relaxation function with the expression:

$$\phi(t) = \beta t^{-n} \qquad (5)$$

von Schweidler gives in this case the values of the infinite integrals of (4), as:

$$\left.\begin{aligned} A &= \omega^{n-1}\beta\Gamma(1-n)\cos\frac{(1-n)\pi}{2} \\ &\text{converges for } o < n < 1 \\ B &= \omega^{n-1}\beta\Gamma(1-n)\sin\frac{(1-n)\pi}{2} \\ &\text{converges for } o < n < 2 \end{aligned}\right\} \qquad (6)$$

In all the cases mentioned above the values of n given lie between 0 and 1. Benedict[5] shows from experiments with solid dielectrics the same type of agreement up to 100 milliseconds. Whitehead,[4] however, has reported several cases of values of $n > 1$, which introduces a serious objection to the use of the type of relaxation function given in equation (5). As an example, in tests on commercial abietic acid over a range of temperature, Whitehead[9] has found that the discharge current-time relation is of the t^{-n} type with values of n ranging from 0.1 to 2.5.

In spite of the wide spread experimental agreement with the t^{-n} expression it is found to be inadequate for the accurate prediction of the a-c. behavior for the following reasons:

1. Analytically, when $n > 1$, the value of the A integral in (6) above does not converge and it becomes impossible to compute the increase in capacity due to absorption.

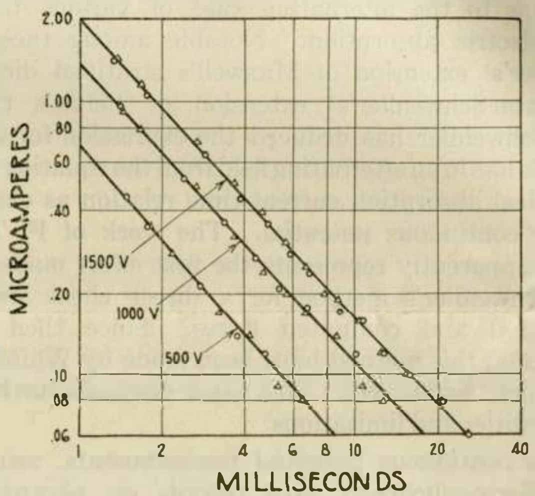

FIG. 2—VARIATION WITH VOLTAGE OF CHARGE AND DISCHARGE CURRENT-TIME RELATIONS AT 60°C. DRY PAPER SPECIMEN A7. ILLUSTRATES AGREEMENT WITH t^{-n} EXPRESSION FOR VALUES OF $n > 1$.

—○— Charge
—△— Discharge

2. Examination of the expression for the B integral discloses a mode of variation of power factor with frequency which is contrary to both theory and experiment. The computed power factor will decrease, remain constant, or increase with frequency depending on whether the value of n is less than one, equal to one, or greater than one, but in no case will it show a power

factor maximum as required by experience based on well accepted theories.

3. Experimentally, the negative power of time calling as it does for infinite current at zero time, gives rise to computed losses which are usually in excess of the measured losses.

4. Finally, the t^{-n} expression is by no means general, for noted departures are found in many cases.

Fig. 2 gives the current-time relations at three voltages for a specimen of dry, unimpregnated paper showing agreement with t^{-n} expression for $n > 1$. Dielectric loss in dry paper specimens is entirely due to reversible absorption as evinced by the equality of charge and discharge currents. The irreversible conduction current even at 60 deg. cent. is too small to be detected by the amplifier-oscillograph. Fig. 3 gives the corresponding discharge current-time relations for the same paper specimen after impregnation with an oil whose electrical properties are described later. Note in this case the decided departure of the absorption (discharge) current-time relation from the t^{-n} expression. We find in general, that if a capacitor exhibits charge and discharge curves which are identical, that is, if its dielectric loss is due entirely to reversible absorption, then the empirical current-time relation obeys the negative power law of equation (5). If, on the other hand, a capacitor shows measurable irreversible conduction, or if it is made to acquire it, either through elevation in temperature or as in the case of a paper specimen, through impregnation, then the observed discharge current-time relation will differ materially from the simple t^{-n} expression.

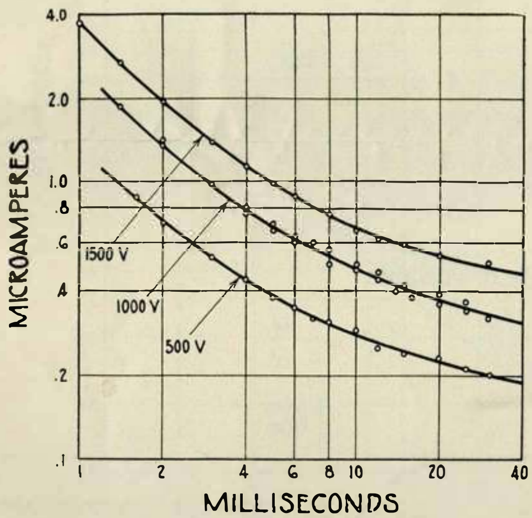

FIG. 3–VARIATION WITH VOLTAGE OF DISCHARGE CURRENT TIME RELATIONS AT 60°C. IMPREGNATED PAPER SPECIMEN C7. ILLUSTRATES DEPARTURE FROM t^{-n} EXPRESSION.

The Method of Three Exponentials

A more general expression for the relaxation function which is not open to the analytical objections of the t^{-n} type is a series of exponentials of the form

$$\phi(t) = \beta_1 e^{-\alpha_1 t} + \beta_2 e^{-\alpha_2 t} + \ldots + \beta_n e^{-\alpha_n t} \tag{7}$$

for which the A and B integrals of (4) become

$$\left.\begin{aligned} A &= \frac{\alpha_1\beta_1}{\omega^2+\alpha_1^2} + \frac{\alpha_2\beta_2}{\omega^2+\alpha_2^2} + \ldots + \frac{\alpha_n\beta_n}{\omega^2+\alpha_n^2} \\ B &= \frac{\omega\beta_1}{\omega^2+\alpha_1^2} + \frac{\omega\beta_2}{\omega^2+\alpha_2^2} + \ldots + \frac{\omega\beta_n}{\omega^2+\alpha_n^2} \end{aligned}\right\} \tag{8}$$

Hopkinson, Steinmetz, Wagner, and others have found that three terms of equation (7) usually will suffice to express the empirical equation for the observed relaxation function.

The resolution of a given observed current-time relation into the sum of three exponentials is indeterminate and the two constants of each term may take wide ranges of values. Analytically this means that a set of six constants corresponding to the three exponentials is so chosen that the resultant equation will be satisfied by any six given points of the observed current-time relation. This implies six simultaneous equations involving transcendental functions, the solution of which can only be obtained by graphical means. To illustrate with a concrete example, consider the 1,500-volt curve of Fig. 2 and resolve it into three exponentials by the method of successive residuals as outlined by J. Lipka.[10]

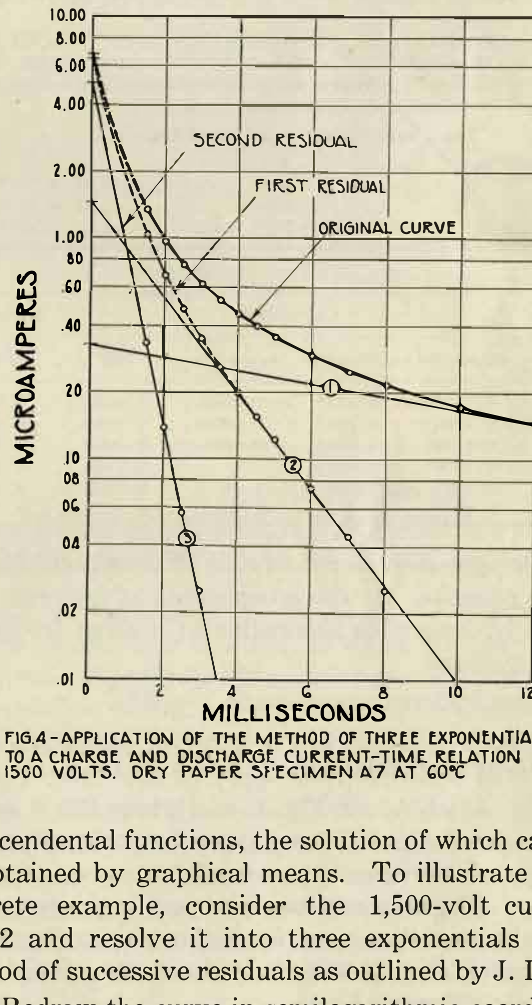

FIG. 4–APPLICATION OF THE METHOD OF THREE EXPONENTIALS TO A CHARGE AND DISCHARGE CURRENT-TIME RELATION AT 1500 VOLTS. DRY PAPER SPECIMEN A7 AT 60°C

1. Redraw the curve in semilogarithmic coordinates; that is, the logarithm of the current against the time in seconds. This is shown by the original curve in Fig. 4.

2. At any arbitrarily chosen value of time, say 12 milliseconds in this case, draw a tangent to the curve terming this straight line the *first exponential.*

3. By subtracting the corresponding ordinates of the first exponential from the original curve obtain the *first residual* (see Fig. 4). Draw a tangent to this curve and note that its position is no longer arbitrary, but is defined by the later part of the curve with points lying in a straight line. This straight line is termed the *second exponential.*

4. By subtracting the corresponding ordinates of the second exponential from the first residual obtain the *second residual,* which this time is completely defined by a series of points lying in a straight line and constituting the last and *third exponential.* Obviously, if the second residual had not been expressible as a straight line it would have been necessary to continue the method to a fourth or even a fifth exponential. In practise, however, this is seldom the case.

5. For each exponential the intercept on the current axis is a measure of the constant multiplier β and its slope a measure of the inverse time constant α. This permits the computation of the empirical current-time relation and its direct comparison with the observed curve. Table I gives such a comparison showing remarkable agreement for the range of time investigated.

TABLE I—COMPARISON OF OBSERVED AND CALCULATED VALUES OF CURRENT. THE METHOD OF THREE EXPONENTIALS. CHARGE AND DISCHARGE CURRENT-TIME RELATION AT 1,500 VOLTS

Dry Paper Specimen A-7 at 60 deg. cent.

Exponential	α	$i\,(o)$
(1)	69	0.33×10^{-6}
(2)	500	1.45×10^{-6}
(3)	1,787	5.00×10^{-6}

(t) Time in seconds	Current: microamperes — Calculated $i_1(o)e^{-\alpha_1 t}$	$i_2(o)e^{-\alpha_2 t}$	$i_3(o)e^{-\alpha_3 t}$	$i(t)$	Observed $i(t)$
0.002	0.2875	0.5335	0.1402	0.9612	0.960
0.004	0.2504	0.1962	0.0039	0.4505	0.450
0.006	0.2181	0.0722	0.0001	0.2904	0.290
0.008	0.1900	0.0266	—	0.2166	0.215
0.010	0.1655	0.0098	—	0.1753	0.171
0.012	0.1442	0.0036	—	0.1478	0.145

From equation (3) the components of current contributed by reversible absorption at a given frequency and voltage are:

$$\left.\begin{aligned} &\text{Quadrature component} \quad I_r'' = AI_0'' \\ &\text{In phase component} \quad I_r' = BI_0'' \end{aligned}\right\} \quad (\mathbf{9})$$

where $I_0'' = \omega E C_0$, see Fig. 1, and where the A and B integrals are computed for a given frequency from the constants of the three exponentials by means of equations (**8**). The above values now permit the computation of the contribution of reversible absorption to the a-c. behavior, since A and B are simple functions of the α's and β's. Thus we have:

$$\left.\begin{aligned} &\text{Phase defect angle } \tan \Psi_r = \frac{B}{1 + A} \\ &\text{Dielectric loss} \quad W_r = \omega E^2 C_0 B \\ &\text{Apparent capacitance } C' = C_0(1 + A) \end{aligned}\right\} \quad (\mathbf{10})$$

Equations (**10**) show the dependence of the computed a-c. behavior on the values of the A and B integrals. These, however, for a given dielectric at a given temperature are functions of the constants α and β and also of the frequency. It therefore becomes important to examine how wide a range of choice of the constants α and β as related to the frequency is permissible for agreement of computed and measured values. Assume therefore for simplicity a current-time relation of a single exponential term and examine the theoretical variation with frequency of the A and B integrals which now become:

$$\left.\begin{aligned} A(\omega) &= \frac{\alpha\beta}{\omega^2 + \alpha^2} \\ B(\omega) &= \frac{\omega\beta}{\omega^2 + \alpha^2} \end{aligned}\right\} \quad (\mathbf{11})$$

By change of variable

$$e^u = \frac{\omega}{\alpha} \quad (\mathbf{12})$$

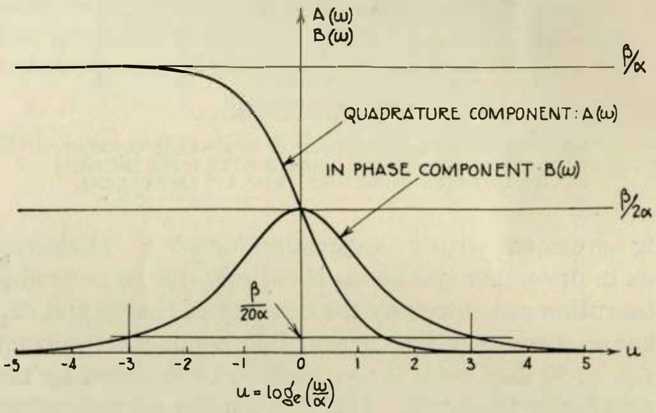

FIG. 5—VARIATION WITH FREQUENCY OF THE IN PHASE AND QUADRATURE COMPONENTS OF A SINGLE EXPONENTIAL.

