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The Natural Interest Rate Under Transitory Shocks: Evidence from Mexico

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Este trabajo es para mi papá, quien me enseñó el trabajo duro y disciplina. Para mi mamá, quien ha sido mi mayor apoyo y por darme amor incondicional. Para mis hermanos José Emiliano y Fernando Manuel. Por mis abuelos Juana, Fernando, y Manuel. En memoria de mi abuela María Elena y su eterno cariño.
Agradezco a mi asesor, Dr. Stephen McKnight, por sus invaluables aportaciones.

Abstract

This thesis estimates the Mexican natural interest rate from 1991 to 2022. Over the period analyzed, the economy has experienced a number of transitory shocks that have affected the nominal interest rate, inflation, and the real GDP growth. Using a Time-Varying Bayesian Vector Autorregressive Model, this thesis demonstrates that these transitory shocks have affected the behaviour of the natural interest rate. The results show that the natural interest rate have been drastically decreasing since 1995. More recently, since the Great Recession it has remained low. From 2018 to 2020, the natural interest rate has marginally increased. Furthermore, from 2020 to 2022 it has shown an accelerated upward growth. This increase in the natural interest rate is attributed to the latest shocks the Mexican economy has experienced, particularly the Covid-19 pandemic and Russia's invasion of Ukraine.

Resumen

Esta tesis examina la tasa natural de interes Mexicana de 1991 a 2022. En el periodo analizado, la economía ha experimentado *shocks* transitorios que han afectado la tasa nominal de interes, la inflación, y el crecimiento de PIB real. Utilizando un modelo de Vector Autorregresivo Bayesiano que cambia en el tiempo, esta tesis demuestra que los *shock* transitorios más recientes han afectado el comportamiento de la tasa natural de interés. Los resultados demuestran que la ha caído drásticamente desde 1995. Después de la Gran Recesión, permaneció continuamente baja hasta 2018. De 2018 a 2020, la tasa natural de interés incrementó marginalmente. Además, de 2020 a 2022 ha mostrado un crecimiento acelerado. Este crecimiento se atribuye a los últimos *shocks* que la economía Mexicana ha experimentado, particularmente la pandemia causada por la enfermadad Covid-19 y la invasión de Rusia a Ucrania.

Summary

1 Introduction

The natural interest rate, defined as "the real short-term interest rate consistent with output converging to potential output, where potential output is the level of output consistent with stable inflation" [Laubach and Williams, 2003, pg. 1], plays a key role in modern macroeconomics and policy. Consider the following straightforward monetary policy example, where the economy is producing more than the potential output, causing an increase in inflationary pressure. Assuming a deviation from the proposed steady state inflation (inflation target), the nominal interest rate should increase. This generates a chain of events that results in an adjustment in inflation expectations, potential output, and ultimately the natural interest rate. If the real interest rate is below the natural interest rate (henceforth, r^*), monetary policy can be considered expansionary such that, the economy by exceeding the steady state inflation rate, will generate higher inflation resulting in a reduction in the purchasing power of consumers and higher costs of production for firms.

On the other hand, if the real interest rate exceeds the natural rate, monetary policy can be considered contractionary. If output falls below potential output, the economy is not fully exploiting its goods and services resulting in a negative output gap.

As the above example shows, knowing the value of r^* relative to the real interest rate is crucial for policymakers in the setting of monetary policy.[1]. Consequently, there is a large literature that has attempted to estimate r* for several countries[2].

The main objective of this study is to estimate the Mexican natural interest rate from January 1991 to June 2022, and analyze how the most recent transitory shocks have affected it. Mexico has experienced a number of transitory shocks recently. The presidential election of Donald Trump in the U.S.A in November 2016 and the presidential election of AMLO in July 2018 saw a shift to populism in North

[1]Currently, the majority of modernized Central Banks conduct research for estimating its natural interest rate. For further detail, see Bank of Canada 2020, Bank of Mexico 2019, Federal Reserve Bank 2022

[2]See Laubach and Williams [2003], Holston et al. [2017], Carrillo et al. [2018], Lopez-Salido et al. [2020], Evans [2020], Brand et al. [2018], Garnier and Wilhelsem [2005]

America. More recently, in March 2020, the global Covid-19 pandemic occurred and on February 2022 Russia invaded Ukraine.

Given r^* is an unknown variable which data does not record, a state-space model is required to estimate it. The particularity of state-space models is that the desired unobserved variable, r^*, can be computed setting a relationship between an observable variable and the unknown (also called *state*) variable. Defined as the *measurement equation*, this relationship reports the connection between the observable variable and the state variable. The *transition equation* details the "evolution of the state variables as being driven by the stochastic process of innovations" [Pichler, 2007, pg. 2]

Following the notation of Pichler [2007], the transition (1) and measurement (2) equations can be written as:

$$z_t = Bz_{t-1} + w_t \tag{1}$$

$$y_t = Hz_t + v_t \tag{2}$$

where y_t is the observable time series, z_t the state variable, v_t and w_t are the stochastic process of innovations. Jointly referred as the system parameters matrices, H and B can be affected by time or remain constant. H is the state-transition parameter matrix that defines how the state variable changes from period to period. The measurement parameter matrix, B, specifies the transition between the state and observable variables, affected by the innovation processes.

State-space models are increasingly popular in macroeconomics and financial econometrics. The usual variables of interest are the natural interest rate, risk beta coefficient (see Mergner and Bulla [2008], Mergner [2009], Nath and Brooks [2020]), uncertainty models (see Onatski and Williams [2002], Luo et al. [2013]), inflation (see Simone et al. [2000], Basdevant [2003]), yield curves (see Donati and Donati [2008], Joshi [2021]), among others. Thus, this thesis follows the modern literature and computes r^* using a state-space model.

The state-space model used in this thesis is a Time-Varying Bayesian Vector Autoregressive (TV-BVAR) model. This technique allows for quarterly nonlinear relationships, enhancing the statistical properties of the time-series signal extrac-

tion method. The signal extraction relies on the Kalman Filtering process, which separates the transition data (i.e, the natural interest rate) from the measurement variable. Data extraction and filtering grants time varying intercepts. Inference can be done because of the stochastic volatility and Bayesian techniques, generating efficient time varying standard errors.
The benchmark TV-BVAR model of the thesis uses a Moving Average (4) as expected inflation. As a robustness exercise, the thesis also considers an alternative TV-BVAR model that uses expected inflation taken from Banco de Mexico's Expectations Survey (*Encuestas sobre las expectativas de los especialistas en economía del sector privado*) for the period 1Q-1999 to 2Q-2022. This is a survey of private professional forecasters (available at banxico.org.mx/Encuestas sobre las expectativas)
In addition to the TV-BVAR models, the natural interest rate is estimated using traditional band-pass filter methods, from 1991 to 2022. These estimates using traditional techniques grant some comparisons to measure the accuracy of the TV-BVAR model. We estimate the natural interest rate using a standard Hodrick-Prescott (HP) Filter, an adjusted HP-Filter (to avoid potential bias of the HP-Filter), and a Butterworth Filter.

The main results suggest that the Mexican natural interest rate, r^*_m, has been significantly decreasing since the Mexican Peso crisis in 1995. The dotcom crisis caused a slight increase in r^*_m in 2001. Nevertheless, r^*_m decreased during the 2000's until 2016, the one exception being in 2010. Since 2016, r^*_m has been rapidly increasing. By 2Q 2020, at the the Covid-19 pandemic, r^*_m has experienced faster growth. The estimations suggest that lately the real interest rate is above the natural rate, implying that the monetary policy stance of Banxico is contractionary, and the inflationary period will peak in the third quarter of 2022
The robustness model that uses survey data for expected inflation finds lower estimates for the natural interest rate. However r^*_m follows the same upward path as the benchmark model since 2011. Results from the Band-Pass filters find that before the Great Recession r^*_m continuously increased, peaking in early 2005. Since 2Q 2005, r^*_m has been declining. Similar to the TV-BVAR estimates, from 2015 to 2022, r^*_m has been rising.

The thesis is structured as follows. In Section 2, the literature is discussed. Section 3 presents the model and the data employed. Section 4 discuss the benchmark results and Section 5 undertakes a robustness analysis. Finally, Section 6 concludes.

2 Literature Review

Many recent studies have focused on estimating r^* using state-space models. The literature started with Laubach and Williams [2003], who estimated the natural interest rate for the United States using a Kalman Filter (KF) Maximum Likelihood Estimation (MLE) with Stock and Watson [1998] ratios to correct estimates and standard errors from the "pile-up" problem described by Stock [1994]. The pile-up problem suggests that dynamic estimations using MLE are biased and inefficient, because estimations will be biased towards zero. Using their proposed ratios, Laubach and Williams results show that the natural interest rate for the United States has been constantly decreasing over the last forty years.

For Mexico, Magud and Tsounta [2012] used Band-Pass filters, KF/MLE, A Dynamic Taylor Rule, and an Implicit Common Stochastic Trend to estimate r_m^* from 2000 to 2012. The average r_m^* is 2.1% (pg.15). Nevertheless, they report that r_m^* has a downward trend for the analyzed period.

Carrillo et al. [2018], follow the approach of Laubach and Williams [2003] and estimate r_m^* using a KF/MLE with the adjusted ratios, Band-Pass filters, a Taylor rule, Affine term structural Model, and a TV-BVAR from 2000 to 2017. Their estimations also find a downward trend for r_m^* in the analyzed period.

Similar to Carrillo et al. [2018], Sánchez Vargas and López-Herrera [2020] r_m^* estimations rely on a KF/MLE, and a Cointegrate Vector Autoregresive (CVAR) model from 2008 to 2020. Similar to the exiting literature, their results suggest a downward trend for the Mexican natural interest rate. Furthermore, using the KF/MLE and CVAR models, they forecast r_m^* for the next five years. Their results suggest that r_m^* would have remained steady at 0.1%, from 2020 to 2024, giving

Banxico enough space to reduce the nominal interest rate by at least 50 or 100 basis points (pg.13).

Despite the popularity of the KF/MLE estimation method, it is subject to several serious concerns. For instance, results are biased towards zero (i.e, the pile-up problem), so alternative estimator ratios have been used. Buncic [2020] claims that the Median Unbiased Estimator ratio used in Laubach and Williams [2003], and studies that follow their approach, suffers from misspecification in the model second stage, causing spuriousness of the results. The spurious relation can lead the adjustment ratio, λ_z, to be statically insignificant, affecting all the computations.
KF/MLE models set linear relationships for macroeconomic time series, such as real GDP, nominal interest rate, and inflation. However, these variables typically follow nonlinear relationships. This can cause ambiguous conclusions [Maansson, 2014]. Moreover, nonlinear relationships explain better the asymmetries in the response of external shocks [Lee and Pesaran, 1999], such that inference is more robust for transitory and stochastic shocks. In addition, the pile-up problem can be erased without further computations using nonlinear relationships and Bayesian econometric tools.

Given the deficiencies of the KF/MLE method, Time Varying Bayesian Vector Autoregressions (TV-BVAR) have been put forward as an alternative that allow nonlinear relationships, acknowledge the importance of external shocks, and generate unbiased estimations eliminating the pile-up problem [Primiceri, 2005, Kim and Kim, 2013, Ito et al., 2022].

TV-BVAR models "emphasizes the changes in the transmission mechanism, i.e. the way macroeconomic variables respond to shocks" Primiceri [2005, pg. 2]. By allowing time-varying intercepts, transitory shocks can be reflected with nonlinear relationships. Moreover, the time varying variance-covariance matrix allows for efficient inference and heteroskedasticity.

The econometric technique utilized in TV-BVAR models are Bayesian and Markov Chain Monte Carlo algorithms[3], specifically Gibbs sampling. Gibbs sampling evalu-

[3]Primiceri [2005, pg. 7] states "...deal efficiently with the high dimension of the parameter space and the nonlinearities of the model, splitting the original estimation problem in smaller and simpler ones"

ates the results from the lower conditional prior distribution and provides a primary method for assessing the posterior distribution obtained from Bayesian Identification. Hence, this method is good technique to measure r^*.

Using TV-BVAR estimations, Lubik and Matthes [2015a] measure the US r^* over the period 1961 to 2015, extracting the real interest rate from a matrix which components include the real GDP growth rate, the PCE inflation rate, and the real interest rate measured as the current inflation minus expected inflation. Their results show a downward trend since 1985, reaching the lowest level in 2008 and being constant since then. Equally important, their estimates for r^* never became negative, contrasting with Laubach and Williams [2003].

Changing the definition and stating r^* as the long-run real interest rate, Jarocinski [2017] use a TV-BVAR to compute r^* for the Euro Area. The variables used are the real GDP growth, GDP deflator inflation, and distinct euro interest rates. Interestingly, their results dfind that the natural interest rate has been negative in the Euro Area since 2015, with negative rates forecasted until 2025.

For the Mexican natural interest rate, only the papers of Carrillo et al. [2018] and and Rodrigues [2020] (2020) estimate r^*_m using time-varying Bayesian techniques. Carrillo et al. [2018] estimate r^*_m using core inflation, real GDP growth, short-term real interest rate (consistent with Laubach and Williams [2003]), nominal peso-dollar exchange rate, and an additional country risk index (pg.22). Their results exhibit a descending pattern over the last seventeen years. Nevertheless, r^*_m never reached negative levels.

Rodrigues [2020] computations are for the period 1998 to 2018. Defining r^* for several economies as the relationship between inflation, short-run real interest rate, and the short-run nominal interest rate trend, r^*_m falls from 3.4% in 1998 to 2.48% in 2013, remaining steady thereafter. Her estimations are in accordance with Carrillo et al. [2018] because both studies found that from 2000 to the Great Recession (2008), r^*_m continuously decreased. From 2009 to 2013, both studies show a faster decline and slight recovery afterwards.

As mentioned before, TV-BVAR r_m^* computations are rare for Mexico. Moreover, the recent data allows me to investigate the implications of recent events on the Mexican natural interest rate.

3 Model and Data

3.1 A New Keynesian Model

In order to establish an economic relationship between the variables of interest (inflation, real GDP growth rate, and the real interest rate), a Dynamic, Stochastic, General Equilibrium (DSGE) model is needed to help validate the econometric results.

The DSGE model employed in this paper is a closed-economy New Keynesian model which uses a trivariate Vector Autorregressive Model (VAR) developed by Del Negro and Schorfheide [2004]. The distinctiveness of this model depends on permitting backward-looking and forward-looking relationships for all the variables, resulting in better solutions if both dynamics are incorporated (see Giacomini 2013, Wickens 2014, Del Negro and Primiceri 2015).

This approach is taken for two reasons: First, incorporating future-looking dynamics grants external shocks in the model, and second, including both backward-looking and forward-looking components does not disturb the data [Wickens, 2014]. Thus, the subsequent dynamics between output, inflation, and interest rates are taken and derived from Del Negro and Schorfheide [2004][4].

The representative household gets utility from the following lifetime utility function:

$$U_t = E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} \left(\frac{C_s^{1-\tau} - 1}{1-\tau} + \chi log \frac{M_s}{P_s} - h_s \right) \right] \tag{3}$$

where β is the discount factor, C represents consumption, A is the current level of technology, M/P is the money balances, and h is hours worked by the household.

[4]For further details, see Del Negro and Schorfheide [2004]

τ is the risk aversion parameter and χ is a scale factor.
Inflation, can be expressed as:

$$\pi_t = \frac{P_t}{P_{t-1}} \tag{4}$$

The representative household can earn income from supplying labor to firms $W_t h_t$ and can earn interest R from bonds B. Additionally the household receives dividends D_t from firms and pays lump-sum taxes T_t. Therefore, the household budget constraint is given by:

$$C_t + \frac{B_t}{P_t} + \frac{M_t}{P_t} + \frac{T_t}{P_t} = W_t h_t + \frac{M_{t-1}}{P_{t-1}} + R_{t-1}\frac{B_{t-1}}{P_t} + D_t \tag{5}$$

After solving for the First Order Conditions, the optimal choices for the household are the Euler equation (6), money demand equation (7), and labor supply equation (8):

$$\left(\frac{C_t}{A_t}\right)^{-\tau} \frac{1}{A_t P_t} = \beta R_t E_t \left[\left(\frac{C_{t+1}}{A_{t+1]}}\right)^{-\tau} \frac{1}{A_{t+1} P_{t+1}}\right] \tag{6}$$

$$\chi \left(\frac{M_t}{P_t}\right)^{-1} = \left(\frac{C_t}{A_t}\right)^{-\tau} \left(\frac{1}{A_t}\right) \left(\frac{R_t - 1}{R_t}\right) \tag{7}$$

$$1 = \frac{W_t}{A_t} \left(\frac{C_t}{A_t}\right)^{-\tau} \tag{8}$$

Firms are monopolistically competitive with differentiated products, where v is the elasticity of substitution, $P_t(j)$ is the profit-maximizing price for the production $Y_t(j)$. Then, the demand function for firm j can be represented as

$$P_t(j) = \left(\frac{Y_t(j)}{Y_t}\right)^{-1/v} P_t \tag{9}$$

If any firm wants to change prices beyond π_t, and assuming a nominal rigidity over the demand (quadratic term), the cost function is

$$\frac{\phi}{2}\left[\frac{P_t(j)}{P_{t-1}(j)} - \pi^*\right]^2 Y_t(j) \tag{10}$$

where the parameter $\phi \geq 0$ is the degree of price stickiness in the economy, and π^* is the economy state-state inflation rate.