obtain the relations

$$\left.\begin{aligned} A(u) &= \frac{\beta}{2\,\alpha}\, e^{-u} \operatorname{sech} u \\ B(u) &= \frac{\beta}{2\,\alpha} \operatorname{sech} u \end{aligned}\right\} \quad (\mathbf{13})$$

which when plotted as shown in Fig. 5 are useful in showing the symmetry of the functions. The contribution to the in-phase component attains a maximum equal to $\beta/2\alpha$ at a value of frequency for which $\omega = \alpha$. The value of $B(\omega)$ falls off rapidly as the ratio ω/α departs either side from unity. For example, an exponential giving its maximum contribution to the in-phase component at $\omega = 100$ would contribute only one-tenth of the maximum value at either $\omega = 5$ or $\omega = 2{,}000$. This is readily deduced from Fig. 5. The variation with frequency of the in-phase component for a relaxation function expressible as the sum of several exponentials is given by the summation of a number of curves of the type $B(\omega)$ in Fig. 5, corresponding to the several exponentials. We have found that the maximum of a fourth exponential that might be introduced occurs at a frequency far from the frequency being

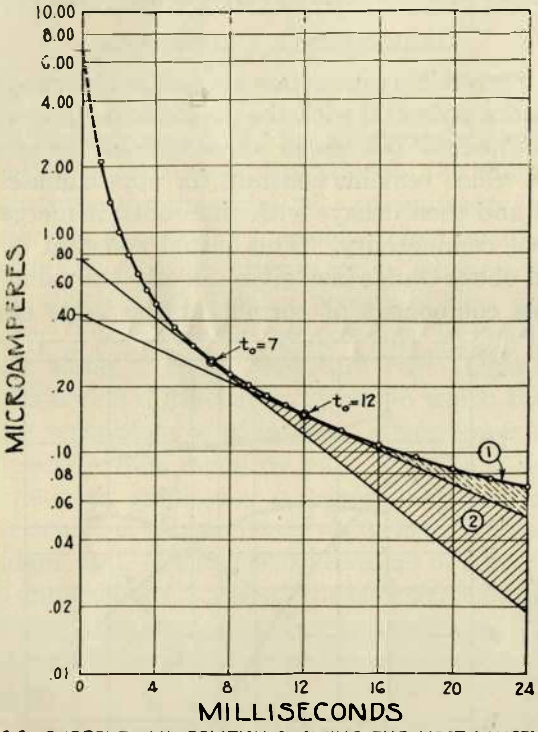

FIG. 6—CURRENT-TIME RELATION SHOWING THE FIRST TANGENT DRAWN AT TWO DIFFERENT VALUES OF "t_o" SHADED AREAS SEGREGATE IN EACH CASE THE NEGLECTED PORTION OF THE CURVE

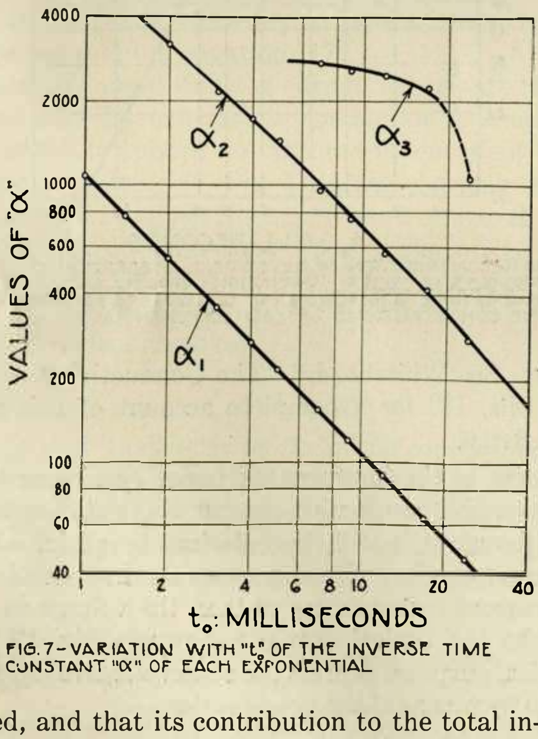

FIG. 7—VARIATION WITH "t_o" OF THE INVERSE TIME CONSTANT "α" OF EACH EXPONENTIAL

studied, and that its contribution to the total in-phase component is negligible. The maximum of each contribution occurs at a value of ω equal to the inverse time constant α of that exponential. Therefore only the exponentials with values of α in the neighborhood of ω at the frequency at which correlation is intended contribute an appreciable amount to the loss component $B(\omega)$. Thus for agreement between measured and computed losses at 60 cycles, where $\omega = 377$, the values of α range approximately from 50 to 2,000. The same reasoning may be applied to examine the time range within which the d-c. measurements will give accurate prediction of the a-c. behavior. It shows that the values of the α's obtained for the three exponentials must range in the neighborhood of the value of ω corresponding to the frequency at which correlation is intended. This means that the range of time covered by the d-c. measurements must be of the same order of magnitude as the period of the alternating wave.

Accuracy of the Method

It has been stated that the resolution of a current-time relation into three exponentials is quite indeterminate. The constants of the three exponentials depend

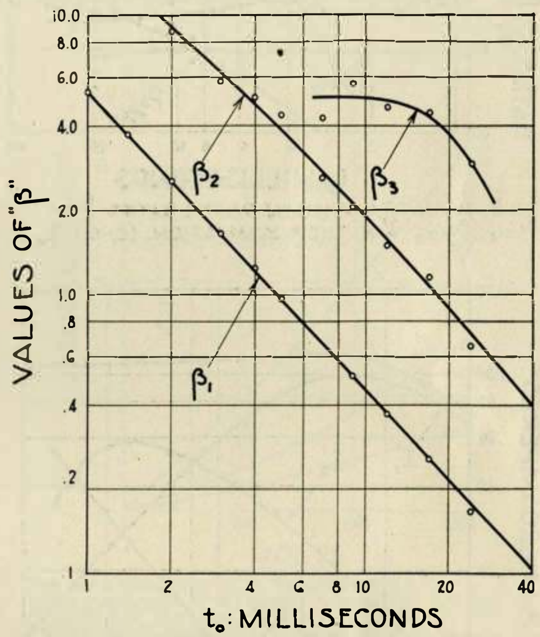

FIG. 8—VARIATION WITH "t_o" OF THE CONSTANT MULTIPLIER "β" OF EACH EXPONENTIAL.

entirely on the value of time t_0, of the empirical discharge current-time relation, at which the first tangent is drawn. Fig. 6 illustrates the point for the current-time relation of Fig. 4 showing the first tangent drawn at two different values of t_0 with the shaded areas indicating in each case the neglected portions of the curve.

Figs. 7, 8 and 9 give respectively for each exponential the variation with t_0 of the inverse time constant α, the constant multiplier β, and their ratio β/α. This ratio is shown in Fig. 5 to be a measure of the maximum contributions to the in-phase and quadrature components with varying frequency. Figs. 7 and 8 are significant in pointing out the definite dependence of the α and β constants on the chosen value of t_0, while their ratio, as shown by Fig. 9, is found to be essentially independent of t_0 at least for the first and second exponentials. The upper curve of this figure gives the summation of the three ratios, which is seen to be independent of t_0. From Fig. 5 and the equation (**10**) for the apparent capacitance it follows that this summation gives a measure of the dielectric constant at zero frequency, ϵ_1, in terms of the geometric dielectric constant, ϵ_0, at infinite frequency. Specifically, the relation is

$$\frac{\epsilon_1 - \epsilon_0}{\epsilon_0} = \frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2} + \frac{\beta_3}{\alpha_3} \qquad \textbf{(14)}$$

That this summation should prove independent of t_0 further corroborates the sufficiency of three exponentials.

While the constants of the three exponentials are definitely functions of t_0, the total contribution to the

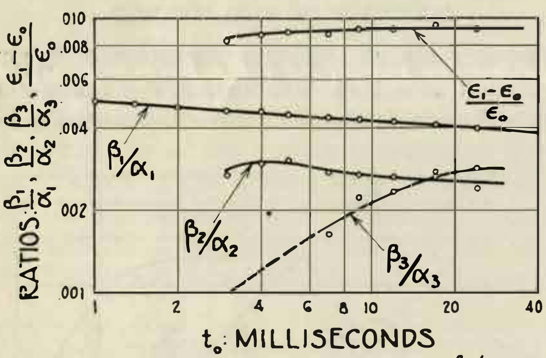

FIG. 9—VARIATION WITH "t_0" OF THE RATIOS β_1/α_1, β_2/α_2, β_3/α_3, AND THEIR SUMMATION, $(\epsilon_1-\epsilon_0)/\epsilon_0$

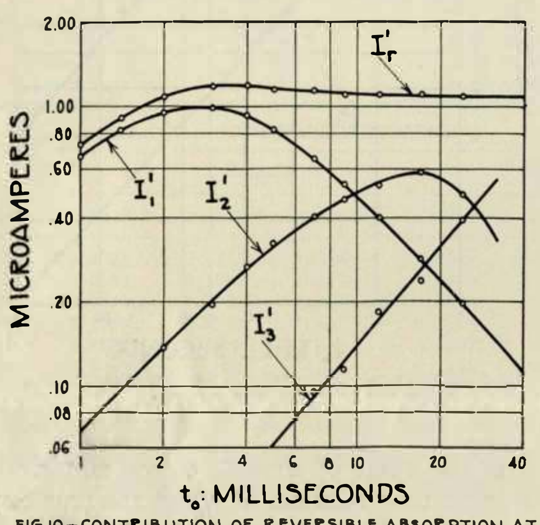

FIG. 10—CONTRIBUTION OF REVERSIBLE ABSORPTION AT 1500 VOLTS, 60 CYCLES. VARIATION WITH "t_0" OF THE IN PHASE COMPONENT OF CURRENT (I'_r) SHOWING THE CONTRIBUTIONS OF EACH EXPONENTIAL.

a-c. behavior, as given by the in-phase and quadrature components of current at a given voltage and frequency, is essentially independent of t_0 for a limited range of values. This is illustrated in Figs. 10 and 11 which show that for t_0 beyond six milliseconds the in-phase and quadrature components are essentially constant even though the separate contributions of each exponential may vary widely. This is particularly true of the in-phase component, Fig. 10, which is a measure of the power factor, and therefore the one most readily checked by experiment. No physical significance is to be attached to the actual numerical values of the constants of the three exponentials. They serve merely as an empirical means of computing the a-c. behavior. It is apparent from Fig. 10 that the optimum value of t_0 lies in the neighborhood of the period of the alternating wave thus corroborating from another point of view the criterion of what constitutes suitable d-c. measurements.

Irreversible Conduction

The irreversible conduction current as observed under continuous potential with the amplifier oscillograph for some of the cable oils tested is characterized by an initial current which remains constant for approximately one second and then decays with time until it merges into the final conductivity. This initial constant current, when it obeys Ohm's law, gives directly the value of the in-phase component of current at the same effective

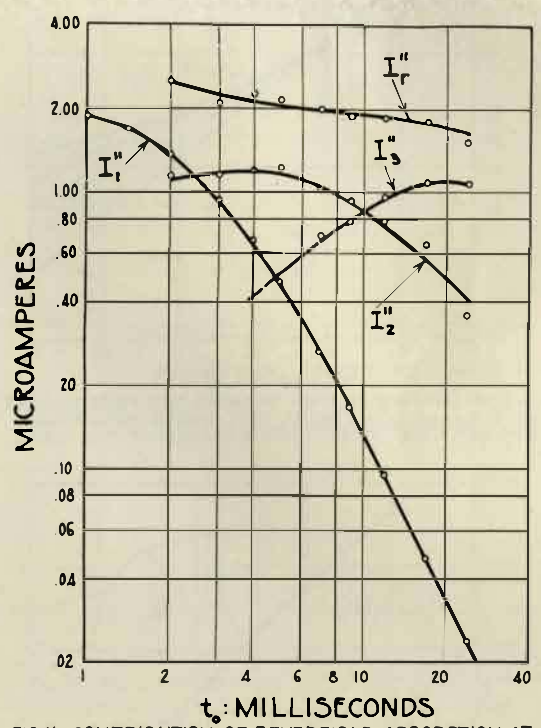

FIG. 11—CONTRIBUTION OF REVERSIBLE ABSORPTION AT 1500 VOLTS, 60 CYCLES. VARIATION WITH "t_0" OF THE QUADRATURE COMPONENT OF CURRENT (I''_r) SHOWING THE CONTRIBUTIONS OF EACH EXPONENTIAL.

voltage. See Whitehead,[11] "The Conductivity of Insulating Oils, II" for a complete account of this initial conductivity.

In some of the impregnated paper specimens tested the irreversible conduction current does not obey Ohm's law, although it is still characterized by an initial constant conductivity. In such cases the irreversible constant current increases faster than the voltage as illustrated by the typical example shown in Fig. 12. For analytical purposes express the observed current-voltage relation by means of the power series:

$$i(e) = ae + be^3 + ce^5 + \ . \ . \ . \qquad \textbf{(15)}$$

in which the coefficients may be directly determined from the observed current-voltage relation. To facilitate this, however, express the above relation thus:

$$i(e) = \gamma \sinh \mu e \qquad \textbf{(16)}$$

where the γ and μ constants are chosen to give the best agreement with the observed curve within the voltage range required. Obviously μ is a function of the temperature alone while γ is a function of the temperature and the dimensions of the capacitor. Expand equation (**16**) into a series

$$i(e) = \gamma \left[\mu e + \frac{(\mu e)^3}{3!} + \frac{(\mu e)^5}{5!} + \ldots \right] \quad (\mathbf{17})$$

in which higher powers than the fifth may be neglected.

To obtain the current-time relation for the alternating case substitute for the value of e in equation (**17**) its expression as a periodic sinusoidal function of time. This results in a power series of sin α, where $\alpha = (\omega t + \theta)$, in which all the higher powers of $\sin\alpha$ may be broken up into harmonic components by means of Fourier's series. Thus, departure from Ohm's law causes distortion of the current wave in which the fundamental term alone is of interest, when power factor and dielectric loss measurements are made with a bridge network employing a tuned vibration galvanometer or an a-c. galvanometer whose field exhibits only the fundamental. Using effective values of current and voltage and calling I_i' (Fig. 1) the effective value of the *fundamental* component of current contributed by irreversible conduction obtain from the above analysis the expression:

$$I_i' = \frac{\gamma \mu E}{2} [1 + \cosh \mu E\,] \quad (\mathbf{18})$$

which is valid only when higher powers than the fifth may be neglected in equation (**17**).

It must be noted that the current contributed to the a-c. behavior by irreversible conduction is exactly in phase with the applied voltage and independent of the frequency. Letting I'' (Fig. 1) be the charging current as measured at a given frequency and voltage, the contribution of irreversible conduction to the a-c. behavior is as follows:

$$\left.\begin{array}{ll} \text{Phase defect angle } \tan \Psi_i = \dfrac{I_i'}{I''} \\ \text{Irreversible loss} \qquad W_i = EI_i' \end{array}\right\} \quad (\mathbf{19})$$

where I_i' is given directly by the initial constant current under continuous potential when Ohm's law is obeyed or is computed by (**18**) when the non-linear current-voltage relation is determined according to equation (**17**). Table II gives the computed variation of power factor with voltage and temperature for the typical example shown in Fig. 12.

TABLE II—VARIATION WITH VOLTAGE AND TEMPERATURE OF POWER FACTOR DUE TO IRREVERSIBLE CONDUCTION. NON-LINEAR CURRENT-VOLTAGE CHARACTERISTIC

Impregnated Paper Specimen C-7

Temp. deg. cent.	γ Amp. $\times 10^{-8}$	μ	Power factor 500 volts	Power factor 1,000 volts	Power factor 1,500 volts
30	9.5	0.00082	0.00016	0.00017	0.00020
45	20.4	0.00076	0.00035	0.00039	0.00047
60	50.8	0.00072	0.00082	0.00090	0.00104

Experimental Procedure

Continuous potential measurements and corresponding 60-cycle a-c. measurements were made on a number of samples of oil, paper, and impregnated paper at voltages of 500, 1,000, and 1,500, and at temperatures of 30, 45, and 60 deg. cent. For a complete description of the constant temperature, drying, impregnating, and measuring tanks and of the test specimens see Whitehead and Kouwenhoven,[12] "Fundamental Properties of Impregnated Paper."

The measurements under continuous voltage include the determination of the charge and discharge current-time relations with the aid of the amplifier-oscillograph. The amplifier increases the normal sensitivity of the electromagnetic oscillograph 10^5 times, giving a deflection on the photographic film of 1 mm. per 4×10^{-8} amperes. The further development of the amplifier oscillograph, as applied to this work, is described by S. K. Waldorf[13] in a separate paper.

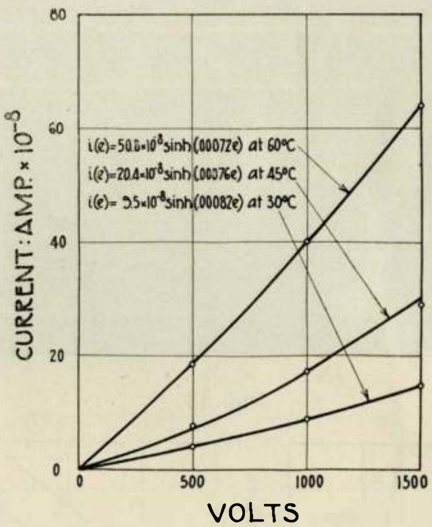

FIG. 12—VARIATION WITH TEMPERATURE OF CURRENT-VOLTAGE RELATIONS FOR THE INITIAL, CONSTANT, IRREVERSIBLE CONDUCTION CURRENT IMPREGNATED PAPER SPECIMEN C7.

The a-c. measurements include the determination of power factor and apparent capacitance using a modified Schering bridge whose power factor sensitivity is $\pm 5 \times 10^{-6}$. The development, theory, and special features of this bridge are described in a separate paper by Kouwenhoven and Baños.[14]

Experimental Results

Paper Specimens. Dry paper specimens reveal a dielectric loss due entirely to reversible absorption; that is, the irreversible conduction current is not measurable as shown by the equality of charge and discharge currents in Fig. 2. Fig. 13 gives a typical pair of oscillograms at a given temperature and voltage. From such oscillograms are computed the current-time relations shown in Figs. 14 and 15 which illustrate the respective variations with voltage and temperature. The insert in Fig. 14 shows how closely Curie's law is obeyed, while the insert in Fig. 15 illustrates the variation of the current ordinates with temperature.

Table III gives the variation with temperature of the α and β constants of the absorption current-time relation for the dry paper specimen before and after impregnation with an oil whose electrical characteristics are described in the next section. Interesting changes

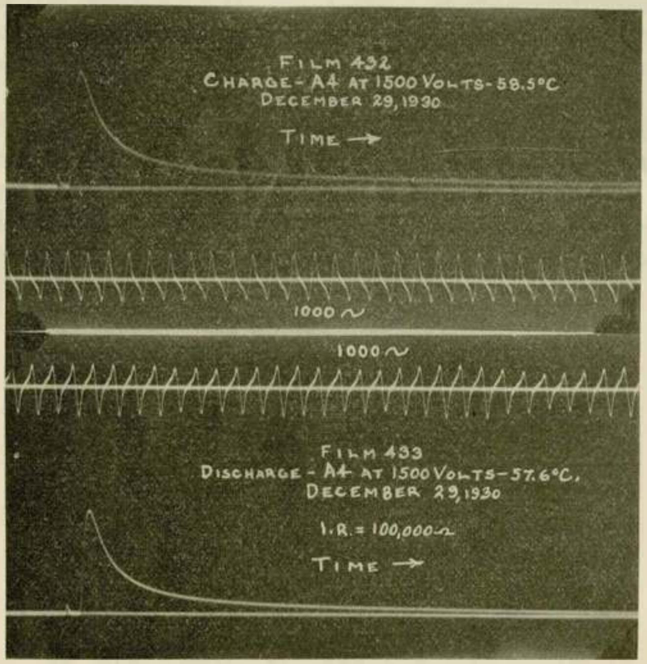

FIG. 13

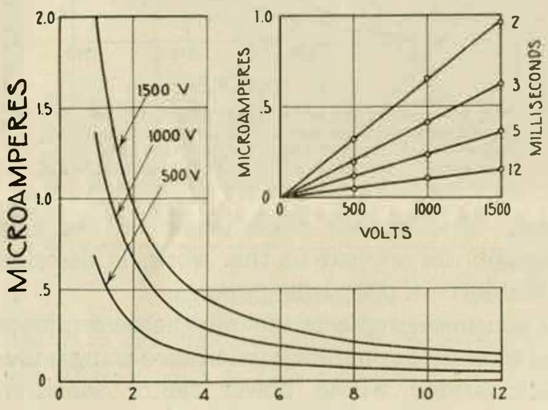

FIG. 14 - VARIATION WITH VOLTAGE OF CHARGE AND DISCHARGE CURRENT-TIME RELATIONS AT 60°C DRY PAPER SPECIMEN A7.

of the constants are noted as result of impregnation. These differences are best appreciated by direct comparison of Figs. 2 and 3. Table IV gives the correlation of a-c. and d-c. measurements for the whole range of temperature and voltage explored. Note that in general the agreement between computed and measured losses is very close.

Oil Specimens. Fig. 16 gives a set of typical charge oscillograms at 1,500 volts and at 30, 45, and 60 deg.

TABLE III—VARIATION WITH TEMPERATURE OF THE AVERAGE "α" AND "β" CONSTANTS IN THE EMPIRICAL CURRENT-TIME RELATION

$$i(t) = EC_0\{\beta_1 e^{-\alpha_1 t} + \beta_2 e^{-\alpha_2 t} + \beta_3 e^{-\alpha_3 t}\}$$

Specimen	Temp. deg.cent.	C_0 $\mu\mu f$	β_1	β_2	β_3	α_1	α_2	α_3
Dry paper A-7.	30	741	0.52	2.52	10.32	112	638	2,160
	45	751	0.40	1.93	6.49	94	614	2,067
	60	760	0.31	1.37	4.09	77	553	1,797
Impregnated paper C-7	30	1,191	0.49	2.33	6.70	95	570	1,827
	45	1,200	0.44	1.92	6.34	59	561	2,030
	60	1,209	0.55	1.32	3.13	30	490	1,635

cent. for the cable oil used to impregnate the above paper specimen. It is noted that the charge current is characterized by a large initial constant current which accounts for the major part of the loss. Superimposed on this initial current is a smaller decaying current function of time which is reversible and, therefore, attributable to dielectric absorption in oil. Fig. 18 shows the variation with temperature of the charge and discharge

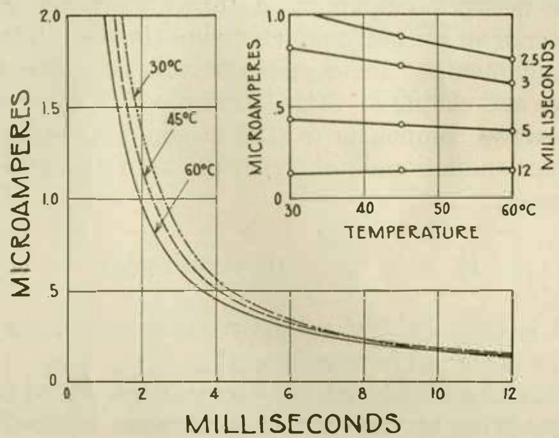

FIG. 15 - VARIATION WITH TEMPERATURE OF CHARGE AND DISCHARGE CURRENT-TIME RELATIONS AT 1500 VOLTS DRY PAPER SPECIMEN A7

current-time relations at 1,500 volts. The equations for discharge show that the small element of dielectric absorption is expressible by a single exponential thus corroborating the results published by Whitehead[11] in "The Conductivity of Insulating Oils, II." Fig. 17 gives the evidence that the initial constant current in the insulating oils tested obeys Ohm's law.

Table V gives the comparison between several measured and corresponding computed losses for one oil specimen B-7. The data are typical and the close agreement shown is further evidence of the soundness of the method used in computing the a-c. behavior.

Impregnated Paper Specimens. Some of the changes brought about on a dry paper specimen by impregnation with an insulating oil have already been pointed out in Fig. 3 and Table III. Fig. 3 shows the effect of the conductivity of the insulating oil in increasing the absorption and changing its character. Fig. 19 gives a typical pair of oscillograms in which the presence of the irreversible conduction current is apparent in the charge record. This fact is better appreciated from the equa-

TABLE IV—ANALYSIS OF POWER FACTOR AND DIELECTRIC LOSS AT 60 CYCLES BY COMPUTATION FROM D-C. CHARACTERISTICS

Paper Specimen A-7

		Measured at 60 cycles			Computed from d-c.		
Temp deg. cent.	Volts	Charging current amp. $\times 10^{-6}$	Power factor	Dielectric loss watt $\times 10^{-6}$	In-phase component of current amp. $\times 10^{-6}$	Power factor	Dielectric loss watt $\times 10^{-6}$
T	E	I''	$\tan \Psi$	W	I_r^1	$\tan \Psi_r$	W_r
60	500	144	0.00237	169	0.340	0.00236	170
	1,000	288	0.00237	676	0.684	0.00237	684
	1,500	432	0.00237	1,518	1.047	0.00242	1,560
	Average	—	0.00237	—	—	0.00238	—
45	500	142	0.00276	197	0.410	0.00288	205
	1,000	285	0.00276	786	0.830	0.00291	830
	1,500	427	0.00276	1,796	1.289	0.00302	1,930
	Average	—	0.00276	—	—	0.00293	—
30	500	141	0.00346	243	0.533	0.00378	266
	1,000	282	0.00346	974	1.024	0.00363	1,024
	1,500	422	0.00346	2,192	1.558	0.00368	2,335
	Average	—	0.00346	—	—	0.00369	—

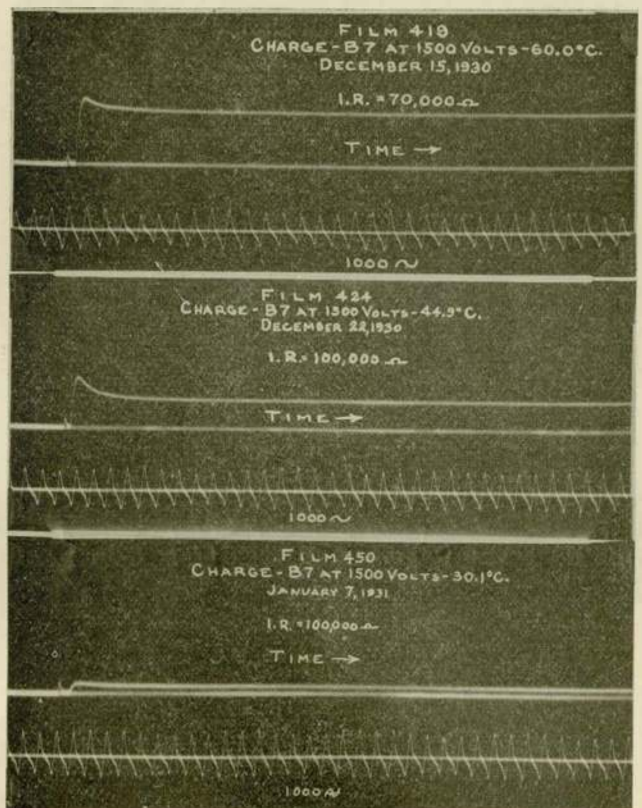

FIG. 16

tions of charge and discharge given in Fig. 20, where the irreversible conduction current is seen to be the *constant* difference between charge and discharge currents. This constant irreversible conduction current does not obey Ohm's law, as shown by Fig. 12, and its contribution to the power factor has already been computed from theoretical considerations and given in Table II.