Assuming that production is a linear relationship between labor and the factor productivity, $Y_t(j) = A_t h_t(j)$, equations (9) and (10) can be expressed as:

$$P_t(j) = \left(\frac{A_t h_t(j)}{A_t h_t}\right)^{-1/v} P_t \tag{11}$$

$$\frac{\phi}{2}\left[\left(\frac{A_t h_t(j)}{A_t h_t}\right)^{1/v}\left(\frac{A_t h_{t-1}(j)}{A_t h_{t-1}}\right)^{1/v}\pi_t - \pi^*\right]^2 A_t h_t(j) \tag{12}$$

where A_t is assumed to follow a unit root process with a stochastic shock, z:

$$lnA_t = ln\alpha + lnA_{t-1} + z_t \tag{13}$$

where the stochastic innovation, z_t, follows a random walk with its own lag, the parameter ρ, and the stochastic error $\epsilon_{z,t}$

$$z = \rho z_{t-1} + \epsilon_{z,t} \tag{14}$$

The central bank sets the nominal interest rate in response to inflation and output deviations from the steady-states r^*, π^*, Y^*(nominal target, inflation and potential output):

$$\frac{R_t}{R^*} = \left(\frac{R_{t-1}}{R^*}\right)^{\rho R}\left[\left(\frac{\pi_t}{\pi^*}\right)^{\psi_1}\left(\frac{Y_t}{Y^*}\right)^{\psi_2}\right]^{1-\rho R}\epsilon_{R,t} \tag{15}$$

where ρR describes the degree of interest rate smoothing, ψ_1 and ψ_2 are the inflation and output response coefficients, respectively, and $\epsilon_{R,t}$ is any exogenous monetary policy shock.

The model collapses to the following market clearing vector, M_c, composed of the nominal interest target, output and inflation target, which can be represented as:

$$\vec{M_c} = [\bar{y}_t \, \bar{\pi}_t \, \bar{R}_t] \tag{16}$$

M_c is constructed from linear relationships between its own lags, stochastic shocks, and static parameters. The log-linear equations for the output, inflation, and nominal interest rate are represented by $\bar{y}_t, \bar{\pi}_t, \bar{R}_t$ are given by the following equations:

$$\bar{y}_t = E_t[\bar{y}_{t+1}] - \tau^{-1}(\bar{R}_t - E[\bar{\pi}_{t+1}]) + \rho_z \frac{1}{\tau} \bar{z}_t \tag{17}$$

$$\bar{\pi}_t = \frac{\alpha}{r^*} E[\bar{\pi}_{t+1}] + k\bar{y}_t \tag{18}$$

$$\bar{R}_t = \rho_R \bar{R}_{t-1} + (1 - \rho_R)(\psi_1 \bar{\pi}_t + \psi_2 \bar{y}_t) + \epsilon_{R,t} \tag{19}$$

where r^* is the steady-state natural interest rate and k is a price adjustment function.

After solving using the Sims algorithm,the econometric log relationships are the following measurement equations for quarterly data:

$$\Delta ln y_t = ln\alpha + \Delta\bar{y}_t + \bar{z}_t + \epsilon_{y,t} \tag{20}$$

$$\Delta ln P_t = ln\pi^* + \bar{\pi}_t + \epsilon_{\pi,t} \tag{21}$$

$$\Delta ln r^*_t = \frac{1}{4} ln R_t + ln\pi^* + \bar{R}_t + \epsilon_{r^*,t} \tag{22}$$

Despite the solid economic relationships presented, the two main characteristics of the DSGE-VAR closed economy model is that the parameters do not change over time or with exogenous stochastic shocks. Moreover, the linear restrictions remove the complex nonlinear relationships between the variables, weakening the model. Thus to circumvent these characteristics, the thesis will employ a time-varying VAR with Bayesian estimators.

3.2 Time-Varying Bayesian VAR Model

In order to improve the weaknesses discussed above and maintain the relationships between variables, a TV-BVAR model is used. This allows the natural interest rate to be estimated allowing for time dependent parameters, which are affected by different exogenous shocks and nonlinear relationships. In this approach, the econometric models can be explained by external shocks and lags, where the economic shocks are modeled through the stochastic errors $(\epsilon_{y,t}, \epsilon_{\pi,t}, \epsilon_{r^*,t})$; this approach has been widely used in the literature.[5]. Moreover, this econometric tool allows "asymmetric movements of variables over the course of the business cycle" [Lubik and Matthes, 2015a, pg. 7].

Lubik and Matthes [2015b] show that DSGE models can be represented as a reduced-form with a TV-BVAR. Thus, this estimation technique is appropriate to uncover the natural interest rate because it allows multiplex relationships that are unknown under conventional VAR models.

The TV-BVAR model is a vector which extracts r^*_m from the short-run real interest rate:

$$M_c = \alpha_t + \beta_t M_{c_{t-1}} + \lambda_t M_{c_{t-2}} \tag{23}$$

where M_c represents the matrix composed of the real interest rate, real GDP growth, and inflation. Notice the model has two lags, $M_{c_{t-1}}$, $M_{c_{t-2}}$, the time varying parameters β_t, λ_t, and the dynamic intercept α_t. These time varying parameters change every quarter, and represent the effect that the lags have over the matrix M_c.

Random Walk Coefficients

The time varying coefficients $(\alpha_t, \beta_t, \lambda_t)$ are set to follow a random walk. Lubik and Matthes [2015b] suggest establishing a prior distribution to these coefficients. Prior distributions "represent the information about an uncertain parameter that is combined with the probability distribution of new data to yield the posterior distribution" [Gelman, 2002, pg. 1634]. Following the existing literature[6], the first 40 observations (1Q 1981:1Q 1991) serve as the prior observations used for correcting the distribution and setting the posterior distribution.

[5]Laubach and Williams [2003], Holston et al. [2017], Carrillo et al. [2018], Lubik and Matthes [2015b], Primiceri [2005], Galí [2015], among other studies incorporate transitory and permanent shocks through the stochastic errors.

[6]Lubik and Matthes [2015a,b], Primiceri [2005]

Posterior Distributions

The posterior distribution is the assigned distribution for the parameters of interest and delivered from Bayesian conditional distributions. The posterior distribution, $p(\theta|y^t)$, is the result from applying Bayes' Theorem to the prior distribution, $p(\theta)$, and the likelihood function, $p(y^t|\theta)$ (see Bernardo [1979], Wasserman and Lafferty [2014], Dall'Aglio [2018] for further details):

$$p(\theta|y^t) = \frac{p(y^t|\theta)p(\theta)}{\int p(y^t|\theta)p(\theta)d\theta} \tag{24}$$

where $\int p(y^t|\theta)p(\theta)d\theta$ is the marginal data density, defined as "the integral of the likelihood function with respect to the prior density of the parameters" [Fuentes-Albero and Melosi, 2013, pg. 1]

The integral likelihood function, despite its theoretical characteristics, is burdensome to compute it. Given the nonlinearities and stochastic volatility parameters present, in his influential study, Primiceri [2005] proposes using Gibbs sampling to fit, evaluate, and efficiently estimate the posterior distribution.

Gibbs Sampling

Without having to determine the probability density, the Gibbs sampler is a method for generating random variables in the absence of a marginal distribution, grounded on Markov Chain Monte Carlo Simulations [Casella and George, 1992]. In this thesis, 55,000 Monte Carlo simulations where conducted to set the Gibbs Sampler, such that the posterior distribution can be estimated. The model results are the posterior draws from this procedure. Moreover, the Gibbs sampler provides smoothed results because it analyzes the entire set, unlike one-tailed estimates that only examine an specific range.

The smoothing process is based on a two-sided Kalman Filter, which establishes the signal extraction relationship (state-space equations) from the data. In this particular model, the state-space model is given by:

$$r^*_{m_{t+1}} = r^*_{m_t} + \epsilon_{t,r^*_m} \tag{25}$$

$$r_m = r^*_{m_t} + \epsilon_{t,r_m} \tag{26}$$

where r_m is the Mexican real interest rate and the stochastic errors, which allow for external shocks. Notice that the main difference between the real interest rate (r_m) and the natural interest rate (r^*_m), is that r^*_m is the short-run real interest rate that converges to the potential output. Then, the Kalman Filter extracts[7] r^*_m from r_m to use it in the Gibbs Sampler and compute the posterior distribution.
The real interest rate (r_m) is defined following the Fischer equation:

$$(1 + r_m) = \frac{1 + i_t}{1 + E_t(\pi_{t+4})}$$

where i_t is the short-run nominal interest rate, and the current expected inflation for the one year ahead same quarter, $E_t(\pi_{t+4})$, delivered from the Moving Average (4).

The main difference between the use of the Kalman Filter in this model compared to the KF/MLE models is the Maximum Likelihood Estimation (MLE). As previously discussed, different ratios have to be adjusted in the MLE due to toward-zero biased estimations. Under the approach taken by this thesis, Kalman Filtering is used in the posterior draws and evaluated by the Gibbs sampling (via Monte Carlo Simulations), surpassing any bias or inefficiency. Consequently, the posterior draws estimations are the base for efficient inference.

Inference

Inference is more efficient using Gibbs sampling given the unobservable state evolution. In addition, Gibbs sampling allows for time varying variance-covariance

[7] Additional information for the extracting process can be considered in Engle and Watson [1987], Pei et al. [2019], Poncela et al. [2021]

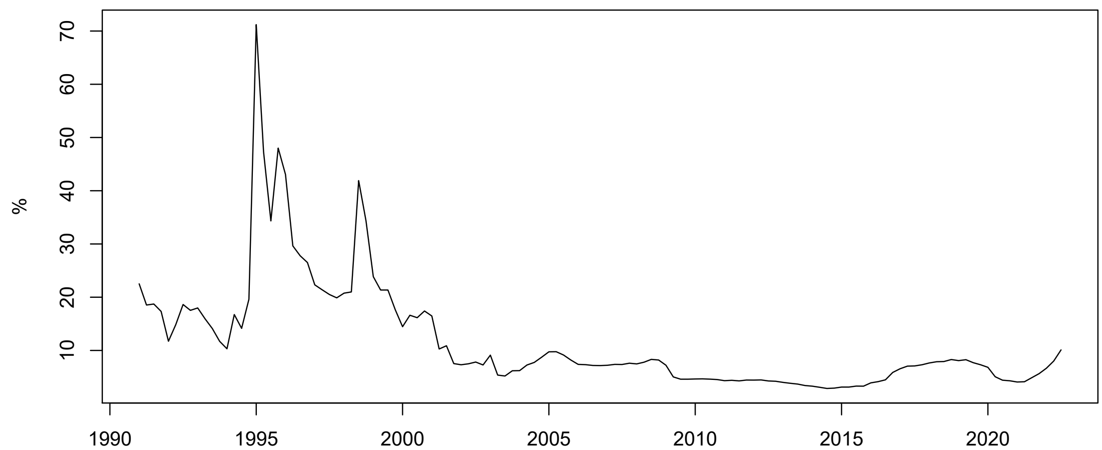
stochastic estimates [Casella and George, 1992, Primiceri, 2005]. To compute the variance-covariance matrix, first, the estimations are delivered in the prior distribution. After that, the Gibbs sampler draws the variance matrix for the model[8].

3.3 Data

The data used in the analysis is as follows. The short-run nominal interest, i_t, is the quarterly three-month CETES interest rate, taken given the data availability and because this rate has been the reference for investors and Banco de Mexico, before the central bank adopted an inflation targeting policy regime in 2001. As shown in Figure 1, the CETES 3 month rate experienced its peak in 1995, because of the Mexican Peso Crisis. This currency crisis was caused by large fiscal deficits and a negative current account, financed by short term bonds. These bonds were paid in dollars, causing a fragile scenario for international reserves. In 1994, this financing scheme became unstable, causing a 81% decline in international reserves. Then, the country experienced massive capital flights and a currency devaluation [Cárdenas Sánchez, 2015]. Thus, from 1995 to 2000, the country offered a high nominal interest rate as a response to the capital flight. Since 2001, the CETES 3 month rate has been relatively steady, with a slight decrease after the Great Recession and the most recent increases during 2021 and 2022.

[8]The derivation of the variance-covariance matrix for this model is beyond the scope of this thesis. The reader is referred to Primiceri [2005], Lubik and Matthes [2015b], Gorgi et al. [2017], Dendramis et al. [2021]

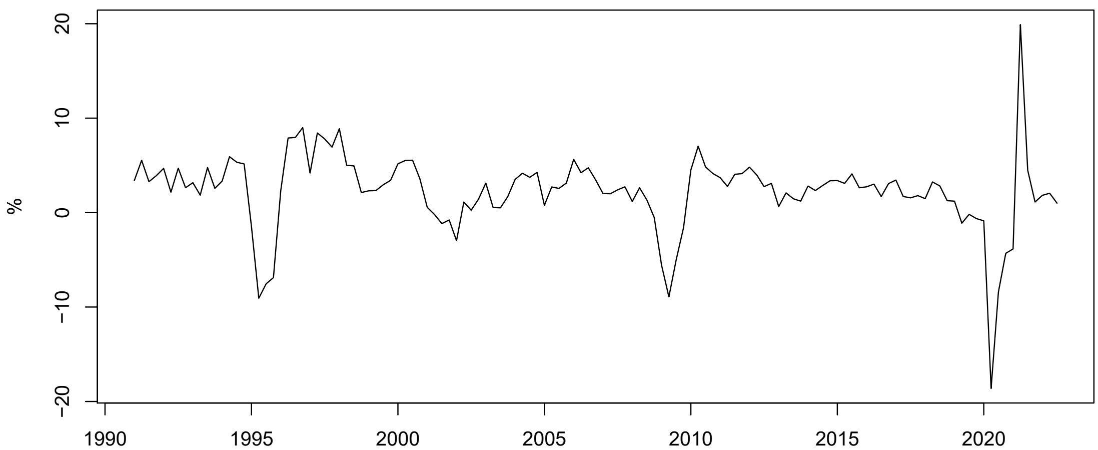
Figure 1: CETES 3 Month Interest Rate



The real quarterly GDP growth and quarterly inflation (computed with the CPI) are taken from the National Institute of Statistics, Geography and Informatics (INEGI).The data sample annual average growth rate is 2.24%, and the major economic crises that occurred during the sample period: The Mexican Peso crisis (1994/1995), Dotcom crisis (2000/01), Great Recession (2008), and the Covid-19 pandemic crisis (2020/21) explain the steep downturns in the Figure 2.

This Figure reveals the biggest decrease, and increase, the real GDP growth rate over the last thirty years. In first quarter of 2020, at the beginning of the Covid-19 pandemic, the (-)18.62% decrease has been the largest reduction in the growth rate of real GDP in Mexico over the last 30 years. This slowdown was caused due to the sudden production stop, the massive dismiss the jobs suffered, and the lack of an expansive fiscal policy to support aggregate demand [Ahmed Hannan et al., 2020, Vázquez Muñoz et al., 2021]. Nevertheless, one year later, the Mexican economy recorded the highest quarterly recovery over the last century, with an 19.90% annualized quarterly growth rate.