Summary of Specimens. As an example of the insight afforded by this method of analysis, Fig. 21 gives the power factor as a function of temperature for the oil, paper, and impregnated paper specimens. The shaded areas represent the total increase in power factor with impregnation. Through the analysis of the discharge current-time relations it is possible to resolve this increase into two components:

1. The increase in reversible absorption brought about by impregnation.

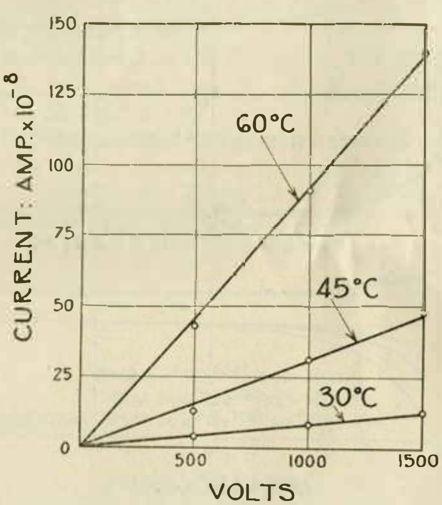

FIG.17—VARIATION OF INITIAL CONSTANT CURRENT WITH VOLTAGE. OIL SPECIMEN B7.

2. The creation of the additional contribution of irreversible conduction totally absent before impregnation and directly attributable to the conductivity of the impregnating oil.

ACKNOWLEDGMENT

Grateful appreciation is expressed to the Utilities Research Commission of Illinois, whose financial sup-

TABLE V—ANALYSIS OF POWER FACTOR AND DIELECTRIC LOSS AT 1,500 VOLTS AND 60 CYCLES BY COMPUTATION FROM D-C. CHARACTERISTICS

Oil Specimen B-7

	Measured at 60 cycles			Computed from d-c. characteristics: Reversible absorption			Irreversible conduction			Total	
Temp. deg. cent.	Charging current amp. $\times 10^{-6}$	Power factor	Dielectric loss watt $\times 10^{-6}$	In phase component of current amp. $\times 10^{-6}$	Power factor	Dielectric loss watt $\times 10^{-6}$	Initial constant current amp. $\times 10^{-6}$	Power factor	Dielectric loss watt $\times 10^{-6}$	Power factor	Dielectric loss watt $\times 10^{-6}$
T	I''	$\tan\psi$	W	I_r'	$\tan\psi_r$	W_r	I_i'	$\tan\psi_i$	W_i	$\tan\psi_o$	W_o
60	420	0.00346	2,179	0.065	0.00016	98	1.309	0.00312	1.965	0.00328	2.063
	421	0.00346	2,184	0.145	0.00035	218	1.251	0.00297	1,880	0.00332	2,098
	422	0.00374	2,357	0.102	0.00024	153	1.349	0.00320	2,025	0.00344	2,178
45	425	0.00116	739	0.029	0.000067	43	0.475	0.001117	713	0.00118	756
30	430	0.00037	239	0.027	0.000064	41	0.124	0.000288	186	0.00035	227
	430	0.00034	219	0.018	0.000042	27	0.135	0.000313	202	0.00035	229
	429	0.00035	225	0.019	0.000043	28	0.120	0.000280	180	0.00032	208
	430	0.00032	206	0.022	0.000051	33	0.128	0.000298	192	0.00035	225
	431	0.00032	210	0.032	0.000074	48	0.105	0.000244	158	0.00032	206
Average	430	0.00034	220	0.024	0.000055	36	0.122	0.000285	184	0.00034	219

port has made possible the active prosecution of a research on the properties of impregnated paper from which the experimental results here reported have been taken. The authors also wish to acknowledge the interest and careful work of Dr. S. K. Waldorf in the development and operation of the amplifier oscillograph.

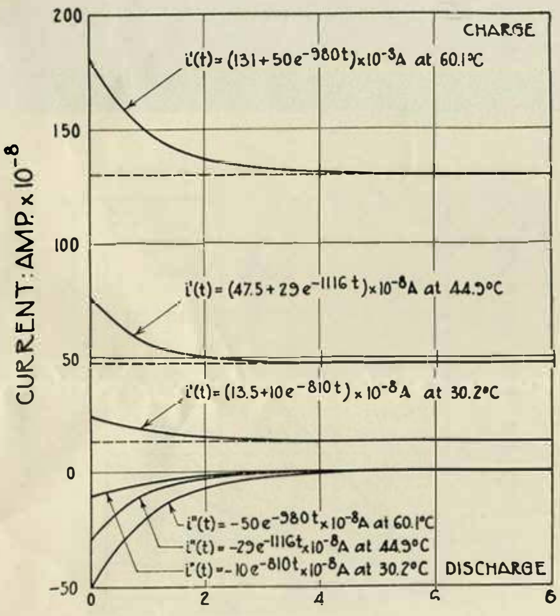

FIG. 18—VARIATION WITH TEMPERATURE OF CHARGE AND DISCHARGE CURRENT-TIME RELATIONS AT 1500 VOLTS. OIL SPECIMEN B7

CONCLUSIONS

1. It is shown that for any type of dielectric the loss, power factor and capacity at 60 cycles may be accurately predicted from the charge and discharge currents under continuous potential.

2. The method developed is available at any frequency, provided that the continuous potential charge

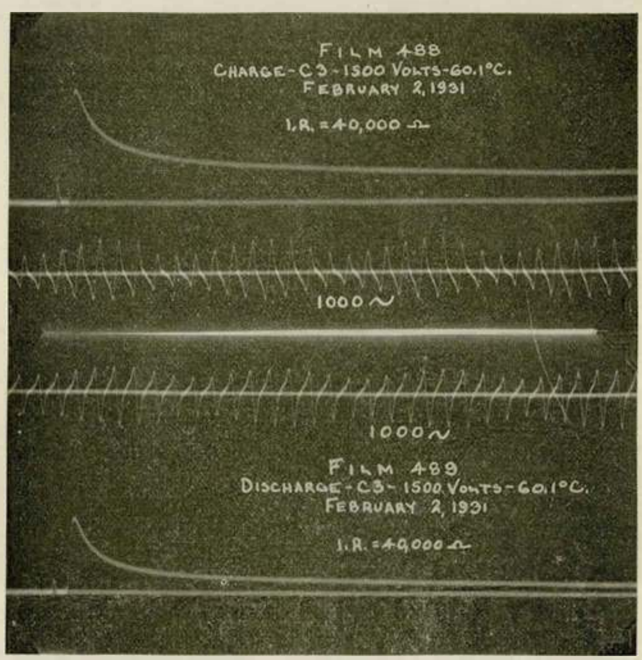

FIG. 19

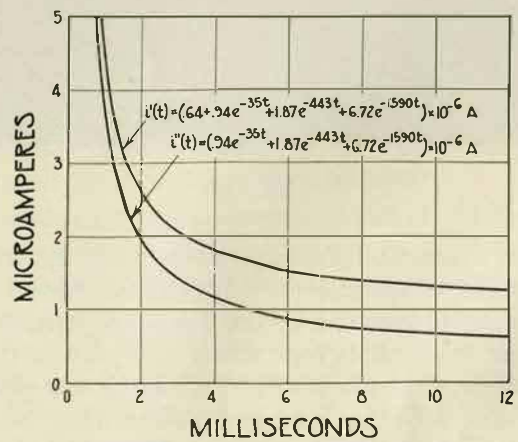

FIG. 20—CHARGE i'(t) AND DISCHARGE i''(t) CURRENT-TIME RELATIONS AT 1500 VOLTS, 60°C. IMPREGNATED PAPER SPECIMEN C7.

and discharge currents may be measured over initial time intervals comparable with the alternating period.

3. The nature of the relaxation function characteristic of reversible absorption in dielectric has been discussed from the experimental and analytical points of view.

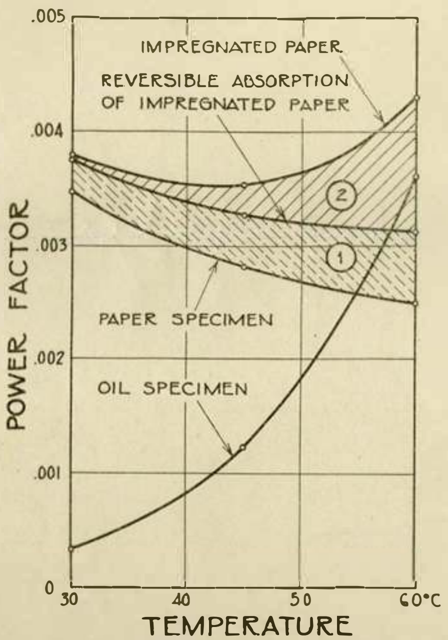

FIG. 21—ANALYSIS OF POWER FACTOR AT 1500 VOLTS. IMPREGNATED PAPER SPECIMEN C7

Increase in Power Factor with Impregnation
(1) Increase in Reversible Absorption
(2) Contribution of Irreversible Conduct...

4. It has been shown that agreement of the relaxation function with the t^{-n} expression is not general, but on the contrary it is confined to dielectrics showing negligible or no irreversible conduction. The inadequacy of the t^{-n} expression has been proved on experimental and analytical reasons. Chief among them is the failure of the t^{-n} expression to predict accurately the variation of dielectric loss with frequency.

5. The method of three exponentials has been developed as a convenient and sufficient expression for the relaxation function. The sufficiency and accuracy of three exponentials has been proved both analytically and experimentally.

6. Irreversible conduction does not obey the principle of superposition. The usual forms of irreversible conduction encountered have been defined and classified. The case where the initial constant current does not obey Ohm's law has been considered analytically in its contribution to the a-c. behavior.

7. The manner in which the complete a-c. behavior of a dielectric is accurately predicted from suitable d-c. measurements has been illustrated by three typical sets of experimental data, paper, oil, and impregnated paper.

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A High Sensitivity Power Factor Bridge

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Synopsis.—This paper describes an a-c. bridge of high precision, used for power factor measurements in one of the dielectric investigations being carried on at the Johns Hopkins University. The bridge, a modified form of the Schering bridge, possesses several novel features and advantages. The detecting instrument is a moving coil a-c. galvanometer; its field excitation is supplied from a phase shifting transformer, which permits obtaining independent ratio and phase angle balances. A shielded transformer electrostatically isolates the galvanometer from the bridge circuits.

This bridge is completely shielded and guarded and the analytical theory of the resulting mesh connection is given in the paper. The mathematical treatment is general and may be applied to any similar network. A full discussion is given of the effects upon the balance relation of various sources of error, for example, as failure to balance guard circuits properly and the presence of leakage resistance between measuring and guard circuits. Experimental verification of the equations is presented.

Power factors of specimens of cable material have been measured ranging from 0.00007 to 0.16 with a maximum variation of ± 0.000005.

THE Schering bridge is probably the most widely used a-c. bridge for the measurement of the capacitances and phase defect angles of dielectrics under high-voltage gradients. In a study of the properties of high-voltage cables carried on in the Electrical Laboratories of the School of Engineering of The Johns Hopkins University we have used a completely shielded and guarded Schering bridge[1] for a number of years.

About two years ago, under the auspices of the Utilities Research Commission, an investigation was begun of the properties of cable compounds and papers and of the characteristics that occur after impregnation. In this work it became necessary to modify the Schering bridge in order to obtain the desired range and sensitivity. The bridge that finally resulted is completely shielded and guarded and possesses several novel features.

A bridge has been developed in which it is possible to balance the guard circuits without the use of a second galvanometer or disturbing in any way the main galvanometer connections. It is shown that in this bridge it is not necessary to balance the guard circuits for phase but only for magnitude.