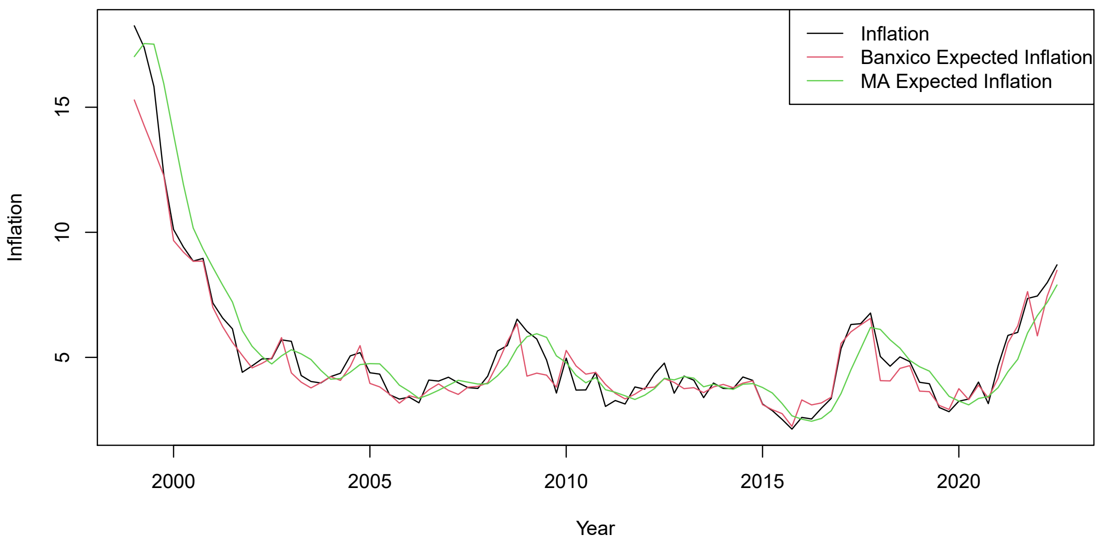
Figure 2: Real GDP Growth Rate



One of the main differences of this thesis to the existing literature relies on the use of current expected inflation, $E_t(\pi_{t+4})$. Similar to Holston et al. [2017], $E_t(\pi_{t+4})$ is estimated with an inflation Moving Average (4)[9].
As a robustness exercise, a second approach is conducted where expected inflation is taken from the Banxico Expectations Survey[10]. It must be made clear that the survey started from January 1999. Figure 3 compares expected inflation estimates using the survey data against the inflation estimates from the Moving Average approach (restricted to same sample period 1999-2022).

[9]The four lags were significant on the ADF test. The test results are given in the Appendix
[10]The related study of Carrillo et al. [2018] uses this data series.

Figure 3: Inflation



Finally Tables 1, 2, and 3 offer summary statistics for the data for the two proxies for expected inflation. On average, MA(4) expected inflation is estimated to be lower using the survey data. Consequently, the real interest rates are on average larger using the survey data.

Table 1: Data Summary Statistics (%). 1991-2022. Expected Inflation: MA(4)

Value	CETES 3-M	Inflation	MA(4) Expected Inf.	MA(4) r_m	Real GDP %
Min.	2.86	2.1308	2.447	-12.6570	-18.6220
Median	7.64	5.01957	4.9234	0.7891	2.73
Average	11.69	8.9378	9.1815	0.6544	2.2402
Max.	71.2	51.9661	44.2286	23.5618	19.901

Table 2: Data Summary Statistics (%). 1999-2022. Expected Inflation: MA(4)

Value	CETES 3-M	Inflation	MA(4) Expected Inf.	MA(4) r_m	Real GDP %
Min.	2.860	2.130	2.447	-3.548	-18.622
Median	7.16	4.274	4.282	1.276	2.407
Average	7.316	5.160	5.300	1.455	1.806
Max.	23.86	18.255	17.545	5.981	19.901

Table 3: Data Summary Statistics (%). 1999-2022. Expected Inflation: Banxico survey data

Value	Banxico Expected Inf.	Banxico r_m
Min.	2.24	-1.99
Median	4.06	1.85
Average	4.9452	2.3714
Max.	15.29	9.47

4 Results

4.1 Time-Varying Bayesian VAR: Benchmark Model

Figure 4 shows the estimations from the TV-BVAR, with 10th and 90th percentile of standard deviation, whereas Table 4 shows the estimated values for r_m^*. Figure 5 compares r_m^* with the real interest rate. Notice that r_m^* never reaches negative rates as r_m does. The main results suggest after the beginning of the Mexican Pesos crisis, the so called "December Error", r_m^* has been constantly falling.

From 1996 to the beginning of the DotCom shock, 2000, the natural interest rate continued to decline over the years. However, the DotCom crisis, another transitory shock, affected r_m^* such that for 2000, 2001, and 2002, the rate remained relatively steady at approximately 2%. After the DotCom shock, r_m^* declined until the Great Recession in 2008. From 2007 to 2008, r_m^* rose 96.446%.

Despite this large increase, the Great Recession did not have a significant impact on r_m^*, because it never reached levels similar to 2008 and remained constantly low. The Mexican economy reached its steady-states (natural interest rate, inflation target and potential output) for almost a decade, from 2009 to 2016.

While these estimations have similarities with Carrillo et al. [2018] and Rodrigues [2020], there are several important differences. First, Rodrigues [2020] shows the same downward trend from 1998 to 2007. However, her estimations are approximately 1% greater than the results reported here. Second, the results of Carrillo et al. [2018] model show a marginal rise in the natural interest rate after the Great Recession, in 2009, contrasting with the large increase computed by this model.

Figure 4: Time Varying BVAR Natural Interest Rate

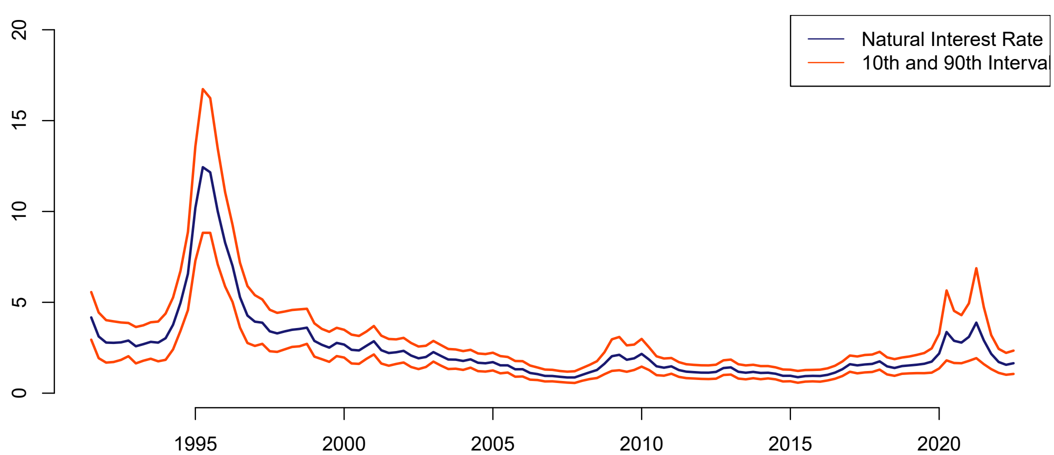
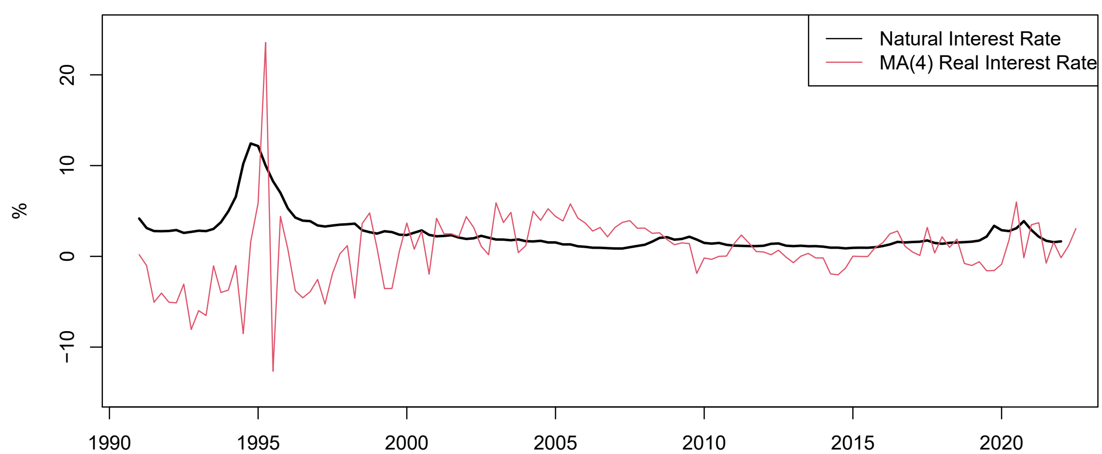


Table 4: Natural Interest Rate Values (%)

Date	r_m^*	Date	r_m^*	Date	r_m^*
1991	4.169407	2002	2.053053	2013	1.1238187
1992	2.770464	2003	1.863588	2014	0.8799161
1993	3.765334	2004	1.532548	2015	1.0083875
1994	12.43778	2005	1.1111135	2016	1.529053
1995	6.999966	2006	0.9013065	2017	1.475148
1996	3.879472	2007	1.1461846	2018	1.564692
1997	3.489829	2008	2.110797	2019	1.736525
1998	2.662739	2009	1.849172	2020	3.366747
1999	2.384954	2010	0.971327	2021	3.097025
2000	2.364834	2011	1.127595	2022	4.82844
2001	2.070923	2012	1.1763042	2023	2.165134

Figure 5: Natural Interest Rate and Real Interest Rate



Third, our results suggest that r_m^* has been increasing since 2015. Carrillo et al. [2018] and Rodrigues [2020] argue the same pattern, with different years (2013 and 2015, respectively). Interestingly, the model estimations are different from Sánchez Vargas and López-Herrera [2020] computations, as they suggest that the r_m^* has been decreasing since 2008. The main difference relies from 2015 to their forecasted dates. Sánchez Vargas and López-Herrera [2020] computations suggest that r_m^* remain steady from 2020 until 2024. However, this model suggests that r_m^* has been rising significantly since 2020. The estimates from this study are preferred because it takes into account the recent transitory shocks that have hit the Mexican economy.

From 2013 to 2016, r_m^* is estimated to have increased at overly low rate, passing from 1.1238187% to 1.529053%. Nevertheless, passing 2017, the year the former president Donald Trump assumed office, r_m^* has been constantly increasing. In 2018, when AMLO was elected and assumed office, r_m^* continued rising. These changes can be accredited to the shocks that real GDP suffered in those years.

In the beginning of the Covid-19 pandemic, r_m^* started to rapidly grow in 2020, 2021, and 2022. This growth is attributed to the substantial increase inflation has had because of the supply-side congestion and issues. Furthermore, the Russian invassion to Ukraine has worsened the inflationary pressures, suggesting that potential output may not have returned to its pre-Covid 19 level. Moreover, the proposed forecast suggest that the "heated" economy will peak in 2022, and slowly descend through 2023. According to the most recent estimates, Banxico has been successful in raising the nominal interest rate sufficiently that the real interest rate now exceeds the natural interest rate.

Given the importance the latest transitory shock have in this thesis, the model parameters from 1Q 2016 to 3Q 2023 are presented in Table 5 (Appendix offers parameters from 1Q 2011 to 4Q 2015).

The time-varying parameters show that from 2016 to 2018, the lags firstly had a constant and compact positive impact on r_m^*, resulting in an almost imperceptible increase. From 2019 to 2020, the parameters are both positive and then negative, provoking a small change. Nevertheless, from 2021 to the forecast 3Q 2022, the parameters show a positive sign, raising r_m^*. The dynamic parameters suggests that from 4Q2022 to the forecast 3Q 2022, r_m^* will decrease. It is important to realize that pre-pandemic levels will be not reached in 2023.

Table 5: Time Varying Parameters 2016-2023.

Date	α_t	β_t	λ_t
1Q 2016	0.57682942	0.12327989	0.05690374
2Q 2016	0.57267750	0.12335141	0.05681657
3Q 2016	0.57294380	0.12357915	-0.05687764
4Q 2016	0.5737379	0.12333663	-0.05688689
1Q 2017	0.57414549	0.12350951	-0.05680053
2Q 2017	0.56888753	0.12379176	-0.05676589
3Q 2017	0.57303209	0.12376493	-0.05676693
4Q 2017	0.57622163	0.12373906	-0.05669860
1Q 2018	0.58449791	0.12377107	-0.0566212
2Q 2018	0.5955721	-0.12348896	-0.05662522
3Q 2018	0.5955721	-0.12348896	-0.05658335
4Q 2018	0.5980659	-0.123029447	-0.05604627
1Q 2019	0.6007690	-0.1229599	-0.05658335
2Q 2019	0.6082545	-0.12261062	-0.05658626
3Q 2019	0.6128403	-0.1223764	-0.05663427
4Q 2019	0.6186482	-0.12190784	-0.05667456

Date	α_t	β_t	λ_t
1Q 2020	0.6193888	-0.11182877	-0.05074152
2Q 2020	0.6201590	-0.05211078	-0.01679546
3Q 2020	0.6183596	-0.01426347	-0.00607134
4Q 2020	0.6267074	0.01203096	0.00567608
1Q 2021	0.63105730	0.11480209	0.06778421
2Q 2021	0.63900323	0.14953675	0.04174056
3Q 2021	0.64167187	0.17286580	0.04873796
4Q 2021	0.65162152	0.10167845	0.04173866
1Q 2022	0.06424201	0.06459642	0.08674316
2Q 2022	0.6533068	0.102150879	0.04171421
3Q 2022	0.6623454	0.122354626	0.05171459
4Q 2022	0.6780409	-0.010376629	-0.0515421
1Q 2023	0.6890078	0.00417097	0.008690513
2Q 2023	0.6719936	-0.05213883	-0.01942642
3Q 2023	0.7087219	-0.01221041	-0.04691381

5 Robustness

5.1 Alternative Data Series

For robustness, expected inflation is proxied using survey data from Banco de Mexico. Recalling the Fischer equation, the MA(4) Inflation is substituted for the Banxico expected inflation. Letting $E_t^B[\pi_{t+4}]$ represent the Banxico expected inflation measure, the short-run real interest rate is defined as:

$$(1 + r_m) = \frac{1 + i_t}{1 + E_t^B[\pi_{t+4}]}$$

.

Given $E_t^B[\pi_{t+4}]$ started being published in 1999, now the priors of the model are from 1Q 1999 to 1Q 2010. Following the same methodology, Figure 6 presents the evolution r_m^* has had under the alternative measure for expected inflation. Table 6 presents the values and Table 7 shows the time-varying parameters.

Figure 6: TV−BVAR Natural Interest Rate

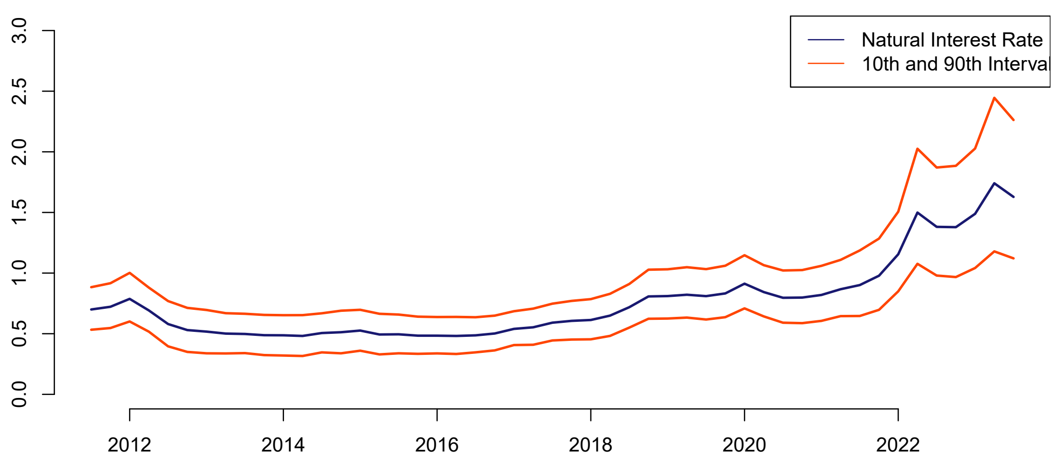


Table 6: Alternative Natural Interest Rate Values (%)

Date	r_m^*
2011	0.6998144%
2012	0.5797798%
2013	0.5010466%
2014	0.4917233 %
2015	0.4936663 %
2016	0.5026344 %
2017	0.5528819 %
2018	0.6492389%
2019	0.8214008%
2020	0.9234476 %
2021	0.9012967%
2022	1.4886636%
2023	1.6280652%

Table 7: Alternative Time Varying Parameters 2016-2023.