The Bridge

A diagram of the bridge connections is given in Fig. 1, where C_1 is the specimen and C_2 the air capacitor. The latter is adjustable and its high-tension plate is so mounted that any leakage currents from it flow directly to ground and not into its guard circuit.[2] Both the specimen and the air capacitor are provided with guard ring electrodes. The leads from their main or measuring electrodes to the points A and B of the bridge are one-eighth inch (3. mm.) brass rods shielded by one inch (2.54 cm.) diameter tubes. The central conductors are supported at approximately three-foot (1 meter) intervals by small hard rubber washers. Each shielding tube is connected to its proper guard ring as may be seen from the figure.

The capacitance between the main electrode with its lead and the guard ring with its shield is 470 $\mu\mu$f for the specimen and 490 $\mu\mu$f for the air capacitor. It is essential for accurate measurement that the insulation resistance of these two capacitances be maintained *equal to infinity,* as measured with the Megger.

The physical arrangement of the apparatus is such that the specimen and air capacitor may be quickly

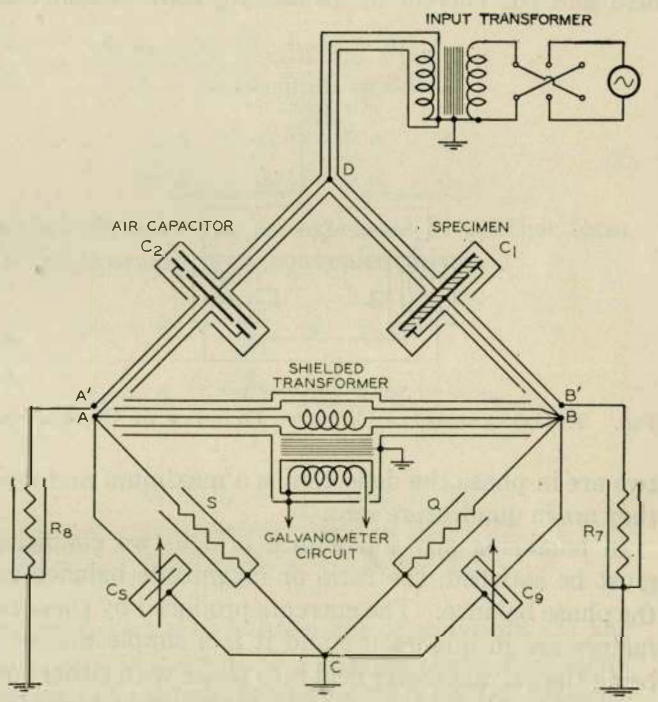

FIG. 1—GALVANOMETER CIRCUIT—DIAGRAM OF CONNECTIONS

interchanged. It is also a simple matter to disconnect the specimen and substitute an air capacitor in its place for making check measurements.

One feature of this bridge, originally used by Hartshorn[3] in low-voltage work, is the use of two variable air capacitors Cs and Cq in parallel with the S and Q arms respectively. The capacitance Cq consists of the variable capacitor shown in the figure plus the capacitance to ground of the detector arm shield, which is connected to the point B of the bridge. In normal operation Cq is kept at a fixed value and balance is obtained

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by varying Cs, a variable calibrated air capacitor. The use of the capacitor Cq permits the neutralization of the fixed capacitance of the bridge and the measurement of very small values of power factor.

As Cq includes certain fixed capacitances it is necessary to determine its value. This is accomplished by substituting an air capacitor for the specimen, and balancing by adjusting Cs, using a 1 to 1 bridge. Then the two high-voltage air capacitors are interchanged and the balance checked. The same value of Cs should hold for both cases to within the sensitivity of the bridge, a phase angle whose cosine is $\pm\ 5 \times 10^{-6}$. Under these conditions and a ratio of one to one the value of Cq equals that of Cs, the calibrated air capacitor.

The detector is a Leeds and Northrup a-c. moving coil type of galvanometer, which is connected to the bridge through an electrostatically shielded transformer, as shown in Fig. 2. The magnetic field of the galvanometer is excited from the secondary of a phase shifting transformer. The deflection of the galvanometer depends upon the phase relation between its magnetic field and the current in its moving coil. When these

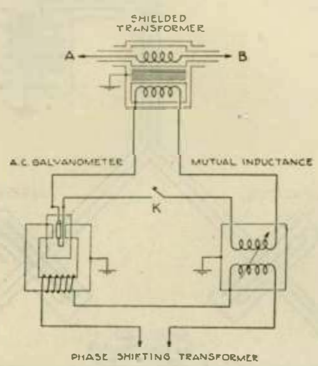

Fig. 2—Galvanometer Circuit—Diagram of Connections

two are in phase the deflection is a maximum and when they are in quadrature zero.

In balancing any impedance bridge two conditions must be satisfied, the ratio or magnitude balance and the phase balance. The currents produced by these two factors are in quadrature and it is a simple matter to bring the galvanometer field into phase with either component by adjusting the position of the secondary of the phase shifting transformer. This feature enables the operator not only to balance the bridge for either ratio or phase angle separately and independently, but also to adjust properly the magnitude of the guard resistances, R_7 and R_8, without either disturbing the connections of the main galvanometer or the use of a second detector.

A variable mutual inductance is connected in series with the galvanometer to neutralize the e.m.f. induced in its moving coil by the a-c. field. The value of the mutual inductance is adjusted so as to keep the galvanometer reading on the scale, that is, by making the electrical and mechanical zeros coincide. Balance of the bridge is obtained when there is no change in the galvanometer deflection[4] upon reversal of the input e.m.f.

The simple balance relations of this bridge are: ratio

$$C_1 = C_2\, S/Q \tag{1}$$

and the phase angle ψ_1, of the specimen

$$\psi_1 = \omega\, Cs\, S - \omega\, Cq\, Q \tag{2}$$

Where ω equals $2\ \pi$ times the frequency.

The power supply for the bridge and the primary of the phase shifting transformer are taken from a three phase "General Electric Company" sine wave generator driven by a motor supplied by the University storage battery. The sensitivity of the bridge is ample and for phase angle corresponds to an angle whose cosine is $5. \times 10^{-6}$. Values of capacitance ranging from 40 $\mu\mu$f to 1,500 $\mu\mu$f and specimen power factors from 0.007 to 16 per cent have been successfully measured. The bridge is usually operated at 60 cycles and 1,500 volts alternating current.

Operation of the Bridge

In the operation of the bridge the first step is the proper adjustment of the phase shifting transformer. This is attained in the following manner:

Using the specimen, C_1, and air capacitor, C_2, in the high-voltage arms, and having set the ratio arms Q and S to 5,000 ohms each, as tentative values, low-voltage is applied to the bridge. Then the operator proceeds to find by trial the phase shifter setting which will give a minimum deflection when the ratio arms are upset by, say, 1,000 ohms in 5,000. This gives the approximate phase shifter setting for the phase angle balance. The phase shifter is then thrown through 90 deg. to the "ratio" setting and the ratio balance roughly determined. The process is then repeated until it is possible to have an unbalance of several thousand ohms in the ratio arms remain undetected when the phase shifter is in the "phase angle" setting. These adjustments give accurately the phase angle and ratio settings of the secondary of the phase shifting transformer.

In balancing the bridge all four low-voltage arms, R_8, S, Q, and R_7, are set at some tentative values. With the phase shifter on the ratio setting and Q at 5,000 ohms, S is adjusted to the nearest ohm. If Q and S differ materially from each other the spacing of the standard air capacitor is varied to bring the bridge closer to a one to one ratio.

Once a satisfactory ratio of Q and S is obtained, the operator proceeds to balance the guard circuit of the specimen. With the phase shifting transformer in the ratio position the operator connects the points $B - B'$ of the bridge by means of a short lead and then adjusts the guard resistance, R_7, until the upset in balance caused by this procedure disappears. With R_7 at its proper value for magnitude balance, the operator removes the short circuit from across $B - B'$, places it on $A - A'$ and proceeds to adjust R_8 in the same manner as R_7. These adjustments of R_7 and R_8 are for magnitude only and not for phase. The magnitude balance is thus obtained using the main galvanometer as a detector, but without disturbing its connections in any way. The proof of the correctness of this procedure will be discussed later.

After the proper settings of R_7 and R_8 are obtained, the ratio balance is once more checked; usually the balance is not off more than five ohms in 5,000, unless the preliminary settings of R_7 and R_8 were considerably out.

The full test voltage is now applied to the bridge and the operator obtains the final ratio balance, after rechecking the guard resistances for magnitude balance. The phase shifter is then rotated through 90 deg. to the phase angle position and the phase balance obtained by adjusting the variable air capacitor, C_s.

The power factor of the specimen is calculated in accordance with equation (2). It will be shown that this simple relation gives the correct value of the power factor, despite the fact that the guards have been balanced for magnitude only; provided the leakage resistances existing between the guarding systems and their respective measuring circuits are infinity.

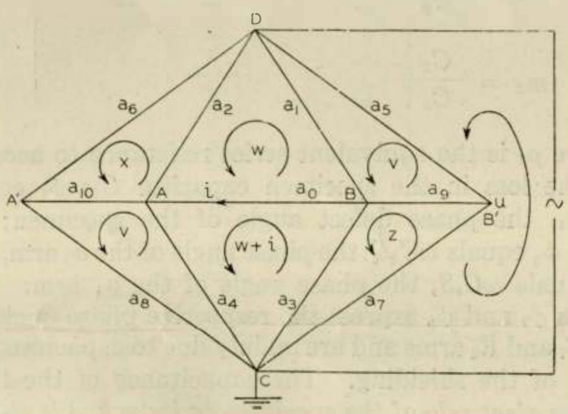

FIG. 3—GENERALIZED MESH DIAGRAM

MESH DESIGN

The mesh diagram of the bridge is given in Fig. 3. This diagram is a general one and may be applied to almost any shielded bridge. The complex impedance operators are represented by $\bar{a}_1$, $\bar{a}_2$, etc. Where $\bar{a}_1$, $\bar{a}_2$, $\bar{a}_3$ and $\bar{a}_4$ represent the measuring circuits of the bridge, and $\bar{a}_5$, $\bar{a}_6$, $\bar{a}_7$, and $\bar{a}_8$ the guard circuits. The operator $\bar{a}_1$ belongs to the measuring circuit of the specimen and $\bar{a}_5$ to its guard. In like manner $\bar{a}_2$ and $\bar{a}_6$ belong to the measuring and guard circuits of the air capacitor, respectively. The operator $\bar{a}_9$ represents the impedance existing between the measuring and guard circuits of the specimen and $\bar{a}_{10}$ performs the same function for the air capacitor circuit. The operator $\bar{a}_0$ expresses the impedance of the galvanometer arm of the bridge.

Using the cyclic currents of Maxwell[5] we obtain a set of six simultaneous equations. Table I below gives the coefficients of the cyclic currents of the generalized mesh.

TABLE I—THE COEFFICIENTS OF THE CYCLIC CURRENTS

Mesh	i	v	w	x	y	z	u
$A\,A'\,D$	0	0	$-\bar{a}_2$	$(\bar{a}_2 + \bar{a}_6 + \bar{a}_{10})$	$-\bar{a}_{10}$	0	0
$A\,B\,D$	$-\bar{a}_0$	$-\bar{a}_1$	$(\bar{a}_1 + \bar{a}_2)$	$-\bar{a}_2$	0	0	0
$B\,B'\,D$	0	$(\bar{a}_1 + \bar{a}_5 + \bar{a}_9)$	$-\bar{a}_1$	0	0	$-\bar{a}_9$	$\bar{a}_5$
$A\,A'\,C$	$-\bar{a}_4$	0	$-\bar{a}_4$	$-\bar{a}_{10}$	$(\bar{a}_4 + \bar{a}_8 + \bar{a}_{10})$	0	0
$A\,B\,C$	$(\bar{a}_0 + \bar{a}_3 + \bar{a}_4)$	0	$(\bar{a}_3 + \bar{a}_4)$	0	$-\bar{a}_4$	$-\bar{a}_3$	0
$B\,B'\,C$	$-\bar{a}_3$	$-\bar{a}_9$	$-\bar{a}_3$	0	0	$(\bar{a}_3 + \bar{a}_7 + \bar{a}_9)$	$\bar{a}_7$

The current, i, through the galvanometer circuit may be obtained by the solution of the simultaneous equations by means of determinants. For balance the galvanometer current must equal zero and this will only be fulfilled when the determinant for the numerator is zero.