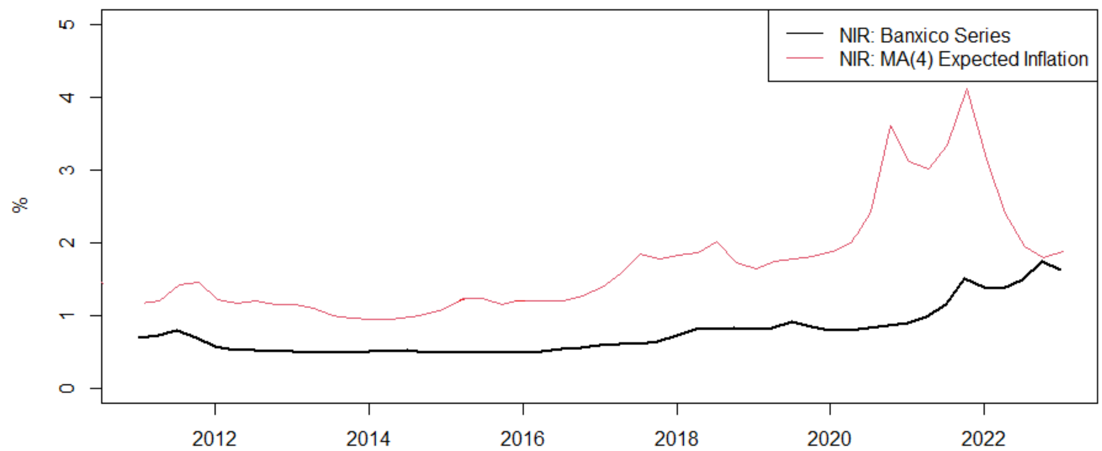
Date	α_t	β_t	λ_t
1Q 2016	0.06728113	-0.13180807	-0.02898623
2Q 2016	0.06739478	-0.13185831	-0.02898805
3Q 2016	0.06783157	-0.13189307	-0.02899778
4Q 2016	0.06782229	-0.13189914	-0.02913173
1Q 2017	0.06677387	-0.13193161	-0.02900851
2Q 2017	0.06670471	-0.13194562	-0.02911481
3Q 2017	0.0667116	-0.13193720	-0.02900959
4Q 2017	0.06718003	-0.03200084	-0.02899225
1Q 2018	0.06704945	0.01172181	0.00109973
2Q 2018	0.06658718	0.03201569	0.02914466
3Q 2018	0.06614405	0.05703207	0.02902015
4Q 2018	0.06598905	0.07196945	0.03217978
1Q 2019	0.06563155	0.10194459	0.03926124
2Q 2019	0.06486464	0.10319986	0.04428993
3Q 2019	0.06434361	0.11200598	0.04940040
4Q 2019	0.06447681	0.11198062	0.04953952

Date	α_t	β_t	λ_t
1Q 2020	0.06465452	0.13193490	0.04960975
2Q 2020	0.06438324	0.14198290	0.05170964
3Q 2020	0.06470994	0.14586873	0.05669844
4Q 2020	0.06495826	0.14610345	0.06141681
1Q 2021	0.0646011	0.15331474	0.06997125
2Q 2021	0.06448223	0.15582173	0.05687304
3Q 2021	0.06444874	0.15612067	0.06000062
4Q 2021	0.06452485	0.15848709	0.06796243
1Q 2022	0.06465101	0.15233171	0.05817718
2Q 2022	0.06537411	0.1604007	0.05651455
3Q 2022	0.06040719	0.16844278	0.05041439
4Q 2022	0.06535409	-0.01172985	0.00650241
1Q 2023	0.06445228	-0.13978902	-0.0301422
2Q 2023	0.06456409	-0.13174443	-0.0497303
3Q 20223	0.06461577	-0.1121295	-0.04972705

Figure 7 compares both TV-BVAR estimates for the natural interest rate using the two alternative measures of expected inflation. Although both proxies of expected inflation imply a natural interest rate that follows a similar path, the estimates from the benchmark model, based on a MA(4) inflation, are significantly larger (on average) than the estimates that use the survey data. This result has significant implications for the conduct of monetary policy. Using the estimated values for 2022, the natural rate is found to be 4.82844% for the benchmark model and only 1.6991684% when survey data is used.

In the latter case, the central bank only needs to increase the nominal interest rates by a small amount to get the real interest rate above the natural rate in response to higher inflation. However, according to the estimates obtained from the benchmark model, a much more aggressive monetary tightening is required.

Figure 7: TV-BVAR NIR



5.2 Band-Pass Filters

Following several authors (see, e.g, Larsen et al. [2003], Manrique Simón and Marqués Sevillano [2004], Krustev [2019]), r_m^* can be estimated by decomposing the real interest rate's trend using Band-Pass (B-P) filters. These filters rely completely on statistical methods because no economic composition is imposed in the estimation. B-P Filters compute r^* is by extracting the growth component from the cyclical component from the real interest rate. To do so, several filters have been utilized to extract the cycle, the most prominent being the Hodrick-Prescott (HP) Filter, HP-adjusted filters, and Butterworth filter.

The HP filter estimates r_m^* by splitting it into two components: trend (growth) and cycle. Equation (27) sets the real interest rate as the sum of the trend component plus the cyclical.

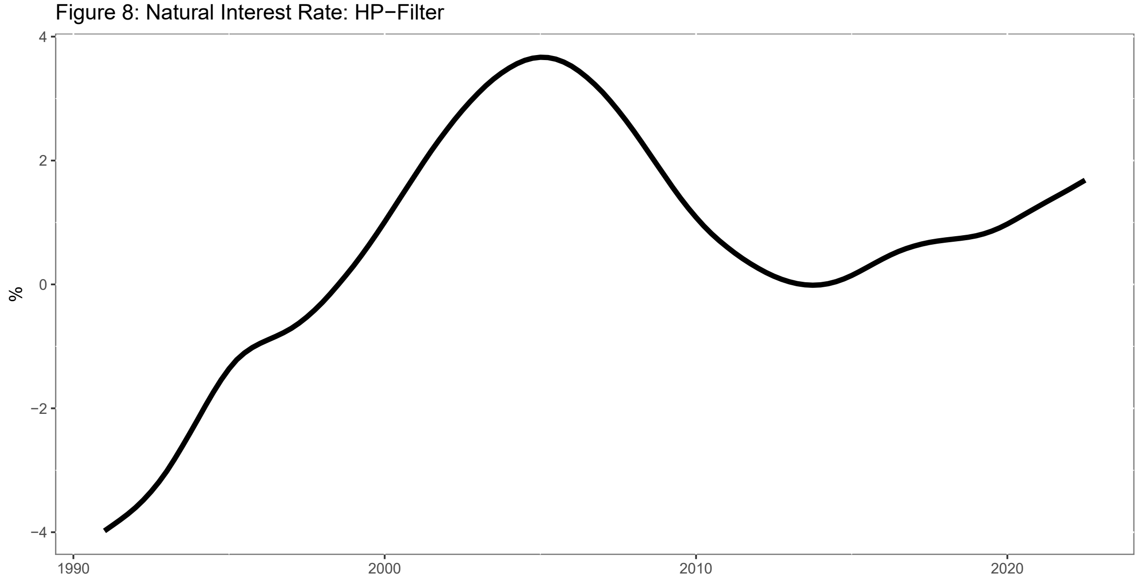
$$r_m = r_m^{tr} + r_m^c \tag{27}$$

The relationship between r_m and r_m^* is such that, using the smoothing parameter λ, an estimate for the natural interest rate is obtained from the trend component:

$$r_m^* = r_m^{tr} \tag{28}$$

Figure 8 estimates r_m^* using HP-filtering setting λ=1,600 without additional parameters[11]. Complete values can be found in the Appendix.

Figure 8: Natural Interest Rate: HP−Filter



Using the HP filter, r_m^* has reached negative levels, reaching a minimum -4.93952060% in 1991. Contrarily to the TV-BVAR estimations, from 1991 until 2005, it expanded to the maximum 3.66546636%. After the Great Recession, it had been constantly decreasing until 2014 at -0.00680%. Nevertheless, the HP estimations reached its minimum in 2014, reporting a -0.00680205. Interestingly, the trend after the Great Recession follows the path from the TV-BVAR models and Carrillo et al. [2018],

[11]Setting $\lambda = 1,600$ is consistent with quarterly data

Rodrigues [2020] estimates. From 2020 to 2022, the curve has became steeper, suggesting an increase in the Mexican nominal interest rate. Values validate this rise, because of the 0.56392892% differential between the dates. These higher values suggest that the latest shocks the economy has faced have increased r_m^*.

There is a large literature criticising the use of the HP filter regarding its biases, spuriousness, and lack of modern estimations [Bruchez, 2003, Hamilton, 2017, Phillips and Shi, 2019]. To deal with the Hodrick-Prescott bias, Magud and Tsounta [2012] suggest forecasting the next 18 months using an univarate ARIMA. Using an AIC algorith, the best fitted model for the forecast is an ARIMA(3,1,1), with a 95% confidence interval. Figure 9 shows that even with this correction, the results are similar (confidence interval shaded in blue).