Solving this determinant and equating to zero the complete balance relation is obtained, equation (3), for the generalized network, namely

$$\frac{\bar{a}_1\,\bar{a}_4}{\bar{a}_2\,\bar{a}_3} = \frac{\bar{a}_5\,\bar{a}_9 + \bar{a}_7\,(\bar{a}_1 + \bar{a}_5 + \bar{a}_9)}{\bar{a}_7\,\bar{a}_9 + \bar{a}_5\,(\bar{a}_3 + \bar{a}_7 + \bar{a}_9)} \cdot \frac{\bar{a}_8\,\bar{a}_{10} + \bar{a}_6\,(\bar{a}_4 + \bar{a}_8 + \bar{a}_{10})}{\bar{a}_6\,\bar{a}_{10} + \bar{a}_8\,(\bar{a}_2 + \bar{a}_6 + \bar{a}_{10})} \tag{3}$$

Equation (3) may also be expressed in another form, (4), which is usually more convenient to use.

$$\frac{\bar{a}_1\,\bar{a}_4}{\bar{a}_2\,\bar{a}_3} = \frac{\bar{a}_5 + \bar{a}_7 + \dfrac{\bar{a}_7}{\bar{a}_9}\,(\bar{a}_1 + \bar{a}_5)}{\bar{a}_5 + \bar{a}_7 + \dfrac{\bar{a}_5}{\bar{a}_9}\,(\bar{a}_3 + \bar{a}_7)} \cdot \frac{\bar{a}_6 + \bar{a}_8 + \dfrac{\bar{a}_6}{\bar{a}_{10}}\,(\bar{a}_4 + \bar{a}_8)}{\bar{a}_6 + \bar{a}_8 + \dfrac{\bar{a}_8}{\bar{a}_{10}}\,(\bar{a}_2 + \bar{a}_6)} \tag{4}$$

The terms on the right hand side of equations (3) and (4) are correction terms that express the factors introduced into the simple bridge relation by the presence of the guard circuits. These equations may be expressed in the form of

$$\frac{\bar{a}_1\,\bar{a}_4}{\bar{a}_2\,\bar{a}_3} = \bar{k}_1\,\bar{k}_2 \tag{5}$$

Where
$$\bar{k}_1 = \frac{\bar{a}_5 + \bar{a}_7 + \dfrac{\bar{a}_7}{\bar{a}_9}\,(\bar{a}_1 + \bar{a}_5)}{\bar{a}_5 + \bar{a}_7 + \dfrac{\bar{a}_5}{\bar{a}_9}\,(\bar{a}_3 + \bar{a}_7)}$$

and
$$\bar{k}_2 = \frac{\bar{a}_6 + \bar{a}_8 + \dfrac{\bar{a}_6}{\bar{a}_{10}}\,(\bar{a}_4 + \bar{a}_8)}{\bar{a}_6 + \bar{a}_8 + \dfrac{\bar{a}_8}{\bar{a}_{10}}\,(\bar{a}_2 + \bar{a}_6)}$$

It is interesting to note that if the guard circuits are balanced with respect to their corresponding bridge circuits, *i. e.*,

$$\frac{\bar{a}_1}{\bar{a}_3} = \frac{\bar{a}_5}{\bar{a}_7} \text{ and } \frac{\bar{a}_2}{\bar{a}_4} = \frac{\bar{a}_6}{\bar{a}_8}$$

that the generalized expression reduces to the simple bridge relation given in

$$\frac{\bar{a}_1\,\bar{a}_4}{\bar{a}_2\,\bar{a}_3} = 1 \tag{6}$$

If no guard circuits are used in the bridge or if the impedances expressed by $\bar{a}_9$ and $\bar{a}_{10}$ are equal to infinity, equation (4) again reduces to (6) the simple balance relation for a wheatstone network.

Equation (**6**) contains the complex operators of the four arm impedance bridge; each operator having the general form $R \pm jX$. The balance of the bridge requires two adjustments, ratio and phase[6] as stated in the introduction. The ratio and phase balance relations, readily deduced from equation (**6**), are given (7) below:

$$\left.\begin{aligned} \frac{Z_1 Z_4}{Z_2 Z_3} &= 1 \\ (\phi_1 + \phi_4) - (\phi_2 + \phi_3) &= 0 \end{aligned}\right\} \tag{7}$$

Where Z_1, Z_2, etc., are absolute values and ϕ_1, ϕ_2, etc., are the corresponding phase angles of their respective complex impedance operators.

In like manner the correction terms $\bar{k}_1$ and $\bar{k}_2$ of equation (**5**) are complex quantities for which K_1 and K_2 are their respective absolute values and ϕ_{K1} and ϕ_{K2} their phase angles. For the guarded and shielded bridge the general balance relations are (8)

$$\left.\begin{aligned} \frac{Z_1 Z_4}{Z_2 Z_3} &= K_1 K_2 \\ (\phi_1 + \phi_4) - (\phi_2 + \phi_3) &= \phi_{K1} + \phi_{K2} \end{aligned}\right\} \tag{8}$$

Correction Terms

It is evident from equation (8) that the simple ratio and phase angle balance relations mentioned under the description of the bridge must be modified to take care of the correction terms, $\bar{k}_1$ and $\bar{k}_2$ introduced by the presence of the guarding and shielding circuits and that these terms must be evaluated.

It is well to point out here that these correction terms are actually present in every four arm impedance bridge. In unshielded bridges their evaluation is impossible. The use of guards and shields serves to give definite values to these terms and to make their accurate determination feasible.

In order to evaluate the correction terms we must substitute the values of the general operators given below into equation (5)

$$\left.\begin{aligned}
\bar{a}_1 &= \rho_1 - \frac{j}{\omega C_1} = \frac{1}{\omega C_1}(\psi_1 - j) \\
\bar{a}_2 &= \frac{-j}{\omega C_2} \\
\bar{a}_3 &= \frac{1}{1/Q + j\,\omega C_q} = Q\,(1 - j\,\phi_q) \\
\bar{a}_4 &= \frac{1}{1/S + j\,\omega C_s} = S\,(1 - j\,\phi_s) \\
\bar{a}_5 &= \frac{1}{\omega C_5}(\psi_5 - j) = \frac{m_1}{\omega C_1}(\psi_1 - j) \\
\bar{a}_6 &= -\frac{j}{\omega C_6} = -\frac{j\,m_2}{\omega C_6} \\
\bar{a}_7 &= R_7\,(1 - j\,\phi_7) = m_1\,Q\,(1 - j\,\phi_7) \\
\bar{a}_8 &= R_8\,(1 - j\,\phi_8) = m_2\,S\,(1 - j\,\phi_8) \\
\bar{a}_9 &= \frac{1}{1/W_9 + j\,\omega C_9} \\
\bar{a}_{10} &= \frac{1}{1/W_{10} + j\,\omega C_{10}} \\
m_1 &= \frac{C_1}{C_5} \\
m_2 &= \frac{C_2}{C_6}
\end{aligned}\right\} \tag{9}$$

Where ρ_1 is the equivalent series resistance to account for the loss in the specimen capacitor C_1; ψ_1 equals $\omega C_1 \rho_1$, the phase defect angle of the specimen; the angle ϕ_q equals $\omega C_q Q$, the phase angle of the a_3 arm, and ϕ_s equals $\omega C_s S$, the phase angle of the a_4 arm. The angles ϕ_7 and ϕ_8 express the respective phase angles of the R_7 and R_8 arms and are mainly due to capacitance to earth of the shielding. The capacitance of the high-tension electrode of the specimen to its guard is C_5 and ψ_5 its phase defect angle, and the capacitance of the high side of the air capacitor to its guard is C_6. In the operators we have assumed that the phase defect angles of the specimen and its guard are the same, namely that ψ_1 equals ψ_5, and also that both the air capacitor C_2 and its guard C_6 have zero phase defect angles.[2] The capacitance existing between the measuring circuit and the shielding circuit of the specimen is C_9 and the corresponding capacitance for the air capacitor is C_{10}. Both of these capacitances, C_9 and C_{10} have leakage resistances, W_9 and W_{10} respectively. In normal operation these leakage resistances W_9 and W_{10} must equal infinity as already mentioned.

Substituting the values of the general operators into the equations for the correction terms we obtain the corresponding complex values of $\bar{k}_1$ and $\bar{k}_2$ equations (**10**) and (**11**)

$$\bar{k}_1 = \frac{\left\{\Phi\left[1+\frac{C_9}{C_1}(1+m_1)\right] - \frac{Q}{W_9}\phi_7(1+m_1) + \psi_1\left[1+\frac{Q}{W_9}(1+m_1)\right]\right\} - j\left\{1+\frac{Q}{W_9}(1+m_1)+\Phi\phi_7\left[1+\frac{C_9}{C_1}(1+m_1)\right] - \frac{C_9}{C_1}\Phi\psi_1(1+m_1)\right\}}{\left\{\Phi\left[1+\frac{C_9}{C_1}(1+m_1)\right] - \frac{Q}{W_9}\phi_7(1+m_1)+\psi_1\left[1+\frac{Q}{W_9}(1+m_1)\right]\right\} - \left\{\frac{Q}{W_9}(\phi_q-\phi_7)\right\} - j\left\{1+\frac{Q}{W_9}(1+m_1)+\Phi\phi_7\left[1+\frac{C_9}{C_1}(1+m_1)\right]-\frac{C_9}{C_1}\Phi\psi_1(1+m_1)\right\} - j\left\{\frac{C_9}{C_1}\Phi(\phi_q-\phi_7)\right\}} \tag{10}$$

$$\bar{k}_2 = \frac{\left\{\Phi\left[1+\frac{C_{10}}{C_2}(1+m_2)\right] - \frac{S}{W_{10}}\phi_8(1+m_2)\right\} - \left\{\frac{S}{W_{10}}(\phi_s-\phi_8)\right\} - j\left\{1+\frac{S}{W_{10}}(1+m_2)+\Phi\phi_8\left[1+\frac{C_{10}}{C_2}(1+m_2)\right]\right\} - j\left\{\frac{C_{10}}{C_2}\Phi(\phi_s-\phi_8)\right\}}{\left\{\Phi\left[1+\frac{C_{10}}{C_2}(1+m_2)\right] - \frac{S}{W_{10}}\phi_8(1+m_2)\right\} - j\left\{1+\frac{S}{W_{10}}(1+m_2)+\Phi\phi_8\left[1+\frac{C_{10}}{C_2}(1+m_2)\right]\right\}} \tag{11}$$

In equations (**10**) and (**11**) the term expressed by Φ is

$$\Phi = \omega C_2 S = \omega C_1 Q$$

A study of the expressions for the correction terms shows that the majority of the terms in the numerators and denominators are alike. They differ, however, only in certain terms in both their real and imaginary parts.

These complex expressions for k_1 and k_2 are, therefore, of the general form

$$\bar{k}_1 = \frac{\alpha_1 - j\beta_1}{(\alpha_1 - \gamma_1) - j(\beta_1 + \delta_1)} \tag{12}$$

and

$$\bar{k}_2 = \frac{(\alpha_2 - \gamma_2) - j(\beta_2 + \delta_2)}{\alpha_2 - j\beta_2} \tag{13}$$

Where the α terms denote those that are alike in their real parts in both the numerators and denominators and the β terms apply to those imaginary terms in the respective numerators and denominators that correspond. The γ and δ terms in (**12**) and (**13**) include only those terms by which the respective numerators and denominators differ.

The absolute values and the phase angles of the correction terms introduced by shielding and guarding the bridge may be expressed as follows:

$$K_1 = \sqrt{\frac{\alpha_1^2 + \beta_1^2}{(\alpha_1 - \gamma_1)^2 + (\beta_1 + \delta_1)^2}} \tag{14}$$

$$K_2 = \sqrt{\frac{(\alpha_2 - \gamma_2)^2 + (\beta_2 + \delta_2)^2}{\alpha_2^2 + \beta_2^2}} \tag{15}$$

and

$$\phi_{K1} = +\frac{\alpha_1\delta_1 + \beta_1\gamma_1}{\alpha_1^2 + \beta_1^2 - \alpha_1\gamma_1 + \beta_1\delta_1} \tag{16}$$

$$\phi_{K2} = -\frac{\alpha_2\delta_2 + \beta_2\gamma_2}{\alpha_2^2 + \beta_2^2 - \alpha_2\gamma_2 + \beta_2\delta_2} \tag{17}$$

Taking into account the phase angles of the correction terms, we may write the generalized expression (**8**) for the power factor of the specimen as follows:

$$\psi_1 = \omega C_s S - \omega C_q Q - (\phi_{K1} + \phi_{K2}) \tag{18}$$

Analysis of the Correction Terms

In the analysis of the correction terms four cases that involve the accuracy and method of operation of the bridge are discussed.

Case 1

Case 1 considers the effect of failure to balance the guard circuits for phase. It belongs to the normal operation of the bridge in which the guard circuits are balanced for magnitude only, and the leakage resistances W_9 and W_{10} are assumed equal to infinity. In this case the normal operators are the general operators given in equation (**9**) with the exceptions of $\bar{a}_9$ and $\bar{a}_{10}$, which here become:

$$\bar{a}_9 = -\frac{j}{\omega C_9}$$

$$\bar{a}_{10} = -\frac{j}{\omega C_{10}}$$

Case 2

Case 2 considers the phase angle error introduced by the failure to balance the guard circuits for magnitude as well as phase, provided that the leakage resistances W_9 and W_{10} still remain infinite. Let θ_7 represent the ratio of the actual value of guard resistance R_7 to the proper value for correct magnitude balance, that is:

$$\theta_7 = \frac{R_7}{m_1 Q}$$

Similarly $$\theta_8 = \frac{R_8}{m_2 S}$$

Then, the conditions expressed under this case modify the general operators as follows:

$$\bar{a}_7 = \theta_7 m_1 Q (1 - j\phi_7) \qquad \bar{a}_9 = -\frac{j}{\omega C_9}$$

$$\bar{a}_8 = \theta_8 m_2 S (1 - j\phi_8) \qquad \bar{a}_{10} = -\frac{j}{\omega C_{10}}$$

in which θ_7 and θ_8 obviously become unity when the guard circuits are properly balanced for magnitude.

Case 3

Case 3 considers the phase angle error introduced by the presence of leakage resistances W_9 and W_{10} in the capacitances C_9 and C_{10}, between the measuring circuits and their guards, provided that the guard resistances have the proper value for magnitude balance. In this case, the general operators of equation (**9**) hold.