The third B-P filter used to estimate the natural interest rate is the Butterworth filter. This method is employed due to the benefits it has, such as avoiding contamination spill on decomposing the trends, "appropiate to short trended sequences" [Pollock, 2016, pg. 13]. Gomez [2001] proves that a HP-Filter is a manageable Butterworth filter. In addition, applying this filter "reduces the amount of noise...decreasing the risk of inducing spurious results" [Harvey and Trimbur, 2003, pg. 365]. Harvey and Trimbur [2003] hold that the Butterworth filter produces smooth results optimals for non observable variables. Figure 10 depicts the estimates for r_m^* using the Butterworth filter. Values are available from the Appendix.

Figure 9. Natural Interest Rate: HP−Filter Magud and Tsounta Correction

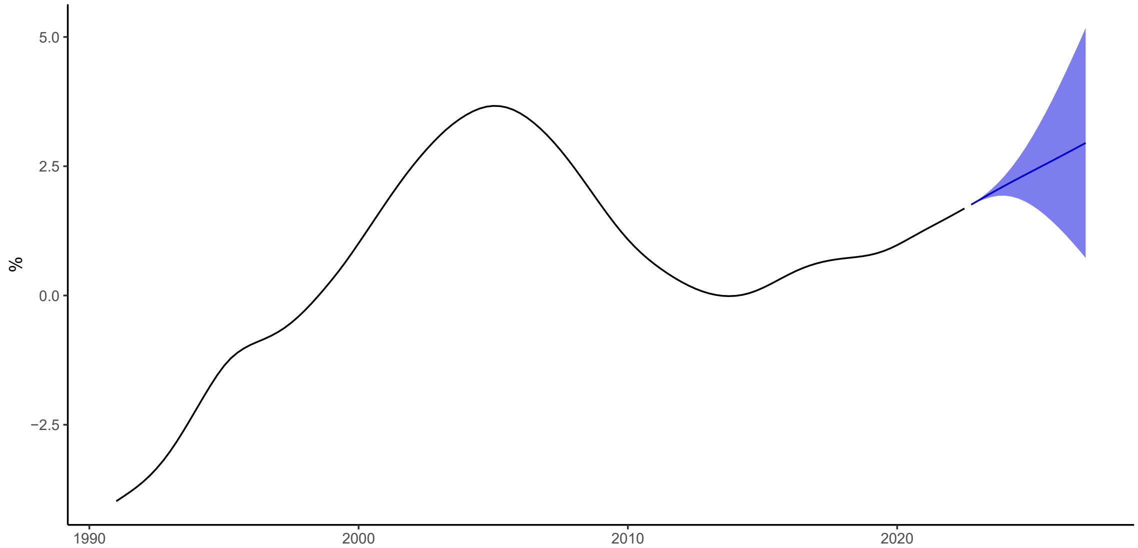


Figure 10: Butterworth NIR

Figure 10 demonstrates an unique pattern. From 1991 to 1993, the Butterworth r_m^*reached its minimum at -3.939843% and recovered until 1995. In 1997, another low was captured. However, similar to the other B-P filters, r_m^* continuously increased until 2005, and then rapidly decrease until 2015. Moreover, the external shocks are more evident in this B-P Filter because after the presidential elections, r_m^* marginally increased in 2016 to then decrease in 2018.

Despite estimating different values, the B-P filters show the same pattern: After 2005, r_m^* had slowly decreased until 2008. After the Great Recession, it drastically decreased until 2013. From 2013 to mid 2016, it had continuously risen. From 2020 onward, it has been increasing. As discussed, in all the models and filters, r_m^* has shown a rapidly rise since 2020, due to the effects the latest transitory shocks have had on the Mexican natural interest rate.

6 Conclusion

Defining the natural interest rate as the interest rate where output converges to its potential level implying stable inflation, this thesis estimates the Mexican natural interest rate for the period 1991 to 2022, using a time-varying Bayesian vector autoregressive model. The results suggest that the Mexican natural interest rate, has been affected in different ways by the transitory shocks that the economy has experienced over the last 30 years.

Since 1995, the natural interest rate is estimated to have been falling and remained almost constant after the DotCom crisis and the Great Recession until 2016. Since 2016, the natural interest rate has started to rise. The recent economic shocks, the Covid-19 pandemic and the Russian invasion to Ukraine, has resulted in a rapid increase in the Mexican natural interest rate and estimations from the thesis forecast that potential output will not reach pre-Covid 19 levels in either 2022 or 2023. The thesis also shows that the estimates obtained for the natural interest rate are highly sensitive to the data used to measure expected inflation in Mexico. Using a Moving Average (4) measure for expected inflation, the thesis estimates that the natural interest rate is roughly double the estimate when survey data is used as a measure for expected inflation. Finally, the thesis has shown that estimates for the natural interest rate using a variety of Band-Pass filter techniques, support the argument of a recent large increase in the Mexican natural interest rate.

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8 Appendix

Augmented Dickey-Fuller Test (Inflation 4 lags)

Dickey-Fuller Coefficient	Lag Order	p-value
-3.8529	4 lags	0.09812

Appendix 2: Time Varying Parameters

The parameters describe the evolution from 2011 to 2015 was continually decreasing at very low levels. In addition, the first lag affects more the estimation than the second, for all the estimations.

Date	α_t	β_t	λ_t
1Q 2011	0.6725780	-0.12645650	-0.05939948
2Q 2011	0.66545278	-0.12586096	-0.05165173
3Q 2011	0.66518699	-0.12496379	-0.05894729
4Q 2011	0.66514330	-0.57743565	- 0.05165620
1Q 2012	0.64518422	-0.12478654	-0.05883855
2Q 2012	0.6441170	-0.12423351	-0.05872306
3Q 2012	0.63800692	-0.12424928	-0.05851781
4Q 2012	0.6327213	-0.1239312	-0.05835449
1Q 2013	0.62668378	-0.12381390	-0.05814202
2Q 2013	0.62056234	-0.12357499	-0.05801014
3Q 2013	0.61972057	-0.12320048	-0.05789137
4Q 2013	0.61269980	-0.12303275	-0.05776478
1Q 2014	0.60913890	-0.12289395	-0.05761465
2Q 2014	0.60154564	-0.12266495	-0.05741741
3Q 2014	0.59810531	-0.12247525	-0.05735679
4Q 2014	0.59160127	-0.12248842	-0.05728005
1Q 2015	0.58394268	-0.12244077	-0.05716231
2Q 2015	0.58258843	-0.12259403	-0.05703865
3Q 2015	0.57793242	-0.12280850	-0.05699638
4Q 2015	0.57587673	-0.12298718	-0.05692178

Appendix 3: Hodrick-Prescott values

Date	r_m^*	Date	r_m^*
1991	-4.93952060	2012	0.26018322
1992	-4.14862266	2013	0.04213820
1993	-3.25295370	2014	-0.00680205
1994	-2.22636613	2015	0.14457429
1995	-1.32387584	2016	0.41255347
1996	-0.88661339	2017	0.61882060
1997	-0.64988339	2018	0.71620075
1998	-0.24781576	2019	0.78532948
1999	0.32350612	2020	0.97477086
2000	1.02101008	2021	1.25997789
2001	1.78603462	2022	1.53869978
2002	2.49954722		
2003	3.08318815		
2004	3.49118750		
2005	3.66546636		
2006	3.52353393		
2007	3.10076041		
2008	2.47582880		
2009	1.74731195		
2010	1.08310594		
2011	0.60193452		

Appendix 4: Butterworth Estimates

Date	r_m^*	Date	r_m^*
1991	-2.847351	2012	0.488283
1992	-3.556488	2013	0.117071
1993	-3.939843	2014	-0.132903
1994	-2.180897	2015	0.754703
1995	-0.039083	2016	0.759481
1996	-0.642139	2017	0.948034
1997	-1.323089	2018	0.71620075
1998	-0.506335	2019	0.4528531
1999	0.32350612	2020	0.7376412
2000	0.227174	2021	1.300152
2001	1.7723652	2022	1.511333
2002	2.45621		
2003	2.912776		
2004	3.227274		
2005	3.673729		
2006	3.456102		
2007	3.029136		
2008	2.499749		
2009	1.6085315		
2010	0.803646		
2011	0.733298		