Case 4

Case 4 considers the conditions and ratio errors that exist when the corners of the bridge are connected together, *i. e.*, A to A', as is customary in balancing the guard resistances R_7 and R_8 (See Operation). Under these conditions the operators involved become:

$$\bar{a}_7 = \theta_7 m_1 Q (1 - j\phi_7)$$
$$\bar{a}_8 = \theta_8 m_2 S (1 - j\phi_8)$$
$$\bar{a}_9 = 0$$
$$\bar{a}_{10} = 0$$

To obtain the phase angle and ratio errors produced by the conditions as outlined in these four cases, substitute the corresponding operators in the expressions for $\bar{k}_1$ and $\bar{k}_2$ (**5**) and determine the respective values of α, β, γ and δ terms (**12**) to (**17**) inclusive for each case.

Table II below gives for the four cases the values of the phase angle and ratio errors, together with the modifying operators causing the errors, and the corresponding expressions for the α, β, γ, and δ terms. . In working out the values shown in this table the results

TABLE II

Case	a_7	a_9	α_1	β_1
1	$m_1 Q (1 - j\phi_7)$	$-\frac{j}{\omega C_9}$	$\Phi\left[1 + \frac{C_9}{C_1}(1 + m_1)\right] + \Psi_1$	1
2	$\theta_7 m_1 Q (1 - j\phi_7)$	$-\frac{j}{\omega C_9}$	$\theta_7 \Phi\left[1 + \frac{C_9}{C_1}(1 + m_1)\right] + \Psi_1$	1
3	$m_1 Q (1 - j\phi_7)$	$\frac{1}{\frac{1}{W_9} + j\omega C_9}$	$\Phi\left[1 + \frac{C_9}{C_1}(1 + m_1)\right] - \frac{Q}{W_9}\phi_7 (1 + m_1) + \Psi_1\left[1 + \frac{Q}{W_9}(1 + m_1)\right]$	$1 + \frac{Q}{W_9}(1 + m_1)$
4	$\theta_7 m_1 Q (1 - j\phi_7)$	0	$-\phi_7$	1

Case	a_8	a_{10}	α_2	β_2
1	$m_2 S (1 - j\phi_8)$	$-\frac{j}{\omega C_{10}}$	$\Phi\left[1 + \frac{C_{10}}{C_2}(1 + m_2)\right]$	1
2	$\theta_8 m_2 S (1 - j\phi_8)$	$-\frac{j}{\omega C_{10}}$	$\theta_8 \Phi\left[1 + \frac{C_{10}}{C_2}(1 + m_2)\right]$	1
3	$m_2 S (1 - j\phi_8)$	$\frac{1}{\frac{1}{W_{10}} + j\omega C_{10}}$	$\Phi\left[1 + \frac{C_{10}}{C_2}(1 + m_2)\right] - \frac{S}{W_{10}}\phi_8 (1 + m_2)$	$1 + \frac{S}{W_{10}}(1 + m_2)$
4	$\theta_8 m_2 S (1 - j\phi_8)$	0	$-\phi_8$	1

were simplified by neglecting the product of two phase angles when added to or subtracted from unity.

In most cases this entails an approximation of less than one part in 10,000 of the factor involved, and is, therefore, fully warranted since it means an approximation in the phase angle relation of the bridge of the order of one part in one hundred million.

Experimental Verification of Equations

Case 1

A study of the phase angle and absolute value errors, Table II, shows that the failure to balance the guard circuits for phase causes no error in the ratio balance since K_1 and K_2 here equal unity, and only a negligible phase angle error.

TABLE II

γ_1	δ_1	K_1	ϕ_{k1}
0	$-\frac{C_9}{C_1}\Phi(\phi_q - \phi_7)$	1	$+\frac{C_9}{C_1}\left[1+\frac{C_9}{C_1}(1+m_1)\right]\Phi^2(\phi_q - \phi_7)$
$\frac{C_9}{C_1}\Phi(\theta_7 - 1)$	$\frac{C_9}{C_1}\Phi(\phi_q - \theta_7\phi_7) + \frac{C_9}{C_1}\Phi\Psi_1(\theta_7 - 1)$	1	$+\frac{C_9}{C_1}\Phi(\theta_7 - 1)$
$\frac{Q}{W_9}(\phi_q - \phi_7)$	$\frac{C_9}{C_1}\Phi(\phi_q - \phi_7)$	1	$+\frac{(\phi_q - \phi_7)}{\frac{W_9}{Q} + (1+m_1)}$
$\frac{(\phi_q - \theta_7\phi_7)}{\theta_7(1+m_1)}$	$-\frac{(\theta_7 - 1)}{\theta_7(1+m_1)}$	$\frac{\theta_7(1+m_1)}{(1+2m_1\theta_7+m_1^2\theta_7^2)^{\frac{1}{2}}}$	$+\frac{(\phi_q - \phi_7)}{1+\theta_7 m_1}$
γ_2	δ_2	K_2	ϕ_{k2}
0	$\frac{C_{10}}{C_2}\Phi(\phi_s - \phi_8)$	1	$-\frac{C_{10}}{C_2}\left[1+\frac{C_{10}}{C_2}(1+m_2)\right]\Phi^2(\phi^8 - \phi_8)$
$\frac{C_{10}}{C_2}\Phi(\theta_8 - 1)$	$\frac{C_{10}}{C_2}\Phi(\phi_s - \theta_8\phi_8)$	1	$-\frac{C_{10}}{C_2}\Phi(\theta_8 - 1)$
$\frac{S}{W_{10}}(\phi_s - \phi_8)$	$\frac{C_{10}}{C_2}\Phi(\phi_s - \phi_8)$	1	$-\frac{(\phi_s - \phi_8)}{\frac{W_{10}}{S} + (1+m_2)}$
$\frac{(\phi_s - \theta_8\phi_8)}{\theta_8(1+m_2)}$	$-\frac{(\theta_8 - 1)}{\theta_8(1+m_2)}$	$\frac{(1+2m_2\theta_8+m_2^2\theta_8^2)^{\frac{1}{2}}}{\theta_8(1+m_2)}$	$-\frac{(\phi_s - \phi_8)}{1+\theta_8 m_2}$

The latter is shown to be true in the following typical numerical example of the measurement of the power factor at 500 volts 60 cycles of an impregnated paper specimen at 30 deg. cent. Complete data are given below.

Standard Air Capacitor	Specimen
$C_2 = 1110 \times 10^{-12}$ farad	$C_1 = 1159 \times 10^{-12}$ farad
$C_s = 2815 \times 10^{-12}$ farad	$C_q = 889 \times 10^{-12}$ farad
$C_{10} = 489 \times 10^{-12}$ farad	$C_9 = 469 \times 10^{-12}$ farad
$R_8 = 9909$ ohms	$R_7 = 8216$ ohms
$S = 5219$ ohms	$Q = 5000$ ohms
$m_2 = R_8/S = 1.899$	$m_1 = R_7/Q = 1.643$
$\phi_s = \omega C_s S = 0.00554$	$\phi_q = \omega C_q Q = 0.00168$

$$\Phi = \omega C_1 Q = \omega C_2 S = 0.00218$$

$$\psi_1 = \omega C_s S - \omega C_q Q = 0.00386$$

Substituting the proper values in the expressions for ϕ_{K1} and ϕ_{K2} of Case 1, Table II, and assuming as an extreme case that ϕ_7 and ϕ_8 are zero, the following phase angle errors are obtained:

$$\phi_{K1} = +\ 6.7 \times 10^{-9}$$

$$\phi_{K2} = -\ 26.3 \times 10^{-9}$$

These are seen to be negligible when compared with the phase angle sensitivity of our bridge $\pm 5 \times 10^{-6}$.

This numerical example is only one of many that could be presented and fully justifies our method of operating the bridge in which no attempt is made to balance the guard circuit for phase. It might also be stated in further justification of our practise that the capacitances to earth of the shields which are in parallel with R_7 and R_8, respectively, serve to reduce this error still further.

Case 2

In Case 2 where the guards are not properly balanced for either phase or magnitude it is evident that the absolute values of K_1 and K_2 are again unity and that ratio errors are absent.

It may be seen from Table II, Case 2, that the phase angle errors ϕ_{K1} and ϕ_{K2} are linear functions of the magnitude unbalance factors θ_7 and θ_8. To verify these equations experimentally the following test was made:

The bridge was balanced normally as in the numerical example given under Case 1, keeping the air capacitor side of the bridge properly balanced for magnitude, we determined the value of ϕ_{K1} corresponding to different values of the guard resistance, R_7. The ratio, θ_7, of the actual value of the guard resistance to its true value was varied from zero to two. The results are given in Fig. 4 and it is evident that a linear relation exists between ϕ_{K1} and θ_7.

As a further check a similar test was performed to determine the value of ϕ_{K2} when θ_8 was varied, keeping

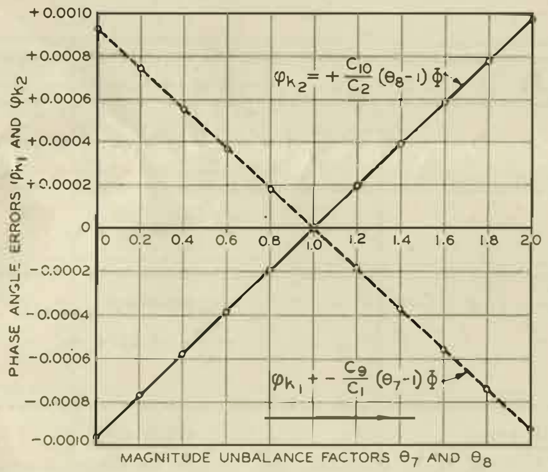

Fig. 4—Experimental Verification of Case 2

θ_7 equal to unity. The results of this run are also plotted in Fig. 4 and we again obtain linear relation between the two quantities.

Obviously, from such experimentally determined curves it is possible to compute the values of the cross-capacitances C_9 and C_{10}.

As a verification of these relations, the cross-capacitances, C_9 and C_{10}, were measured separately using a 1,000-cycle capacitance bridge. The results given below in Table III show a good agreement and serve further to prove the correctness of the equations.

TABLE III—COMPARISON OF CALCULATED AND MEASURED VALUES FOR THE CROSS-CAPACITANCES C_9 AND C_{10}

Capacitance	Capacitance computed from Fig. 4	Capacitance measured at 1,000 cycles
C_9	.469	.478
C_{10}	.489	.472

It is clearly evident from the equations that the phase angles errors are reduced by keeping C_9 and C_{10} small. The experimental results of Case 2 verify the mathematical relation and also show the importance of accurately balancing the guard resistances for magnitude.

Case 3

This case considers the phase angle errors caused by lack of phase angle balance in the guard circuits acting through the presence of leakage resistances W_9 and W_{10}. As in the two preceding cases, the ratio balance still remains unaffected when the guards are properly balanced for magnitude. The following test was performed to obtain experimental verification of the equations involved; Table II, Case 3:

1. The bridge was balanced in the normal way, with W_9 and W_{10} equal to infinity, the air capacitor was adjusted to one to one ratio with the specimen to within five ohms in 5,000, and the guard capacitance C_6 was varied to make the ratios m_1 and m_2 equal to each other to within 0.1 per cent.
2. The specimen guard circuit was then balanced for phase as well as magnitude using an additional variable capacitor in parallel with R_7.
3. An inductance L_8, of approximately 0.5 henry, was introduced in series with the air capacitor guard resistance R_8, and this resistance was decreased by an amount equal to the resistance of the inductance coil, thus preserving the magnitude balance but greatly increasing the phase difference $(\phi_s - \phi_8)$. It must be noted here that the insertion of L_8 giving the arm R_8 a large inductive phase angle caused no measurable power factor error when the guard of the air capacitor was balanced for magnitude only and not for phase.
4. The actual test was then performed, introducing the resistances W_9 and W_{10} which were kept at all times equal to each other, and which were varied from 100,000 ohms to zero, determining at each step the phase angle error caused by the phase angle difference $(\phi_s - \phi_8)$.

In this test it was necessary to keep W_9 equal to W_{10} to preserve the one to one ratio throughout, thereby eliminating errors due to residual angles in the resistance

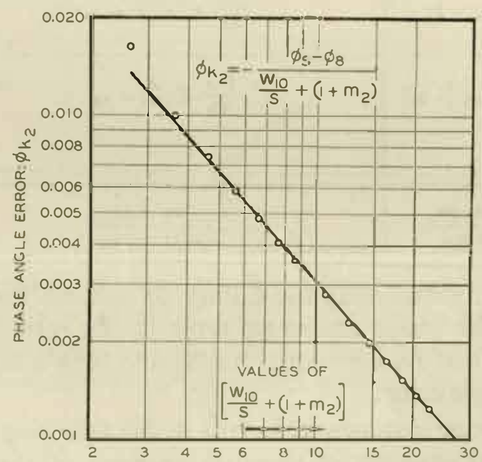

Fig. 5—Experimental Verification of Case 3

boxes. It was, therefore, necessary to balance the specimen guard circuit for phase as well as magnitude to confine the error studied to the phase difference $(\phi_s - \phi_8)$ alone, and it was deemed advisable to introduce an inductance in guard resistance R_8 to accentuate the error and make its measurement more accurate.

The phase angle error under consideration is

$$\phi_{K2} = \frac{\phi_s - \phi_8}{\dfrac{W_{10}}{S} + (1 + m_2)} \tag{19}$$

which may be written in the form

$$\log\phi_{K2} = \log(\phi_s - \phi_8) - \log\left[\frac{W_{10}}{S} + (1+m_2)\right] \quad (20)$$

It is readily seen from (20) that plotting in logarithmic coordinates the phase angle error ϕ_{K2} against the variable

$$\left[\frac{W_{10}}{S} + (1 + m_2)\right]$$

gives a straight line of which the slope is unity for axes of equal moduli. The results of experiment are plotted in Fig. 5 which shows a remarkable agreement between the results of experiment and the above theoretical considerations. It is interesting to note that ϕ_7 and ϕ_8 may be obtained by this test if desired.

Case 4

This case is of importance because it represents the conditions pertaining to the balancing of the guard circuits as described under operation. It considers the ratio error introduced by a given unbalance in the guard circuit acting through a short circuit between the measuring circuit and its guard.

As stated under the operation of the bridge, all four low-voltage arms R_7, Q, S, and R_8 are set at some tentative values, and the operator adjusts Q and S to obtain the preliminary ratio balance. We will confine our attention to the specimen side of the bridge and assume that the preliminary ratio balance is obtained with the tentative guard resistance R_7 off from its correct value by a magnitude unbalance, factor θ_7. Under these conditions the ratio error caused by short-circuiting $B - B'$ is

$$K_1 = \frac{\theta_7\,(1 + m_1)}{\sqrt{1 + 2\,m_1\,\theta_7 + m_1^2\,\theta_7^2}} = \frac{\theta_7\,(1 + m_1)}{1 + m_1\,\theta_7} \quad (21)$$

This unbalance in ratio results in a deflection of the galvanometer. Obviously from equation (21), when θ_7 is made equal to one, K_1 becomes unity and the ratio error disappears. In operating the bridge the guard resistance R_7 is varied until the ratio unbalance disappears as already explained. Under these conditions

$$\frac{R_7}{Q} = \frac{C_1}{C_5} = m_1$$

Similarly when the air capacitor side of the bridge is adjusted K_2 equals 1 and

$$\frac{R_8}{S} = \frac{C_2}{C_6} = m_2$$

From the analysis of this case it is evident that our method of adjusting the guard resistance without disturbing in any way the main galvanometer connection is fully justified.

Conclusions

The conclusions may be summarized as follows:

1. In a completely guarded and shielded bridge it is advisable to keep the cross-capacitances existing between the measuring circuits and the guard circuits low.
2. In such a bridge it is of prime importance that the conductances of the cross-capacitances be maintained at zero.
3. We have developed a method employing a separately excited moving coil a-c. galvanometer that makes it possible to balance the guard circuits using the main galvanometer as a detector, without disturbing the connections of the main galvanometer in any way.
4. The equations for the correction terms introduced by the shielding and guarding system are developed and their values are determined for the constants of our bridge.
5. We have shown by both mathematical and experimental proofs that the error introduced in our bridge by failure to balance the guards for phase angle is negligible.

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Baltimore, Maryland, U. S. A.
Julio 5, 1932

Señor Fernando Torreblanca
Consulado General de Mexico
París, Francia

Muy querido tío:

Con mucho gusto recibí tu carta donde me dices que tan pronto como regreses a México tratarás de mi asunto y te esforzarás en obsequiar mis deseos. No sabes cuanto te agradezco estas palabras que presagian la realización de mis anhelos y esperanzas de completar mis estudios para el doctorado y de regresar a México para aprovechar en bien de mi patria los conocimientos adquiridos.

Como sabes, el empleo que tenía en la Universidad de Hopkins concluyó el mes pasado. Con esto nuestra situación es bien precaria pues no cuento en lo absoluto con otros medios de vida. Por esta razón mi esposa y yo nos vamos a vivir a Washington con mi papá durante las semanas que falten para tu regreso a México. Esperamos de tu bondad que para entonces sea posible arreglar que mis servicios sean utilizados por nuestra embajada en Berlín.

Hazme favor de decirme si de regreso piensan pasar por Nueva York ó Washington, pues de ser así quiero aprovechar la ocasión para hablarte acerca de mis estudios en Berlín y de los planes que tengo para el futuro. De esta manera podrás aconsejarme acerca de estos planes, pues al solicitar tu ayuda para llevarlos a cabo quiero contar con to aprobación.

Con cariñosos saludos de nuestra parte para tí y los tuyos se despide tu sobrino que te estima.

Alfredo Baños, Jr.

P. S. - Puedes dirigir tu contestación a cargo de mi papá en la embajada ó a 3033 16th. St., N.W., Washinton, D. C., U.S.A.

Enterado. Como ya te dije antes, a mi regreso a Mexico procuraré ver la manera de obsequiar tus deseos, pero desde ahora te digo que no debes contar con esto como un hecho pues ya ves que a pesar de la buena voluntad muchas veces no basta para lograr lo que se desea.

23

SERVICIO DIPLOMÁTICO MEXICANO

GINEBRA

Clinique Mon Repos
Mont Pélerin a 12 de agosto de 1932.
Suiza.

Sr. D. Alfredo BAÑOS

W a s h i n g t o n .-

Mi querido sobrino:

Con el interés y el afecto de siempre me impuse de tu grata fechada el 5 de julio.

Como ya te dijé antes, a mi regreso a México procuraré ver la manera de obsequiar tus deseos, pero aunque no quiero desanimarte, creo preferible decirte, desde ahora, que no debes contar con ésto como un hecho, pues a pesar de que yo tengo la mejor buena voluntad, ya sabes que muchas veces ésta no basta para lograr lo que se desea.

Seguramente regresaré a México por New York, y si estas en ésta, tendré mucho gusto en verte.

Con cariñosos recuerdos de nosotros para todos ustedes, se despide afectuosamente tu tío.

F. TORREBLANCA.

29

Telegramm – Télégramme – Telegramma

MONT-PÉLERIN SUR VEVEY 25.VIII.32 – 8 – 11

LE MONT-PÉLERIN	von – de – da	Washington d.c	№ 619
Aufgegeben den – Consigné le – Consegnato il 24/8 1932		Stunde – Heure – Ora 21.40	Wörter – Mots – Parole 15

Erhalten – Reçu – Ricevuto 25/8

Befördert – Transmis – Trasmesso

von – de – da	Stunde – Heure – Ora	Name – Nom – Nome	nach – à – a	Stunde – Heure – Ora	Name – Nom – Nome
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Via W.U.R.S. № 138

L.C.O. Fernando Torreblanca

LE MONT-PÉLERIN clinique

Mon Repos

Please cable soon date

arrival New-York

alfredo Baños Contreras

Embajada en Washington

No me es posible
desde ahora saber la
fecha de mi regreso a
México. Esto depende de
la mejoría que obtenga
en mi salud.

N° 2. – A 5. – (148 × 210). – Qu 070.

25

SERVICIO DIPLOMÁTICO MEXICANO

GINEBRA

"Mon Repos" Mont Pélerin, Vevey. Suiza.
a 29 de Agosto de 1932.

Señor Dn Alfredo Baños hijo.
Embajada de México en
WASHINGTON. E.U.A.

Mi querido sobrino:

Recibí tu cable del 24 último, en el que me pides que te telegrafíe la fecha de mi llegada a Nueva-York. Desgraciadamente no me es posible, desde ahora, saber cuando podré efectuar mi regreso a México y, mucho menos, fijar fecha. Esto depende de la mejoría que obtenga en mi salud, pues aun cuando voy mejorando, esta mejoría es lenta y me obliga, muy a pesar mio, a permanecer más tiempo del que yo esperaba. No obstante, tan pronto como esté yo en condiciones de complacerte, lo haré con toda oportunidad.

Con mis afectuosos recuerdos para todos, recibe los de tu tío que bien te estima.

26

El Paso, Texas, U.S.A.
Octubre 3, 1932.

Sr. Fernando Torreblanca,
Clinique Mon Repos,
Mont Pélerin, Vevey, Suiza.

Muy querido tío:

Me refiero a tus cartas del 12 y 29 del mes de agosto. Por tu última me enteré con pena que la mejoría de tu salud no ha sido tan rápida como tu deseabas y que por esta razón permanecerás en Europa más tiempo. Deseo sinceramente que al recibo de ésta te encuentres si es posible completamente restablecido.

Como sabes, desde el último de junio estoy sin empleo. Después de haber exhaustado todas las posibilidades de conseguir trabajo en numerosas universidades y planteles de investigaciones científicas, mi esposa y yo nos vimos obligados por lo precario de nuestra situación a irnos a casa de mi papá en Washington. Ahí esperábamos tu pronto regreso con la esperanza de ver nuestros deseos favorecidos.

No deseando permanecer en casa tanto tiempo sin tener trabajo aprovechamos la oportunidad que se nos presentó de hacer el viaje hasta El Paso en automóvil de un amigo; aquí piensa permanecer mi esposa en casa de su mamá mientras voy a México a buscar empleo. Como comprendes, es de todo punto necesario que consiga cuanto antes algun trabajo, pues carezco en lo absoluto de medios de subsistencia.

Así que llegue a México son mis deseos exponerle al Lic. Primo Villa Michel, Secretario de Industria, el mismo proyecto de trabajos en Geofísica Aplicada que propuse a la Fundación Guggenheim; ambiciono al puesto de ingeniero encargado de estudios geofísicos y exploraciones especiales. Comprendo, sin embargo, que sin tu ayuda no llegaré muy lejos y por esto te ruego me escribas una carta de recomendación para el Lic. Villa Michel. Te suplico encarecidamente accedas a este deseo desde luego pues pienso salir para México el 5 de octubre. Al pedirte este nuevo favor invoco las distinciones y honores que tu ya conoces he recibido como estudiante y como profesional.

Porsupuesto que los estudios que ambiciono hacer en Alemania servirán para que me perfeccione en estos trabajos; de modo que si es compatible con el empleo que deseo obtener en México, contando con tu valiosa ayuda en cuanto regreses, quiero solicitar que la Secretaría de Industria me comisione como agregado en nuestra legación en Berlín para completar dichos estudios.

Mi esposa y yo te enviamos cariñosos saludos para tí y tu familia junto con nuestros mejores deseos para tu pronto alivio. Tu sobrino que mucho te aprecia.

Alfredo Baños, Jr.

P. S. - En México me tendrás a tus órdenes en casa de María y Elvira, Ave. Providencia Num. 303-F, Colonia del Valle.

Recibí tu carta del [illegible] [illegible]

[illegible] me hablas [illegible] [illegible]

[illegible] de [illegible] [illegible] en tu casa

Con recuerdos para [illegible]

[illegible] los amigos [illegible] [illegible]

[illegible] [illegible] [illegible]

[illegible] [illegible] [illegible]

[illegible] extendida [illegible]

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27

París,
~~México~~, 19 de Octubre de 1932

Señor don
Alfredo Baños, jr,
Ave. Providencia, 303 -F
Colonia del Valle,
México, - D. F.

Mi querido sobrino:

Recibí tu carta fechada el 3 de los corrientes y accediendo a tus deseos te envío adjunta la carta que me pides para el Señor Lic. Primo Villa Michel, esperando pueda servirte y obtengas algún resultado.

Con recuerdos para todos en tu casa, como siempre quedo tu tío que te quiere

1 Anexo

FT/cs.

28

París,

~~México~~, 19 de Octubre de 1932

Señor Lic. don
Primo Villa Michel,
Secretario de Industria,
Comercio y Trabajo,
México.- D. F.

Muy estimado amigo:

Mi sobrino el joven Alfredo Baños jr., me escribe pidiéndome una carta de presentación y recomendación ante usted, la que no he tenido inconveniente en extenderle.

Mi citado sobrino fué pensionado por nuestra Secretaría de Educación y en vista de sus excelentes estudios el Estado de Texas continuó ayudándolo concediéndole una beca hasta terminar su carrera de Ingeniero electricista, habiendo obtenido su título en forma muy satisfactoria. El está deseoso de que sus servicios puedan ser aprovechados por alguna de las dependencias de nuestro Gobierno aunque ello signifique un sacrificio por su parte, al designársele en cualquier empleo modesto, pues desea también patentizar el reconocimiento que siente hacia nuestro Gobierno por la ayuda que éste le impartió en el desarrollo de sus estudios.

Si esto fuese posible para usted le aseguro que por parte del joven Baños tendría esa Secretaría un servidor muy eficiente.

Lo saluda con el afecto de siempre y me repito a sus órdenes como su atento amigo y s.s.

FT/cs.

29

SECRETARIO DE INDUSTRIA COMERCIO Y TRABAJO
MEXICO.

Noviembre 16, 1932.

Sr. Fernando Torreblanca,
A/c. de la Legación de México.
PARIS, FRANCIA.

Muy estimado amigo:

Por conducto de su apreciable sobrino, el señor Ing. Alfredo Baños Jr., recibí la atenta carta de usted de fecha 19 de octubre próximo pasado, habiéndome impuesto de la valiosa recomendación que de él se sirve usted hacerme, a propósito de sus deseos de trabajar en esta Secretaría.

Con el fin de obsequiarlos tan pronto como se presente una oportunidad, he tomado buena nota de los datos que usted me proporciona acerca del interesado y, mientras tanto, me repito como siempre, suyo afectísimo, atento amigo y seguro servidor.

P. V. Michel

Primo Villa Michel.

src.bra.